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Neutrosophic Cubic Fuzzy Dombi Hamy Mean Operators with Application to Multi-Criteria Decision Making

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Abstract. The aim of the paper is to find most optimistic results from among uncertain information or vague data. We theoretically use the notion of neutrosophic cubic sets to create enhanced decision-making models for multi-criteria. The advantage of neutrosophic cubic sets is that it comprehends the knowledge of neutrosophic sets and interval valued neutrosophic sets. Aggregation operators are used to retrieve the core information from a collection of data. So, this research executes aggregation operators for neutrosophic cubic sets dynamically. In this paper we avail the aid of hamy mean and dombi operations to establish fuzzy dombi hamy mean aggregation operators for neutrosophic cubic sets. This paper also explains the algebraic sum and scalar multiplication operations. A decision making methodology has been generated to prove the necessity of the proposed operators. Finally an illustration is provided from a real life decision making situation.

Keywords: Fuzzy Sets; Neutrosophic Cubic Sets; Hamy Mean; Dombi Operations; Aggregations Operators; MCDM

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1. Introduction

Every aspect of human life involves making decisions and most of the times human brain takes decision after processing information that comes to it in an incomplete and imprecise form. Hence we thrive to understand the fuzziness of the information in order to take decisions. To fulfil this need, in the year 1965 Zadeh's scientific studies revealed the concept of fuzzy sets [1] and it is used to reduce fuzziness on difficult decision situations. The evolution of this notion of fuzzy sets with their operations has been presented in the literature [2–5]. Neutrosophic sets are a novel extended form of fuzzy sets and were initiated by F Smarandache [6]. Neutrosophic sets handle ambiguity using three membership types. Functions namely the functions of truthness, indeterminacy and falsehood membership provide a more general way of measuring vagueness. Further, Y.B. Jun et al. [7] and M. Ali et al. [8] effectively utilized the concept of cubic fuzzy sets to neutrosophic sets in order to introduce neutrosophic cubic fuzzy sets (NCFSs) with some basic operations and NCFSs deal with uncertain information in the form of intervals followed by single valued neutrosophic data. So this notion is a more general way to handle NSs. It is evident from the literature that the concept of decision making is one of the significant research areas in the field of neutrosophic sets. More recently, Ajay, D., et al. [9] used this notion with the help of weighted neutrosophic cubic fuzzy Bonferroni geometric mean aggregation operators.

In fuzzy mathematics aggregation operators play a significant role and it is more useful in aggregating knowledge that is involved in decision making systems. Various types of neutrosophic cubic aggregation operators available in the literature are, namely, Weighted Neutrosophic Cubic Cuzzy Bonferroni Geometric Mean ($WNCFBGM_w^{u,v}$) operator [9], Neutrosophic Cubic Dombi Weighted Arithmetic and Geometric Average (NCDWAA, NCDWGA) operators [10], Linguistic Neutrosophic Cubic Number Generalized Weighted Heronian Mean (LNCNGWHM) operator [11], Neutrosophic Cubic Einstein Weighted Geometric (NCEWG) operator [12], Neutrosophic Cubic Heronian Mean (NCHM) operator [13], Neutrosophic Cubic Einstein Ordered Weighted Geometric (NCEOWG) operator [14], New Operators on Interval Valued Neutrosophic Sets [15] and still there is a need for more efficient aggregation operators for huge underivable neutrosophic data.

Some of the measures on neutrosophic sets are utilized in decision making models. For example, Ajay D., et al. [16] introduced a new decision making approach based on bipolar neutrosophic similarity and entropy measures. Similarly, Lu, Z., et.al. [17] introduced neutrosophic cubic cosine similarity measure and applied in the field of multi criteria decision making. Abdel-Basset et al. [18] ultilized bipolar neutrosophic sets to MCDM with Analytic hierarchy process (AHP) and Technique in order of preference by similarity to ideal solution (TOPSIS). Multi-objective optimisation on the basis of simple ratio (MOOSRA) method has been developed based on single valued triangular numbers which has been used to personnel selection [19].

