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Decomposition of Single-Valued Neutrosophic Ideal Continuity via Fuzzy Idealization

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Abstract: The aim of this paper is to introduce various types of r-single-valued neutrosophic open sets based on the single-valued neutrosophic ideals in Šostak Sense. Different mappings of single-valued continuity and ideal continuity based on the r-single-valued neutrosophic ideal openness are defined and many implications between them are investigated with counterexamples illustrated.

Keywords: r-single-valued neutrosophic open set; r-single-valued neutrosophic ideal closed set; single-valued neutrosophic continuous mappings and single-valued neutrosophic ideal continuous mappings.

1. Introduction

In the classic text, Kuratowski [1] dealt with the genesis of the concept of ideal in general topological spaces. This area of study is approached by many others and hence some sorts of ideals arise as one goes further in mathematics such as the ideal of finite subsets of $\tilde{\mathfrak{X}}$, the ideal of nowhere dense sets and ideal of meager sets. Many topologists introduced distinct types of operators as regards ideals, compatibility property, compactness module an ideal and other concedes. Vaidyanathaswamy [2] introduced the concept of local function of $\tilde{\mathcal{I}}$ in relation to $\tilde{\tau}$. The notion of fuzzy ideal and the concept of fuzzy local function of $\tilde{\mathcal{I}}$ with respect to $\tilde{\tau}$ had been introduced and examined by Sarkar [3]. Besides, the notion of compatibility of fuzzy ideals with fuzzy topologies had been introduced and studied by Sarkar. In [4], Šostak initiated a new definition of fuzzy topology, which is termed "fuzzy topology in Šostak sense", as an extension of both crisp topology and Chang's fuzzy topology, in the logic that not only the objects are fuzzified, but also the axiomatics. Šostak [5-7] presented some rules and explained how such an extension can be realized. Saber et al [8] familiarized and considered the notion of fuzzy ideal and the concept of fuzzy local function of $\tilde{\mathcal{I}}$ in respect of $\tilde{\tau}$ in Šostak sense. Saber et al [9-13] provided several rules and displayed how such an extension can be acquired.

Thus, Smarandache [14] generalizes almost all the existing logics like, fuzzy logic, intutionistic fuzzy logic etc. After this, many researchers used neutrosophic sets and logic in topological spaces, such as Das et al. [15], Fatimah et al. [16], Riaz et al. [17], Porselvi et al. [18], Singh et al. [19]. In recent times, Abdel-Basset et al. have studied a novel neutrosophic approach [20-23] in many areas, in other words, information and communication technology. In the meantime, Salama et al. [24, 25] investigated the notions of generalized neutrosophic set ($\mathcal{NS}$) and Intuitionistic neutrosophic set ($\mathcal{IFS}$). Respectively, Hur et al [26, 27] brought to light classifications neutrosophic H-set ($\mathcal{N}\text{h}\mathcal{S}$) and ($\mathcal{N}\mathbb{C}\mathcal{S}$) as well as neutrosophic crisp as they scrutinized them in a universe topological position. Still, Salama and Alblowi [28] displayed neutrosophic topology in as much as they claimed a number of its features. Wang et al [29], among many others, shaped the single-valued neutrosophic set concept. Presently, Kim et al grappled with a neutrosophic partition single-value, neutrosophoic equivalence relation single-value and neutrosophic relation single-value. The notion of single-valued neutrosophic ideal, single-valued neutrosophic ideal open local function and single-valued neutrosophic ideal open compatible are explored in (2020) by Saber et al [30, 31].

This paper is arranged as follows. Preliminaries of single-value neutrosophic sets and single-valued neutrosophic topology are reviewed in Section **2.** In Section **3** and **4**, we obtained very important relevant topics and results such as single-valued neutrosophic ideal closed sets in Šostak sense and single-valued neutrosophic ideal continuous ($\mathcal{SVNJ}-$continuous) mappings, single-valued neutrosophic continuous ($\mathcal{SVN}-$continuous) mappings and investigated several characterizations of these crucial topics and ideas. These mappings are obviously considered to be generalizations of fuzzy ideal continuous mappings, introduced by Saber et al [32] In Section 5, we obtained very important relevant topics and results such as single-valued neutrosophic ideal closed sets (r-**SVNSO**) in Šostak sense and $\mathcal{SVNJ}-$continuous. We have arrived to notable definitions theorems, and counter examples in detailed analysis to examine some of their substantial characteristics and to explore the best results and imports. We can safely claim that diverse decisive concepts in single-valued neutrosophic topology were established and generalized in this article. Distinct aspects like continuous and ideal continuous which have a major effect on the overall topology's notions were also considered.

Original aspects and credits of this article juxtaposed to pertinent recent research on groups related to it are very worthwhile. This study deals with continuous and ideal continuous of single-valued neutrosophic topological spaces ($\mathcal{SVNTS}$) in Šostak sense. The great import of this study is the introduction of the concept of *r*-single-valued neutrosophic open (r-**SVNSO**). The researchers secure some of its basic properties. Moreover, as an application, we give a multicriteria decision making for the combining effects of certain enzymes on chosen DNA.

2. Preliminaries

Here, in this section, we consider the fundamental concepts of single valued neutrosophic sets (briefly, $\mathcal{SVNS}$), single valued neutrosophic topological spaces (briefly, $\mathcal{SVNTS}$) and single-valued neutrosophic ideals (briefly, $\mathcal{SVNJ}$). Although Section **2** is considered as a background for the material included in this paper.

Definition 2.1 [33] Suppose that $\tilde{\mathfrak{X}}$ is a non empty set, then $\mathcal{S}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\tilde{\mathfrak{X}}\}$, is called a neutrosophic set (briefly, $\mathcal{NS}$) in $\tilde{\mathfrak{X}}$, where, $\tilde{\mu}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\gamma}_{\mathcal{S}}$ and the degree of non-membership (namely $\tilde{\mu}_{\mathcal{S}}(\omega)$), the degree of indeterminacy (namely $\tilde{\eta}_{\mathcal{S}}(\omega)$), and the degree of membership (namely $\tilde{\gamma}_{\mathcal{S}}(\omega)$), for all $\omega\in\tilde{\mathfrak{X}}$ to the set $\mathcal{S}$.

A neutrosophic set $\mathcal{S}=\langle\omega.,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}:\omega\in\tilde{\mathfrak{X}}\rangle$, can be identified as $\langle\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle$ in $\rfloor^{-}0,1^{+}\lfloor$ in $\tilde{\mathfrak{X}}$.

Definition 2.2 [35] Suppose that $\mathcal{S}$ and $\mathcal{E}$ are $\mathcal{NS}'$s of the form $\mathcal{S}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\tilde{\mathfrak{X}}\}$ and $\mathcal{E}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\tilde{\mathfrak{X}}\}$ Then, $\mathcal{S}\subseteq\mathcal{E}$, iff for every $\omega\in\tilde{\mathfrak{X}}$.

$$\inf\tilde{\eta}_{\mathcal{S}}(\omega)\geq\inf\tilde{\eta}_{\mathcal{E}}(\omega),\quad \inf\tilde{\mu}_{\mathcal{S}}(\omega)\geq\inf\tilde{\mu}_{\mathcal{E}}(\omega)\text{ and }\inf\tilde{\gamma}_{\mathcal{E}}(\omega)\leq\inf\tilde{\gamma}_{\mathcal{E}}(\omega),$$
$$\sup\tilde{\eta}_{\mathcal{S}}(\omega)\geq\sup\tilde{\eta}_{\mathcal{E}}(\omega),\ \sup\tilde{\mu}_{\mathcal{S}}(\omega)\geq\sup\tilde{\mu}_{\mathcal{S}}(\omega)\ \ and\ \sup\tilde{\gamma}_{\mathcal{S}}(\omega)\leq\sup\tilde{\gamma}_{\mathcal{E}}(\omega).$$

Definition 2.3 [29] Suppose that $\mathcal{S}$ is a space of points (objects) with a generic element in $\widetilde{\mathfrak{T}}$ denoted by ω. Then, $\mathcal{S}$ is called a single-valued neutrosophic set (briefly, $\mathcal{SVNS}$) in $\widetilde{\mathfrak{T}}$, if $\mathcal{S}$ has the form $\mathcal{S} = \langle \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}} \rangle$, where $\tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}}: \widetilde{\mathfrak{T}} \to [0.1]$.

In this case, $\tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}}$ are called truth-membership mapping, indeterminacy-membership mapping, falsity-membership mapping, respectively, and we will denote the set of all $\mathcal{SVNS}'s$ in $\widetilde{\mathfrak{T}}$ as $I^{\widetilde{\mathfrak{T}}}$.

Moreover, we will refer to the Null (empty) $\mathcal{SVNS}$ (resp. the absolute (universe) $\mathcal{SVNS}$) in $\widetilde{\mathfrak{T}}$ as 0_N (resp. 1_N) and define by $0_N = \langle 0,1,1 \rangle$ (resp. $1_N = \langle 1,0,0 \rangle$) for each $\omega \in \widetilde{\mathfrak{T}}$.

