Neutrosophic Sets and Systems, Vol. 38, 2020
University of New Mexico

Pairwise Neutrosophic-*b*-Open Set in Neutrosophic Bitopological Spaces

Suman Das[1], and Binod Chandra Tripathy[2*]

[1,2]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.

E-mail: [1]suman.mathematics@tripurauniv.in, and [2]binodtripathy@tripurauniv.in, tripathybc@yahoo.com

***Correspondence:** tripathybc@yahoo.com Tel.: (+91-9864087231))

Abstract: In this paper we introduce the notion of neutrosophic-*b*-open set, pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces. We have investigated some of their basic properties and established relation between the other existing notions.

Keywords: *Neutrosophic set; Neutrosophic topology; Neutrosophic bitopology; Neutrosophic-b-open set.*

---

1. Introduction

Smarandache (1998) introduced the notion of neutrosophic set as a generalization of intuitionistic fuzzy set. The concept of neutrosophic topological space was introduced by Salama and Alblowi (2012a). Salama and Alblowi (2012b) introduced the concept of generalized *neutrosophic set* and generalized neutrosophic topological space. Thereafter Ozturk and Ozkan (2019) introduce the concept of neutrosophic bitopologoical space. The concept of *b*-open sets in topological space was introduced by Andrijevic (1996). Ebenanjar, Immaculate, and Wilfred (2018) introduced neutrosophic *b*-open sets in neutrosophic topological spaces. Thangavelu and Thamizharsi (2011) introduce the concept of b_t-open sets in bitopological spaces. In this paper, we introduce the notion of pairwise neutrosophic *b*-open set in neutrosophic bitopological spaces.

2. Preliminaries and some properties

Definition 2.1. [Smarandache, 2005] Let X be a non-empty set. Then H, a neutrosophic set (NS in short) over X is denoted as follows:

$H = \{(y, T_H(y), I_H(y), F_H(y)): y \in X$ and $T_H(y), I_H(y), F_H(y) \in\ ]^-0,1^+[\}$, where $T_H(y)$, $I_H(y)$ and $F_H(y)$ are the degree of truthness, indeterminacy and falseness respectively.

There is no restriction on the sum of $T_G(y)$, $F_G(y)$ and $I_G(y)$, so

$^{-}0 \leq T_H(y) + I_H(y) + F_H(y) \leq 3^{+}$.

Definition 2.2. [Smarandache, 2005] Let $H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in X\}$ be a *neutrosophic set* over X. Then the complement of H is defined by $H^c = \{(y, 1-T_H(y), 1-I_H(y), 1-F_H(y)) : y \in X\}$.

Definition 2.3. [Smarandache, 2005] A neutrosophic set $H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in X\}$ is contained in the other neutrosophic set $K = \{(y, T_K(y), I_K(y), F_K(y)) : y \in X\}$ (i.e. $H \subseteq K$) if and only if $T_H(y) \leq T_K(y)$, $I_H(y) \geq I_K(y)$, $F_H(y) \geq F_K(y)$, for each $y \in X$.

Definition 2.4. [Smarandache, 2005] If $H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in X\}$ and $K = \{(y, T_K(y), I_K(y), F_K(y)) : y \in X\}$ are any two *neutrosophic sets* over X, then $H \cup K$ and $H \cap K$ is defined by
$H \cup K = \{(y, T_H(y) \vee T_K(y), I_H(y) \wedge I_K(y), F_H(y) \wedge F_K(y)) : y \in X\}$;
$H \cap K = \{(y, T_H(y) \wedge T_K(y), I_H(y) \vee I_K(y), F_H(y) \vee F_K(y),) : y \in X\}$.

Here we can construct two neutrosophic set 0_N and 1_N over X as follows:

1) $0_N = \{(y, 0, 0, 1) : y \in X\}$;
2) $1_N = \{(y, 1, 0, 0) : y \in X\}$.

The neutrosophic set 0_N is known as neutrosophic null set and neutrosophic set 1_N is known as neutrosophic whole set over X. Also, 0_N and 1_N over X have three other types of representation too. Clearly, $0_N \subseteq 1_N$.