One of the most recent generalization of neutrosophic sets is plithogenic set (PSs) which was introduced by Smarandache [20]. The elements of these PSs is characterized by one or more attributes, and each attribute may have many values. Using the notion of PSs, best-worst method has been implemented in supply chain problem [21] and also plithogenic multi criteria decision making (MCDM) approach has been introduced based on neutrosophic AHP, TOPSIS and Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) methods [22,23]. Moreover, Plithogenic n-super hypergraph is used in multi alternative decision making and it is a new perpective to deal with certain types of graphs for practical applications [24].

The available literature suggests that the decision making models are mainly focused on aggregation operators, rather than the studies on similarity measures. The introduction of neutrosophic set theory led to wide range of research areas like neutrosophic graph theory [25], Neutrosophic topology [26], etc. and more recently neutrosophic set theory has been used in finite automata [27]. Many real time applications exist under neutrosophic sets [28–35]. More recently on this pandemic, Health-Fog framework universal system has been introduced with the help of deep learning and neutrosophic classifiers to confront Covid-19 [36].

The Dombi and Hamy mean operators are efficient and flexible aggregation tools to handle information fusion in MCDM. Shi et al. [37] applied the dombi aggregation operators to neutrosophic cubic sets in order to make decision over uncertainty. They have investigated some of the properties of aggregation operators and illustrated a numerical problem in detail by changing the paramerter values between 1 to 5. Recently, Liu et al. [38] combined the conventional Hamy mean to traditional power operator in interval valued neutrosophic sets and introduced interval valued power neutrosophic mean operators. However, it is clear that some research have been done on dombi and hamy mean operators separately and both of them have not yet been combined to formulate MCDM mechanism.

The main focus of this research is to find a new aggregation operator based on neutrosophic cubic sets with the help of combined dombi hamy mean operators. Two decision making methods are developed using score function, similarity measure and aggregation operators.

Table 1. Some notations with their descriptions

Notations	Descriptions
$\overset{*}{V}$, $\check{v}_i$	Universal set , Element of $\overset{*}{V}$
$\mathbb{S}$, $\underset{\sim}{\mathbb{N}}$	Fuzzy set, Neutrosophic set
$T_{\underset{\sim}{\mathbb{N}}}$, $I_{\underset{\sim}{\mathbb{N}}}$, $F_{\underset{\sim}{\mathbb{N}}}$	Truth, Indeterminacy and False membership functions
$\widetilde{T}_{\underset{\sim}{\mathbb{N}}} = [T^L_{\underset{\sim}{\mathbb{N}}}, T^U_{\underset{\sim}{\mathbb{N}}}]$	Interval valued functions with respect to lower and upper bound

The framework of the rest of the paper is organized with five sections that follow. Section 2 addresses the basic definitions, operations and measures of similarity on neutrosophic cubic sets. The next section deals with hamy and dombi operations on neutrosophic cubic sets. Further, section 4 introduces neutrosophic cubic fuzzy dombi hamy mean aggregation operators with weighted values. The section 5 of the paper describes algorithms of the proposed MCDM methods with suitable real life illustrations. Finally the conclusion of the research is made available in section 6.

2. **Preliminaries of Neutrosophic Cubic Sets (NCSs) and their operations**

In this section, we briefly review some basic concepts about NCSs, Dombi and Hamy Mean operators. Some of the notations description are given in table.1

Definition 2.1. [1] If $\check{v}$ is a particular element of universe of discourse $\overset{*}{V}$, then a fuzzy set $\mathbb{S}$ is defined by a fuzzy membership function ($\mu_{\mathbb{S}}$) which associates to each $\check{v}$ a membership value in the closed unit interval of zero and one. i.e. $\mu_{\mathbb{S}}(\check{v}) : X \to [0,1]$

Definition 2.2. [6] Let $\underset{\sim}{\mathbb{N}}_j = \left\{ \left(T_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), I_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), F_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i)\right) | \check{v}_i \in \overset{*}{V} \right\}$ be a neutrosophic set (Ns), where $\left\{T_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), I_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), F_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i) \in [0,1]\right\}$ are called truth, indeterminacy and falsity functions, respectively. This can be represented by $\underset{\sim}{\mathbb{N}}_j = (T_{\underset{\sim}{\mathbb{N}}_j}, I_{\underset{\sim}{\mathbb{N}}_j}, F_{\underset{\sim}{\mathbb{N}}_j})$.

