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On New Types of Weakly Neutrosophic Crisp Continuity

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Abstract: The article processes the conceptualizations of neutrosophic crisp α-open and neutrosophic crisp semi-α-open sets to define some new types of weakly "neutrosophic crisp continuity" essentially, neutrosophic crisp α*-continuous, neutrosophic crisp α**-continuous, neutrosophic crisp semi-α-continuous, neutrosophic crisp semi-α*-continuous and neutrosophic crisp semi-α**-continuous functions. Also, we shall explain the relationships between these types of weakly neutrosophic crisp continuity and the concepts of neutrosophic crisp continuity.

Keywords: Neutrosophic crisp α*-continuous, neutrosophic crisp α**-continuous, neutrosophic crisp semi-α-continuous, neutrosophic crisp semi-α*-continuous, and neutrosophic crisp semi-α**-continuous functions.

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1. Introduction

In 2014, Salama et al. [1] performed the abstraction of neutrosophic crisp topological space (concisely, $NCTS$). Al-Hamido et al. [2] submitted the intellect of neutrosophic crisp semi-α-closed sets in $NCTS$s. Abdel-Basset et al. [3-8] gave a novel neutrosophic approach. Maheswari et al. [9] presented gb-closed sets and gb-continuity in aspects of the neutrosophic theory. Banupriya et al. [10] investigated the notion of αgs continuity and αgs irresolute maps in the sense of neutrosophic view. In [11], Dhavaseelan et al. exhibited the theme of neutrosophic α^m-continuity. Al-Hamido et al. [15] introduced neutrosophic crisp topology via N-topology. Imran et al. [16] introduced and studied the thought of neutrosophic generalized alpha generalized continuity. Hanif PAGE et al. [17] presented neutrosophic generalized homeomorphism. This paper aspires to lay on new types of weakly neutrosophic crisp continuity, for instance, neutrosophic crisp α*-continuous, neutrosophic crisp α**-continuous, neutrosophic crisp semi-α-continuous, neutrosophic crisp semi-α*-continuous and neutrosophic crisp semi-α**-continuous functions. Likewise, we shall explain the relationships between these types of weakly neutrosophic crisp continuity and the concepts of neutrosophic crisp continuity.

2. Preliminaries

For the whole of the disquisition, $(\mathbb{X},\Gamma_1),(\mathbb{Y},\Gamma_2)$, and $(\mathbb{Z},\Gamma_3)$ (merely $\mathbb{X},\mathbb{Y}$, and $\mathbb{Z}$) habitually intend $NCTS$s. Let C be a neutrosophic crisp set (shortly, NCS) in $NCTS$ $(\mathbb{X},\Gamma_1)$ and denote its complement by C^c. Indicate the neutrosophic crisp open set as NC-OS, and the neutrosophic crisp closed set (its complement) as NC-CS in $NCTS$ $(\mathbb{X},\Gamma_1)$. Additionally, we refer to the neutrosophic crisp closure and neutrosophic crisp interior of C via $NCcl(C)$ and $NCint(C)$, correspondingly.

Definition 2.1 [1]: Assume that nonempty particular understudy space $\mathbb{X}$ has mutually disjoint subsets $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$. A NCS $\mathcal{C}$ with form $\mathcal{C} = \langle \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \rangle$ is called an object.

Definition 2.2: For any NCS $\mathcal{C}$ in $NCTS$ $(\mathbb{X}, \Gamma_1)$, we have
(i) if $\mathcal{C} \subseteq NCint(NCcl(NCint(\mathcal{C})))$, then it is called a neutrosophic crisp α-open set and symbolize by NCα-OS. Furthermore, its complement is named neutrosophic crisp α-closed set and signified by NCα-CS. Likewise, we reveal the collection consisting of all NCα-OSs in $\mathbb{X}$ with NCαO$(\mathbb{X})$. [12]
(ii) if $\mathcal{C} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{C}))))$, then it is said to be a neutrosophic crisp semi-α-open set and indicated via NCSα-OS. Moreover, its complement is known as a neutrosophic crisp semi-α-closed set and referred with NCSα-CS. Besides, we mentioned the collection of all NCSα-OSs in $\mathbb{X}$ through NCSαO$(\mathbb{X})$. [2]

Proposition 2.3 [12]: For any NCS $\mathcal{C}$ in $NCTS$ $(\mathbb{X}, \Gamma_1)$, then $\mathcal{C} \in NC\alpha O(\mathbb{X})$ iff we have at least a NC-OS $\mathcal{D}$ satisfying $\mathcal{D} \subseteq \mathcal{C} \subseteq NCint(NCcl(\mathcal{D}))$.