The neutrosophic topological space is defined as follows:

Definition 2.5. [Salama & Alblowi, 2012a] Let X be a non-empty fixed set and τ be the family of some *NSs* over X. Then τ is said to be a neutrosophic topology (*NT* in short) on X if the following properties holds:

1. $0_N, 1_N \in \tau$,
2. $T_1, T_2 \in \tau \Rightarrow T_1 \cap T_2 \in \tau$,
3. $\cup_{i \in \Delta} T_i \in \tau$, for every $\{T_i : i \in \Delta\} \subseteq \tau$.

Then the pair (X, τ) is called a neutrosophic topological space (*NTS* in short). The members of τ are called neutrosophic-open set (*NOS* in short). A NS D is called a neutrosophic-closed set (*NCS* in short) in (X, τ) if and only if D^c is a neutrosophic-open set.

Example 2.1. Let $X = \{z_1, z_2\}$and let
$G = \{(z_1, 0.6, 0.5, 0.3), (z_2, 0.6, 0.7, 0.3) : z_1, z_2 \in X\}$
$H = \{(z_1, 0.5, 0.6, 0.8), (z_2, 0.4, 0.9, 0.8) : z_1, z_2 \in X\}$
$K = \{(z_1, 0.6, 0.6, 0.3, (z_2, 0.4, 0.8, 0.6) : z_1, z_2 \in X\}$ be three *NSs* over X.
Then clearly the family $\tau = \{0_N, 1_N, G, H, K\}$ is a *NT* on X.

Example 2.2. Let $X = \{z_1, z_2, z_3\}$and let
$L = \{(z_1, 0.6, 0.7, 0.4), (z_2, 0.5, 0.6, 0.8), (z_3, 0.5, 0.5, 0.4) : z_1, z_2, z_3 \in X\}$

K ={(z_1,0.4,0.9,0.8), (z_2,0.3,0.7,0.8), (z_3,0.4,0.6,0.8): z_1, z_2, $z_3 \in X$ }
J={(z_1,0.4,0.9,0.9), (z_2,0.2,0.8,0.9), (z_3,0.3,0.5,0.8): z_1, z_2, $z_3 \in X$ }
be three *NSs* over *X*.
Here the collection τ={0_N,1_N , *L*, *K*, *J*} is not a *neutrosophic topology* on *X* because $K \cap J \notin \tau$.

Definition 2.5. [Salama & Alblowi, 2012a] Let (X,τ) be a NTS and *H* be a *NS* over *X*. The neutrosophic-interior (in short N_{int}) and neutrosophic-closure (in short N_{cl}) of *H* are defined by
$N_{int}(H) = \cup\{P : P$ is an NOS in X and $P \subseteq H\}$;
$N_{cl}(H) = \cap\{Q : Q$ is an NCS in X and $H \subseteq Q\}$.

Proposition 2.1. [Salama & Alblowi, 2012a] Let *C*, *D* are two neutrosophic subsets of (X, τ). Then the following properties hold:

1) $C \subseteq N_{cl}(C)$;
2) $N_{int}(C) \subseteq C$;
3) $N_{int}(C) \subseteq N_{cl}(C)$;
4) $C \subseteq D \Rightarrow N_{int}(C) \subseteq N_{int}(D)$;
5) $C \subseteq D \Rightarrow N_{cl}(C) \subseteq N_{cl}(D)$;
6) $N_{cl}(0_N) = 0_N$;
7) $N_{int}(1_N) = 1_N$;
8) $N_{cl}(C \cup D) = N_{cl}(C) \cup N_{cl}(D)$;
9) $N_{int}(C \cup D) \supset N_{int}(C) \cup N_{int}(D)$;
10) $N_{int}(C \cap D) = N_{int}(C) \cap N_{int}(D)$;
11) $N_{cl}(C \cap D) \subset N_{cl}(C) \cap N_{cl}(D)$;
12) *C* is neutrosophic closed if and only if $N_{cl}(C)=C$;
13) *C* is neutrosophic open if and only if $N_{int}(C)=C$.

The neutrosophic bitopological space is defined as follows:

Definition 2.6. [Ozturk & Ozkan, 2019] Assume that (X, τ_1) and (X, τ_2) be two different *NTSs*. Then the triplet (X, τ_1, τ_2) is called a *neutrosophic bitopological space* (*NBTS* in short).