Definition 2.3. [6] Let $\underset{\sim}{\mathbb{N}}_j = \left\{ \left(\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i)\right) | \check{v}_i \in \overset{*}{V} \right\}$ be an interval neutrosophic set in $\overset{*}{V}$, where $\left\{\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i) \in [0,1]\right\}$ is called truth, indeterminacy and falsity function in $\overset{*}{V}$, respectively. This can be represented by $\underset{\sim}{\mathbb{N}}_j = (\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j})$. For convience, we denote $\underset{\sim}{\mathbb{N}}_j = (\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j})$ by $\underset{\sim}{\mathbb{N}}_j = \left(\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j} = [T^L_{\underset{\sim}{\mathbb{N}}_j}, T^U_{\underset{\sim}{\mathbb{N}}_j}], \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j} = [I^L_{\underset{\sim}{\mathbb{N}}_j}, I^U_{\underset{\sim}{\mathbb{N}}_j}], \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j} = [F^L_{\underset{\sim}{\mathbb{N}}_j}, F^U_{\underset{\sim}{\mathbb{N}}_j}]\right)$.

Definition 2.4. [6] Let $\underset{\sim}{\mathbb{N}}_j = \left\{ \left(\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i)\right); T_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), I_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i), F_{\underset{\sim}{\mathbb{N}}_j}(\check{v}_i) | \check{v}_i \in \overset{*}{V} \right\}$ be a neutrosophic cubic sets in $\overset{*}{V}$, in which $\widetilde{T}_{\underset{\sim}{\mathbb{N}}_j} = [T^L_{\underset{\sim}{\mathbb{N}}_j}, T^U_{\underset{\sim}{\mathbb{N}}_j}], \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j} = [I^L_{\underset{\sim}{\mathbb{N}}_j}, I^U_{\underset{\sim}{\mathbb{N}}_j}], \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j} = [F^L_{\underset{\sim}{\mathbb{N}}_j}, F^U_{\underset{\sim}{\mathbb{N}}_j}]$ is an interval valued neutrosophic set in $\overset{*}{V}$ simply denoted by $\underset{\sim}{\mathbb{N}}_j = \left\langle \widetilde{T}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j}, T_{\underset{\sim}{\mathbb{N}}_j}, I_{\underset{\sim}{\mathbb{N}}_j}, F_{\underset{\sim}{\mathbb{N}}_j} \right\rangle$, $[0,0] \leq \widetilde{T}_{\underset{\sim}{\mathbb{N}}_j} + \widetilde{I}_{\underset{\sim}{\mathbb{N}}_j} + \widetilde{F}_{\underset{\sim}{\mathbb{N}}_j} \leq [3,3]$ and $0 \leq T_{\underset{\sim}{\mathbb{N}}_j} + I_{\underset{\sim}{\mathbb{N}}_j} + F_{\underset{\sim}{\mathbb{N}}_j} \leq 3$.

Definition 2.5. [7] Let $C_j = \left\{ (\check{v}_i, \widetilde{\mu}(\check{v}_i), \mu(\check{v}_i)) | \check{v}_i \in \overset{*}{V} \right\}$ be a cubic fuzzy set in $\overset{*}{V}$ in which $\widetilde{\mu}$ is interval fuzzy set in $\overset{*}{V}$, i.e., $\widetilde{\mu} = [\mu^L, \mu^U]$ and μ is a fuzzy set in $\overset{*}{V}$.