Proposition 2.4 [14]: Every NC-OS is a NCα-OS, but the opposite is not valid in general.

Proposition 2.5 [2]: In a $NCTS$ $(\mathbb{X}, \Gamma_1)$, the next assertions stand, but not vice versa:
(i) All NC-OSs are NCSα-OSs.
(ii) All NCα-OSs are NCSα-OSs.

Definition 2.6 [1]: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, we called it a neutrosophic crisp continuous and denoted by NC-continuous iff for all NC-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NC-OS from $\mathbb{X}$.

Theorem 2.7 [1]: A function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is NC-continuous iff $\eta^{-1}(NCint(\mathcal{D})) \subseteq NCint(\eta^{-1}(\mathcal{D}))$ for every $\mathcal{D} \subseteq \mathbb{Y}$.

Definition 2.8 [1]: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, we named it a neutrosophic crisp open and indicated via NC-open iff for all NC-OSs $\mathcal{C}$ from $\mathbb{X}$, then its image $\eta(\mathcal{C})$ is a NC-OS from $\mathbb{Y}$.

Definition 2.9 [13]: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, we said it a neutrosophic crisp α-continuous and referred through NCα-continuous iff for all NC-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NCα-OS from $\mathbb{X}$.

Proposition 2.10 [14]: Every NC-continuous function is a NCα-continuous, but the opposite is not valid in general.

3. Weakly Neutrosophic Crisp Continuity Functions

Definition 3.1: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, we call it as

(i) a neutrosophic crisp α*-continuous and denoted by NCα*-continuous iff for all NCα-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NCα-OS from $\mathbb{X}$.
(ii) a neutrosophic crisp α**-continuous and indicated via NCα**-continuous iff for all NCα-OS $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NC-OS from $\mathbb{X}$.

Definition 3.2: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, we named it as
(i) a neutrosophic crisp semi-α-continuous and referred through NCSα-continuous iff for all NC-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NCSα-OS from $\mathbb{X}$.
(ii) a neutrosophic crisp semi-α*-continuous and symbolize by NCSα*-continuous iff for all NCSα-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NCSα-OS from $\mathbb{X}$.
(iii) a neutrosophic crisp semi-α**-continuous and signified via NCSα**-continuous iff for all NCSα-OSs $\mathcal{D}$ from $\mathbb{Y}$, then its inverse image $\eta^{-1}(\mathcal{D})$ is a NC-OS from $\mathbb{X}$.

Theorem 3.3: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a function, then the next declarations are same:
(i) η is a NCSα-continuous.
(ii) its inverse image of each NC-CS from $\mathbb{Y}$ is NCSα-CS from $\mathbb{X}$.
(iii) $\eta(NCint(NCcl(NCint(NCcl(\mathcal{C}))))) \subseteq NCcl(\eta(\mathcal{C}))$, for each $\mathcal{C} \in \mathbb{X}$.
(iv) $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{D}))))) \subseteq \eta^{-1}(NCcl(\mathcal{D}))$, for each $\mathcal{D} \in \mathbb{Y}$.
Proof:
[(i) ⟹ (ii)] Suppose $\mathcal{D}$ is a NC-CS from $\mathbb{Y}$. This implies that $\mathcal{D}^c$ stands a NC-OS. Hence $\eta^{-1}(\mathcal{D}^c)$ is a NCSα-OS from $\mathbb{X}$. In other words, $(\eta^{-1}(\mathcal{D}))^c$ stands a NCSα-OS from $\mathbb{X}$. Thus $\eta^{-1}(\mathcal{D})$ is a NCSα-CS in $\mathbb{X}$.
[(ii) ⟹ (iii)] Let $\mathcal{C} \in \mathbb{X}$, then $NCcl(\eta(\mathcal{C}))$ stays a NC-CS from $\mathbb{Y}$. Hence $\eta^{-1}(NCcl(\eta(\mathcal{C})))$ is NCSα-CS in $\mathbb{X}$. Thus we have $\eta^{-1}(NCcl(\eta(\mathcal{C}))) \supseteq NCint(NCcl(NCint(NCcl(\eta^{-1}(NCcl(\eta(\mathcal{C}))))))) \supseteq NCint(NCcl(NCint(NCcl(\mathcal{C}))))$.
Or $NCcl(\eta(\mathcal{C}))) \supseteq \eta(NCint(NCcl(NCint(NCcl(\mathcal{C})))))$.
[(iii) ⟹ (iv)] Since $\mathcal{D} \in \mathbb{Y}, \eta^{-1}(\mathcal{D}) \in \mathbb{X}$.So, we have by our hypothesis the corresponding notation $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{D}))))) \subseteq NCcl(\eta(\eta^{-1}(\mathcal{D}))) \subseteq NCcl(\mathcal{D})$, and that leads us to this fact $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{D}))))) \subseteq \eta^{-1}(NCcl(\mathcal{D}))$.
[(iv) ⟹ (i)] Let $\mathcal{D}$ be a NC-OS of $\mathbb{Y}$. Let $\mathcal{C} = \mathcal{D}^c$ and $\mathcal{D} = \eta^{-1}(\mathcal{C})$ by (iii) we have $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{C}))))) \subseteq NCcl(\mathcal{C}) = \mathcal{C}$.
That is $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{D}^c))))) \subseteq \eta^{-1}(\mathcal{D}^c)$. Or $NCint(NCcl(NCint(NCcl(\eta^{-1}(\mathcal{D}))))) \supseteq \eta^{-1}(\mathcal{D})$. Hence $\eta^{-1}(\mathcal{D})$ is a NCSα-OS in $\mathbb{X}$ and thus η be there a NCSα-continuous. ■

