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Neutrosophic Fuzzy Strong Bi-ideals of Near-Subtraction Semigroups

Siva Ranjini J [1]*, Mahalakshmi V [2]

[1] Research Scholar (Reg No. 19222012092003),P.G & Research Department of Mathematics,A.P.C.Mahalaxmi College for Women,Thoothukudi,Tamilnadu,India; e-mail@sivaranjini@apcmcollege.ac.in
[2] Assistant Professor, P.G & Research Department of Mathematics, A.P.C.Mahalaxmi College for Women, Thoothukudi Tamilnadu,India; e-mail@mahalakshmi@apcmcollege.ac.in
Affiliated to Manonmanian Sundaranar University, Tirunelveli, Tamilnadu, India.
* Correspondence: e-mail@sivaranjini@apcmcollege.ac.in

Abstract: The theory of Neutrosophy fuzzy set is the extension of the fuzzy set that deals with imprecise and indeterminate data Neutrosophy is a new branch of Philosophy. We already conceptualized the Neutrosophic fuzzy bi-ideals of Near –subtraction Semigroups(NFBI).In this article, We extend our study to strong bi-ideals. We examine some of its fundamentals and algebraic structures. Our aim of this manuscript are given as follows:

(i)To explore the new ideas in Neutrosophic fuzzy Near-subtraction semigroups of said bi-ideals and strong bi-ideals.

(ii)To examine the some basic properties and fundamentals.

(iii)Also expand the direct product and regularity of Neutrosophic fuzzy strong bi-ideals(NFSBI) of a Near- Subtraction Semigroups.

Keywords: Neutrosophic Fuzzy sub algebra, Neutrosophic fuzzy X-sub algebra, Neutrosophic fuzzy bi-ideal, Neutrosophic fuzzy strong bi-ideal.

1.Introduction

The fuzzy set was first introduced by L.A. Zadeh [18] .It was conceptualized the grade of truth values belonging to a unit interval.The fuzzy sub nearrings and fuzzy ideals of near-rings was introduced by Abou zaid[1]. V.Chinnadurai and S.Kadalarasi[4] examined the direct product of fuzzy subnearring, fuzzy ideal and fuzzy R-subgroups. Atanassov[3] expanded the intuitionstic fuzzy set to deal with complicated version.It explained the truth and false membership functions.It may applicable in various fields such as medicine, decision making techniques.

Later, Florentin Smarandache[13]introduced the concept of Neutrosophy. Neutrosophy is an extension of fuzzy logic in which indeterminancy also included. In Neutrosophic logic, we may have truth membership functions, false membership function and indeterminate functions. This idea of neutrosophic set have a remarkable achievement in various fields like medical diagnosis, image processing, decision making problem,robotics and so on. I.Arockiarani[8] consider the neutrosophic set with value from the subset of [0,1] and extended the research in fuzzy neutrosophic set. We gained inspiration from the advantages of Neutrosophy fuzzy set.J.Sivaranjini, V. Mahalakshmi[10]examined the concept of fuzzy bi-ideals in Near-Subtraction Semigroups.The results obtained are entirely more beneficial to the researchers.

2. Preliminaries

The aim of this section is to recall some basic definitions.

2.1 Definition[7]

A non-empty set X together with the binary operation '-' and'•' is said to be a right(left) ***near-subtraction semigroup*** if it satisfies the following.

(i)(X ,-)is a subtraction algebra (ii)(X ,•)is a semigroup (iii)(p-q)r=pr-qr for all p,q,r in X. It is clear that 0p=0 for all p in X. Similarly, we can define for left near-subtraction semigroup.

2.2 Definition[12]

A ***Neutrosophic Fuzzy*** Set S on the universe of discourse X Characterised by a truth membership function $T_S(p)$, a indeterminacy function $I_S(p)$ and a non- membership function $F_S(p)$ is defined as S={<p, $T_S(p)$, $I_S(p)$, $F_S(p)$>/pϵX} where T_S, I_S, F_S:X→[0,1] and 0≤ $T_S(p)$+ $I_S(p)$+ $F_S(p)$≤3.

2.3 Definition[12]

If V is said to be ***Neutrosophic fuzzy sub algebra*** of a near Subtraction Semigroup X , then it satisfies the following conditions:

(i)$T_V(p-q) \geq \min\{T_V(p), T_V(q)\}$ (ii)$I_V(p-q) \leq \max\{I_V(p), I_V(q)\}$

(iii)$F_V(p-q) \leq \max\{F_V(p), F_V(q)\}$ for all p,q, in V.