Example 2.3. Let X={z_1, z_2} and let
U_1={(z_1,0.6,0.5,0.4), (z_2,0.8,0.7,0.6): z_1, $z_2 \in X$ },
U_2={(z_1,0.4,0.6,0.5), (b,0.7,0.8,o.8): z_1, $z_2 \in X$ },
U_3={(z_1,0.4,0.6,0.8), (z_2,0.6,0.9,0.8): z_1, $z_2 \in X$ },
U_4={(z_1,0.6,0.8,0.7), (z_2,0.4,0.6,0.7): z_1, $z_2 \in X$ },
U_5={(z_1,0.8,0.4,0.5), (z_2,0.6,0.4,0.5): z_1, $z_2 \in X$ },
U_6={(z_1,0.7,0.5,0.6), (z_2,0.6,0.5,0.5): z_1, $z_2 \in X$ } are six *NSs* over *X*.
Then clearly τ_1={0_N, 1_N, U_1, U_2, U_3} and τ_2={0_N, 1_N , U_4, U_5, U_6} are two different *NTs* on *X*. So the triplet (X, τ_1, τ_2) is a *neutrosophic bitopological space*.

Definition 2.7. [Ozturk & Ozkan, 2019] Let (X, τ_1, τ_2) be an *neutrosophic bitopological space*. Then H, a *neutrosophic set* over X is called a pairwise open set in (X, τ_1, τ_2) if there exist a open set G_1 in τ_1 and a open set G_2 in τ_2 such that $H= G_1 \cup G_2$.

Remark 2.2. Let G be a neutrosophic subset of a *neutrosophic bitopological space* (X, τ_1, τ_2). Then we shall use the following notations:

1) $N_{cl}^{i}(G)$= τ_i -neutrosophic-closure of G (i=1, 2);
2) $N_{int}^{i}(G)$= τ_i -neutrosophic-interior of G (i=1, 2).

3. τ_i -neutrosophic-b-open set:

Definition 3.1. Let (X, τ_1, τ_2) be an *neutrosophic bitopological space*. Then P, a *NS* over X is called

1) τ_i -neutrosophic-semi-open if and only if $P \subseteq N_{cl}^{i}N_{int}^{i}(P)$;
2) τ_i -neutrosophic-pre-open if and only if $P \subseteq N_{int}^{i}N_{cl}^{i}(P)$;
3) τ_i -neutrosophic-b-open if and only if $P \subseteq N_{cl}^{i}N_{int}^{i}(P) \cup N_{int}^{i}N_{cl}^{i}(P)$.

Remark 3.1. In a *neutrosophic bitopological space* (X, τ_1, τ_2), a *NS P* over X is called a τ_i-neutrosophic-b-closed set if and only if its complement is τ_i-neutrosophic-b-open set.

We formulate the following results based on the above definitions.

Proposition 3.1. In a *neutrosophic bitopological space* (X, τ_1, τ_2), if P is τ_i-neutrosophic-semi-open (τ_i -neutrosophic-pre-open), then P is τ_i neutrosophic-b-open.

Proposition 3.2. In a *neutrosophic bitopological space* (X, τ_1, τ_2), the union of two τ_i-neutrosophic-b-open set is a τ_i-neutrosophic-b-open set.

4. τ_{ij} -neutrosophic-b-open set:

Definition 4.1. Assume that (X, τ_1, τ_2) be a *neutrosophic bitopological space*. Then P, a NS over X is called

1) τ_{ij} -neutrosophic-semi-open if and only if $P \subseteq N_{cl}^{i}N_{int}^{j}(P)$;
2) τ_{ij} -neutrosophic-pre-open if and only if $P \subseteq N_{int}^{j}N_{cl}^{i}(P)$;
3) τ_{ij} -neutrosophic-b-open if and only if $P \subseteq N_{cl}^{i}N_{int}^{j}(P) \cup N_{int}^{j}N_{cl}^{i}(P)$.

Remark 4.1. A *neutrosophic set P* over X is called a τ_{ij} -neutrosophic-b-closed set if and only if P^c (complement of P) is τ_{ij} -neutrosophic-b-open set in (X, τ_1, τ_2).

Definition 4.2. Assume that (X, τ_1, τ_2) be a *neutrosophic bitopological space*. Then a *neutrosophic set G* over X is said to be a

1) τ_{ij} -neutrosophic-p-set if and only if $N_{cl}^{i}N_{int}^{j}(G) \subseteq N_{int}^{i}N_{cl}^{j}(G)$;

2) Contra τ_{ij} -neutrosophic-*p*-set if and only if $N_{cl}^{j}N_{int}^{i}(G) \subseteq N_{int}^{i}N_{cl}^{j}(G)$;
3) τ_{ij} -neutrosophic-*q*-set if and only if $N_{int}^{j}N_{cl}^{i}(G) \subseteq N_{cl}^{i}N_{int}^{j}(G)$.
4) Contra τ_{ij} -neutrosophic-*q*-set if and only $N_{int}^{i}N_{cl}^{j}(G) \subseteq N_{cl}^{i}N_{int}^{j}(G)$.