Proposition 3.4:
(i) all NC-continuous functions are NCSα-continuous, but the opposite is not valid in general.
(ii) all NCα-continuous functions are NCSα-continuous, but the opposite is not exact in general.
Proof:
(i) Suppose $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is a NC-continuous function, and $\mathcal{D}$ be a NC-OS from $\mathbb{Y}$. Next $\eta^{-1}(\mathcal{D})$ remains a NC-OS from $\mathbb{X}$. Since any NC-OS is a NCSα-OS, $\eta^{-1}(\mathcal{D})$ stays a NCSα-OS from $\mathbb{X}$. Thus η exists a NCSα-continuous function.

(ii) Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a NCα-continuous function and $\mathcal{D}$ be a NC-OS from $\mathbb{Y}$. Subsequently $\eta^{-1}(\mathcal{D})$ happens a NCα-OS from $\mathbb{X}$. Since any NCα-OS is NCSα-OS, $\eta^{-1}(\mathcal{D})$ stays a NCSα-OS from $\mathbb{X}$. Thus η is a NCSα-continuous function. ■

Example 3.5: Suppose $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$. Then $\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle\}$ be neutrosophic crisp topologies (shortly, $NCTs$) on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly. Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle$, $\eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle, \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$. Then η is a NC α-continuous function but not NC-continuous since $\langle\{u\}, \phi, \phi\rangle$ is NC-OS but $\eta^{-1}(\langle\{u\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ which is not NC-OS in $\mathbb{X}$. Also, η is a NCSα-continuous function but not NC-continuous, since $\langle\{u\}, \phi, \phi\rangle$ is NC-OS in $\mathbb{Y}$ but $\eta^{-1}(\langle\{u\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ is not NC-OS from $\mathbb{X}$.

Example 3.6: Suppose $\mathbb{X} = \{p, q, r\}$. Then $\Gamma = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q\}, \phi, \phi\rangle, \langle\{p, q\}, \phi, \phi\rangle\}$ be a NCT on $\mathbb{X}$.

Define the function $\eta: (\mathbb{X}, \Gamma) \longrightarrow (\mathbb{X}, \Gamma)$ by $\eta(\langle\{p\}, \phi, \phi\rangle) = \langle\{p\}, \phi, \phi\rangle, \eta(\langle\{q\}, \phi, \phi\rangle) = \eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{q\}, \phi, \phi\rangle$. It is easily seen that η is a NCSα-continuous function but not NCα-continuous, since $\langle\{q\}, \phi, \phi\rangle$ is NC-OS in $\mathbb{X}$ but $\eta^{-1}(\langle\{q\}, \phi, \phi\rangle) = \langle\{q, r\}, \phi, \phi\rangle$ is not NCα-OS in $\mathbb{X}$.

