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A Study on Neutrosophic Bitopological Group

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Abstract. In this paper we try to introduce neutrosophic bitopological group. We try to investigate some new definition and properties of neutrosophic bitopological group.

Keywords: Neutrosophic Group; Neutrosophic Topological Group; Neutrosophic Bitopological Group.

1. Introduction

In 1965, Zadeh [1] defined the concept of fuzzy set (FS). With the help of FS, defined the concept of membership function and explained the idea of uncertainty. In 1986, Atanassov [4] generalised the concept of FS and introduced the degree of non-membership as an independent component and proposed the intuitionistics fuzzy set (IFS). After that many researchers defined various new concepts on generalisation of FS. Smarandache [2, 3] introduced the degree of indeterminacy as independent component and discovered the neutrosophic set (NS).

After the generalisation of FS many researchers have applied the generalisation of fuzzy set theory in many branches of Science and Technology. Chang [6] introduced the fuzzy topology space (FTS). Coker [7] defined the concept of Intuitionistic fuzzy topological space (IFTS). In 1963, Kelly [5] defined the study of bitopological spaces. Kandil et al [9] discussed on fuzzy bitopological space (FBTS). Lee et al [8] discussion on some properties of Intuitionistic fuzzy bitopological space (IFBTS).

In 2012, Salama and Alblowi [18] introduced the concept of neutrosophic set (NS) and neutrosophic topological space (NTS) and in 2018, Riad K. Al-Hamido, [28, 29] defined the concept of Neutrosophic Crisp Bi-Topological Spaces and Crisp Tri-Topological Spaces. Narmada Devi R. et al [27] discussed on separation axioms in an ordered neutrosophic bitopological space (NBTS). Ozturk and Ozkan discussion on neutrosophic bitopological spaces.

In 2015, Sumathi and Arockiarani [10], defined the fuzzy neutrosophic group (FNG). After defining neutrosophic group (NG) Sumathi and Arockiarani [22] defined the concept of topological group structure of neutrosophic set. Currently, neutrosophic set has been applying by researchers in various field of Science and Technology as a tool for getting more appropriate result. Abdel-Basset et al. [12] has applied neutrosophic as a tool on group discussion making framework. Abdel-Basset et al. [13] done on work in solving chain problem using base-worst method based on novel plithogenic model. Smarandache [39] extended neutrosophic set to neutrosophic Overset, Neutrosophic Underset and Neutrosophic Offset. Smarandache [39], in 2016, introduced Neutrosophic Tripolar Set, Neutrosophic Multipolar Set, Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph. Salama [40] studied some properties of topological space of rough sets with tools for data mining. Broumi et al. [41] introduced the concept of rough neutrosophic sets. Parimala et al. [42] studied on $\alpha\omega$-closed sets and its connectedness in neutrosophic topological spaces. Pamucar and Bozanic [30] (2019) used single-valued neutrosophic sets to propose projection-based multi-attributive border approximation area comparison (MABAC) method. In 2018, Liu et al. [31] studied on new extension of decision-making trial and evaluation laboratory method (DEMATEL). Guo et al. [32] (2017) extended the rough set model to neutrosophic environment and used to multi-attribute decision making (MADM) problem. Nie et al. (2017) [33] studied the Weighted Aggregates Sum Product Assessment (WASPAS) method in the context of interval neutrosophic sets (INS). Ye (2016) [34] introduced interval neutrosophic hesitant fuzzy set (INHFS). Pamucar et al. (2018, 2019) [35, 36] studied application of linguistic neutrosophic number. Karaaslan (2020) [37] introduced type-2 single valued neutrosophic set along with some distance measures. Neutrosophic number is used by Maiti et al. (2019) [38] to solve multi-objective linear programming problem. Abdel-Basset et al. [24] recently studied integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries. Abdel-Basset et al. [26] observed on a novel framework to evaluate innovation value proposition for smart product-service systems.

NS is used to control uncertainty by using truth membership function, indeterminacy membership function and falsity membership function. Whereas FS is used to control uncertainty by using membership function only. NS is used indeterminacy as an independent measure of the membership and non-membership function. As a result, NS is considered as a generalization of FS and intuitionistic fuzzy set (IFS) and shows more better result. NS is more necessary to manage the real-life information which are uncertain and inconsistent in nature. In various problem FS and IFS can not completely assured due to in exact inconsistent characteristic. Therefore, NS shows more rational to design the membership function. By observing this we are going to do our research and try to study neutrosophic bitopological group (NBTG) by using NS and try to prove some of their properties.

2. Preliminaries

2.1. Definition:[18]

A NS A on a universe of discourse X is defined as $A = \big\{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\big\}$, where $\mathcal{T}, \mathcal{I}, \mathcal{F} : X \to [0,1]$. Note that $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

2.2. Definition:[21, 18]

The complement of NS A is denoted by A^c and is defined as $A^c(x) = \big\{\langle x, \mathcal{T}_{A^c}(x) = \mathcal{F}_A(x), \mathcal{I}_{A^c}(x) = 1 - \mathcal{I}_A(x), \mathcal{F}_{A^c}(x) = \mathcal{T}_A(x)\rangle : x \in X\big\}$.

2.3. Definition:[21, 18]

Let $X \neq \phi$ and $A = \big\{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\big\}$, $B = \big\{\langle x, \mathcal{T}_B(x), \mathcal{I}_B(x), \mathcal{F}_B(x)\rangle : x \in X\big\}$, are NSs. Then

(i) $A \wedge B = \big\{\langle x, min(\mathcal{T}_A(x), \mathcal{T}_B(x)), min(\mathcal{I}_A(x), \mathcal{I}_B(x)), max(\mathcal{F}_A(x), \mathcal{F}_B(x)), \rangle : x \in X\big\}$

(ii) $A \vee B = \big\{\langle x, max(\mathcal{T}_A(x), \mathcal{T}_B(x)), max(\mathcal{I}_A(x), \mathcal{I}_B(x)), min(\mathcal{F}_A(x), \mathcal{F}_B(x)), \rangle : x \in X\big\}$

(iii) $A \leq B$ if for each $x \in X, \mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \leq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$.