Theorem 4.1. In a *neutrosophic bitopological space* (X, τ_1, τ_2),

1) if G is τ_i-neutrosophic-closed and τ_{ij}-neutrosophic-pre-open then G is τ_{ij}-neutrosophic-semi-open.
2) if G is τ_j-neutrosophic-open and τ_{ij}-neutrosophic-semi-open then G is τ_{ij}-neutrosophic-pre-open.

Proof:

1) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and G be a *neutrosophic set* over X, which is both τ_i-neutrosophic-closed and τ_{ij}-neutrosophic-pre-open. So, we have

$$G= N_{cl}^{i}(G) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{and } G\subseteq N_{int}^{j}N_{cl}^{i}(G) \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

From eq (2) we have $G\subseteq N_{int}^{j}N_{cl}^{i}(G)$

$= N_{int}^{j}(G)$ [by eq (1)]

$\Rightarrow G\subseteq N_{int}^{j}(G) \subseteq N_{cl}^{i}N_{int}^{j}(G)$

$\Rightarrow G\subseteq N_{cl}^{i}N_{int}^{j}(G)$

Hence, G is a τ_{ij}-neutrosophic-semi-open set in (X, τ_1, τ_2).

2) Let (X, τ_1, τ_2) be a neutrosophic bitopological space and G be a *NS* over X, which is both τ_j-neutrosophic-open and τ_{ij}-neutrosophic-semi-open. So, we have

$$G=N_{int}^{j}(G) \quad \ldots\ldots\ldots\ldots\ldots..(3)$$

$$\text{and } G\subseteq N_{cl}^{i}N_{int}^{j}(G) \quad \ldots\ldots\ldots\ldots\ldots.(4)$$

From eq (4) we have

$G\subseteq N_{cl}^{i}N_{int}^{j}(G)$

$= N_{cl}^{i}(G)$ [by eq (3)]

$\Rightarrow G\subseteq N_{cl}^{i}(G)$

$\Rightarrow N_{int}^{j}(G) \subseteq N_{int}^{j}N_{cl}^{i}(G)$

$\Rightarrow G = N_{int}^{j}(G) \subseteq N_{int}^{j}N_{cl}^{i}(G)$ [since $G= N_{int}^{j}(G)$]

$\Rightarrow G \subseteq N_{int}^{j}N_{cl}^{i}(G)$

Hence, G is a τ_{ij}-neutrosophic-pre-open set in (X, τ_1, τ_2).

Theorem 4.2. Let (X, τ_1, τ_2) be a neutrosophic bitopological space. If A is τ_{ij}-neutrosophic-semi-open (τ_{ij}-neutrosophic-pre-open), then A is τ_{ij}-neutrosophic-*b*-open.

Proof: Let us assume that A is τ_{ij}-neutrosophic-semi-open set in a neutrosophic bitopological space (X,τ_1,τ_2). Then $A\subseteq N_{cl}^{i}N_{int}^{j}(A)$.

Now, $A \subseteq N_{cl}^{i} N_{int}^{j}(A)$

$\Rightarrow A \subseteq N_{cl}^{i} N_{int}^{j}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$.

Therefore, A is τ_{ij}-neutrosophic-b-open in (X, τ_1, τ_2).

Similarly, we can show that if A is τ_{ij}-neutrosophic-pre-open set in (X, τ_1, τ_2) then it is τ_{ij}-neutrosophic-b-open.

Theorem 4.3. Let (X, τ_1, τ_2) be a neutrosophic bitopological space.

1) If A is τ_{ij}-neutrosophic-b-open, contra τ_{ji}-neutrosophic-p-set then A is τ_{ij} -neutrosophic-pre-open set;
2) If A is τ_{ij}-neutrosophic-b-open, contra τ_{ij}-neutrosophic-q-set then A is τ_{ij} -neutrosophic-semi-open set;
3) If A is τ_{ij}-neutrosophic-b-open, τ_{ij}-neutrosophic-p-set and contra τ_{ji}-neutrosophic-q-set then A is τ_{ji}-neutrosophic-b-open set;
4) If A is τ_{ij}-neutrosophic-q-set (τ_{ij}-neutrosophic-p-set) then A^c is contra τ_{ji}-neutrosophic-p-set (contra τ_{ij}-neutrosophic-q-set).