Remark 3.7: The concepts of NC-continuity and NCα*-continuity are independent, for examples.

Example 3.8: Suppose $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$. Then
$\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q, r\}, \phi, \phi\rangle, \langle\{p, q, r\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle\}$ be NCTs on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly. Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle$, $\eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle$, $\eta(\langle\{r\}, \phi, \phi\rangle) = \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$. Then η is a NC-continuous function but not NCα*-continuous, since $\langle\{u, v\}, \phi, \phi\rangle$ is NCα-OS in $\mathbb{Y}$ but $\eta^{-1}(\langle\{u, v\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ is not NCα-OS in $\mathbb{X}$.

Example 3.9: Assume $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$. Then $\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle\}$ be NCTs on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly. Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle$, $\eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle, \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$. Then η is a NCα*-continuous function but not NC-continuous, since $\langle\{u\}, \phi, \phi\rangle$ is NC-OS in $\mathbb{Y}$, but $\eta^{-1}(\langle\{u\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ is not NC-OS in $\mathbb{X}$.

Theorem 3.10:

(i) If a function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is NC-open, NC-continuous, and bijective, then η is a NCα*-continuous.

(ii) A function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is NCα*-continuous iff $\eta: (\mathbb{X}, NC\alpha O(\mathbb{X})) \longrightarrow (\mathbb{Y}, NC\alpha O(\mathbb{Y}))$ is a NC-continuous.

Proof:

(i) Let $\mathcal{D} \in NC\alpha O(\mathbb{Y})$, to prove that $\eta^{-1}(\mathcal{D}) \in NC\alpha O(\mathbb{X})$, i.e., $\eta^{-1}(\mathcal{D}) \subseteq NCint(NCcl(NCint(\eta^{-1}(\mathcal{D}))))$. Let $r \in \eta^{-1}(\mathcal{D}) \Longrightarrow \eta(r) \in \mathcal{D}$. Hence $\eta(r) \in NCint(NCcl(NCint(\mathcal{D})))$ (since $\mathcal{D} \in NC\alpha O(\mathbb{Y})$). Therefore, at least NC-OS $\mathcal{H}$ from $\mathbb{Y}$ where $\eta(r) \in \mathcal{H} \subseteq NCcl(NCint(\mathcal{D}))$. Then $r \in \eta^{-1}(\mathcal{H}) \subseteq$

$\eta^{-1}(NCcl(NCint(\mathcal{D})))$, but $\eta^{-1}(NCcl(NCint(\mathcal{D}))) \subseteq NCcl(\eta^{-1}(NCint(\mathcal{D})))$ (since η^{-1} is a NC-continuous, which is equivalent to η is a NC-open and bijective). Then $r \in \eta^{-1}(\mathcal{H}) \subseteq NCcl(\eta^{-1}(NCint(\mathcal{D})))$. Hence $r \in \eta^{-1}(\mathcal{H}) \subseteq NCcl(\eta^{-1}(NCint(\mathcal{D}))) \subseteq NCcl(NCint(\eta^{-1}(\mathcal{D})))$ (since η is a NC-continuous). Hence $r \in \eta^{-1}(\mathcal{H}) \subseteq NCcl(NCint(\eta^{-1}(\mathcal{D})))$, but $\eta^{-1}(\mathcal{H})$ remains a NC-OS from $\mathbb{X}$ (because η be present a NC-continuous). Therefore, $r \in NCint(NCcl(NCint(\eta^{-1}(\mathcal{D}))))$. Hence $\eta^{-1}(\mathcal{D}) \subseteq NCint(NCcl(NCint(\eta^{-1}(\mathcal{D})))) \Rightarrow \eta^{-1}(\mathcal{D}) \in NC\alpha O(\mathbb{X}) \Rightarrow \eta$ is a $NC\alpha^*$-continuous.
(ii) The proof of (ii) is easily. ■

Theorem 3.11: A function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is a $NCS\alpha^*$-continuous iff $\eta: (\mathbb{X}, NCS\alpha O(\mathbb{X})) \longrightarrow (\mathbb{Y}, NCS\alpha O(\mathbb{Y}))$ is a NC-continuous.
Proof: Obvious. ■

Remark 3.12: The concepts of NC-continuity and $NCS\alpha^*$-continuity are independent, for examples.