Definition 2.4 [29]. Let $\mathcal{S} = \{\langle \omega, \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}} \rangle : \omega \in \widetilde{\mathfrak{T}}\}$ be an $\mathcal{SVNS}$ on $\widetilde{\mathfrak{T}}$. The complement of the set $\mathcal{S}$ $(in\ sort, \mathcal{S}^c)$ maybe defined as, for all $\omega \in \widetilde{\mathfrak{T}}$

$$\tilde{\gamma}_{\mathcal{S}^c}(\omega) = \tilde{\mu}_{\mathcal{S}}(\omega), \quad \tilde{\eta}_{\mathcal{S}^c}(\omega) = 1 - \tilde{\eta}_{\mathcal{S}}(\omega) \ and \ \tilde{\mu}_{\mathcal{S}^c}(\omega) = \tilde{\gamma}_{\mathcal{S}}(\omega).$$

Definition 2.5 [34]. Let $\mathcal{S}, \mathcal{E} \in \mathcal{SVNS}(\widetilde{\mathfrak{T}})$. Then,

1. $\mathcal{S} \subseteq \mathcal{E}$, if, for every $\omega \in \widetilde{\mathfrak{T}}$,

$$\tilde{\eta}_{\mathcal{S}}(\omega) \geq \tilde{\eta}_{\mathcal{E}}(\omega), \ \tilde{\mu}_{\mathcal{S}}(\omega) \geq \tilde{\mu}_{\mathcal{E}}(\omega) \ and \ \tilde{\gamma}_{\mathcal{S}}(\omega) \leq \tilde{\gamma}_{\mathcal{E}}(\omega),$$

2. we say $\mathcal{S} = \mathcal{E}$ if $\mathcal{S} \subseteq \mathcal{E}$ and $\mathcal{S} \supseteq \mathcal{E}$.

Definition 2.6 [35]. Let $\mathcal{S}, \mathcal{E} \in \mathcal{SVNS}(\widetilde{\mathfrak{T}})$. Then,

1. $\mathcal{S} \cap \mathcal{E}$ is a $\mathcal{SVNS}$ in $\widetilde{\mathfrak{T}}$ defined as:

$$\mathcal{S} \cap \mathcal{E} = (\tilde{\gamma}_{\mathcal{S}} \cap \tilde{\gamma}_{\mathcal{E}}, \ \tilde{\eta}_{\mathcal{S}} \cup \tilde{\eta}_{\mathcal{E}}, \ \tilde{\mu}_{\mathcal{S}} \cup \tilde{\mu}_{\mathcal{E}}).$$

Where, $(\tilde{\mu}_{\mathcal{S}} \cup \tilde{\mu}_{\mathcal{E}})(\omega) = \tilde{\mu}_{\mathcal{S}}(\omega) \cup \tilde{\mu}_{\mathcal{E}}(\omega)$ and $(\tilde{\gamma}_{\mathcal{S}} \cap \tilde{\gamma}_{\mathcal{E}})(\omega) = \tilde{\gamma}_{\mathcal{S}}(\omega) \cap \tilde{\gamma}_{\mathcal{E}}(\omega)$, for all $\omega \in \widetilde{\mathfrak{T}}$.

2. $\mathcal{S} \cup \mathcal{E}$ is an $\mathcal{SVNS}$ on $\widetilde{\mathfrak{T}}$ defined as:

$$\mathcal{S} \cup \mathcal{E} = (\tilde{\gamma}_{\mathcal{S}} \cup \tilde{\gamma}_{\mathcal{E}}, \ \tilde{\eta}_{\mathcal{S}} \cap \tilde{\eta}_{\mathcal{E}}, \ \tilde{\mu}_{\mathcal{S}} \cap \tilde{\mu}_{\mathcal{E}}).$$

Definition 2.7 [28] Let $\mathcal{S} \in \mathcal{SVNS}(\widetilde{\mathfrak{T}})$. Then.

1. The intersection of $\{\mathcal{S}_j : j \in \triangle\}$ (briefly, $\bigcap_{j\in\triangle} \mathcal{S}_J$) is $\mathcal{SVNS}$ over $\widetilde{\mathfrak{T}}$ defined as: for all $\omega \in \widetilde{\mathfrak{T}}$,

$$\left(\bigcap_{J\in\triangle} \mathcal{S}_j\right)(\omega) = \left(\bigcap_{j\in\triangle} \tilde{\gamma}_{\mathcal{S}_j}(\omega), \ \bigcup_{j\in\triangle} \tilde{\eta}_{\mathcal{S}_j}(\omega), \ \bigcup_{j\in\triangle} \tilde{\mu}_{\mathcal{S}_j}(\omega)\right).$$

2. The union of $\{\mathcal{S}_j : j \in \triangle\}$ (briefly, $\bigcup_{j\in\triangle} \mathcal{S}_j$) is $\mathcal{SVNS}$ over $\widetilde{\mathfrak{T}}$ defined as: for all $\{\mathcal{S}_j : j \in \triangle\}$,

$$\left(\bigcup_{j\in\triangle}\mathcal{S}_j\right)(\omega)=\left(\bigcup_{j\in\triangle}\tilde{\gamma}_{\mathcal{S}_j}(\omega),\quad \bigcap_{j\in\triangle}\tilde{\eta}_{\mathcal{S}_j}(\omega),\quad \bigcap_{j\in\triangle}\tilde{\mu}_{\mathcal{S}_j}(\omega)\right).$$

Definition 2.8 [30]. Suppose that $t,s,k\in I_0$ and $s+t+k\leq 3$. A single-valued neutrosophic point (briefly, $\mathcal{SVNP}$) $x_{,t,k}$ of $\tilde{\mathfrak{T}}$ is the $\mathcal{SVNS}$ in $I^{\tilde{\mathfrak{T}}}$ for every $\omega\in\mathcal{S}$, defined by

$$x_{s,t,k}(\omega)=\begin{cases}(s,t,k), & if\ \ x=\omega,\\ (0,1,1), & if\ \ x\neq\omega.\end{cases}$$

A $\mathcal{SVNP}$ $x_{s,t,k}$ is said to belong to a $\mathcal{SVNS}$ $\mathcal{S}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\tilde{\mathfrak{T}}\}\in I^{\tilde{\mathfrak{T}}}$, (notion: $x_{s.t.p}\in\mathcal{S}$ iff $s<\tilde{\gamma}_{\mathcal{S}},\ t\geq\tilde{\eta}_{\mathcal{S}}$ and $k\geq\tilde{\mu}_{\mathcal{S}}$), and the set off all $\mathcal{SVNP}$ in $\tilde{\mathfrak{T}}$ denoted by $\mathcal{SVNP}(\tilde{\mathfrak{T}})$.

Definition 2.9 [36] Suppose that $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ be the collection of $\mathcal{SVNS}s$ over $\tilde{\mathfrak{T}}$; then $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ is called $\mathcal{SVNT}$ on $\tilde{\mathfrak{T}}$ if $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ satisfies the following axioms:

1. $\tilde{\tau}^{\tilde{\gamma}}(\underline{0})=\tilde{\tau}^{\tilde{\gamma}}(\underline{1})=1$ and $\tilde{\tau}^{\tilde{\eta}}(\underline{0})=\tilde{\tau}^{\tilde{\eta}}(\underline{1})=\tilde{\tau}^{\tilde{\mu}}(\underline{0})=\tilde{\tau}^{\tilde{\mu}}(\underline{1})=0$,
2. $\tau^{\tilde{\gamma}}(\mathcal{S}\cap\mathcal{E})\geq\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S})\cap\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E})$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}\cap\mathcal{E})\leq\tau^{\tilde{\eta}}(\mathcal{S})\cup\tilde{\tau}^{\tilde{\eta}}(\mathcal{E})$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}\cap\mathcal{E})\leq\tilde{\tau}^{\tilde{\mu}}(\mathcal{S})\cup\tilde{\tau}^{\tilde{\mu}}(\mathcal{E})$, for every $\mathcal{S},\mathcal{E}\in I^{\tilde{\mathfrak{T}}}$,
3. $\tilde{\tau}^{\tilde{\gamma}}(\cup_{j\in\triangle}\mathcal{S}_j)\geq\cap_{j\in\triangle}\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j)$, $\tilde{\tau}^{\tilde{\eta}}(\cup_{i\in\triangle}\mathcal{S}_j)\leq\cup_{j\in\triangle}\tau^{\tilde{\eta}}(\mathcal{S}_j)$ and $\tilde{\tau}^{\tilde{\mu}}(\cup_{j\in\triangle}\mathcal{S}_j)\leq\cup_{j\in\triangle}\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)$, for every $\{\mathcal{S}_j,\ j\in\triangle\}\in I^{\tilde{\mathfrak{T}}}$.

The triplet $(\tilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ is called $\mathcal{SVNTS}$, where $\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}}:I^{\tilde{\mathfrak{T}}}\to I$. Occasionally, we will write $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ for $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ and it will cause no ambiguity.

Theorem 2.10 [30] Let $(\tilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$. Then, for all $\mathcal{S}\in I^{\tilde{\mathfrak{T}}}$ and $r\in I_0$, we can define operator $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}:I^{\tilde{\mathfrak{T}}}\times I_0\to I^{\tilde{\mathfrak{T}}}$ as follows:

$$C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}.r)=\cap\{\mathcal{E}\in I^{\tilde{\mathfrak{T}}}:\ \mathcal{S}\leq\mathcal{E},\ \tilde{\tau}^{\tilde{\gamma}}(\underline{1}-\mathcal{E})\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\underline{1}-\mathcal{E})\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\underline{1}-\mathcal{E})\leq 1-r\}.$$

Then, $(\tilde{\mathfrak{T}},C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}})$ is an $\mathcal{SVNCS}$.