2.4. Definition:[21]

Let X and Y be two non empty sets and let ϕ be a function from a set X to a set Y. Let $A = \big\{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\big\}$, $B = \big\{\langle y, \mathcal{T}_B(y), \mathcal{I}_B(y), \mathcal{F}_B(y)\rangle : y \in Y\big\}$ be NS in X and Y. Then

(i) $\phi^{-1}(B)$, the preimage of B under ϕ is the NS in X defined by

$$\phi^{-1}(B) = \big\{\langle x, \phi^{-1}(\mathcal{T}_B)(x), \phi^{-1}(\mathcal{I}_B)(x), \phi^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\big\}$$

where for all $x \in X, \phi^{-1}(\mathcal{T}_B)(x) = \mathcal{T}_B(f(x)), \phi^{-1}(\mathcal{I}_B)(x) = \mathcal{I}_B(f(x)), \phi^{-1}(\mathcal{F}_B)(x) = \mathcal{F}_B(f(x))$.

(ii) The image of A under ϕ denoted by $\phi(A)$ is a NS in Y defined by
$\phi(A) = \big(\phi(\mathcal{T}_A), \phi(\mathcal{I}_A), \phi(\mathcal{F}_A)\big)$, where for each $u \in Y$,

$$\phi(\mathcal{T}_A)(u) = \begin{Bmatrix} \bigvee_{x\in\phi^{-1}(u)} \mathcal{T}_A(x), & \text{if } \phi^{-1}(u) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$\phi(\mathcal{I}_A)(u) = \begin{Bmatrix} \bigvee_{x\in\phi^{-1}(u)} \mathcal{I}_A(x), & \text{if } \phi^{-1}(u) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$\phi(\mathcal{F}_A)(u) = \begin{Bmatrix} \bigvee\limits_{x \in \phi^{-1}(u)} \mathcal{F}_A(x), & \text{if } \phi^{-1}(u) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix}.$$

2.5. **Definition:[19]**

Let $\alpha, \beta, \gamma \in [0,1]$ and $\alpha + \beta + \gamma \leq 3$. A neutrosophic point $x_{(\alpha,\beta,\gamma)}$ of X is the NS in X defined by

$$x_{(\alpha,\beta,\gamma)}(u) = \begin{Bmatrix} (\alpha,\beta,\gamma), & if\ x = u \\ (0,0,1), & if\ x \neq u \end{Bmatrix}; \text{for each } u \in X.$$

A neutrosophic point is said to belong to a NS $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$ in X denoted by $x_{(\alpha,\beta,\gamma)} \in A$ if $\alpha \leq \mathcal{T}_A(x), \beta \leq \mathcal{I}_A(x)$ and $\gamma \geq \mathcal{F}_A(x)$.

3. **Neutrosophic Group**

3.1. **Definition:[10]**

Let $(X, \circ)$ be a group and let A be a neutrosophic group (NG) in X. Then A is said to be a NG in X if it satisfies the following conditions:

(i) $\mathcal{T}_A(xy) \geq \mathcal{T}_A(x) \wedge \mathcal{T}_A(y)$, $\mathcal{I}_A(xy) \geq \mathcal{I}_A(x) \wedge \mathcal{I}_A(y)$ and $\mathcal{F}_A(xy) \leq \mathcal{F}_A(x) \vee \mathcal{F}_A(y)$,
(ii) $\mathcal{T}_A(x^{-1}) \geq \mathcal{T}_A(x)$, $\mathcal{I}_A(x^{-1}) \geq \mathcal{I}_A(x)$, and $\mathcal{F}_A(x^{-1}) \leq \mathcal{F}_A(x)$.

3.2. **Definition:[22]**

Let X be a group and let $\mathcal{G}$ be NG in X and e be the identity of X. We define the NS $\mathcal{G}_e$ by

$$\mathcal{G}_e = \{x \in X : \mathcal{T}_\mathcal{G}(x) = \mathcal{T}_\mathcal{G}(e), \mathcal{I}_\mathcal{G}(x) = \mathcal{I}_\mathcal{G}(e), \mathcal{F}_\mathcal{G}(x) = \mathcal{F}_\mathcal{G}(e)\}.$$

We note for a NG $\mathcal{G}$ in a group X, for every $x \in X : \mathcal{T}_\mathcal{G}(x^{-1}) = \mathcal{T}_\mathcal{G}(x)$, $\mathcal{I}_\mathcal{G}(x^{-1}) = \mathcal{I}_\mathcal{G}(x)$ and $\mathcal{F}_\mathcal{G}(x^{-1}) = \mathcal{F}_\mathcal{G}(x)$. Also for the identity e of the group $X : \mathcal{T}_\mathcal{G}(e) \geq \mathcal{T}_\mathcal{G}(x), \mathcal{I}_\mathcal{G}(e) \geq \mathcal{I}_\mathcal{G}(x)$, and $\mathcal{F}_\mathcal{G}(e) \leq \mathcal{F}_\mathcal{G}(x)$.

3.3. **Proposition:**

Let $\mathcal{G}$ be a NG in a group X. Then for all $x, y \in X$,

(1) $\mathcal{T}_\mathcal{G}(xy^{-1}) = \mathcal{T}_\mathcal{G}(e) \Rightarrow \mathcal{T}_\mathcal{G}(x) = \mathcal{T}_\mathcal{G}(y)$
(2) $\mathcal{I}_\mathcal{G}(xy^{-1}) = \mathcal{I}_\mathcal{G}(e) \Rightarrow \mathcal{I}_\mathcal{G}(x) = \mathcal{I}_\mathcal{G}(y)$
(3) $\mathcal{F}_\mathcal{G}(xy^{-1}) = \mathcal{F}_\mathcal{G}(e) \Rightarrow \mathcal{F}_\mathcal{G}(x) = \mathcal{F}_\mathcal{G}(y)$

3.4. **Proposition:**

Let X be a group. Then the following statements are equivalent;

(i) $\mathcal{G}$ is neutrosophic group in X.

(ii) For all $x, y \in X, \mathcal{T}_{\mathcal{G}}(xy^{-1}) \geq \mathcal{T}_{\mathcal{G}}(x) \wedge \mathcal{T}_{\mathcal{G}}(y), \mathcal{I}_{\mathcal{G}}(xy^{-1}) \geq \mathcal{I}_{\mathcal{G}}(x) \wedge \mathcal{I}_{\mathcal{G}}(y), \mathcal{F}_{\mathcal{G}}(xy^{-1}) \leq \mathcal{F}_{\mathcal{G}}(x) \vee \mathcal{F}_{\mathcal{G}}(y)$.