Example 3.13: Suppose $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$.
Then $\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q, r\}, \phi, \phi\rangle, \langle\{p, q, r\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle\}$ be NCTs on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly. Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle, \eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle, \eta(\langle\{r\}, \phi, \phi\rangle) = \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$. It is easily seen that η is a NC-continuous function but not $NCS\alpha^*$-continuous, since $\langle\{u, v\}, \phi, \phi\rangle$ is $NCS\alpha$-OS in $\mathbb{Y}$ but $\eta^{-1}(\langle\{u, v\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ is not $NCS\alpha$-OS in $\mathbb{X}$.

Example 3.14: Assume $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$. Then $\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle\}$ be NCTs on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly.
Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle$, $\eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle, \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$. Then η is a $NCS\alpha^*$-continuous function but not NC-continuous, since $\langle\{u\}, \phi, \phi\rangle$ is NC-OS in $\mathbb{Y}$, but $\eta^{-1}(\langle\{u\}, \phi, \phi\rangle) = \langle\{p, q\}, \phi, \phi\rangle$ is not NC-OS in $\mathbb{X}$.

Proposition 3.15: Every $NC\alpha^*$-continuous function is a $NC\alpha$-continuous and $NCS\alpha$-continuous; however, the reverse generally is not valid.
Proof: Assume $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is a $NC\alpha^*$-continuous function and let $\mathcal{D}$ be any NC-OS from $\mathbb{Y}$. Then we have $\mathcal{D}$ as a $NC\alpha$-OS from $\mathbb{Y}$ [from proposition 2.4]. Since η is a $NC\alpha^*$-continuous, then $\eta^{-1}(\mathcal{D})$ considers a $NC\alpha$-OS from $\mathbb{X}$. Thus, η stands a $NC\alpha$-continuous. Also, η is a $NCS\alpha$-continuous. ■

Example 3.16: Let $\mathbb{X} = \{p, q, r, s\}$.
Then $\Gamma = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q\}, \phi, \phi\rangle, \langle\{p, q\}, \phi, \phi\rangle, \langle\{p, q, r\}, \phi, \phi\rangle\}$ be a NCT on $\mathbb{X}$. Define the function $\eta: (\mathbb{X}, \Gamma) \longrightarrow (\mathbb{X}, \Gamma)$ by $\eta(\langle\{p\}, \phi, \phi\rangle) = \langle\{p\}, \phi, \phi\rangle, \eta(\langle\{q\}, \phi, \phi\rangle) = \eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{s\}, \phi, \phi\rangle$, $\eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{r\}, \phi, \phi\rangle$. It is easily seen that η is a $NC\alpha$-continuous function but not $NC\alpha^*$-continuous, since $\langle\{p, q, r\}, \phi, \phi\rangle$ is $NC\alpha$-OS in $\mathbb{X}$, but $\eta^{-1}(\langle\{p, q, r\}, \phi, \phi\rangle) = \langle\{p, s\}, \phi, \phi\rangle$ is not $NC\alpha$-OS in $\mathbb{X}$.

Example 3.17: Let $\mathbb{X} = \{p, q, r\}$. Then $\Gamma = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q\}, \phi, \phi\rangle, \langle\{p, q\}, \phi, \phi\rangle\}$ be a NCT on $\mathbb{X}$. Define a function $\eta: (\mathbb{X}, \Gamma) \longrightarrow (\mathbb{X}, \Gamma)$ by $\eta(\langle\{p\}, \phi, \phi\rangle) = \langle\{p\}, \phi, \phi\rangle, \eta(\langle\{q\}, \phi, \phi\rangle) = \eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{q\}, \phi, \phi\rangle$. It is easily seen that η is a NCSα-continuous function but not NCα*-continuous, since $\langle\{q\}, \phi, \phi\rangle$ is NCα-OS in $\mathbb{X}$, but $\eta^{-1}(\langle\{q\}, \phi, \phi\rangle) = \langle\{q, r\}, \phi, \phi\rangle$ is not NCα-OS in $\mathbb{X}$.

Definition 3.18: A function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is called $\mathcal{M}$-function iff $\eta^{-1}(NCint(NCcl(\mathcal{D}))) \subseteq NCint(NCcl(\eta^{-1}(\mathcal{D})))$, for every NCα-OS $\mathcal{D}$ from $\mathbb{Y}$.

Theorem 3.19: If $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is a NCα-continuous function and $\mathcal{M}$-function, then η is a NCα*-continuous.