Definition 2.11 [30] A map $\tilde{\mathfrak{J}}^{\tilde{\gamma}},\tilde{\mathfrak{J}}^{\tilde{\eta}},\tilde{\mathfrak{J}}^{\tilde{\mu}}:I^{\tilde{\mathfrak{T}}}\to I$ is called $\mathcal{SVNJ}$ on $\tilde{\mathfrak{T}}$ if it satisfies the following three conditions:

1. $\tilde{\mathfrak{J}}^{\tilde{\eta}}(\underline{0})=\tilde{\mathfrak{J}}^{\tilde{\mu}}(\underline{0})=0$ and $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\underline{0})=1$,
2. If $\mathcal{S}\leq\mathcal{E}$ then $\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{E})\geq\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S})$, $\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{E})\geq\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S})$ and $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{E})\leq\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S})$, for all $\mathcal{S},\mathcal{E}\in I^{\tilde{\mathfrak{T}}}$.
3. $\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{E})\cup\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{E})$, $\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S})\cup\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{E})$ and $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S}\cup\mathcal{E})\geq\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S})\cap\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{E})$, for all $\mathcal{S},\mathcal{E}\in I^{\tilde{\mathfrak{T}}}$.

The triple $(\tilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathfrak{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is called a single-valued neutrosophic ideal topological space (briefly, $\mathcal{SVNJTS}$).

Definition 2.12 [30] Let $(\tilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a $\mathcal{SVNTS}$ for each $\mathcal{S}\in I^{\tilde{\mathfrak{T}}}$. Then the single-valued neutrosophic ideal open local function $\mathcal{S}_r^{\star}(\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathfrak{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ of $\mathcal{S}$ is the union of all single-valued neutrosophic points $x_{s.t.p}$ such that if $\mathcal{E}\in Q_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(x_{s,t,p},r)$ and $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{D})\geq r,\ \tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{D})\leq 1-r,\ \tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{D})\leq$

$1-r$, then there is at least one $\omega \in \widetilde{\mathfrak{X}}$ for which $\tilde{\gamma}_{\mathcal{E}}(\omega)+\tilde{\gamma}_{\mathcal{S}}(\omega)-1>\tilde{\gamma}_{\mathcal{D}}(\omega)$, $\tilde{\eta}_{\mathcal{E}}(\omega)+\tilde{\eta}_{\mathcal{S}}(\omega)-1\leq \tilde{\eta}_{\mathcal{D}}(\omega)$ and $\tilde{\mu}_{\mathcal{E}}(\omega)+\tilde{\mu}_{\mathcal{S}}(\omega)-1\leq \tilde{\mu}_{\mathcal{D}}(\omega)$.

Occasionally, we will write $\mathcal{S}_r^\star$ for $\mathcal{S}_r^\star(\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ and it will be no ambiguity.

Remark 2.13 [30] Suppose that $(\widetilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is an $\mathcal{SVNJTS}$ and $\mathcal{S}\in I^{\widetilde{\mathfrak{X}}}$. Then we obtain;

$$Cl^\star_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r)=\mathcal{S}\cup\mathcal{S}_r^\star,\qquad int^\star_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r)=\mathcal{S}\wedge[((\mathcal{H}^c)_r^\star)^c].$$

Theorem 2.14 [30] Let $(\widetilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a $\mathcal{SVNJTS}$ and $\tilde{\mathcal{J}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ be a $\mathcal{SVNJ}$ on $\widetilde{\mathfrak{X}}$. Then

1. If $\mathcal{S}\leq\mathcal{E}$, then $\mathcal{S}_r^\star\leq\mathcal{E}_r^\star$;
2. If $\tilde{\mathcal{J}}_1^{\tilde{\gamma}}\leq\tilde{\mathcal{J}}_2^{\tilde{\gamma}}$, $\tilde{\mathcal{J}}_1^{\tilde{\eta}}\geq\tilde{\mathcal{J}}_2^{\tilde{\eta}}$ and $\tilde{\mathcal{J}}_1^{\tilde{\mu}}\geq\tilde{\mathcal{J}}_2^{\tilde{\mu}}$, then $\mathcal{S}_r^\star(\tilde{\mathcal{J}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})\geq\mathcal{S}_r^\star(\tilde{\mathcal{J}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$,
3. $\mathcal{S}_r^\star=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^\star,r)\leq\mathbb{C}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r)$,
4. $(\mathcal{S}_r^\star)_r^\star\leq\mathcal{S}_r^\star$,
5. $(\mathcal{S}_r^\star\vee\mathcal{E}_r^\star)=(\mathcal{S}\vee\mathcal{E})_r^\star$,
6. If $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{E})\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E})\leq 1-r$, and $\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{E})\leq 1-r$ then $(\mathcal{S}\vee\mathcal{E})_r^\star=\mathcal{S}_r^\star\vee\mathcal{E}_r^\star=\mathcal{S}_r^\star$,
7. If $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E})\geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{E})\leq 1-r$, and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{E})\leq 1-r$, then $(\mathcal{E}\wedge\mathcal{S}_r^\star)\leq(\mathcal{E}\wedge\mathcal{S})_r^\star$,
8. $(\mathcal{S}_r^\star\wedge\mathcal{E}_r^\star)\geq(\mathcal{S}\wedge\mathcal{E})_r^\star$.

3. Single-Valued Neutrosophic Ideal Closed Sets in Šostak Sense

The aim of this section is to define the r-single-valued neutrosophic ideal open (briefly, r-**SVNIO**), r-single valued neutrosophic semi-open (briefly, r-**SVNSO**), r-single-valued neutrosophic β-open (briefly, r-**SVN$\boldsymbol{\beta}$O**) and r-single-valued neutrosophic pre-open sets (briefly, r-**SVNPO**) in the sense of Šostak.

Definition 3.1. A single-valued neutrosophic set $\mathcal{S}$ of an $\mathcal{SVNJTS}$ $(\widetilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is called:

1. r-**SVNIO** if $\mathcal{S}\leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^\star,r)$, for $r\in I_0$,
2. r-**SVNSO** if $\mathcal{S}\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r),r)$, for every $r\in I_0$,
3. r-**SVN$\boldsymbol{\beta}$O** if for every $r\in I_0$ $\mathcal{S}\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r),r),r)$,
4. r-**SVNPO** if $\mathcal{S}\leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},r),r)$, for every $r\in I_0$.

The complement of r-**SVNIO** (resp. r-**SVNSO**, r-**SVN$\boldsymbol{\beta}$O**, r-**SVNPO**) is called r-**SVNIC** (resp. r-**SVNSC**, r-**SVN$\boldsymbol{\beta}$C**, r-**SVNPC**).

Remark 3.2. r-**SVNO** and r-**SVNIO** are independent notions

Example 3.3. Let $\widetilde{\mathfrak{X}}=\{a,b\}$. Define $\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3,\mathcal{D}_1,\mathcal{D}_2,\mathcal{D}_3\in I^{\widetilde{\mathfrak{X}}}$ as follows:

$\mathcal{E}_1=\langle(0\cdot5,0\cdot5),(0\cdot5,0\cdot5),(0\cdot5.0\cdot5)\rangle$, $\mathcal{E}_2=\langle(0\cdot4,0\cdot3),(0\cdot4,0\cdot1),(0\cdot1.0\cdot2)\rangle$,
$\mathcal{E}_3=\langle(0\cdot1,0\cdot3),(0\cdot4,0\cdot1),(0\cdot5,0\cdot4)\rangle$, $\mathcal{D}_1=\langle(0\cdot4,0\cdot4),(0\cdot4,0\cdot3),(0\cdot2,0\cdot2)\rangle$,
$\mathcal{D}_2=\langle(0\cdot2,0\cdot2),(0\cdot2,0\cdot2),(0\cdot1,0\cdot1)\rangle$, $\mathcal{D}_3=\langle(0\cdot1,0\cdot1),(0\cdot1,0\cdot1),(0\cdot1,0\cdot1)\rangle$,

Define $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$, $\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}: I^{\widetilde{\mathfrak{X}}}\to I$ as follows:

$$\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = (0,1,1), \\ 1, & if\ \mathcal{S} = (1,0,0), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_1, \end{cases} \qquad \tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = (0,1,1), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{D}_1, \\ \frac{2}{3}, & if\ \underline{0} < \mathcal{S} < \mathcal{D}_1, \end{cases}$$

$$\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = (0,1,1), \\ 0, & if\ \mathcal{S} = (1,0,0), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_2, \end{cases} \qquad \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = (0,1,1), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{D}_2, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{D}_2, \end{cases}$$

$$\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = (0,1,1), \\ 0, & if\ \mathcal{S} = (1,0,0), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_3, \end{cases} \qquad \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = (0,1,1), \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{D}_2, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{D}_2. \end{cases}$$

Then, $\mathcal{E}_1$ is $\frac{1}{2}$-**SVNIO** but $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_1) = \frac{1}{2}$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_1) = 1 \nleq \frac{1}{2}$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_1) = 1 \nleq \frac{1}{2}$ is not $\frac{1}{2}$-**SVNO**.