3.5. **Definition:[10]**

Let $\phi : X \to Y$ be a group homomorphism and let A be a NG in a group X. Then A is said to be neutrosophic-invariant if for any $x, y \in X$,$\mathcal{T}_A(x) = \mathcal{T}_A(y)$, $\mathcal{I}_A(x) = \mathcal{I}_A(y)$ and $\mathcal{F}_A(x) = \mathcal{F}_A(y)$. It is clear that if A is neutrosophic invariant then $f(A) \in$ NG (Y). For each $A \in$ neutrosophic group (X), let $X_A = \{x \in X : \mathcal{T}_A(x) = \mathcal{T}_A(e), \mathcal{I}_A(x) = \mathcal{I}_A(e), \mathcal{F}_A(x) = \mathcal{F}_A(e)\}$. Then it is clear that X_A is a subgroup of X. For each $a \in X$, let $r_a : X \to X$ and $l_a : X \to X$ be the right and left translations of X into itself, defined by $r_a(x) = xa$ and $l_a(x) = ax$, respectively for each $x \in X$.

3.6. **Definition:[18]**

Let X be a non empty set and A neutrosophic topology is a family $\mathfrak{T}$ of neutrosophic subsets of X satisfying the following axioms:

(i) $0_A, 1_A \in \mathfrak{T}$

(ii) $G_1 \cap G_2 \in \mathfrak{T}$ for any $G_1, G_2 \in \mathfrak{T}$

(iii) $\bigcup G_i \ \forall \{G_i : i \in J\} \subseteq \mathfrak{T}$

In this case the pair $(X, \mathfrak{T})$ is called a neutrosophic topological space (NTS) and any neutrosophic set in $\mathfrak{T}$ is known as neuterosophic open set. The elements of $\mathfrak{T}$ are called open neutrosophic sets, a neutrosophic set F is neutrosophic closed set if and only if it C(F) is neutrosophic open set.

3.7. **Definition:[19]**

Let $(X, \mathfrak{T})$ be a NTS and A be a NS in X. Then the induced neutrosophic topology on A is the collection of NSs in A which are the intersection of netrosophic open sets in X with A. Then the pair $(A, \mathfrak{T}_A)$ is called a neutrosophic subspace of $(X, \mathfrak{T})$. The induced neutrosophic topology is denoted by $\mathfrak{T}_A$.

4. Neutrosophic Continuity

It is known by [4] that $f : (X, \mathfrak{T}_X) \to (Y, \mathfrak{T}_Y)$ is neutrosophic continuous if the preimage of each neutrosophic open set in Y is neutrosophic open set in X.

4.1. **Theorem:**

Let $(X, \mathfrak{T}_X)$ and $(Y, \mathfrak{T}_Y)$ be two NTGs and $f : (X, \mathfrak{T}_X) \to (Y, \mathfrak{T}_Y)$ be a mapping , then f is neutrosophic continuous if and only if f is neutrosophic continuous at neutrosophic point

$x_{(\alpha,\beta,\gamma)}$, for each $x \in X$.

5. Neutrosophic Bitopological Spaces

5.1. Definition:[23]

Let $(X, \mathfrak{T}_1)$ and $(X, \mathfrak{T}_2)$ be the two neutrosophic topologies on X. Then $(X, \mathfrak{T}_1, \mathfrak{T}_2)$ is called a neutrosophic bitopological space (In short NBTS).

5.2. Definition:[23]

Let $(X, \mathfrak{T}_1, \mathfrak{T}_2)$ be a NBTS. A NS $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$ over X is said to a pairwise neutrosophic open set in $(X, \mathcal{T}_1, \mathcal{T}_2)$ if there exist a NS $A_1 = \{\langle x, \mathcal{T}_{A_1}(x), \mathcal{I}_{A_1}(x), \mathcal{F}_{A_1}(x)\rangle : x \in X\}$ in $\mathcal{T}_1$ and a NS $A_2 = \{\langle x, \mathcal{T}_{A_2}(x), \mathcal{I}_{A_2}(x), \mathcal{F}_{A_2}(x)\rangle : x \in X\}$ in $\mathcal{T}_2$ such that $A = A_1 \cup A_2 = \{\langle x, min(\mathcal{T}_{A_1}(x), \mathcal{T}_{A_2}(x)), min(\mathcal{I}_{A_1}(x), \mathcal{I}_{A_2}(x)), max(\mathcal{F}_{A_1}(x), \mathcal{F}_{A_2}(x))\rangle : x \in X\}$.

6. Neutrosophic Topological Groups

6.1. Definition:[22]

Let X be a group and $\mathcal{G}$ be a NG on X. Let $\mathfrak{T}^{\mathcal{G}}$ be a neutrosophic topology on $\mathcal{G}$ then $(\mathcal{G}, \mathfrak{T}^{\mathcal{G}})$ is said to be neutrosophic topological group (In short NTG) if the following conditions are satisfied:

(i) The mapping $\psi : (\mathcal{G}, \mathfrak{T}^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}^{\mathcal{G}})$ defined by $\psi(x, y) = xy$, for all $x, y \in X$, is relatively neutrosophic continuous.

(ii) The mapping $\mu : (\mathcal{G}, \mathfrak{T}^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}^{\mathcal{G}})$ defined by $\mu(x) = x^{-1}$, for all $x \in X$, is relatively neutrosophic continuous.

6.2. Definition:[18]

Let X be a group and U, V be two NSs in X. We define the product UV of NS U, V and the inverse V^{-1} of V as follows:

$$UV(x) = \{\langle x, \mathcal{T}_{UV}(x), \mathcal{I}_{UV}(x), \mathcal{F}_{UV}(x)\rangle : x \in X\}$$

where

$$\mathcal{T}_{UV}(x) = sup\{min\{\mathcal{T}_U(x_1), \mathcal{T}_V(x_1)\}\}$$

$$\mathcal{I}_{UV}(x) = sup\{min\{\mathcal{I}_U(x_1), \mathcal{I}_V(x_1)\}\}$$

$$\mathcal{F}_{UV}(x) = sup\{min\{\mathcal{F}_U(x_1), \mathcal{F}_V(x_1)\}\}$$

where $x = x_1.x_2$ and for $V = \{x\langle \mathcal{T}_V(x), \mathcal{I}_V(x), \mathcal{F}_V(x)\rangle : x \in X\}$, we have

$V^{-1} = \{\langle x, \mathcal{T}_V(x^{-1}), \mathcal{I}_V(x^{-1}), \mathcal{F}_V(x^{-1})\rangle : x \in X\}$.