Proof: Let $\mathcal{C}$ be any NCα-OS of $\mathbb{Y}$, then we have at least a NC-OS $\mathcal{D}$ from $\mathbb{Y}$ where $\mathcal{D} \subseteq \mathcal{C} \subseteq NCint(NCcl(\mathcal{D}))$. Since η is $\mathcal{M}$-function, we have $\eta^{-1}(\mathcal{D}) \subseteq \eta^{-1}(\mathcal{C}) \subseteq \eta^{-1}(NCint(NCcl(\mathcal{D}))) \subseteq NCint(NCcl(\eta^{-1}(\mathcal{D})))$. By proposition 2.3, we have $\eta^{-1}(\mathcal{C})$ is a NC α-OS. Hence, η is a NCα*-continuous. ■

Remark 3.20: The concepts of NCα*-continuity and NCSα*-continuity are independent as the following examples show.

Example 3.21: Assume $\mathbb{X} = \{p, q, r, s\}$ and $\mathbb{Y} = \{u, v, w\}$.

Then $\Gamma_1 = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q\}, \phi, \phi\rangle, \langle\{p, q\}, \phi, \phi\rangle, \langle\{p, q, r\}, \phi, \phi\rangle\}$ and $\Gamma_2 = \{\phi_N, \mathbb{Y}_N\} \cup \{\langle\{u\}, \phi, \phi\rangle, \langle\{v\}, \phi, \phi\rangle, \langle\{u, v\}, \phi, \phi\rangle\}$ be NCTs on $\mathbb{X}$ and $\mathbb{Y}$, correspondingly. Define the function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{v\}, \phi, \phi\rangle, \eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{w\}, \phi, \phi\rangle$ and $\eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{u\}, \phi, \phi\rangle$. It is easily seen that η is a NCS α*-continuous function but not NC α*-continuous, since $\langle\{v\}, \phi, \phi\rangle$ is NC α-OS in $\mathbb{Y}$ but $\eta^{-1}(\langle\{v\}, \phi, \phi\rangle) = \langle\{p, s\}, \phi, \phi\rangle$ is not NCα-OS in $\mathbb{X}$.

Example 3.22: Suppose $\mathbb{X} = \{p, q, r, s\}$.

Then $\Gamma = \{\phi_N, \mathbb{X}_N\} \cup \{\langle\{p\}, \phi, \phi\rangle, \langle\{q\}, \phi, \phi\rangle, \langle\{p, q\}, \phi, \phi\rangle, \langle\{p, q, r\}, \phi, \phi\rangle\}$ be a NCT on $\mathbb{X}$. Define the function $\eta: (\mathbb{X}, \Gamma) \longrightarrow (\mathbb{X}, \Gamma)$ via $\eta(\langle\{p\}, \phi, \phi\rangle) = \eta(\langle\{q\}, \phi, \phi\rangle) = \langle\{q\}, \phi, \phi\rangle$, $\eta(\langle\{r\}, \phi, \phi\rangle) = \langle\{s\}, \phi, \phi\rangle, \eta(\langle\{s\}, \phi, \phi\rangle) = \langle\{r\}, \phi, \phi\rangle$. It is easily seen that η is a NCα*-continuous function but not NCSα*-continuous, since $\langle\{p, r\}, \phi, \phi\rangle$ is NCSα-OS in $\mathbb{X}$, but $\eta^{-1}(\langle\{p, r\}, \phi, \phi\rangle) = \langle\{s\}, \phi, \phi\rangle$ is not NCSα-OS in $\mathbb{X}$.

Theorem 3.23: If a function $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ is NCα*-continuous, NC-open and bijective, then it is NCSα*-continuous.

Proof: Let $\eta: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ be a NCα*-continuous, NC-open and bijective. Let $\mathcal{D}$ be a NCSα-OS in $\mathbb{Y}$. Then we have at least a NCα-OS say $\mathcal{P}$ where $\mathcal{P} \subseteq \mathcal{D} \subseteq NCcl(\mathcal{P})$. Therefore $\eta^{-1}(\mathcal{P}) \subseteq \eta^{-1}(\mathcal{D}) \subseteq \eta^{-1}(NCcl(\mathcal{P})) \subseteq NCcl(\eta^{-1}(\mathcal{P}))$ (since η is a NC-open), but $\eta^{-1}(\mathcal{P}) \in NC$α$O(\mathbb{X})$ (since η is a NC α*-continuous). Hence $\eta^{-1}(\mathcal{P}) \subseteq \eta^{-1}(\mathcal{D}) \subseteq NCcl(\eta^{-1}(\mathcal{P}))$. Thus, $\eta^{-1}(\mathcal{D}) \in NCS$α$O(\mathbb{X})$. Therefore, η is a NCSα*-continuous. ■