Proof:

1) Let A be both τ_{ij}-neutrosophic-b-open and contra τ_{ji}-neutrosophic-p-set in a neutrosophic bitopological space (X, τ_1, τ_2).

Then, we have $A \subseteq N_{cl}^{i} N_{int}^{j}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$ ……….(5)

and $N_{cl}^{i} N_{int}^{j}(A) \subseteq N_{int}^{j} N_{cl}^{i}(A)$ ……....(6)

From eqs (5) & (6) we get

$A \subseteq N_{cl}^{i} N_{int}^{j}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$

$\subseteq N_{int}^{j} N_{cl}^{i}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$

$= N_{int}^{j} N_{cl}^{i}(A)$

$\Rightarrow A \subseteq N_{int}^{j} N_{cl}^{i}(A)$

Therefore, A is τ_{ij}-neutrosophic-pre-open set in (X, τ_1, τ_2).

2) The proof is analogous to the proof of part (1), so omitted.
3) Let A be τ_{ij}-neutrosophic-b-open, τ_{ij}-neutrosophic-p-set and contra τ_{ji}-neutrosophic-q-set in a neutrosophic bitopological space (X, τ_1, τ_2). Then we have

$A \subseteq N_{cl}^{i} N_{int}^{j}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$, …………(7)

$N_{cl}^{i} N_{int}^{j}(A) \subseteq N_{int}^{i} N_{cl}^{j}(A)$ …………(8)

and $N_{int}^{j} N_{cl}^{i}(A) \subseteq N_{cl}^{j} N_{int}^{i}(A)$ …………..(9)

From eq (7) we get $A \subseteq N_{cl}^{i} N_{int}^{j}(A) \cup N_{int}^{j} N_{cl}^{i}(A)$

$\subseteq N_{int}^{i} N_{cl}^{j}(A) \cup N_{cl}^{j} N_{int}^{i}(A)$ [by eqs (8) & (9)]

$\Rightarrow A \subseteq N_{int}^{i} N_{cl}^{j}(A) \cup N_{cl}^{j} N_{int}^{i}(A)$

Therefore, A is τ_{ji}-neutrosophic-b-open set in (X, τ_1, τ_2).

Theorem 4.4. In a neutrosophic bitopological space (X, τ_1, τ_2)

1) if A is τ_{ij}-neutrosophic-semi-open and τ_{ij}-neutrosophic-p-set then A is τ_{ji}-neutrosophic-pre-open;
2) If A is τ_{ji}-neutrosophic-semi-open and contra τ_{ji}-neutrosophic-p-set then A is τ_{ji}-neutrosophic-pre-open.

Proof:

1) Let (X, τ_1, τ_2) be a *neutrosophic bitopological space* and A is both τ_{ij}-neutrosophic-semi-open and τ_{ji}-neutrosophic-p-set.

   Since, A is τ_{ij}-neutrosophic-semi-open, so we have

   $$A \subseteq N^i_{cl} N^j_{int}(A) \qquad \text{………………(10)}$$

   Since, A is τ_{ji}-neutrosophic-p-set, so

   $$N^i_{cl} N^j_{int}(A) \subseteq N^i_{int} N^j_{cl}(A) \qquad \text{……………….(11)}$$

   From eqs (10) & (11), we've got

   $$A \subseteq N^i_{int} N^j_{cl}(A).$$

   Hence, A is τ_{ji} -neutrosophic-pre-open in (X, τ_1, τ_2).
2) The proof is analogous to the proof of the first part, so omitted.

Theorem 4.5. Let (X, τ_1, τ_2) be an neutrosophic bitopological space.

1) If A is τ_{ij}-neutrosophic-p-set and τ_{ji}-neutrosophic-q-set then $N^i_{cl} N^j_{int}(A) \subseteq N^j_{cl} N^i_{int}(A)$;
2) If A is contra τ_{ij}-neutrosophic-p-set and contra τ_{ij}-neutrosophic-q-set then $N^j_{cl} N^i_{int}(A) \subseteq N^i_{cl} N^j_{int}(A)$.