7. Main Results:

7.1. Definition:

Let X be a group and $\mathcal{G}$ be a NG on X. Let $\mathfrak{T}_1^{\mathcal{G}}, \mathfrak{T}_2^{\mathcal{G}}$ be two neutrosophic topologies on $\mathcal{G}$ then $(\mathcal{G}, \mathfrak{T}_1^{\mathcal{G}}, \mathfrak{T}_2^{\mathcal{G}})$ is said to be neutrosophic bitopological group (In short NBTG) if the following conditions hold good:

(i) The mapping $\psi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ defined by $\psi(x, y) = xy$, for all $x, y \in X$, is relatively neutrosophic i-continuous for each i=1, 2.

(ii) The mapping $\mu : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ defined by $\mu(x) = x^{-1}$, for all $x \in X$, is relatively neutrosophic i-continuous for each i=1, 2.

7.2. Definition:

Let $\mathcal{G}$ be a NG of a group X. Then for fixed $a \in X$, the left translation $l_a : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ for each $i = 1, 2$; is defined by $l_a(x) = ax$, for all $x \in X$, where $ax = \{\langle a, \mathcal{T}_i^{\mathcal{G}}(ax), \mathcal{I}_i^{\mathcal{G}}(ax), \mathcal{F}_i^{\mathcal{G}}(ax)\rangle : x \in X\}$ for each $i = 1, 2$.
Similarly, the right translation $r_a : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ for each $i = 1, 2$; is defined by $r_a(x) = xa$, for all $x \in X$, where $ax = \{\langle a, \mathcal{T}_i^{\mathcal{G}}(xa), \mathcal{I}_i^{\mathcal{G}}(xa), \mathcal{F}_i^{\mathcal{G}}(xa)\rangle : x \in X\}$ for each $i = 1, 2$.

7.3. Lemma:

Let X be a group with NBTG $\mathcal{G}$ in X with two neutrosophic topologies $\mathfrak{T}_1, \mathfrak{T}_2$. Then for each $a \in \mathcal{G}_e$, the translation l_a and r_a are relatively neutrosophic homeomorphism of $(\mathcal{G}, \mathfrak{T}_1^{\mathcal{G}}, \mathfrak{T}_2^{\mathcal{G}})$ into itself.

Proof: From Proposition 3.11 [10], we have $l_a[\mathcal{G}] = \mathcal{G}$ and $r_a[\mathcal{G}] = \mathcal{G}$, for all $a \in \mathcal{G}_e$ and let $h : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$, for each $i = 1, 2$; defined by $h(x) = (a, x)$ for each $x \in X$. Then $r_a : \psi \circ h$. Since $a \in \mathcal{G}_e$, $\mathcal{T}_i^{\mathcal{G}}(a) = \mathcal{T}_i^{\mathcal{G}}(e)$, $\mathcal{I}_i^{\mathcal{G}}(a) = \mathcal{I}_i^{\mathcal{G}}(e)$, and $\mathcal{F}_i^{\mathcal{G}}(a) = \mathcal{F}_i^{\mathcal{G}}(e)$, for each $i = 1, 2$. Thus $\mathcal{T}_i^{\mathcal{G}}(a) \geq \mathcal{T}_i^{\mathcal{G}}(x)$, $\mathcal{I}_i^{\mathcal{G}}(a) \geq \mathcal{I}_i^{\mathcal{G}}(x)$, and $\mathcal{F}_i^{\mathcal{G}}(a) \leq \mathcal{F}_i^{\mathcal{G}}(x)$, for each $x \in X$. It follows from proposition 3.34 [11] that $\phi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each $i = 1, 2$. By the hypothesis ψ is relatively neutrosophic i-continuous for each $i = 1, 2$,. So r_a is relatively neutrosophic i-continuous for each $i = 1, 2$. Moreover $r_a^{-1} = r_{a^{-1}}$. Similarly we are shown the relatively neutrosophic i-continuous for each $i = 1, 2$ of $l_a^{-1} = l_{a^{-1}}$.

7.4. Theorem:

Let $\mathcal{G}$ be a NBTG on X with $\mathfrak{T}_1, \mathfrak{T}_2$ two neutrosophic topologies. Let U be a neutrosophic open set of $(\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ for each i=1, 2 and $x \in \mathcal{G}_e$, then xU and Ux are neutrosophic open set.

Proof: Since U is neutrosophic open set of $\mathcal{G}$ and $x \in \mathcal{G}_e$, $\lambda : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \rightarrow (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is neutrosophic homeomorphism for each i=1, 2. This implies that $l_x(U) = xU$ is neutrosophic open set in $\mathcal{G}$. Similarly Ux is neutrosophic open set in $\mathcal{G}$.

7.5. **Lemma:**

Let X be a group and let $\mathcal{G}$ be NBTG in X. Then

(i) The inverse function $\phi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \rightarrow (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ defined by $\phi(x) = x^{-1}$, for all $x \in X$ is relatively neutrosophic i-continuous homeomorphism for each i=1, 2.

(ii) The inner automorphism $\lambda : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \rightarrow (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ defined by $\lambda(g) = aga^{-1} = \{\langle g, \mathcal{T}_i^{\mathcal{G}}(aga^{-1}), \mathcal{I}_i^{\mathcal{G}}(aga^{-1}), \mathcal{F}_i^{\mathcal{G}}(aga^{-1})\rangle\}$, where $g \in X$ and $a \in \mathcal{G}_e$ is relatively neutrosophic homeomorphism for each i=1,2.