2.4 Definition[14]

A near- subtraction Semigroup X is said to be ***left permutable*** if pqr=qpr for all p,q,r in X.

2.5 Definition[12]

Let S and V be any two Neutrosophic Fuzzy Sets of X and pϵ X. Then

(1)S∪V={<p, $T_{S\cup V}$ (p), $I_{S\cup V}$ (p), $F_{S\cup V}$ (p)>/pϵX}

(i) $T_{S\cup V}$ (p)=max{$T_S(p)$, $T_V(p)$} (ii)$I_{S\cup V}$(p)=min{ $I_S(p)$, $I_V(p)$} (iii)$F_{S\cup V}$ (p)=min{ $F_S(p)$, $F_V(p)$}

(2)$S \cap V$={<p, $T_{S\cap V}$ (p), $I_{S\cap V}$ (p), $F_{S\cap V}$ (p)>/pϵX}where,

(i) $T_{S\cap V}$ (p)=min{ $T_S(p)$, $T_V(p)$} (ii)$I_{S\cap V}$ (p)=max{ $I_S(p)$, $I_V(p)$} (iii)$F_{S\cap V}$ (p)=max{ $F_S(p)$, $F_V(p)$}

2.6Definition[10]

An fuzzy sub algebra is deal to be ***fuzzy bi-ideal*** of X if μ (pqr) ≥ min {μ (p) , μ(r)} where p,q, r in X.

2.7 Definition[10]

A Neutrosophic Fuzzy Sub algebra S in a near Subtraction Semigroup X is said to be ***Neutrosophic Fuzzy Bi-ideal*** of X if it satisfies the following conditions:

(i) $T_S(pqr) \geq \min\{T_S(p), T_S(r)\}$

(ii) $I_S(pqr) \leq \max\{I_S(p), I_S(r)\}$

(iii) $F_S(pqr) \leq \max\{F_S(p), F_S(r)\}$ for all p,q,r ϵX

2.7 Definition[10]

A Neutrosophic fuzzy set S of X is said to be ***Neutrosophic fuzzy right(left)X-sub algebra*** of X if

(i)$T_S(p\text{-}q) \geq \min\{T_S(p), T_S(q)\}$;$T_S(pq) \geq T_S(p)$[$T_S(pq) \geq T_S(q)$]
(ii)$I_S(p\text{-}q) \leq \max\{I_S(p), I_S(q)\}$; $I_S(pq) \leq I_S(p)$ [$I_S(pq) \leq I_S(q)$]
(iii)$F_S(p\text{-}q) \leq \max\{F_S(p), F_S(q)\}$;$F_S(pq) \leq F_S(p)$, [$F_S(pq) \leq F_S(q)$] for all p,q, in X.

2.8 Definition[14]

Let S and V be any two Neutrosophic Fuzzy subsets of Near Subtraction Semigroups X and Y respectively. Then the ***direct product is*** defined by
$S\times V=\{<(p,q), T_{S\times V}(p,q), I_{S\times V}(p,q), F_{S\times V}(p,q)>/p \in X, q \in Y\}$where,
$T_{S\times V}(p,q)=\min\{T_S(p),T_V(q)\}$;$I_{S\times V}(p,q)=\max\{I_S(p),I_V(q)\}$;$F_{S\times V}(p,q)=\max\{F_S(p),F_V(q)\}$

3. Neutrosophic Fuzzy Strong Bi-ideals of Near-Subtraction Semigroups

The aim of this section is to explore the idea of this concept.

3.1.Definition

A Neutrosophic Fuzzy Bi-Ideal S of X is said to be ***Neutrosophic Fuzzy Strong Bi- Ideal*** (NFSBI) of X if it satisfies the following conditions:

(i)$T_S(pqr) \geq \min\{T_S(q), T_S(r)\}$ (ii)$I_S(pqr) \leq \max\{I_S(q), I_S(r)\}$ (iii)$F_S(pqr) \leq \max\{F_S(q), F_S(r)\}$ for all p,q,r ϵX.