2.1. *Operations on NCSs*

The algebraic addition and scalar multiplication on NCSs are discussed. Essential outcome of exponential multiplication that provides the basis for the concept of Dombi Hamy mean aggregation operators in neutrosophic cubic sets is based on these definitions.

Definition 2.6. The sum and product of the two neutrosophic cubic sets (NCSs), $\underset{\sim}{\mathbb{N}}_1 = \left\langle \widetilde{T}_{\underset{\sim}{\mathbb{N}}_1}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_1}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_1}, T_{\underset{\sim}{\mathbb{N}}_1}, I_{\underset{\sim}{\mathbb{N}}_1}, F_{\underset{\sim}{\mathbb{N}}_1} \right\rangle$, where $\widetilde{T}_{\underset{\sim}{\mathbb{N}}_1} = [T^L_{\underset{\sim}{\mathbb{N}}_1}, T^U_{\underset{\sim}{\mathbb{N}}_1}]$, $\widetilde{I}_{\underset{\sim}{\mathbb{N}}_1} = [I^L_{\underset{\sim}{\mathbb{N}}_1}, I^U_{\underset{\sim}{\mathbb{N}}_1}]$, $\widetilde{F}_{\underset{\sim}{\mathbb{N}}_1} = [F^L_{\underset{\sim}{\mathbb{N}}_1}, F^U_{\underset{\sim}{\mathbb{N}}_1}]$, and

$\underset{\sim}{\mathbb{N}}_2 = \left\langle \widetilde{T}_{\underset{\sim}{\mathbb{N}}_2}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_2}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_2}, T_{\underset{\sim}{\mathbb{N}}_2}, I_{\underset{\sim}{\mathbb{N}}_2}, F_{\underset{\sim}{\mathbb{N}}_2} \right\rangle$, where $\widetilde{T}_{\underset{\sim}{\mathbb{N}}_2} = [T^L_{\underset{\sim}{\mathbb{N}}_2}, T^U_{\underset{\sim}{\mathbb{N}}_2}], \widetilde{I}_{\underset{\sim}{\mathbb{N}}_2} = [I^L_{\underset{\sim}{\mathbb{N}}_2}, I^U_{\underset{\sim}{\mathbb{N}}_2}], \widetilde{F}_{\underset{\sim}{\mathbb{N}}_2} = [F^L_{\underset{\sim}{\mathbb{N}}_2}, F^U_{\underset{\sim}{\mathbb{N}}_2}]$ are defined as follows.

$$\begin{aligned} \widetilde{\mathbb{N}}_1 \oplus \underset{\sim}{\mathbb{N}}_2 = \Big\{ & \Big[T^L_{\widetilde{\mathbb{N}}_1} + T^L_{\underset{\sim}{\mathbb{N}}_2} - T^L_{\underset{\sim}{\mathbb{N}}_1} T^L_{\underset{\sim}{\mathbb{N}}_2}, T^U_{\underset{\sim}{\mathbb{N}}_1} + T^U_{\underset{\sim}{\mathbb{N}}_2} - T^U_{\underset{\sim}{\mathbb{N}}_1} T^U_{\underset{\sim}{\mathbb{N}}_2}\Big], \\ & \Big[I^L_{\underset{\sim}{\mathbb{N}}_1} + I^L_{\underset{\sim}{\mathbb{N}}_2} - I^L_{\underset{\sim}{\mathbb{N}}_1} I^L_{\underset{\sim}{\mathbb{N}}_2}, I^U_{\underset{\sim}{\mathbb{N}}_1} + I^U_{\underset{\sim}{\mathbb{N}}_2} - I^U_{\underset{\sim}{\mathbb{N}}_1} I^U_{\underset{\sim}{\mathbb{N}}_2}\Big], \\ & \Big[F^L_{\underset{\sim}{\mathbb{N}}_1} F^L_{\underset{\sim}{\mathbb{N}}_2}, F^U_{\underset{\sim}{\mathbb{N}}_1} F^U_{\underset{\sim}{\mathbb{N}}_2}\Big]; \big\langle T_{\underset{\sim}{\mathbb{N}}_1} T_{\underset{\sim}{\mathbb{N}}_2}, I_{\underset{\sim}{\mathbb{N}}_1} I_{\underset{\sim}{\mathbb{N}}_2}, F_{\underset{\sim}{\mathbb{N}}_1} + F_{\underset{\sim}{\mathbb{N}}_2} - F_{\underset{\sim}{\mathbb{N}}_1} F_{\underset{\sim}{\mathbb{N}}_2} \big\rangle \Big\} \end{aligned} \tag{1}$$