Proof: (i) Clearly ϕ is one-to-one. Since $\phi(\mathcal{G}) = \{\langle x, \phi(\mathcal{T}_i^{\mathcal{G}}(x)), \phi(\mathcal{I}_i^{\mathcal{G}}(x)), \phi(\mathcal{F}_i^{\mathcal{G}}(x))\rangle : x \in \mathcal{G}\}$ for each i=1, 2 where

$$\begin{aligned}
\phi(\mathcal{T}_i^{\mathcal{G}}(x)) &= \begin{Bmatrix} \vee_{y \in \phi^{-1}(x)} \mathcal{T}_i^{\mathcal{G}}(y), & \text{if } \phi^{-1}(x) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix} \\
&= \begin{Bmatrix} \mathcal{T}_i^{\mathcal{G}}(x^{-1}), & \text{if } \phi^{-1}(x) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix} \\
&= \begin{Bmatrix} \mathcal{T}_i^{\mathcal{G}}(x), & \text{if } \phi^{-1}(x) \neq 0 \\ 0, & \text{otherwise} \end{Bmatrix}
\end{aligned}$$

Also, $\phi(\mathcal{I}_i^{\mathcal{G}}(x)) = \mathcal{I}_i^{\mathcal{G}}(x)$ and $\phi(\mathcal{F}_i^{\mathcal{G}}(x)) = \mathcal{F}_i^{\mathcal{G}}(x)$
Thus $\phi(\mathcal{G}) = \{\langle x, \mathcal{T}_i^{\mathcal{G}}(x), \mathcal{I}_i^{\mathcal{G}}(x), \mathcal{F}_i^{\mathcal{G}}(x)\rangle : x \in \mathcal{G}\}$, for each i= 1, 2. Also ϕ is neutrosophic i-continuous for each i=1,2 by definition because $(\mathcal{G}, \mathfrak{T}_1^{\mathcal{G}}, \mathfrak{T}_2^{\mathcal{G}})$ is NBTG. Since $\phi^{-1}(x) = x^{-1}$ is relatively neutrosophic i-continuous for each i=1, 2. Hence for every $x \in X$, ϕ is relatively neutrosophic open. Thus ϕ is relatively neutrosophic homeomorphism.

(ii) Since r_a and l_a are relatively neutrosophic homeomorphism and $r_a^{-1} = r_{a^{-1}}$. The inner automorphism λ is a composition $r_{a^{-1}}$ and l. Hence λ is a relative neutosophic homeomorphism.

7.6. **Theorem:**

Let $\mathcal{G}$ be a NBTG in a group X and e be the identity of X. If $a \in \mathcal{G}_e$ and N is a neighbourhood of e such that $\mathcal{T}_i^N(e) = 1$, $\mathcal{I}_i^N(e) = 1$, $\mathcal{F}_i^N(e) = 0$ for each i=1, 2 then aN is a nbd of a such that $aN(a) = 1_N$.

Proof: Since N is a nbd of e such that $\mathcal{T}_i^N = 1$, $\mathcal{I}_i^N = 1$, $\mathcal{F}_i^N = 0$ for each i=1, 2; there exists a neutrosophic open set U such that $U \subseteq N$ and $\mathcal{T}_i^U(e) = \mathcal{T}_i^N(e) = 1$, $\mathcal{I}_i^U(e) = \mathcal{I}_i^N(e) = 1$,

$\mathcal{F}_i^U(e) = \mathcal{F}_i^N(e) = 0$, for each i=1, 2. Let $l_a : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ be a left translation defined by $l_a(g) = ag$, for each $g \in X$ and i=1, 2. Then l_a is neutrosophic homeomorphism. Then aU is a neutrosophic open set. Now,

$$\begin{aligned}
aU(a) &= \{\langle a, \mathcal{T}_i^{aU}(a), \mathcal{I}_i^{aU}(a), \mathcal{F}_i^{aU}(a)\rangle\}, \text{for each } i = 1, 2.\\
&= \{\langle a, \mathcal{T}_i^{U}(aa^{-1}), \mathcal{I}_i^{U}(aa^{-1}), \mathcal{F}_i^{U}(aa^{-1})\rangle\}\\
&= \{\langle a, \mathcal{T}_i^{U}(e), \mathcal{I}_i^{U}(e), \mathcal{F}_i^{U}(e)\rangle\}\\
&= \{\langle a, 1, 1, 0\rangle\}\\
\text{Also, } aN(x) &= \{\langle x, \mathcal{T}_i^{aN}(x), \mathcal{I}_i^{aN}(x), \mathcal{F}_i^{aN}(x)\rangle : x \in X\}, \text{for each } i = 1, 2.\\
&= \{\langle x, \mathcal{T}_i^{N}(a^{-1}x), \mathcal{I}_i^{N}(a^{-1}x), \mathcal{F}_i^{N}(a^{-1}x)\rangle : x \in X\}\\
&\geq \{\langle x, \mathcal{T}_i^{U}(a^{-1}x), \mathcal{I}_i^{U}(a^{-1}x), \mathcal{F}_i^{U}(a^{-1}x)\rangle : x \in X\}\\
&= \{\langle x, \mathcal{T}_i^{aU}(x), \mathcal{I}_i^{aU}(x), \mathcal{F}_i^{aU}(x)\rangle\}\\
&= aU(x)\\
aN(x) &\geq aU(x); \text{for each } x \in X.\\
\text{and } aN(a) &= \{\langle a, \mathcal{T}_i^{aN}(a), \mathcal{I}_i^{aN}(a), \mathcal{F}_i^{aN}(a)\rangle\}, \text{for each } i = 1, 2.\\
&= \{\langle a, \mathcal{T}_i^{N}(aa^{-1}), \mathcal{I}_i^{N}(aa^{-1}), \mathcal{F}_i^{N}(aa^{-1})\rangle\}\\
&= \{\langle a, \mathcal{T}_i^{N}(e), \mathcal{I}_i^{N}(e), \mathcal{F}_i^{N}(e)\rangle\}\\
&= \{\langle a, 1, 1, 0\rangle\}\\
\Rightarrow aN(a) &= \{\langle a, 1, 1, 0\rangle\}
\end{aligned}$$

Thus, there exist a neutrosophic open set aU such that $aU \subseteq aN$ and $aU(a) = aN(a) = \{\langle a, 1, 1, 0\rangle\}$.

7.7. **Proposition:**

Let X be a group and $\mathcal{G}$ be a NBTG on X with $\mathfrak{T}_1, \mathfrak{T}_2$ two neutrosophic topologies. Let $\lambda : X \times X \to X$ be the function defined by $\lambda(g, h) = gh^{-1}$ for any $g, h \in X$. Then $\mathcal{G}$ is a NBTG in X iff the function $\lambda : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2.