Proof:

1) Let (X, τ_1, τ_2) be a *neutrosophic bitopological space* and A be both τ_{ij}-neutrosophic-p-set and τ_{ji}-neutrosophic-q-set. Then, we have

   $$N^i_{cl} N^j_{int}(A) \subseteq N^i_{int} N^j_{cl}(A) \qquad \text{……………..(12)}$$

   $$\text{and} \quad N^i_{int} N^j_{cl}(A) \subseteq N^j_{cl} N^i_{int}(A) \qquad \text{……………..(13)}$$

From eqs (12) & (13), we get

$$N^i_{cl} N^j_{int}(A) \subseteq N^j_{cl} N^i_{int}(A).$$

2) Let (X, τ_1, τ_2) be a *neutrosophic bitopological space* and A be both contra τ_{ij}-neutrosophic-p-set and contra τ_{ij}-neutrosophic-q-set. Then, we have

   $$N^j_{cl} N^i_{int}(A) \subseteq N^i_{int} N^j_{cl}(A) \qquad \text{……………..(14)}$$

   $$N^i_{int} N^j_{cl}(A) \subseteq N^i_{cl} N^j_{int}(A) \qquad \text{……………..(15)}$$

From eqs (14) & (15), we get

$$N_{cl}^{j}N_{int}^{i}(A) \subseteq N_{cl}^{i}N_{int}^{j}(A).$$

5. Pairwise τ_{ij} -*b*-open:

Definition 5.1. A *neutrosophic set H* is said to be pairwise τ_{ij}-neutrosophic-semi-open set (pairwise τ_{ij}-neutrosophic-pre-open set) in a neutrosophic bitopological space (X, τ_1, τ_2) if $H=K\cup L$, where K is a τ_{ij}-neutrosophic-semi-open set (τ_{ij}-neutrosophic-pre-open set) and L is a τ_{ji}-neutrosophic-semi-open set (τ_{ji}-neutrosophic-pre-open set) in (X, τ_1, τ_2).

Definition 5.2. A *neutrosophic set H* is said to be pairwise τ_{ij}-neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) if $H=K\cup L$, where K is a τ_{ij}-neutrosophic-*b*-open set and L is a τ_{ji}-neutrosophic-*b*-open set in (X, τ_1, τ_2).

Theorem 5.1. The union of two pairwise τ_{ij}-neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) is again a pairwise τ_{ij}-neutrosophic-*b*-open set.

Proof: Let A, B be two pairwise τ_{ij} -neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2). Then there exists two τ_{ij}-neutrosophic-*b*-open set G_1, G_2 and two τ_{ji}-neutrosophic-*b*-open set H_1, H_2 such that $A= G_1\cup H_1$ and $B= G_2\cup H_2$.

Since, G_1, G_2 are τ_{ij}-neutrosophic-*b*-open set so

$$G_1 \subseteq N_{cl}^{i}N_{int}^{j}(G_1) \cup N_{int}^{j}N_{cl}^{i}(G_1) \qquad \text{............(16)}$$

and $$G_2 \subseteq N_{cl}^{i}N_{int}^{j}(G_2) \cup N_{int}^{j}N_{cl}^{i}(G_2) \qquad \text{..............(17)}$$

Since, H_1, H_2 are τ_{ji}-neutrosophic-*b*-open set so

$$H_1 \subseteq N_{cl}^{j}N_{int}^{i}(H_1) \cup N_{int}^{i}N_{cl}^{j}(H_1) \qquad \text{............(18)}$$

and $$H_2 \subseteq N_{cl}^{j}N_{int}^{i}(H_2) \cup N_{int}^{i}N_{cl}^{j}(H_2) \qquad \text{............(19)}$$

Now, we have

$$G_1 \cup G_2 \subseteq N_{cl}^{i}N_{int}^{j}(G_1) \cup N_{int}^{j}N_{cl}^{i}(G_1) \cup N_{cl}^{i}N_{int}^{j}(G_2) \cup N_{int}^{j}N_{cl}^{i}(G_2) \quad \text{[using eqs (16) \& (17)]}$$
$$=N_{cl}^{i}N_{int}^{j}(G_1) \cup N_{cl}^{i}N_{int}^{j}(G_2) \cup N_{int}^{j}N_{cl}^{i}(G_1) \cup N_{int}^{j}N_{cl}^{i}(G_2)$$
$$\subseteq N_{cl}^{i}(N_{int}^{j}(G_1) \cup N_{int}^{j}(G_2)) \cup N_{int}^{j}(N_{cl}^{i}(G_1) \cup N_{cl}^{i}(G_2))$$
$$\subseteq N_{cl}^{i}(N_{int}^{j}(G_1 \cup G_2)) \cup N_{int}^{j}(N_{cl}^{i}(G_1 \cup G_2))$$

$\Rightarrow G_1\cup G_2$ is a τ_{ij} -neutrosophic-*b*-open set.