Lemma 3.4. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a $\mathcal{SVNJTS}$. Then,

1. any union of **r-SVNIO** sets is **r-SVNIO**,
2. any intersection of **r-SVNIC** sets is r-**SVNIC**.

Proof

1. Let $\{\mathcal{S}_j,\ j \in \triangle\}$ is a family of r-**SVNIOs**. Then, we obtain, $\mathcal{S}_j \leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((\mathcal{S}_r^\star)_j, r)$, and hence for each $\omega \in \tilde{\mathfrak{T}}$,

$$\bigvee_{j\in\triangle} \tilde{\gamma}_{\mathcal{S}_j}(\omega) \leq \bigvee_{j\in\triangle} \tilde{\gamma}_{int_{\tilde{\tau}^{\tilde{\gamma}}}}(\mathcal{S}_r^\star, r)(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}^{\tilde{\gamma}}}}(\bigvee_{j\in\triangle} (\mathcal{S}_j)_r^\star, r)(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}^{\tilde{\gamma}}}}((\bigvee_{j\in\triangle} \mathcal{S}_j)_r^\star, r)(\omega),$$

$$\bigvee_{j\in\triangle} \tilde{\eta}_{\mathcal{S}_j}(\omega) \geq \bigvee_{j\in\triangle} \tilde{\eta}_{int_{\tilde{\tau}^{\tilde{\eta}}}}(\mathcal{S}_r^\star, r)(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}^{\tilde{\eta}}}}(\bigvee_{j\in\triangle} (\mathcal{S}_j)_r^\star, r)(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}^{\tilde{\eta}}}}\left(\left(\bigvee_{j\in\triangle} \mathcal{S}_j\right)_r^\star, r\right)(\omega),$$

$$\bigvee_{j\in\triangle} \tilde{\mu}_{\mathcal{S}_j}(\omega) \geq \bigvee_{j\in\triangle} \tilde{\mu}_{int_{\tilde{\tau}^{\tilde{\mu}}}}(\mathcal{S}_r^\star, r)(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}^{\tilde{\mu}}}}(\bigvee_{j\in\triangle} (\mathcal{S}_j)_r^\star, r)(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}^{\tilde{\mu}}}}((\bigvee_{j\in\triangle} \mathcal{S}_j)_r^\star, r)(\omega).$$

Therefore, $\vee_{j\in\triangle} \mathcal{S}_j \leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\vee_{j\in\triangle} \mathcal{S}_j, r)$ Hence, $\vee_{j\in\triangle} \mathcal{S}_j$ is r-**SVNIO**.

2. Similarly to (1).

Proposition 3.5. Suppose that $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a $\mathcal{SVNJTS}$. Then,

1. If $\mathcal{S}$ is r-**SVNIO**, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}) \leq 1 - r$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{E}) \leq 1 - r$, then, $\mathcal{S} \cap \mathcal{E}$ is r-**SVNIO**.
2. If $\mathcal{S}$ is r-**SVNIC**, $\tilde{\tau}^{\tilde{\gamma}}(\underline{1} - \mathcal{E}) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\underline{1} - \mathcal{E}) \leq 1 - r$ and $\tilde{\tau}^{\tilde{\mu}}(\underline{1} - \mathcal{E}) \leq 1 - r$, then, $\mathcal{S} \cup \mathcal{E}$ is r-**SVNIC**.
3. If $\mathcal{S}$ is both r-**SVNIO** and r-**SVNSC** sets, then $\mathcal{S} = int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^\star, r)$.
4. If $\mathcal{S}$ is r-**SVNIO** and $\mathcal{S} \leq \mathcal{E} \leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)$, then $\mathcal{E}$ is an r-**SVN$\boldsymbol{\beta}$O** set.
5. $\mathcal{S} \cap int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^\star, r)$ is an r-**SVNIO** set.
6. If $\mathcal{S}$ is r-**SVNIO**, then $\mathcal{S} \cap C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}, r) \leq (\mathcal{S} \cap \mathcal{E})_r^\star$, for every $\mathcal{E}$ is r-**SVNSO**.
7. If $\mathcal{S}$ is r-**SVNIO**, $\tilde{\tau}^{\star\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{R}^c) \geq r$, $\tilde{\tau}^{\star\tilde{\eta}\tilde{\eta}\tilde{\mu}}(\mathcal{R}^c) \leq 1 - r$ and $\tilde{\tau}^{\star\tilde{\mu}\tilde{\eta}\tilde{\mu}}(\mathcal{R}^c) \leq 1 - r$, then

$int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) = int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^*_r,r)$.

8. If $\mathcal{S}$ is r-**SVNIC**, then $\mathcal{S} \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r))^\star_r$.

Proof.

1. Since $\mathcal{S}$ is r-**SVNIO** and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}) \leq 1-r$, for each $\omega \in \tilde{\mathfrak{T}}$,

$$\tilde{\gamma}_{\mathcal{E}\wedge\mathcal{S}}(\upsilon) \leq \tilde{\gamma}_{\mathcal{E}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r)}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{E}\wedge\mathcal{S}^\star_r),r)}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{E}\wedge\mathcal{S})^\star_r,r)}(\omega).$$

$$\tilde{\eta}_{\mathcal{E}\wedge\mathcal{S}}(\omega) \geq \tilde{\eta}_{\mathcal{E}\wedge int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}((\mathcal{E}\wedge\mathcal{S}^\star_r),r)}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}((\mathcal{E}\wedge\mathcal{S})^\star_r,r)}(\omega).$$

$$\tilde{\mu}_{\mathcal{E}\wedge\mathcal{S}}(\omega) \geq \tilde{\mu}_{\mathcal{E}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}((\mathcal{E}\wedge\mathcal{S}^\star_r),r)}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}((\mathcal{E}\wedge\mathcal{S})^\star_r,r)}(\omega)$$

Thus $\mathcal{E}\wedge\mathcal{S} \leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}((\mathcal{E}\wedge\mathcal{S})^\star_r,r)$. Hence, $\mathcal{E}\wedge\mathcal{S}$ is an r-**SVNIO** set.

2. It is easily proved by the same manner.

3. Since $\mathcal{S}$ is both r-**SVNIO** and r-**SVNSC**, then for each $\omega \in \tilde{\mathfrak{T}}$ and for each $\omega \in \tilde{\mathfrak{T}}$ (Theorem 2.14.(3)), we have

$$\tilde{\gamma}_{\mathcal{S}}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r)}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}\left(C_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S},r),r\right)}(\omega) \leq \tilde{\gamma}_{\mathcal{S}}(\omega),$$

$$\tilde{\eta}_{\mathcal{S}}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}\left(C_{\tilde{\tau}\tilde{\eta}}(\mathcal{S},r),r\right)}(\omega) \geq \tilde{\eta}_{\mathcal{S}}(\omega),$$

$$\tilde{\mu}_{\mathcal{S}}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}(C_{\tilde{\tau}\tilde{\eta}}(\mathcal{S},r),r)}(\omega) \geq \tilde{\mu}_{\mathcal{S}}(\omega).$$

Thus, $\mathcal{S} = int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^\star_r,r)$.

4. Similarly to (3).

5. Since, $int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^\star_r,r) = \mathcal{S}^\star_r \cap int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^\star_r,r)$, for each $\omega \in \tilde{\mathcal{S}}$ and as we obtained by Theorem 2.14(7), such that for each $\omega \in \tilde{\mathfrak{T}}$. Then we have,

$$\tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r)}(\omega) \leq \tilde{\gamma}_{(\mathcal{S}\cap int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r))^\star_r}(\omega), \quad \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r.r)}(\omega) \geq \tilde{\eta}_{(\mathcal{S}\cap int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^\star_r,r))^\star_r}(\omega),$$

$$\tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\mu}_{(\mathcal{S}\cap int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^\star_r,r))^\star_r}(\omega).$$

Thus,

$$\tilde{\gamma}_{\mathcal{S}\cap int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r)}(\omega) \leq \tilde{\gamma}_{\mathcal{R}\cap int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{S}\cap int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r))^\star_r,r)}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{S}\cap int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^\star_r,r))^\star_r,r)}(\omega),$$

$$\tilde{\eta}_{\mathcal{R}\cap int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r,r)}(\omega) \geq \tilde{\eta}_{\mathcal{S}\cap int_{\tilde{\tau}\tilde{\eta}}((\mathcal{S}\cap int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r,r))^\star_r,r)}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}((\mathcal{S}\cap int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^\star_r,r))^\star_r,r)}(\omega),$$

$$\tilde{\mu}_{\mathcal{R}\cap int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^{\star}_{r}.,)}(\omega) \geq \tilde{\mu}_{\mathcal{R}\cap int_{\tilde{\tau}\tilde{\mu}}((\mathcal{S}\cap int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^{\star}_{r},r))^{\star}_{r},r)}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}((\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^{\star}_{r},r))^{\star}_{r},r)}(\omega).$$

Hence, $\mathcal{S} \cap int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^{\star}_{r}, r) \leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}((\mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^{\star}_{r}, r))^{\star}_{r},$. Therefore, $\mathcal{S} \cap int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^{\star}_{r}, r)$ is r-**SVNIO**.