Proof: The function $\mu : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is neutrosophic relatively i-continuous for each i=1, 2; by the corollary to Proposition 3.28 [11]. Also since $\mathcal{G}$ is a NBTG in X by the Definition [7.1] $\psi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2. Then $\beta : \psi \circ \mu : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2.

Conversely, let $\beta : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2. If e is the identity element of X, then $\mathcal{T}_i^{\mathcal{G}}(e) \geq \mathcal{T}_i^{\mathcal{G}}(g)$, $\mathcal{I}_i^{\mathcal{G}}(e) \geq \mathcal{I}_i^{\mathcal{G}}(g)$ and $\mathcal{F}_i^{\mathcal{G}}(e) \leq \mathcal{F}_i^{\mathcal{G}}(g)$ for all $g \in X$. By the Proposition 3.34 [11], the function $\phi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ defined by $\phi(h) = (e, h)$ is relatively neutrosophic i-continuous for each i=1, 2. Thus the function $\alpha = \beta \circ \phi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2. The function $\mu : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2 by the corollary to Proposition 3.28 [11]. Thus $\psi = \beta \circ \mu : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2. Therefore $\mathcal{G}$ is a NBTG in X.

7.8. **Proposition:**

Let $\phi : X \to Y$ be a group homomorphism and $\mathfrak{T}_1, \mathfrak{T}_2$ and $\mathcal{U}_1, \mathcal{U}_2$ be the neutrosophic topologies on X and Y respectively, where $\mathfrak{T}_i$ is the inverse image of $\mathcal{U}_i$ under ϕ and let $\mathcal{G}$ be a NBTG in Y. Then the inverse image $\phi^{-1}(\mathcal{G})$ of $\mathcal{G}$ is a NBTG in X.

Proof: Consider the function $\alpha : X \times X \to X$ defined by $\alpha(g_1, g_2) = g_1 g_2^{-1}$ for any $g_1, g_2 \in X$. We have to prove that the function $\alpha : \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right) \times \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right) \to \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right)$ is relatively neutrosophic i-continuous for each i=1, 2. Since $\mathfrak{T}_i$ is the inverse image of $\mathcal{U}_i$ under ϕ, $\phi : (X, \mathfrak{T}_i) \to (X, \mathcal{U}_i)$ is the neutrosophic i-continuous for each i=1, 2. Also, $\phi\big(\phi^{-1}(\mathcal{G})\big) \subset \mathcal{G}$. By Proposition 3.9 [11], $\phi : \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right) \to \left(\mathcal{G}, \mathcal{U}_i^{\mathcal{G}}\right)$ is relatively neutrosophic i-continuous for each i=1, 2. Let $U = \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}$. Then there exist a $V = \mathcal{U}_i^{\mathcal{G}}$ such that $\phi^{-1}(V) = U$. Let $(g_1, g_2) \in X \times X$. Then

$$\begin{aligned}
\mathcal{T}_i^{\alpha^{-1}(U)}(g_1, g_2) &= \alpha^{-1}\left(\mathcal{T}_i^{U}\right)(g_1, g_2) = \mathcal{T}_i^{U}\left(\alpha(g_1, g_2)\right) = \mathcal{T}_i^{U}\left(g_1, g_2^{-1}\right), \text{for each } i = 1, 2.\\
&= \mathcal{T}_i^{\phi^{-1}(V)}(g_1, g_2^{-1}) = \phi^{-1}_{(\mathcal{T}_i^{V})}(g_1, g_2^{-1}) = \mathcal{T}_i^{V}\left(\phi(g_1, g_2^{-1})\right)\\
&= \mathcal{T}_i^{V}\left(\phi(g_1), \phi(g_2^{-1})\right) = \mathcal{T}_i^{V}\left(\phi(g_1), \big(\phi(g_2)\big)^{-1}\right)
\end{aligned}$$

Thus $\mathcal{T}_i^{\alpha^{-1}(U)}(g_1, g_2) = \mathcal{T}_i^{V}\left(\phi(g_1), \big(\phi(g_2)\big)^{-1}\right)$

Similarly we have $\mathcal{I}_i^{\alpha^{-1}(U)}(g_1, g_2) = \mathcal{I}_i^{V}\left(\phi(g_1), \big(\phi(g_2)\big)^{-1}\right)$ and $\mathcal{F}_i^{\alpha^{-1}(U)}(g_1, g_2) = \mathcal{F}_i^{V}\left(\phi(g_1), \big(\phi(g_2)\big)^{-1}\right)$ for each i=1,2. By the hypothesis, the function $\beta : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ given by $\beta(h_1, h_2) = h_1 h_2^{-1}$ for any $h_1, h_2 \in Y$ is relatively neutrosophic i-continuous for each i=1, 2. By corollary to the Proposition 3.28 [11] the product function $\phi \times \phi : \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right) \times \left(\phi^{-1}(\mathcal{G}), \mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\right) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is the neutrosophic i-continuous for each i=1, 2. Now, let $(g_1, g_2) \in X \times X$. Then

$$\mathcal{T}_i^{V}\left(\phi(g_1), \big(\phi(g_2)\big)^{-1}\right) = \mathcal{T}_i^{\beta^{-1}(V)}\big(\phi(g_1), \phi(g_2)\big) = \mathcal{T}_i^{(\phi\times\phi)^{-1}(\beta^{-1}(V))}\big(g_1, g_2\big),$$

$$\mathcal{I}_i^V\Big(\phi(g_1),\big(\phi(g_2)\big)^{-1}\Big)=\mathcal{I}_i^{\beta^{-1}(V)}\big(\phi(g_1),\phi(g_2)\big)=\mathcal{I}_i^{(\phi\times\phi)^{-1}(\beta^{-1}(V))}\big(g_1,g_2\big)$$
$$\text{and } \mathcal{F}_i^V\Big(\phi(g_1),\big(\phi(g_2)\big)^{-1}\Big)=\mathcal{F}_i^{\beta^{-1}(V)}\big(\phi(g_1),\phi(g_2)\big)=\mathcal{F}_i^{(\phi\times\phi)^{-1}(\beta^{-1}(V))}\big(g_1,g_2\big)$$
$$\text{for each } i=1,2.$$

$$\text{Thus } \alpha^{-1}(U)\cap\Big(\phi^{-1}(\mathcal{G})\times\phi^{-1}(\mathcal{G})\Big)=(\phi\times\phi)^{-1}\big(\beta^{-1}(V)\big)\cap\Big(\phi^{-1}(\mathcal{G})\times\phi^{-1}(\mathcal{G})\Big)$$
$$=[\beta\circ(\phi\times\phi)]^{-1}(V)\cap\Big(\phi^{-1}(\mathcal{G})\times\phi^{-1}(\mathcal{G})\Big).$$