Further, we have

$$H_1 \cup H_2 \subseteq N_{cl}^{j}N_{int}^{i}(H_1) \cup N_{int}^{i}N_{cl}^{j}(H_1) \cup N_{cl}^{j}N_{int}^{i}(H_2) \cup N_{int}^{i}N_{cl}^{j}(H_2) \quad \text{[using eqs (18) \& (19)]}$$

$$=N_{cl}^{j}N_{int}^{i}(H_1) \cup N_{cl}^{j}N_{int}^{i}(H_2) \cup N_{int}^{i}N_{cl}^{j}(H_1) \cup N_{int}^{i}N_{cl}^{j}(H_2)$$
$$\subseteq N_{cl}^{j}(N_{int}^{i}(H_1) \cup N_{int}^{i}(H_2)) \cup N_{int}^{i}(N_{cl}^{j}(H_1) \cup N_{cl}^{j}(H_2))$$
$$\subseteq N_{cl}^{j}(N_{int}^{i}(H_1 \cup H_2)) \cup N_{int}^{i}(N_{cl}^{j}(H_1 \cup H_2))$$

$\Rightarrow H_1\cup H_2$ is a τ_{ji}-neutrosophic-*b*-open set.

Hence, $A\cup B=(G_1\cup H_1)\cup (G_2\cup H_2)= (G_1\cup G_2)\cup(H_1\cup H_2)=G\cup H.$

Therefore there exists a τ_{ij}-neutrosophic-*b*-open set $G=(G_1\cup G_2)$ and a τ_{ji}-neutrosophic-*b*-open set $H=(H_1\cup H_2)$ such that $A\cup B= G\cup H$. Hence $A\cup B$ is a pairwise τ_{ij}-neutrosophic-*b*-open set. Thus the union of two pairwise τ_{ij}-neutrosophic-*b*-open set in a neutrosophic bitopological space (X, τ_1, τ_2) is again a pairwise τ_{ij}-neutrosophic-*b*-open set.

Theorem 5.2. In a neutrosophic bitopological space (X, τ_1, τ_2), every pairwise τ_{ij}-neutrosophic-semi open set (pairwise τ_{ij}-neutrosophic-pre-open set) is a pairwise τ_{ij}-neutrosophic-*b*-open set.

Proof: Let G be a pairwise τ_{ij}-neutrosophic-semi-open set (pairwise τ_{ij}-neutrosophic-pre-open set). Then there exist a τ_{ij}-neutrosophic-semi-open set A (τ_{ij}-neutrosophic-pre-open set A) and a τ_{ji}-neutrosophic-semi-open set B (τ_{ji}-neutrosophic-pre-open set B) such that $G=A\cup B$.

In theorem 4.2, it is clearly shown that every τ_{ij}-neutrosophic-semi-open set (τ_{ij}-neutrosophic-pre-open set) is a τ_{ij}-neutrosophic-*b*-open set and every τ_{ji}-neutrosophic-semi-open set (τ_{ji}-neutrosophic-pre-open set) is a τ_{ji}-neutrosophic-*b*-open set. So A be τ_{ij}-neutrosophic-*b*-open and B be τ_{ij}-neutrosophic-*b*-open set. Therefore, there exist a τ_{ij}-neutrosophic-*b*-open set A and a τ_{ij}-neutrosophic-*b*-open set B such that $G=A\cup B$. Hence, G is a pairwise τ_{ij}-neutrosophic-*b*-open set. Thus every pairwise τ_{ij}-neutrosophic-semi-open set (pairwise τ_{ij}-neutrosophic-pre-open set) is a pairwise τ_{ij}-neutrosophic-*b*-open set.

6. Conclusion

In this article, we studied neutrosophic-*b*-open set, pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces and investigate their basic properties. By defining neutrosophic-*b*-open set, pairwise neutrosophic-*b*-open set, we prove some theorems on neutrosophic bitopological spaces and some examples are given.