6. Let $\mathcal{E}$ be r-**SVNSO**. Then $C_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}.r) = C_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}.r).r)$ and by Theorem 2.14(3,7), for each $\omega \in \tilde{\mathfrak{T}}$ we have,

$$\begin{aligned}\tilde{\gamma}_{\mathcal{S}\wedge C_{\tilde{\tau}\tilde{\gamma}}(\mathcal{E},r)} &\leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^{\star}_{r},r)\wedge C_{\tilde{\tau}\mathcal{T}}(int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{E},r),r)}\\ &\leq \tilde{\gamma}_{C_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}^{\star}_{r}\wedge int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{E},r),.r))}\\ &\leq \tilde{\gamma}_{C_{\tilde{\tau}\tilde{\gamma}}((\mathcal{S}\wedge\mathcal{E})^{\star}_{r},r)} = \tilde{\gamma}_{(\mathcal{S}\wedge\mathcal{E})^{\star}_{r}},\end{aligned}$$

$$\begin{aligned}\tilde{\eta}_{\mathcal{S}\wedge C_{\tilde{\tau}\tilde{\eta}}(\mathcal{E},r)} &\geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^{\star}_{r},r)\wedge C_{\tilde{\tau}\tilde{\eta}}(int_{\tilde{\tau}\tilde{\eta}}(\mathcal{E},r),r)}\\ &\geq \tilde{\eta}_{C_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}^{\star}_{r}\wedge int_{\tilde{\tau}\tilde{\eta}}((\mathcal{E},r),r))}\\ &\geq \tilde{\eta}_{C_{\tilde{\tau}\tilde{\eta}}((\mathcal{S}\wedge\mathcal{E})^{\star}_{r},r)} = \tilde{\eta}_{(\mathcal{S}\wedge\mathcal{E})^{\star}_{r}},\end{aligned}$$

$$\begin{aligned}\tilde{\mu}_{\mathcal{S}\wedge C_{\tilde{\tau}\tilde{\mu}}(\mathcal{E},r)} &\geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}(.,r)\wedge C_{\tilde{\tau}\tilde{\mu}}(int_{\tilde{\tau}\tilde{\mu}}(\mathcal{E},r),r)}\\ &\geq \tilde{\mu}_{C_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}^{\star}_{r}\wedge int_{\tilde{\tau}\tilde{\mu}}((\mathcal{E},r),r))}\\ &\geq \tilde{\mu}_{C_{\tilde{\tau}\tilde{\mu}}((\mathcal{S}\wedge\mathcal{E})^{\star}_{r},r)} = \tilde{\mu}_{(\mathcal{S}\wedge\mathcal{E})^{\star}_{r}}.\end{aligned}$$

7. Similarly to (6).

8. Let $\mathcal{S}$ be r-**SVNIC**. Then, $\mathcal{S}^{c} \leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}((\mathcal{S}^{c})^{\star}_{r}, r)$. Since, $\mathcal{S}^{\star}_{r} \leq \mathbb{C}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r)$, by Theorem 2.14(3),

$$\begin{aligned}\mathcal{S}^{c} &\leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}((\mathcal{S}^{\star}_{r})^{c}r)\\ &\leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^{c}, r), r)\\ &= (C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}\gamma_{\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r))^{c}.\end{aligned}$$

Then, $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}\gamma_{\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r) \leq \mathcal{S}$. Thus, $int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r))^{\star}_{r} \leq C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r), r) \leq \mathcal{S}$.

Theorem 3.6. Suppose that $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a $\mathcal{SVNJTS}$, for each $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{T}}}$. Define the operator $\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}: I^{\tilde{\mathfrak{T}}} \times I_0 \to I$ as follows:

$$\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r) = \cap\{\mathcal{E} \in I^{\tilde{\mathfrak{T}}} | \mathcal{S} \leq \mathcal{E},\ \mathcal{E}\ is\ r - \boldsymbol{SVNIC}\ set\}.$$

Then, for each $r \in I_0$ the operator $\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ satisfies the following conditions:

1. $\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\langle 0,1,1\rangle, r) = \langle 0,1,1\rangle$,
2. $\mathcal{S} \leq \mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r)$,
3. $\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r) \vee \mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r) \leq \mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S} \vee \mathcal{E}, r)$,
4. $\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r), r) = \mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r)$,
5. $\mathcal{S}$ is r-**SVNIC**, iff $\mathcal{S} = \mathcal{JC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r)$,

6. If $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r)$ is r-**SVNIC**, then $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{IC}_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r),r) = \mathcal{IC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r),r) = C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r)$.

Proof. It is trivial.

Theorem 3.7. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a $\mathcal{SVNJTS}$, for each $\mathcal{S},\mathcal{E} \in I^{\tilde{\mathfrak{T}}}$, we define the operator $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}: I^{\tilde{\mathfrak{T}}} \times I_0 \to I$ as follows:

$$\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}r) = \bigcap\{\mathcal{E} \in I^{\tilde{\mathfrak{T}}} | \mathcal{S} \leq \mathcal{E},\ \mathcal{E}\ is\ r - \boldsymbol{SVNIO}\ set\}.$$

Then

1. $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}^c,r) = (\mathcal{IC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r))^c$,
2. $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) \leq \mathcal{S} \leq \mathcal{IC}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r)$,
3. $\mathcal{S}$ is r-**SVNIO** iff $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) = \mathcal{S}$,
4. $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) = \mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r)$,
5. $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) = \langle 0,1,1\rangle$ if and only if $int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r) = \langle 0,1,1\rangle$,

Proof.

(1), (2) and (3) are trivial form the definition of $\mathcal{J}int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ and $\mathcal{IC}_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$.

(4) By Theorem 2.14(7), we have

$$\tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r)} = \tilde{\gamma}_{\mathcal{S}_r^\star \wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r)} \leq \tilde{\gamma}_{(\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r))_r^\star},$$

$$\tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}_r^\star,r)} = \tilde{\eta}_{\mathcal{S}_r^\star \wedge int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}_r^\star,r)} \geq \tilde{\eta}_{(\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}_r^\star,r))_r^\star},$$

$$\tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}_r^\star,r)} = \tilde{\mu}_{\mathcal{S}_r^{\star\tilde{\mu}} \wedge int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}_r^\star,r)} \geq \tilde{\mu}_{(\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}_r^\star,r))_r^\star}.$$

This implies that

$$\tilde{\gamma}_{\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r)} \leq \tilde{\gamma}_{\mathcal{S}^{\tilde{\gamma}} \wedge int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r))_r^\star,r)} \leq \tilde{\gamma}_{int_{\tilde{\tau}\tilde{\gamma}}((\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\gamma}}(\mathcal{S}_r^\star,r))_r^\star,r)},$$

$$\tilde{\eta}_{\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}_r^\star,r)} \geq \tilde{\eta}_{\mathcal{S}\wedge(\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\eta}}((\mathcal{S}_r^\star,r))_r^\star,r)} \geq \tilde{\eta}_{int_{\tilde{\tau}\tilde{\eta}}((\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\eta}}(\mathcal{S}_r^\star,r))_r^\star,r)},$$

$$\tilde{\mu}_{\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}_r^\star,r)} \geq \tilde{\mu}_{\mathcal{S}\wedge(\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\mu}}((\mathcal{S}_r^\star,r))_r^\star,r)} \geq \tilde{\mu}_{int_{\tilde{\tau}\tilde{\mu}}((\mathcal{S}\wedge int_{\tilde{\tau}\tilde{\mu}}(\mathcal{S}_r^\star,r))_r^\star,r)}.$$

Thus, $\mathcal{S} \wedge int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r)$ is r-**SVNIO**, then $\mathcal{S} \wedge int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r) \leq \mathcal{J}int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r)$.
For each $\mathcal{E}$ is r-**SVNIO** set and $\mathcal{E} \leq \mathcal{S}$ then by Theorem 2.14(1), we have $\mathcal{E}_r^\star \leq \mathcal{S}_r^\star$, and so, $int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}_r^\star,r) \leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r)$,

$$\mathcal{E} \leq \mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}_r^\star,r) \leq \mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r).$$

Thus, $\mathcal{J}int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) \leq \mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r)$.

(5) Let $\mathcal{J}int_{\tau\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) = \langle 0,1,1\rangle$. Then, $\mathcal{S} \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r) = \langle 0,1,1\rangle$, implies that $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S} \wedge int_{\tilde{\tau}}(\mathcal{S}_r^\star,r),r) = \langle 0,1,1\rangle$ and $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) \wedge int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^\star,r) = \langle 0,1,1\rangle$, by Theorem 2.14(3), $C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S},r) \wedge$

$int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^{\star}, r) = int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^{\star}, r) = \langle 0,1,1 \rangle$.

On the other hand, let $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^{\star}, r) = \langle 0,1,1 \rangle$. Then $\mathcal{S} \wedge int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_r^{\star}, r) = \langle 0,1,1 \rangle$. Hence, by (2), $\mathcal{I}int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r) = \langle 0,1,1 \rangle$.