3.2 Example

Assume that X={0,p,q,r} in which '-' and '•' defined by

-	**0**	**p**	**q**	**R**
0	**0**	**0**	**0**	**0**
p	**P**	**0**	**p**	**0**
q	**Q**	**q**	**0**	**0**
r	**R**	**q**	**p**	**0**

•	**0**	**P**	**q**	**r**
0	**0**	**0**	**0**	**0**
P	**0**	**Q**	**0**	**q**
Q	**0**	**0**	**0**	**0**
R	**0**	**Q**	**0**	**q**

Consider the Fuzzy set S:X→[0,1] be a fuzzy subset of X defined by

$T_S(0)=.7$ $T_S(p)=.5$ $T_S(q)=.3$ $T_S(r)=.2$;$I_S(0)=.3$ $I_S(p)=.4$ $I_S(q)=.6$ $I_S(r)=.8$;$F_S(0)=.2$ $F_S(p)=.3$ $F_S(q)=.7$ $F_S(r)=.9$.

3.3Theorem

Consider S=(T_S, I_S, F_S) to be a NFSBI of X iff XTT⊆T(XII⊇I,XFF⊇F)

Proof: Assume that S is a NFSBI of X. Let p,q,l,m,aϵX.

Consider a=pq and p=lm. We already prove that T is a NFBI X[10]. Therefore

$XTT(a)=sup_{a=pq}\{min\{(XT)(p), T(q)\}\}$

$$=sup_{a=pq}\{min\{sup_{p=lm}\{min\{X(l),T(m)\},T(q)\}=sup_{a=pq}\{min\{sup_{p=lm}\{T(m)\},T(q)\}$$

Since T is a NFBIof X.

$$=sup_{a=pq}min\{T(m),T(q)\}\leq sup_{p=lmq}\{T(lmq)\}=T(lmq)=T(a)$$

We have, XTT⊆T. Conversely, Assume that XTT⊆T

If a cannot expressed as a=pq then, XTT(a)=0≤T(a) .In both cases XTT⊆T. Choose p,q,r,a,b,c ϵX such that a=pqr. Then

$T(pqr)=T(a)\geq XTT(a)$

$$= sup_{a=bc}min\{(XT)(b),T(c)\}\geq min\{X(p),T(q),T(r)\}=min\{T(q),T(r)\}$$

$XII(a)=inf_{a=pq}\{max\{(XI)(p), I(q)\}\}$

$$=inf_{a=pq}\{max\{inf_{p=lm}\{max\{X(l),I(m)\},I(q)\}$$

$$=inf_{a=pq}\{max\{inf_{p=lm}\{I(m)\},I(q)\}$$

Since I is a NFSBI of X.

$$=inf_{a=pq}max\{I(m),I(q)\}\geq inf_{p=lmq}\{I(lmq)\}=I(lmq)=I(a)$$

We have, IXI⊇I. If a cannot expressed as a=pq then XII(a)=0≥I(a).In both cases, XII⊇I

Conversely, Assume that XII⊇I.Choose p,q,r,a,b,c ϵX such that a=pqr. Then

$I(pqr)=I(a)\leq XII(a)$

$$= inf_{a=bc}max\{(XI)(b),I(c)\} \leq max\{X(p),I(q),I(r)\}=max\{I(q),I(r)\}$$

$FXF(a)=inf_{a=pq}\{max\{(XF)(p), F(q)\}\}$

$$=inf_{a=pq}\{max\{inf_{p=lm}\{max\{X(l),F(m)\},F(q)\}$$

$=inf_{a=pq}\{\max\{inf_{p=lm}\{F(m)\},F(q)\}$

Since F is a Neutrosophic Fuzzy strong bi-ideal of X.

$=inf_{a=pq}\max\{F(m),F(q)\}\geq inf_{p=lmq}\{F(lmq)\}=F(lmq)=F(a)$

Hence $FXF\supseteq F$ If a cannot expressed as a=pq then $XFF(a)=0\geq F(a)$.In both cases, $XFF\supseteq F$

Conversely, Assume $FXF\supseteq F$.Choose p,q,r,a,b,c ϵX such that a=pqr.Then

$F(pqr)=F(a)\leq XFF(a)$

$= inf_{a=bc}\max\{(XF)(b),F(c)\}\leq \max\{X(p),F(q),F(r)\}=\max\{F(q),F(r)\}$

3.4 Theorem

The Direct Product of any two NFSBI of a Near-Subtraction Semigroups is again a NFSBI of X×Y.

Proof:

Consider S and V be any two NFSBI of X and Y respectively.We already prove that S×V is a NFBI of X×Y[10].

Now $p=(p_1,p_2)$ $q=(q_1,q_2)$ $r=(r_1,r_2)\epsilon X\times Y$ respectively.