Remark 3.24: Let $\eta_1: (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ and $\eta_2: (\mathbb{Y}, \Gamma_2) \longrightarrow (\mathbb{Z}, \Gamma_3)$ be two functions, then:

(i) If η_1 and η_2 are NC α-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ need not to be a NCα-continuous.

(ii) If η_1 and η_2 are NCS α-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ need not to be a NCSα-continuous.

Theorem 3.25: Let $\eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Y}, \Gamma_2)$ and $\eta_2 : (\mathbb{Y}, \Gamma_2) \longrightarrow (\mathbb{Z}, \Gamma_3)$ be two functions, then:

(i) If η_1 is NC α-continuous and η_2 is NC -continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα-continuous.

(ii) If η_1 is NC α*-continuous and η_2 is NC α-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα-continuous.

(iii) If η_1 and η_2 are NCα*-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα*-continuous.

(iv) If η_1 and η_2 are NCSα*-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCSα*-continuous.

(v) If η_1 and η_2 are NCα**-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα**-continuous.

(vi) If η_1 and η_2 are NCSα**-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCSα**-continuous.

(vii) If η_1 is NC α**-continuous and η_2 is NC α*-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα**-continuous.

(viii) If η_1 is NC α**-continuous and η_2 is NC α-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NC-continuous.

(ix) If η_1 is NC α-continuous and η_2 is NC α**-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα*-continuous.

(x) If η_1 is NC -continuous and η_2 is NC α**-continuous, then $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα**-continuous.

Proof:

(i) Assume $\mathcal{F}$ considers a NC-OS from $\mathbb{Z}$. Since η_2 is a NC-continuous, ${\eta_2}^{-1}(\mathcal{F})$ is a NCα-OS in $\mathbb{Y}$. Since η_1 is a NC α-continuous, ${\eta_1}^{-1}({\eta_2}^{-1}(\mathcal{F})) = (\eta_2 \circ \eta_1)^{-1}(\mathcal{F})$ is a NC α-OS in $\mathbb{X}$. Thus, $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ exists a NCα-continuous.

(ii) Let $\mathcal{F}$ be a NC-OS in $\mathbb{Z}$. Subsequently η_2 stands a NCα-continuous, and ${\eta_2}^{-1}(\mathcal{F})$ stays a NCα-OS from $\mathbb{Y}$. Since η_1 is a NCα*-continuous, ${\eta_1}^{-1}({\eta_2}^{-1}(\mathcal{F})) = (\eta_2 \circ \eta_1)^{-1}(\mathcal{F})$ is a NCα-OS in $\mathbb{X}$. Thus, $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα-continuous.

(iii) Let $\mathcal{F}$ be a NCα-OS in $\mathbb{Z}$. Since η_2 is a NCα*-continuous, ${\eta_2}^{-1}(\mathcal{F})$ is a NCα-OS in $\mathbb{Y}$. Since η_1 is a NCα*-continuous, ${\eta_1}^{-1}({\eta_2}^{-1}(\mathcal{F})) = (\eta_2 \circ \eta_1)^{-1}(\mathcal{F})$ is a NCα-OS in $\mathbb{X}$. Thus, $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα*-continuous.

(iv) Let $\mathcal{F}$ be a NCSα-OS in $\mathbb{Z}$. Since η_2 is a NCSα*-continuous, ${\eta_2}^{-1}(\mathcal{F})$ is a NCSα-OS in $\mathbb{Y}$. Since η_1 is a NCSα*-continuous, ${\eta_1}^{-1}({\eta_2}^{-1}(\mathcal{F})) = (\eta_2 \circ \eta_1)^{-1}(\mathcal{F})$ is a NCSα-OS in $\mathbb{X}$. Thus, $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCSα*-continuous.