4. Single-Valued Neutrosophic Ideal Continuous Mappings

We introduce the notions of single-valued neutrosophic continuous (briefly, $\mathcal{SVN}$-continuous) (resp. single-valued neutrosophic ideal continuous (briefly, $\mathcal{SVNI}$-continuous), single-valued neutrosophic ideal-open (briefly, $\mathcal{SVNI}$-open), single-valued neutrosophic $\mathcal{I}$-closed (briefly, $\mathcal{SVNI}$-closed), single-valued neutrosophic pre continuous (briefly, $\mathcal{SVNP}$-continuous)) mappings. Also, we obtain new decompositions of $\mathcal{SVN}$-continuous in $\mathcal{SVNITS}$ in Šostak Sense.

Definition 4.1. Suppose that $f: (\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{I}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\tilde{\mathfrak{Y}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a mapping and $r \in I_0$. Then, f is called: $\mathcal{SVNI}$-continuous iff $f^{-1}(\mathcal{S})$ is r-**SVNIO** in $\tilde{\mathfrak{X}}$ for every $\mathcal{E} \in I^{\tilde{\mathfrak{Y}}}$, $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$, $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}) \leq 1-r$,

Definition 4.2. Suppose that $f: (\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\tilde{\mathfrak{Y}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a mapping and $r \in I_0$. Then, f is said to be:

1. $\mathcal{SVNI}$-open iff $f(\mathcal{S})$ is r-**SVNIO** in $\tilde{\mathfrak{Y}}$ for every $\mathcal{E} \in I^{\tilde{\mathfrak{X}}}$, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) \leq 1-r$,
2. $\mathcal{SVNI}$-closed iff $f(\mathcal{S})$ is r-**SVNIC** in $\tilde{\mathfrak{Y}}$ for every $\mathcal{E} \in I^{\tilde{\mathfrak{X}}}$, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$.

Definition 4.3. Suppose that $f: (\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}} \to (\tilde{\mathfrak{Y}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a mapping and $r \in I_0$. Then, f is called:

1. $\mathcal{SVN}$-continuous iff $f^{-1}(\mathcal{S})$ is r-**SVNO** in $\tilde{\mathfrak{X}}$ for every $\mathcal{E} \in I^{\tilde{\mathfrak{Y}}}$, $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}) \leq 1-r$,
2. $\mathcal{SVNP}$-continuous iff $f^{-1}(\mathcal{S})$ is r-**SVNPO** in $\tilde{\mathfrak{X}}$ for every $\mathcal{E} \in I^{\tilde{\mathfrak{Y}}}$, $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}) \leq 1-r$.

Remark 4.4.

1. $\mathcal{SVNI}$-continuous $\Rightarrow$ $\mathcal{SVNP}$-continuous,
2. $\mathcal{SVNL}$-continuous and $\mathcal{SVN}$-continuous are independent.

Example 4.5. Suppose that $\tilde{\mathfrak{X}} = \{a, b\}$. Define $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \in I^{\tilde{\mathfrak{X}}}$ as follows:

$\mathcal{E}_1 = \langle (0\cdot 5, 0\cdot 4), (0\cdot 5, 0\cdot 5), (0\cdot 9, 0\cdot 6)\rangle$, $\mathcal{E}_2 = \langle (0\cdot 4, 0\cdot 4), (0\cdot 1, 0\cdot 1), (0\cdot 1, 0\cdot 1)\rangle$,
$\mathcal{E}_3 = \langle (0\cdot 3, 0\cdot 1), (0\cdot 1, 0\cdot 1), (0\cdot 1.0\cdot 4)\rangle$, $\mathcal{C}_1 = \langle (0\cdot 4, 0\cdot 5), (0\cdot 5, 0\cdot 5), (0\cdot 6, 0\cdot 9)\rangle$,
$\mathcal{C}_2 = \langle (0\cdot 2, 0\cdot 2), (0\cdot 2, 0\cdot 2), (0\cdot 1.0\cdot 1)\rangle$, $\mathcal{C}_3 = \langle (0\cdot 1, 0\cdot 1), (0\cdot 1, 0\cdot 1), (0\cdot 1.0\cdot 1)\rangle$.

Define $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}} : I^{\tilde{\mathfrak{X}}} \to I$ as follows:

$$\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 1, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_1, \end{cases} \qquad \tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 1, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_1, \end{cases}$$

$$\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_2, \end{cases} \qquad \tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_2, \end{cases}$$

$$\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_3, \end{cases} \qquad \tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_3, \end{cases}$$

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_1, \\ \frac{2}{3}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_1, \end{cases} \qquad \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_2, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_2 \end{cases}$$

$$\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_3, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_3. \end{cases}$$

Define $f: (\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\tilde{\mathfrak{J}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ as follows $f(a) = b$ and $f(b) = a$. If $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{C}_1) \geq \frac{1}{2}$, $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{C}_1) \leq 1 - \frac{1}{2}$ and $\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{C}_1) \leq 1 - \frac{1}{2}$. Then $f^{-1}(\mathcal{C}_1) = \langle (0\cdot 5, 0\cdot 4),\ (0\cdot 5, 0\cdot 5),\ (0\cdot 9, 0\cdot 6)\rangle$ is $\frac{1}{2}$-**SVNO** in $\tilde{\mathfrak{X}}$. Thus, f is $\mathcal{SVN}$-continuous. However, it is not $\mathcal{SVNJ}$-continuous.

Theorem 4.6. Let $f: (\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, ^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\tilde{\mathfrak{J}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a mapping and $r \in I_0$. Then the following are equivalent.

1. f is $\mathcal{SVNJ}$-continuous.
2. For any $x_{s,t,k} \in \mathcal{SVNP}(\tilde{\mathfrak{X}})$, $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$ and $\tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$ containing $f(x_{s,t,k})$, there exists r-**SVNIO** set $\mathcal{E}$ such that $x_{s.t.k} \in \mathcal{E}$, $f(\mathcal{E}) \leq \mathcal{S}$.
3. For any $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$, $f^{-1}(\mathcal{S})$ is r-**SVIC** set.
4. $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(\mathcal{S}))^{\star}_r, r) \leq f^{-1}(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}), r)$, for any $\mathcal{S} \in I^{\tilde{\mathfrak{J}}}$.
5. $f(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}^{\star}_r, r)) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{E}), r)$ for any and $\mathcal{E} \in I^{\tilde{\mathfrak{X}}}$.

Proof.

(1) ⇒ (2): For any $x_{s,t,k} \in \mathcal{SVNP}(\tilde{\mathfrak{X}})$, $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}) \geq r$. $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$. By $\mathcal{SVNJ}$-continuity of f we have $\mathcal{E} = f^{-1}(\mathcal{S})$ is an r-**SVNIO** set and for any $\omega \in \tilde{\mathfrak{X}}$

$$s < \tilde{\gamma}_{f^{-1}(\mathcal{S})}(\omega) = \tilde{\gamma}_{\mathcal{E}}(\omega), \quad t \geq \tilde{\eta}_{f^{-1}(\mathcal{S})}(\omega) = \tilde{\eta}_{\mathcal{E}}(\omega), \quad k \geq \tilde{\mu}_{f^{-1}(\mathcal{S})}(\omega) = \tilde{\mu}_{\mathcal{E}}(\omega).$$

Hence, $f(\mathcal{E}) \leq \mathcal{S}$.

(2)⇒(3): Suppose that $\widetilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\widetilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, $\widetilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ and $x_{s.t.k} \in f^{-1}(\underline{1} - \mathcal{s})$, by (2). There exists r-**SVNIO** set $\mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$ and $x_{s,t,k} \in \mathcal{E}$ such that $f(\mathcal{E}) \leq \underline{1} - \mathcal{S}$. Hence, for any $\omega \in \widetilde{\mathfrak{T}}$

$$s < \tilde{\gamma}_{\mathcal{E}}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_r^\star, r)}(\omega) \leq \tilde{\gamma}_{int_{\tilde{\tau}^{\tilde{\gamma}}}((f^{-1}(\mathcal{S}^c))_r^\star, r)}(\omega),$$

$$t \geq \tilde{\eta}_{\mathcal{E}}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_r^\star, r)}(\omega) \geq \tilde{\eta}_{int_{\tilde{\tau}^{\tilde{\eta}}}((f^{-1}(\mathcal{S}^c))_r^\star, r)}(\omega),$$

$$k \geq \tilde{\mu}_{\mathcal{E}}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_r^\star, r)}(\omega) \geq \tilde{\mu}_{int_{\tilde{\tau}^{\tilde{\mu}}}((f^{-1}(\mathcal{S}^c))_r^\star, r)}(\omega).$$

Hence $f^{-1}(\mathcal{S}^c) \leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(\mathcal{S}^c))_r^\star, r)$. Then $f^{-1}(\mathcal{S}^c) = (f^{-1}(\mathcal{S}))^c$ is r-**SVNIO** set in $\widetilde{\mathfrak{T}}$. Thus, $f^{-1}(\mathcal{S})$ is r-**SVNIC** set in $\widetilde{\mathfrak{T}}$.