(i) $T_{S\times V}((p_1,p_2),(q_1,q_2),(r_1,r_2))= T_{S\times V}(p_1q_1r_1,p_2q_2r_2)$

$=\min\{T_S(p_1q_1r_1), T_V(p_2q_2r_2)\}$

$\geq\min\{\min\{T_S(q_1), T_S(r_1)\},\min\{T_V(q_2), T_V(r_2)\}\}$

$=\min\{ T_{S\times V}(q_1,q_2), T_{S\times V}(r_1,r_2)\}$

(ii) $I_{S\times V}((p_1,p_2),(q_1,q_2),(r_1,r_2))= I_{S\times V}(p_1q_1r_1,p_2q_2r_2)$

$=\max\{I_S(p_1q_1r_1), I_V(p_2q_2r_2)\}$

$\leq\max\{\max\{I_S(q_1), I_S(r_1)\},\min\{I_V(q_2), I_V(r_2)\}\}$

$=\max\{ I_{S\times V}(q_1,q_2), I_{S\times V}(r_1,r_2)\}$

(iii) $F_{S\times V}((p_1,p_2),(q_1,q_2),(r_1,r_2))= F_{S\times V}(p_1q_1r_1,p_2q_2r_2)$

$=\max\{F_S(p_1q_1r_1), F_V(p_2q_2r_2)\}$

$\leq\max\{\max\{F_S(q_1), F_S(r_1)\},\min\{F_V(q_2), F_V(r_2)\}\}$

$=\max\{ F_{S\times V}(q_1,q_2), F_{S\times V}(r_1,r_2)\}$

Hence, S×V is a NFSBI of X×Y.

3.5Theorem

If $S\times V=(T_{S\times V}, I_{S\times V}, F_{S\times V})$ be a NFSBI of $X\times Y$ .Then $S\times V=(T_{S\times V}, I_{S\times V}, F^{C}_{S\times V})$ is a NFSBI of $X\times Y$.

Proof:

Consider $S\times V=(T_{S\times V}, I_{S\times V}, F_{S\times V})$ be a NFSBI of $X\times Y$.

Now $p=(p_1,p_2)$ $q=(q_1,q_2)$ $r=(r_1,r_2)\epsilon X\times Y$

By [Theorem 3.4] $T_{S\times V}$, $I_{S\times V}$ and $F_{S\times V}$ are NFSBI of $X\times Y$.

Now it is enough to prove $T_{S\times V}{}^{C}(p_1,p_2)(q_1,q_2)(r_1,r_2)\leq\max\{T_{S\times V}(q_1,q_2), T_{S\times V}(r_1,r_2)\}$

Now, $T^{C}_{S\times V}(p_1,p_2)(q_1,q_2)(r_1,r_2) = 1- T_{S\times V}(p_1,p_2)(q_1,q_2)(r_1,r_2)$

$\leq 1-\min\{T_{S\times V}(q_1,q_2), T_{S\times V}(r_1,r_2)\}$

$=\max\{1- T_{S\times V}(q_1,q_2), 1-T_{S\times V}(r_1,r_2)\}$

$=\max\{T^{C}_{S\times V}(q_1,q_2), T^{C}_{S\times V}(r_1,r_2)\}$

Thus, $S\times V=(T_{S\times V}, I_{S\times V}, F^{C}_{S\times V})$ is a NFSBI of $X\times Y$.

3.6Corollary

If $S\times V=(T_{S\times V}, I_{S\times V}, F_{S\times V})$ be a NFSBI of $X\times Y$.Then $S\times V=(F^{C}_{S\times V}, I_{S\times V}, T^{C}_{S\times V})$ is NFSBI of $X\times Y$.

3.7Corollary

Consider $S\times V=(T_{S\times V}, I_{S\times V}, F_{S\times V})$ be a NFSBI of $X\times Y$.Then $S\times V=(F_{S\times V}{}^{C}, I_{S\times V}, I_{S\times V})$ is a NFSBI of $X\times Y$.

3.8 Theorem

Let X be a Strong regular Near –Subtraction Semigroup. Let $S=(T_S, I_S, F_S)$ be a NFSBI of X,then $XTT=T$, $XII=I$ and $XFF=F$

Proof:

Consider $S=(T_S, I_S, F_S)$ be a NFSBI of X. Choose $p\epsilon X$. Since X is a strong regular near subtraction semigroup there exists a ϵX such that $p=ap^2$.

Now, $XTT(p)=XTT(ap^2)$.