(v) Let $\mathcal{F}$ be a NCα-OS in $\mathbb{Z}$. Since η_2 is a NCα**-continuous, ${\eta_2}^{-1}(\mathcal{F})$ is a NC-OS in $\mathbb{Y}$. Since any NC-OS is a NCα-OS, ${\eta_2}^{-1}(\mathcal{F})$ is a NCα-OS in $\mathbb{Y}$. Since η_1 is a NCα**-continuous, ${\eta_1}^{-1}({\eta_2}^{-1}(\mathcal{F})) = (\eta_2 \circ \eta_1)^{-1}(\mathcal{F})$ is a NC-OS in $\mathbb{X}$. Thus, $\eta_2 \circ \eta_1 : (\mathbb{X}, \Gamma_1) \longrightarrow (\mathbb{Z}, \Gamma_3)$ is a NCα**-continuous. The proof is obvious for others. ■

Remark 3.26: The next figure describes the relationship between various classes of weakly NC-continuous functions:

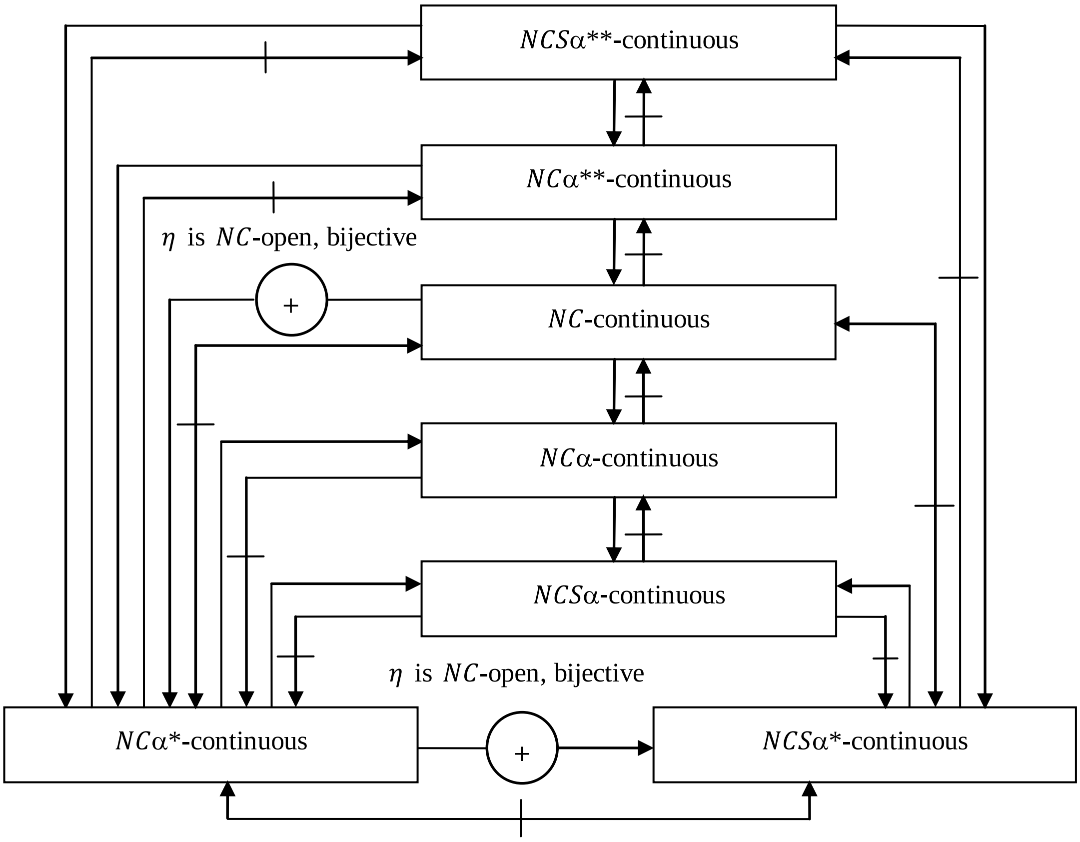

Fig. 1

4. Conclusion

We shall use the concepts of $NC\alpha$-OS and $NCS\alpha$-CS to define several new types of weakly NC-continuity such as; $NC\alpha$*-continuous, $NC\alpha$**-continuous, $NCS\alpha$-continuous, $NCS\alpha$*-continuous and NCS α**-continuous functions. The neutrosophic crisp α-open and neutrosophic crisp semi-α-open sets can be used to derive some new types of weakly NC-open (NC-closed) functions.

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