(3)⇒(4): For any $\mathcal{S} \in I^{\widetilde{\mathfrak{J}}}$ and $r \in I_0$, since $\tilde{\sigma}^{\tilde{\gamma}}((C_{\tilde{\sigma}^{\tilde{\gamma}}}(\mathcal{S}, r))^c) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}((C_{\tilde{\sigma}^{\tilde{\eta}}}(\mathcal{S}, r))^c) \leq 1-r$ and $\tilde{\sigma}^{\tilde{\mu}}((C_{\tilde{\sigma}^{\tilde{\mu}}}(\mathcal{S}, r))^c) \leq 1-r$, by (3), we have $f^{-1}(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r))$ is r-**SVNIC** set. Hence,

$$f^{-1}(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \geq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}\left(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)\right))_r^\star, r) \geq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(\mathcal{S}, r))_r^\star, r),$$

(4)⇒(5): For any $\mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$ and $r \in I_0$. Put $f(\mathcal{E}) = \mathcal{S}$. By (4), we have,

$$C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_r^\star, r) \leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(f(\mathcal{E})))_r^\star, r) \leq f^{-1}(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{E}), r)).$$

It implies $f(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_r^\star, r)) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{E}), r)$.

(5)⇒(1): Let $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{E}) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{E}) \leq 1-r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{E}) \leq 1-r$. Then by (5) and Theorem 2.14(3), we have,

$$f(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(\mathcal{E}^c))_r^\star, r)) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(f^{-1}(\mathcal{E}^c)), r) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}^c, r) = \mathcal{E}^c.$$

Therefore, $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}((f^{-1}(\mathcal{E}^c))_r^\star, r) \leq f^{-1}(\mathcal{E}^c)$. This show that $f^{-1}(\mathcal{E})$ is r-**SVNIO** set. Thus, $\mathcal{SVNJ}$-continuous.

Theorem 4.7. Suppose that $f: (\widetilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{J}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $\mathcal{SVNJ}$-continuous for all $\mathcal{S} \in I^{\widetilde{\mathfrak{J}}}$. $\mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$ and $r \in I_0$. Then the following are holds:

1. $(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r))_r^\star \leq f^{-1}(\mathcal{S}_r^\star)$, for every [r-single valued neutrosophic ⋆-dense-in-itself $\mathcal{S} \leq \mathcal{S}_r^\star$].
2. $f(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}, r))_r^\star \leq (f(\mathcal{E}))_r^\star$, for every [r-single valued neutrosophic ⋆-prefect $(\mathcal{E}_r^\star = \mathcal{E})$].

Proof.

1. For every $\mathcal{S} \in I^{\widetilde{\mathfrak{I}}}$ by Theorem 2.14(3), we obtain $C_{\tilde{\sigma}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^{\star}.r) = \mathcal{S}_r^{\star}$, this implies that, $\tilde{\sigma}^{\tilde{\gamma}}((\mathcal{S}_r^{\star})^c) \geq r$, $\tilde{\tau}^{\tilde{\sigma}}((\mathcal{S}_r^{\star})^c) \geq r$ and $\tilde{\sigma}^{\tilde{\mu}}((\mathcal{S}_r^{\star})^c) \geq r$. Then by Theorem 4.6(3), we have $f^{-1}(\mathcal{S}_r^{\star})$ is r-**SVNIC** set in $\widetilde{\mathfrak{T}}$. Thus, by using Proposition 3.5(8), we have $f^{-1}(\mathcal{S}_r^{\star}) \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{S}_r^{\star}).r))_r^{\star}$. Hence,

$$f^{-1}(\mathcal{S}_r^{\star}) \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{S}_r^{\star}), r))_r^{\star} \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{S}), r))_r^{\star}.$$

2. For every $\mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$ and $r \in I_0$, Put $\mathcal{S} = f(\mathcal{E})$ from (2). Then

$$f^{-1}((f(\mathcal{E}))_r^{\star}) \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(f(\mathcal{E})), r))_r^{\star} \geq (int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}, r))_r^{\star}.$$

It implies $f(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}, r))_r^{\star} \leq (f(\mathcal{E}))_r^{\star}$.

Theorem 4.8. Suppose that $f: (\widetilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{I}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is a $\mathcal{SVNJ}$-continuous for each $r \in I_0$. Then,

1. $f(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}^{\star}}(\mathcal{S}, r), r)) \leq C_{\tilde{\sigma}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f(\mathcal{S}), r)$, for each r-**SVNIO** $\mathcal{S} \in I^{\widetilde{\mathfrak{T}}}$.
2. $int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{E}), r).r) \leq f^{-1}(C_{\tilde{\sigma}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}, r))$, for each [r-r-single valued neutrosophic $\star$-dense-in-itself $\mathcal{E} \in I^{\widetilde{\mathfrak{I}}}$].

Proof.

1. LeT $\mathcal{S} \in I^{\widetilde{\mathfrak{T}}}$ be a r-**SVNPO**. Then $\mathcal{S} \leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^{\star}, r)$. Hence, by Theorem 4.6(5), we obtain

$$\begin{aligned} f\left(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\left(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}, r), r\right)\right) &\leq f(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\left(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^{\star}, r), r\right), .r)) \\ &\leq f(int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^{\star}, r), r)) \\ &\leq f(C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_r^{\star}, r)) \leq C_{\tilde{\sigma}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f(\mathcal{S}), r), \end{aligned}$$

1. Let $\mathcal{E} \in I^{\widetilde{\mathfrak{I}}}$ be r-r-single valued neutrosophic $\star$-dense-in-itself. Then $\mathcal{E} \leq \mathcal{E}_r^{\star}$. By Theorem 4.6(4), we obtain,

$$\begin{aligned} int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\left(C^{\star}_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{E}), r), r\right) &\leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{E}), r), r) \\ &\leq int_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{E}_r^{\star}, r), r), r) \\ &\leq C_{\tilde{\tau}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(f^{-1}(\mathcal{E}_r^{\star}, r), r) \leq f^{-1}(C_{\tilde{\sigma}\tilde{\gamma}\tilde{\eta}\tilde{\mu}}(\mathcal{E}, r)). \end{aligned}$$

Theorem 4.9. A mapping $f: (\widetilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{I}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $\mathcal{SVNJ}$-open iff for every $\mathcal{S} \in I^{\widetilde{\mathfrak{I}}}$ and for each $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}) \leq 1 - r$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{E}) \leq 1 - r$ such that $f^{-1}(\mathcal{S}) \leq \mathcal{E}$, there exists $\mathcal{D} \in I^{\widetilde{\mathfrak{I}}}$ is r-**SVNIC** set containing $\mathcal{S}$ such that $f^{-1}(\mathcal{D}) \leq \mathcal{E}$.

Proof. Obvious.

Theorem 4.10. If $f:(\widetilde{\mathfrak{X}},\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}) \to (\widetilde{\mathfrak{J}},\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}},\mathfrak{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is $\mathcal{SVNJ}$-open, then the following properties are holds:

1. $f^{-1}(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r).r)) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r)$ for all$\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{S}) \geq r.$ $\tilde{\sigma}^{\widetilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\sigma}^{\widetilde{\mu}}(\mathcal{S}) \leq 1-r.$
2. $f^{-1}(Cl^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r)) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{E}),r)$ for all $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{E}) \geq r,$ $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{E}) \leq 1-r$ and $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{E}) \leq 1-r.$

Proof.

1. Since,
$\tilde{\tau}^{\widetilde{\gamma}}((C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r))^c) \geq r,$ $\tilde{\tau}^{\widetilde{\gamma}}((C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r))^c) \leq 1-r,$ $\tilde{\tau}^{\widetilde{\gamma}}((C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r))^c) \leq 1-r$ for each $\mathcal{S} \in I^{\widetilde{\mathfrak{J}}}$. By Theorem 4.9, there exists $\mathcal{D} \in I^{\widetilde{\mathfrak{J}}}$ is r-**SVNIC** set containing $\mathcal{S}$ such that $f^{-1}(\mathcal{D}) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}).r)$. Since $\mathcal{D}^c$ is r-**SVNIO** set, $f^{-1}(\mathcal{D}^c) \leq f^{-1}(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}((\mathcal{D}^c)^{\star}_r,r),$ we obtain,

$$
\begin{aligned}
(f^{-1}(\mathcal{D}))^c \quad &\leq f^{-1}(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}((\mathcal{D}^c)^{\star}_r,r) \leq f^{-1}((int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{D}^c)^{\star}_r,r) \\
&\leq f^{-1}\left(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(Cl^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{D}^c,r),r\right),r\right)\right) \\
&\leq (f^{-1}(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{D},r),r\right),r)))^c.
\end{aligned}
$$

Since $\mathcal{S} \leq \mathcal{D}$, we obtain,

$$
\begin{aligned}
f^{-1}\left(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r),r\right)\right) &\leq f^{-1}(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{D},r),r\right),r)) \\
&\leq f^{-1}(\mathcal{D}) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r).
\end{aligned}
$$

Hence, $f^{-1}(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r),r)) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r).$

2. For each $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{E}) \geq r,$ $\tilde{\sigma}^{\widetilde{\eta}}(\mathcal{E}) \leq 1-r$ and $\tilde{\sigma}^{\widetilde{\mu}}(\mathcal{E}) \leq 1-r$. By (1), we have

$$
\begin{aligned}
f^{-1}(Cl^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r)) &\leq f^{-1}(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r) \leq f^{-1}\left(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r),r\right)\right) \\
&\leq f^{-1}\left(C_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r),r\right)\right) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{E}),r).
\end{aligned}
$$

Theorem 4.11 below, is similarly proved as Theorem 4.10.