So $\alpha^{-1}(U)\cap\Big(\phi^{-1}(\mathcal{G})\times\phi^{-1}(\mathcal{G})\Big)\in\mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\times\mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}$, i.e., $\alpha:\Big(\phi^{-1}(\mathcal{G}),\mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\Big)\times\Big(\phi^{-1}(\mathcal{G}),\mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\Big)\rightarrow\Big(\phi^{-1}(\mathcal{G}),\mathfrak{T}_i^{\phi^{-1}(\mathcal{G})}\Big)$ is a relatively neutrosophic i-continuous for each i=1,2. By Result 3.9 [10], $\phi^{-1}(\mathcal{G})$ is neutrosophic group in X. Hence by Proposition [7.7], $\phi^{-1}(\mathcal{G})$ is NBTG in X.

7.9. **Proposition:**

Let $\phi:X\rightarrow Y$ be a group homomorphism. Let $\mathfrak{T}_1,\mathfrak{T}_2$ and $\mathcal{U}_1,\mathcal{U}_2$ be the neutrosophic topologies on X and Y respectively, where $\mathcal{U}_i$ is the image under ϕ and of $\mathfrak{T}_i$, for each i=1, 2; and let $\mathcal{G}$ be a NBTG in X. If $\mathcal{G}$ is the neutrosophic invariant, then the image $\phi(\mathcal{G})$ of $\mathcal{G}$ is a NBTG in Y.

Proof: Consider the function $\beta:Y\rightarrow Y$ defined by $\beta(h_1,h_2)=h_1h_2^{-1}$ for any $h_1,h_2\in Y$. We have to prove that the function $\beta:\Big(\phi(\mathcal{G}),\mathcal{U}_i^{\phi(\mathcal{G})}\Big)\times\Big(\phi(\mathcal{G}),\mathcal{U}_i^{\phi(\mathcal{G})}\Big)\rightarrow\Big(\phi(\mathcal{G}),\mathcal{U}_i^{\phi(\mathcal{G})}\Big)$ is a relatively neutrosophic i-continuous for each i=1, 2. Suppose $\mathcal{G}$ is a neutrosophic invariant. By the Definition 3.2, $\phi(\mathcal{G})$ is a neutrosophic group in Y. Let $U\in\mathfrak{T}_i$. Also $U\subset\phi^{-1}\big(\phi(U)\big)$. Then there exist a family $\{U_\lambda\}_{\lambda\in\wedge}\subset\mathfrak{T}_i$ such that $\phi^{-1}\big(\phi(U)\big)=\bigcup\limits_{\alpha\in\wedge}U_\alpha$. So $\phi^{-1}\big(\phi(U)\big)\in\mathfrak{T}_i$. Since $\mathcal{U}_i$ is the image of $\mathfrak{T}_i$ under ϕ, $\phi(U)\in\mathcal{U}_i$, for each i=1, 2. So ϕ is neutrosophic i-open. Now, let $U\in\mathfrak{T}_i^{\mathcal{G}}$. Then there exist a $U=U_1\cap\mathcal{G}$. Since $\mathcal{G}$ is neutrosophic invariant, by Proposition 3.12 [10], $\phi(U)=\phi(U_1)\cap\phi(\mathcal{G})$. Since ϕ is neutrosophic i-open, $\phi(U_1)=\mathfrak{T}_i$, for each i=1,2. Then $\phi(U)\in\mathcal{U}_i^{\phi(\mathcal{G})}$, for each i=1,2. Thus $\phi:(\mathcal{G},\mathfrak{T}_i^{\mathcal{G}})\rightarrow\Big(\phi(\mathcal{G}),\mathcal{U}_i^{\phi(\mathcal{G})}\Big)$ is relatively neutrosophic i-open for each i=1,2. By Proposition 3.31 [11], the product function $(\phi\times\phi):(\mathcal{G},\mathfrak{T}_i^{\mathcal{G}})\times(\mathcal{G},\mathfrak{T}_i^{\mathcal{G}})\rightarrow\Big(\phi(\mathcal{G}),\mathcal{U}_i^{\phi(\mathcal{G})}\Big)$ is relative neutrosophic i-open for each i=1,2.

Let $V\in\mathcal{U}_i^{\phi(\mathcal{G})}$ and let $(g_1,g_2)\in X\times X$. Then

$$\mathcal{T}_i^{\beta\circ(\phi\times\phi)^{-1}(V)}(g_1,g_2)=\big[\beta\circ(\phi\times\phi)\big]^{-1}\Big(\mathcal{T}_i^V\Big)(g_1,g_2),\text{for each } i=1,2.$$
$$=\mathcal{T}_i^V\big[\beta\circ(\phi\times\phi)\big](g_1,g_2)=\mathcal{T}_i^V\Big(\phi(g_1),\phi(g_2)\Big)$$
$$=\mathcal{T}_i^V\Big(\phi(g_1),\big(\phi(g_2)^{-1}\big)\Big)=\mathcal{T}_i^V\Big(\phi(g_1),\phi(g_2^{-1})\Big)\text{[Since } \phi \text{ is homomorphism]}$$
$$=\mathcal{T}_i^V\Big(\phi(g_1g_2^{-1})\Big)=\mathcal{T}_i^V\phi\Big(\alpha(g_1,g_2)\Big)=\mathcal{T}_i^V\Big(\phi\circ\alpha(g_1,g_2)\Big)$$

$$= (\phi \circ \alpha)^{-1}\Big(\mathcal{T}_i^V(g_1, g_2)\Big) = \mathcal{T}_i^{(\phi\circ\alpha)^{-1}(V)}(g_1, g_2),$$