(i)$XTT(p)=sup_{p=app}\{\min\{(XT)(ap), T(p)\}\}\geq\min\{XT(ap),T(p)\}$

$=\min\{sup_{ap=lm}\{\min\{X(l),T(m)\},T(p)\}\}$

$\geq\min\{\min\{X(a),T(p)\},T(p)\}=\min\{T(p),T(p)\}=T(p)$

Also we know that $XTT\subseteq T$.From that, $XTT=T$

(ii) XII(p)=$inf_{p=app}$ {max{(XI)(ap), I(p)}}

≤max{XI(ap),I(p)}

=max{$inf_{ap=lm}$ {min{X(l),I(m)},I(p)}}

≤max{max{X(a),I(p)},I(p)}=max{I(p),I(p)}=I(p)

Also we know that XII⊇I.From that, XII=I

(iii) XFF(p)=$inf_{p=app}$ {max{(XF)(ap), F(p)}}

≤max{XF(ap),F(p)}

=max{$inf_{ap=lm}$ {min{X(l),F(m)},F(p)}}

≤max{max{X(a),F(p)},F(p)}=max{F(p),F(p)}=F(p)

Also we know that XFF⊇F.From that, XFF=F

3.9 Theorem

Every left permutable fuzzy right X-sub algebra of X is a NFSBI of X.

Proof:

Consider S=(T_S, I_S, F_S) be a Neutrosophic fuzzy right X-sub algebra of X.First we prove S is a NFBI of X.Choose a,p,q,l,m϶X.Also a=pq,p=lm

TXT(a) =$sup_{a=pq}$ {min{(TX)(p), T(q)}}=$sup_{a=pq}$ {min{$sup_{p=lm}$ {min{T(l),X(m)},T(q)}

=$sup_{a=pq}$ {min{$sup_{p=lm}$ {T(l)},T(q)}=$sup_{a=pq}$ min{T(l),T(q)}

Since T is a Neutrosophic fuzzy right X-sub algebra T(pq)=T((lm)q)≥T(l)

≤$sup_{a=pq}$ min{T(pq),X(q)}sinceX(q)=1=T(pq)=T(a)

Therefore, TXT⊆T

IXI(a)=$inf_{a=pq}${max{(IX)(p), I(q)}}

=$inf_{a=pq}${max{$inf_{p=lm}$ {max{I(l),X(m)},I(q)}

=$inf_{a=pq}${max{$inf_{p=lm}$ {I(l)},I(q)}=$inf_{a=pq}$max{I(l),I(q)}

Since I is a Neutrosophic fuzzy right X-sub algebra I(pq)=I((lm)q)≤I(l)

$\geq inf_{a=pq}$max{I(pq),X(q)}sinceX(q)=0=I(pq)=I(a)

Therefore, IXI⊒I

FXF(a) $=inf_{a=pq}$\{max{(FX)(p), F(q)}}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{max{F(l),X(m)},F(q)}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{F(l)},F(q)} $=inf_{a=pq}$max{F(l),F(q)}

Since I is a Neutrosophic fuzzy right X-sub algebra F(pq)=F((lm)q)≤F(l)

$\geq inf_{a=pq}$max{F(pq),X(q)}sinceX(q)=0=F(pq) =F(a)

Therefore, FXF⊒F

XTT(a) $=sup_{a=pq}$\{min{(XT)(p), T(q)}}$=sup_{a=pq}$\{min{$sup_{p=lm}$\{min{X(l),T(m)},T(q)}

$=sup_{a=pq}$\{min{$sup_{p=lm}$\{T(m)},T(q)}

Since T is a left permutable Neutrosophic Fuzzy right X-Sub algebra of X.T(pq)=T((lm)q)=T(mlq)≥T(m)≤$sup_{p=lmq}$\{min{T(pq),X(q)}.Since X(q)=1=T(pq)=T(a)

XII(a)$=inf_{a=pq}$\{max{(XI)(p), I(q)}}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{max{X(l),I(m)},I(q)}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{I(m)},I(q)}

Since I is a left permutable Neutrosophic Fuzzy right X-sub algebra of X.