Theorem 4.11. If $f:(\widetilde{\mathfrak{X}},\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}) \to (\widetilde{\mathfrak{J}},\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}},\mathfrak{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is $\mathcal{SVNJ}$-closed, then the following properties are holds:

1. $f^{-1}(int_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r).r)) \leq C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),r)$, for each $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\sigma}^{\widetilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\sigma}^{\widetilde{\mu}}(\mathcal{S}) \leq 1-r.$
2. $f^{-1}(int^{\star}_{\tilde{\sigma}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E},r)) \leq int_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{E}),r),$ for each $\tilde{\sigma}^{\widetilde{\gamma}}(\mathcal{E}^C) \geq r,$ $\tilde{\sigma}^{\widetilde{\eta}}(\mathcal{E}^C) \leq 1-r$ and $\tilde{\sigma}^{\widetilde{\mu}}(\mathcal{E}^C) \leq 1-r.$

Theorem 4.12. The following hold for the mappings $f:(\widetilde{\mathfrak{T}},\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})\rightarrow(\widetilde{\mathfrak{J}},\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ and $g:(\widetilde{\mathfrak{J}},\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})\rightarrow(\tilde{\chi},\tilde{\tau}_3^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_3^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$;

1. $g\circ f$ is $\mathcal{SVNJ}$-continuous if f is $\mathcal{SVNJ}$-continuous and g is $\mathcal{SVN}$-continuous,
2. $g\circ f$ is $\mathcal{SVNP}$-continuous if f is $\mathcal{SVNP}$-continuous and g is $\mathcal{SVNJ}$-continuous,
3. $g\circ f$ is $\mathcal{SVNJ}$-open if f and g is $\mathcal{SVNJ}$-open, f is surjective and $g(\mathcal{S}_r^\star)\leq(g(\mathcal{E}))_r^\star$ for each $\mathcal{S}\leq\mathcal{E}$.

Proof. Straightforward.

Remark 4.13. The composition of two $\mathcal{SVNJ}$ -continuous mappings need not to be a $\mathcal{SVNJ}$-continuous.

Example 4.14. Suppose that $\widetilde{\mathfrak{T}}=\{a,b\}$. Define $\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3,\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3\in I^{\widetilde{\mathfrak{T}}}$ as follows:

$\mathcal{E}_1=\langle(0\cdot4,0\cdot4),(0\cdot4,0\cdot4),(0\cdot4,0\cdot4)\rangle$, $\mathcal{E}_2=\langle(0\cdot3,0\cdot3),(0\cdot3,0\cdot3),(0\cdot3,0\cdot3)\rangle$,
$\mathcal{E}_3=\langle(0\cdot2,0\cdot2),(0\cdot2,0\cdot2),(0\cdot2.0\cdot2)\rangle$, $\mathcal{C}_1=\langle(0\cdot2,0\cdot2),(0\cdot2,0\cdot2),(0\cdot2,0\cdot2)\rangle$,

$$\mathcal{C}_2=\langle(0\cdot1,0\cdot1),(0\cdot1,0\cdot1),(0\cdot1.0\cdot1)\rangle,$$

Define $\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\tau}_3^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\jmath}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}:I^{\widetilde{\mathfrak{T}}}\rightarrow I$ as follows:

$$\tilde{\tau}_1^{\tilde{\gamma}}(\mathcal{S})=\begin{cases}1, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 1, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_1,\end{cases}\qquad \tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S})=\begin{cases}1, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 1, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_2,\end{cases}$$

$$\tilde{\tau}_1^{\tilde{\eta}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_1,\end{cases}\qquad \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=,\ \mathcal{E}_2\end{cases}$$

$$\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_1,\end{cases}\qquad \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_2,\end{cases}$$

$$\tilde{\tau}_3^{\tilde{\gamma}}(\mathcal{S})=\begin{cases}1, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 1, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_3,\end{cases}\qquad \tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S})=\begin{cases}1, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 1, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_2,\end{cases}$$

$$\tilde{\tau}_3^{\tilde{\eta}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=\mathcal{E}_3,\end{cases}\qquad \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S})=\begin{cases}0, & if\ \mathcal{S}=\langle 0,1,1\rangle,\\ 0, & if\ \mathcal{S}=\langle 1,1,0\rangle,\\ \frac{1}{2}, & if\ \mathcal{S}=,\ \mathcal{E}_2\end{cases}$$

$$\tilde{\tau}_3^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_3, \end{cases} \qquad \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ 0, & if\ \mathcal{S} = \langle 1,1,0\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_2, \end{cases}$$

$$\tilde{\mathcal{J}}_1^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_1, \\ \frac{2}{3}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_1, \end{cases} \qquad \tilde{\tilde{\mathcal{J}}}_2^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_1, \\ \frac{2}{3}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_2, \end{cases}$$

$$\tilde{\mathcal{J}}_1^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_2, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_1 \end{cases} \qquad \tilde{\mathcal{J}}_2^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_2, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_2 \end{cases}$$

$$\tilde{\mathcal{J}}_1^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_3, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_1. \end{cases} \qquad \tilde{\mathcal{J}}_2^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \langle 0,1,1\rangle, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{C}_3, \\ \frac{1}{4}, & if\ \underline{0} < \mathcal{S} < \mathcal{C}_2. \end{cases}$$

The identity mappings $id_X\colon\left(X, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\tilde{\mathcal{J}}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right) \to (Y, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ and $id_X\colon\left(Y, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\tilde{\mathcal{J}}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right) \to (Z, \tilde{\tau}_3^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ are $\mathcal{SVNJ}$-continuous. But the identity mapping $id_X\colon\left(X, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right) \to (Z, \tilde{\tau}_3^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is not $\mathcal{SVNJ}$-continuous because, $\tilde{\tau}_3^{\tilde{\gamma}}(\mathcal{E}_3) \geq \frac{1}{2}$, $\tilde{\tau}_3^{\tilde{\eta}}(\mathcal{E}_3) \leq 1 - \frac{1}{2}$, $\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{E}_3) \leq 1 - \frac{1}{2}$ *and* $(\mathcal{E}_3)^*_{\frac{1}{2}} = \langle 0,1,1\rangle$ *and* $(\mathcal{E}_3)^*_{\frac{1}{2}} \nleq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\left((\mathcal{E}_3)^*_{\frac{1}{2}}, \frac{1}{2}\right)$.

Propositions (4.13) and (4.14) are similarly proved from Theorems (4.6) and (4.8), respectively.

Proposition 4.13. Let $f\colon(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\tilde{\mathfrak{J}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a mapping. Then, following statements are equivalent.

1. f is $\mathcal{SVNJ}$-continuous.
2. $f^{-1}(\mathcal{S})$ is r-**SVNIC** for each $\tilde{\sigma}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\sigma}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ and $\tilde{\sigma}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$,
3. $f(\mathcal{JC}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)$, for each $r \in I_0$ and $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$,
4. $\mathcal{JC}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{E}, r)) \leq f^{-1}(C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}, r))$, for each $r \in I_0$ and $\mathcal{E} \in I^{\tilde{\mathfrak{J}}}$,
5. $f^{-1}(int_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}, r)) \leq \mathcal{J}int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{E}.r))$, for each $r \in I_0$ and $\mathcal{E} \in I^{\tilde{\mathfrak{J}}}$.

Proof. Obvious.

Proposition 4.14. Let $f: (\widetilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{Z}}, \tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{I}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a mapping. Then, the following statements are hold:

1. f is called $\mathcal{SVNJ}$-closed.
2. $f(\mathcal{IC}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \leq C_{\tilde{\sigma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)$, for each $r \in I_0$ and $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}}$,
3. for any $\mathcal{S} \in I^{\widetilde{\mathfrak{Z}}}$ and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}) \geq r$, $\tilde{\eta}^{\tilde{\gamma}}(\mathcal{E}) \leq 1 - r$ and $\tilde{\mu}^{\tilde{\gamma}}(\mathcal{E}) \leq 1 - r$ such that $f^{-1}(\mathcal{S}) \leq \mathcal{E}$,

there exists a r-**SVNIO** set $\mathcal{D} \in I^{\widetilde{\mathfrak{Z}}}$ with $\mathcal{S} \leq \mathcal{D}$ such that $f^{-1}(\mathcal{D}) \leq \mathcal{E}$.

Proof. Obvious.

6. Conclusions

In this paper, the author has made a study of the r-single-valued neutrosophic ideal open (**r-SVNIO**), the idea of r-single-valued neutrosophic β-open (r-**SVN$\boldsymbol{\beta}$O**) and r-single-valued neutrosophic pre-open sets (r-**SVNPO**) in the sense of Šostak, which are different from the study taken so far and obtained some of their basic properties. Next, the concepts of a single-valued neutrosophic continuous (resp. single-valued neutrosophic ideal continuous, single-valued neutrosophic $\mathcal{J}$-open, single-valued neutrosophic $\mathcal{J}$-closed, single-valued neutrosophic pre continuous) mappings were introduced and studied and too obtained new decompositions of $\mathcal{SVN}$-continuous in $\mathcal{SVNJTS}$ in Šostak Sense.

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Discussion for Further Works:

The theory can be extended in the following natural ways. One may study the properties of single-valued neutrosophic metric topological spaces using the concept of basis defined in this paper;

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