where $\alpha : X \times X \to X$ is the mapping given by $\alpha(g_1, g_2) = g_1 g_2^{-1}$ for each $(g_1, g_2) \in X \times X$. Thus $\mathcal{T}_i^{[\beta\circ(\phi\times\phi)]^{-1}(V)} = \mathcal{T}_i^{(\phi\circ\alpha)^{-1}(V)}$, $\mathcal{T}_i^{(\phi\times\phi)^{-1}[\beta^{-1}(V)]} = \mathcal{T}_i^{\alpha^{-1}(\phi^{-1}(V))}$. Similarly $\mathcal{I}_i^{(\phi\times\phi)^{-1}[\beta^{-1}(V)]} = \mathcal{I}_i^{\alpha^{-1}(\phi^{-1}(V))}$ and $\mathcal{F}_i^{(\phi\times\phi)^{-1}[\beta^{-1}(V)]} = \mathcal{F}_i^{\alpha^{-1}(\phi^{-1}(V))}$. So $(\phi \times \phi)^{-1}[\beta^{-1}(V)] = \alpha^{-1}(\phi^{-1}(V))$. Since $\mathcal{G}$ is NBTG in X, $\alpha : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$ is relatively neutrosophic i-continuous for each i=1, 2. Since $\mathcal{U}_i$ is the image of $\mathfrak{T}_i$ under ϕ, $\phi : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}})$, $\mathfrak{T}_i^{\phi(\mathcal{G})}$ is relatively neutrosophic i-continuous for each i=1, 2. Then $(\phi \times \phi) : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to \Big(\phi(\mathcal{G}), \mathcal{U}_i^{\phi(\mathcal{G})}\Big) \times \Big(\phi(\mathcal{G}), \mathcal{U}_i^{\phi(\mathcal{G})}\Big)$ is relatively neutrosophic i-continuous for each i=1, 2. Thus $(\phi \times \phi) \circ \beta : (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \times (\mathcal{G}, \mathfrak{T}_i^{\mathcal{G}}) \to \Big(\phi(\mathcal{G}), \mathcal{U}_i^{\phi(\mathcal{G})}\Big)$ is relatively neutrosophic i-continuous for each i=1, 2. Since $\mathcal{G}$ is neutrosophic invariant, $(\phi \times \phi)^{-1}\big[\beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big)\big] = (\phi \times \phi)^{-1}\big[\beta^{-1}(V)\big] \cap (\mathcal{G} \times \mathcal{G})$.
So $(\phi \times \phi)^{-1}\big[\beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big)\big] \in \mathfrak{T}_i^{\mathcal{G}} \times \mathfrak{T}_i^{\mathcal{G}}$. Since $(\phi \times \phi)$ is relatively neutrosophic i-open for each i=1, 2; $(\phi \times \phi)(\phi \times \phi)^{-1}\big[\beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big)\big] \in \mathcal{U}_i^{\phi(\mathcal{G})} \times \mathcal{U}_i^{\phi(\mathcal{G})}$ for each i=1, 2. But $(\phi \times \phi)(\phi \times \phi)^{-1}\big[\beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big)\big] = \beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big)$. So $\beta^{-1}(V) \cap \big(\phi(\mathcal{G}) \times \phi(\mathcal{G})\big) \in \mathcal{U}_i^{\phi(\mathcal{G})} \times \mathcal{U}_i^{\phi(\mathcal{G})}$ for each i=1, 2. Hence $\phi(\mathcal{G})$ is a NBTG in Y.

7.10. **Proposition:**

Let X be a group and let $\mathcal{G}$ be a NBTG in X with $\mathfrak{T}_1, \mathfrak{T}_2$ two neutrosophic topologies. N a normal subgroup of X and let f be the canonical homomorphism of X onto the quetient group X/N. If $\mathcal{G}$ is constant on N, then $\mathcal{G}$ is f invariant.

Proof: Suppose $f(x_1) = f(x_2)$ for any $x_1, x_2 \in N$. Then $x_1N = x_2N$. Thus there exist $k_1, k_2 \in N$ such that $x_1k_1 = x_2k_2$. Since $\mathcal{G}$ is a constant on N, $\mathcal{T}_i^{\mathcal{G}}(x) = \mathcal{T}_i^{\mathcal{G}}(e)$, $\mathcal{I}_i^{\mathcal{G}}(x) = \mathcal{I}_i^{\mathcal{G}}(e)$ and $\mathcal{F}_i^{\mathcal{G}}(x) = \mathcal{F}_i^{\mathcal{G}}(e)$ for each i=1, 2 and $x \in X$. Then

$$\begin{aligned}\mathcal{T}_i^{\mathcal{G}}(x_1) &= \mathcal{T}_i^{\mathcal{G}}(x_2k_2k_1^{-1}) \geq \mathcal{T}_i^{\mathcal{G}}(x_2) \wedge \mathcal{T}_i^{\mathcal{G}}(k_2k_1^{-1}) \\ &= \mathcal{T}_i^{\mathcal{G}}(x_2) \wedge \mathcal{T}_i^{\mathcal{G}}(e)(k_2k_1^{-1} \in N) \\ &= \mathcal{T}_i^{\mathcal{G}}(x_2) \\ \text{i.e., } \mathcal{T}_i^{\mathcal{G}}(x_1) &\geq \mathcal{T}_i^{\mathcal{G}}(x_2).\end{aligned}$$

Similarly, we get $\mathcal{T}_i^{\mathcal{G}}(x_2) \geq \mathcal{T}_i^{\mathcal{G}}(x_1)$. Thus $\mathcal{T}_i^{\mathcal{G}}(x_1) = \mathcal{T}_i^{\mathcal{G}}(x_2)$. Similarly we can show that $\mathcal{I}_i^{\mathcal{G}}(x_1) = \mathcal{I}_i^{\mathcal{G}}(x_2)$ and $\mathcal{F}_i^{\mathcal{G}}(x_1) = \mathcal{F}_i^{\mathcal{G}}(x_2)$. Hence $\mathcal{G}$ is f invariant.

8. Conclusion:

The main characteristic of NS is that NS can deal with imprecises as well as inconsistent information which is very helpful to handle the various real-life application. By observing this we have studied NBTG on the basis of NS, so that we can deal with various problem of topological group with respect to NS. In this study we have introduced some new definition of NBTG. We investigated some properties and proved some propositions on NBTG. We hope our work will help in further study of generalised NBTG and also for study of neutrosophic almost topological group and neutrosophic almost bitopological group.

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