I(pq)=I((lm)q)=I(mlq)≤I(m)

$\geq inf_{a=pq}$max{I(pq),X(q)}. Since X(q)=0=I(pq)=I(a)

We have, XII⊒I

XFF(a) $=inf_{a=pq}$\{max{(XF)(p), F(q)}}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{max{X(l),F(m)},F(q)}

$=inf_{a=pq}$\{max{$inf_{p=lm}$\{F(m)},F(q)}

Since F is a left permutable Neutrosophic Fuzzy right X-sub algebra of X. F(pq)=I((lm)q)=F(mlq)≤F(m)

$\geq inf_{a=pq}$max{F(pq),X(q)}. Since X(q)=0=F(pq)=F(a)

We have, FXX⊇I

3.10Theorem

Every left permutable fuzzy left X-sub algebra of X is a NFSBI of X.

Proof: Consider S=(T_S, I_S, F_S) be a Neutrosophic fuzzy left X-sub algebra of X.First we prove S is a NFBI of X.Choose a,p,q,l,mϵX. Also a=pq,p=lm

TXT(a) $=sup_{a=pq}$\{min{(T)(p), XT(q)}}

$=sup_{a=pq}$\{min{T(p), {$sup_{q=lm}$min{X(l)},T(m)}}

$=sup_{a=pq}$\{min{T(p), $sup_{q=lm}$T(m)}=$sup_{a=pq}$min{T(p),T(m)}

Since T is a Neutrosophic fuzzy left X-sub algebra T(pq)=T((pl)m)≥T(m)

$\leq sup_{a=pq}$min{X(p),T(pq)}sinceX(q)=1=T(pq)=T(a)

Therefore, TXT⊆T

IXI(a) $=inf_{a=pq}$\{max{I(p), XI(q)}}=$inf_{a=pq}$\{max {I(p), $inf_{q=lm}$max{X(l),I(m)}

$=inf_{a=pq}$\{max{I(p), {$inf_{p=lm}$I(m)}

$=inf_{a=pq}$max{I(p),I(m)}

Since I is a Neutrosophic fuzzy left X-sub algebra I(pq)=I((pl)m)≤I(m)

$\geq inf_{a=pq}$max{X(p),I(pq)}sinceX(q)=0=I(pq)=I(a)

Therefore, IXI⊇I

FXF(a) $=inf_{a=pq}$\{max{F(p), XF(q)}}

$=inf_{a=pq}$\{max {F(p), $inf_{q=lm}$max{X(l),F(m)}

$=inf_{a=pq}$\{max{F(p), {$inf_{p=lm}$F(m)}=$inf_{a=pq}$max{F(p),F(m)}

Since I is a Neutrosophic fuzzy left X-sub algebra F(pq)=F((pl)m)≤F(m)

$\geq inf_{a=pq}$max{X(p),F(pq)}sinceX(q)=0=F(pq)=F(a)

Therefore, FXF⊇F

XTT(a) $=sup_{a=pq}$\{min{(X)(p), TT(q)}}$=sup_{a=pq}${min{X(p), $sup_{q=lm}$min{T(l),T(m)}}

Since T is a left permutable Neutrosophic Fuzzy left X-Sub algebra of X.T(pq)=T(plm)=T((lp)m)≥T(m)

$\leq sup_{a=pq}${min{X(l),T(pq)}.Since X(l)=1=T(pq) =T(a)

XII(a)$=inf_{a=pq}${max{(X)(p), II(q)}}$=inf_{a=pq}${max{X(p), $inf_{q=lm}$max{I(l),I(m)}}

Since I is a left permutable Neutrosophic Fuzzy left X-Sub algebra of X.I(pq)=I(plm)=I((lp)m)≤I(m)

$\geq inf_{a=pq}${max{X(l),I(pq)}.Since X(l)=0 =I(pq)=I(a)

XFF(a) $=inf_{a=pq}${max{(X)(p), FF(q)}}$=inf_{a=pq}${max{X(p), $inf_{q=lm}$max{F(l),F(m)}}

Since F is a left permutable Neutrosophic Fuzzy left X-Sub algebra of X.F(pq)=F(plm)=F((lp)m)≤F(m)

$\geq inf_{a=pq}${max{X(l),F(pq)}.Since X(l)=0=F(pq)=F(a)

We have, FXX⊇F

3.11Theorem

Every Neutrosophic fuzzy two-sided (left and right) X- sub algebra of X is a NFSBI of X.

Proof: Straight forward

Conclusion

The theory of Neutrosophy fuzzy set is basically the extension of the Intuitionistic fuzzy set. In the present manuscript, we have defined the Union, direct product, Intersection, Homomorphism of Neutrosophic fuzzy Strong Biideal in Near subtraction Semi group In future, we will investigate the Neutrosophy fuzzy Ideals and their fundamentals.

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