.

References

1. Akram, M., Gulzar, H., Smarandache, F. (2019). Neutrosophic soft topological k-algebras, *Neutrosophic Sets and Systems*, 25, 104-124.
2. Andrijevic, D. (1996). On b-open sets. *Mat. Vesnik*, 48, 59-64.
3. Chang, C. L. (1968). Fuzzy Topological Spaces. *Journal of Mathematical Analysis and Applications*. 24, 182-190.
4. Das, R., Smarandache, F. & Tripathy, B. C. (2020). Neutrosophic fuzzy matrices and some algebraic operation, *Neutrosophic Sets and Systems*, 32, 401-409.
5. Das, R. & Tripathy B. C. (2020). Neutrosophic multiset topological space, *Neutrosophic Sets and Systems*, 35, 142-152.
6. Das, S., Das, R., & Tripathy, B. C. (2020). Multi-criteria group decision making model using single-valued neutrosophic set, *LogForum*, 16 (3), 421-429.
7. Das, S., & Pramanik, S. (2020). Generalized neutrosophic b-open sets in neutrosophic topological space, *Neutrosophic Sets and Systems*, 35, 522-530.
8. Dutta, A. & Tripathy, B.C. (2017). On fuzzy *b*-θ open sets in fuzzy topological space, *Journal of Intelligent and Fuzzy Systems*, 32 (1), 137-139.

9. Ebenanjar, E., Immaculate, J., & Wilfred, C. B. (2018). On neutrosophic b-open sets in neutrosophic topological space. *Journal of Physics Conference Series*, 1139 (1), 012062.
10. Kelly, J. C. (1963). Bitopological Spaces. *Proceedings of the London Mathematical Society*, 3 (1), 71-89.
11. Bera, T., & Mahapatra, N. K. (2018). On Neutrosophic Soft Prime Ideal, *Neutrosophic Sets and Systems*, 20, 54-75.
12. Ozturk, T. Y., & Ozkan, A. (2019). Neutrosophic Bitopological Spaces. *Neutrosophic Sets and Systems*, 30, 88-97.
13. Rao, V. V., & Srinivasa, R. (2017). Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology. *International Journal of Chemical Technology Research*, 10 (10), 449-458.
14. Salama, A. A., & Alblowi, S. A. (2012a). Neutrosophic set and neutrosophic topological space. *ISOR Journal of Mathematics*, 3 (4), 31-35.
15. Salama, A. A., & Alblowi, S.A. (2012b). Generalized Neutrosophic Set and Generalized Neutrosophic Topological Space. *Comp. Sci. Engg.*, 2, 129-132.
16. Salama, A. A., Smarandache, F., & Alblowi, S.A. (2014). New neutrosophic crisp topological concepts. *Neutrosophic Sets and Systems*, 2, 50-54.
17. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. *Rehoboth: American Research Press.*
18. Smarandache, F. (2005). Neutrosophic set: a generalization of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24, 287-297.
19. Thangavelu, P., & Thamizharasi, G. (2011). *bt*-open sets and *t*-open sets in bitopological space. *International Journal of Mathematical Archive.*, 2 (5), 681-687.
20. Tripathy, B. C., & Debnath, S. (2015). On fuzzy *b*-locally open sets in bitopological spaces. *Songklanakarin Journal of Science and Technol.*, 37 (1), 93-96.
21. Tripathy, B. C., & Sarma, D. J. (2011). On *b*-Locally Open Sets in Bitopological Spaces. *Kyungpook Mathematical Journal*, 51, 429-433.
22. Tripathy, B. C., & Sarma, D. J. (2012). On pairwise *b*–locally open and pairwise *b*–locally closed functions in bitopological spaces, Tamkang Jour. Math., 43 (4), 533-539.
23. Tripathy, B. C., & Sarma, D. J. (2013). On weakly *b*-continuous functions in bitopological spaces, *Acta Scientiarum Technology*, 35 (3), 521-525.
24. Tripathy, B. C., & Sarma, D. J. (2014). Generalized *b*-closed sets in Ideal bitopological spaces, *Proyecciones J. Math*., 33 (3), 315-324.
25. Tripathy, B. C., & Sarma, D. J. (2017). Pairwise generalized b-R_0 spaces in bitopological spaces. *Proyecciones Journal of Mathematics*, 36 (4), 589-600.

Received: May 20, 2020. Accepted: Nov 21, 2020