$$\begin{aligned} \underset{\sim}{\mathbb{N}}_1 \otimes \underset{\sim}{\mathbb{N}}_2 = \Big\{ & \Big[T^L_{\underset{\sim}{\mathbb{N}}_1} T^L_{\underset{\sim}{\mathbb{N}}_2}, T^U_{\underset{\sim}{\mathbb{N}}_1} T^U_{\underset{\sim}{\mathbb{N}}_2}\Big], \Big[I^L_{\underset{\sim}{\mathbb{N}}_1} I^L_{\underset{\sim}{\mathbb{N}}_2}, I^U_{\underset{\sim}{\mathbb{N}}_1} I^U_{\underset{\sim}{\mathbb{N}}_2}\Big], \Big[F^L_{\underset{\sim}{\mathbb{N}}_1} + F^L_{\underset{\sim}{\mathbb{N}}_2} - F^L_{\underset{\sim}{\mathbb{N}}_1} F^L_{\underset{\sim}{\mathbb{N}}_2}, F^U_{\underset{\sim}{\mathbb{N}}_1} + F^U_{\underset{\sim}{\mathbb{N}}_2} - F^U_{\underset{\sim}{\mathbb{N}}_1} F^U_{\underset{\sim}{\mathbb{N}}_2}\Big]; \\ & \big\langle T_{\underset{\sim}{\mathbb{N}}_1} + T_{\underset{\sim}{\mathbb{N}}_2} - T_{\underset{\sim}{\mathbb{N}}_1} T_{\underset{\sim}{\mathbb{N}}_2}, I_{\underset{\sim}{\mathbb{N}}_1} + I_{\underset{\sim}{\mathbb{N}}_2} - I_{\underset{\sim}{\mathbb{N}}_1} I_{\underset{\sim}{\mathbb{N}}_2}, F_{\underset{\sim}{\mathbb{N}}_1} F_{\underset{\sim}{\mathbb{N}}_2} \big\rangle \Big\} \end{aligned} \tag{2}$$

Definition 2.7. The scalar and exponential multiplication on neutrosophic cubic sets (NCs), $\underset{\sim}{\mathbb{N}}_1 = \left\langle \widetilde{T}_{\underset{\sim}{\mathbb{N}}_1}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}_1}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}_1}, T_{\underset{\sim}{\mathbb{N}}_1}, I_{\underset{\sim}{\mathbb{N}}_1}, F_{\underset{\sim}{\mathbb{N}}_1} \right\rangle$, where $\widetilde{T}_{\underset{\sim}{\mathbb{N}}_1} = [T^L_{\underset{\sim}{\mathbb{N}}_1}, T^U_{\underset{\sim}{\mathbb{N}}_1}], \widetilde{I}_{\underset{\sim}{\mathbb{N}}_1} = [I^L_{\underset{\sim}{\mathbb{N}}_1}, I^U_{\underset{\sim}{\mathbb{N}}_1}], \widetilde{F}_{\underset{\sim}{\mathbb{N}}_1} = [F^L_{\underset{\sim}{\mathbb{N}}_1}, F^U_{\underset{\sim}{\mathbb{N}}_1}]$, and a scalar value ϱ are defined as respectively:

$$\begin{aligned} \varrho \underset{\sim}{\mathbb{N}}_1 = \Big\{ & \Big[1 - (1 - T^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, 1 - (1 - T^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big], \Big[1 - (1 - I^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, 1 - (1 - I^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big], \\ & \Big[(F^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, (F^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big]; \big\langle (T_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, (I_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, 1 - (1 - F_{\underset{\sim}{\mathbb{N}}_1})^{\varrho} \big\rangle \Big\} \end{aligned} \tag{3}$$

$$\begin{aligned} \underset{\sim}{\mathbb{N}}_1^{\varrho} = \Big\{ & \Big[(T^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, (T^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big], \Big[(I^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, (I^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big], \Big[1 - (1 - F^L_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, 1 - (1 - F^U_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}\Big]; \\ & \big\langle 1 - (1 - T_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, 1 - (1 - I_{\underset{\sim}{\mathbb{N}}_1})^{\varrho}, (F_{\underset{\sim}{\mathbb{N}}_1})^{\varrho} \big\rangle \Big\} \end{aligned} \tag{4}$$

2.2. *Similarity Measure of NCSs*

Let $\underset{\sim}{\mathbb{N}}_A$ and $\underset{\sim}{\mathbb{N}}_B$ be two neutrosophic cubic sets in $\overset{*}{V}$. Then, the measure of similarity of $\underset{\sim}{\mathbb{N}}_A$ and $\underset{\sim}{\mathbb{N}}_B$ is defined by $\underset{\sim}{\mathbb{N}}_{SM} : \underset{\sim}{\mathbb{N}}_A(\overset{*}{V}) \times \underset{\sim}{\mathbb{N}}_A(\overset{*}{V}) \to [0,1]$ which satisfies the following condition;

(i). $0 \le \underset{\sim}{\mathbb{N}}_{SM}\left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_B(\check{v}_i)\right) \le 1$

(ii). $\underset{\sim}{\mathbb{N}}_{SM}\left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_B(\check{v}_i)\right) = 1$ iff $\underset{\sim}{\mathbb{N}}_A(\check{v}_i) = \underset{\sim}{\mathbb{N}}_B(\check{v}_i)$

(iii). $\underset{\sim}{\mathbb{N}}_{SM}\left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_B(\check{v}_i)\right) = \underset{\sim}{\mathbb{N}}_{SM}\left(\underset{\sim}{\mathbb{N}}_B(\check{v}_i), \underset{\sim}{\mathbb{N}}_A(\check{v}_i)\right)$

(iv). If $\underset{\sim}{\mathbb{N}}_A(\check{v}_i) \subseteq \underset{\sim}{\mathbb{N}}_B(\check{v}_i) \subseteq \underset{\sim}{\mathbb{N}}_C(\check{v}_i)$, then $\mathbb{N}_{SM}\left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_C(\check{v}_i)\right) \le \left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_B(\check{v}_i)\right)$ and $\mathbb{N}_{SM}\left(\underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_C(\check{v}_i)\right) \le \left(\underset{\sim}{\mathbb{N}}_B(\check{v}_i), \underset{\sim}{\mathbb{N}}_C(\check{v}_i)\right) \forall \underset{\sim}{\mathbb{N}}_A(\check{v}_i), \underset{\sim}{\mathbb{N}}_B(\check{v}_i), \underset{\sim}{\mathbb{N}}_C(\check{v}_i) \in NCSs(\overset{*}{V})$

The similarity measure between two $NCSs$ is expressed as follows:

$$\underset{\sim}{\mathbb{N}}_{SM}\left(\underset{\sim}{\mathbb{N}}_A, \underset{\sim}{\mathbb{N}}_B\right) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{D_s(i)}{9}\right) \tag{5}$$

where

$$\begin{aligned} D_s(i) = \Big(& \left|T^L_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - T^L_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|T^U_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - T^U_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|I^L_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - I^L_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \\ & \left|I^U_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - I^U_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|F^L_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - F^L_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|F^U_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - F^U_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|T_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - T_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| \\ & + \left|I_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - I_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right| + \left|F_{\underset{\sim}{\mathbb{N}}_A}(\check{v}_i) - F_{\underset{\sim}{\mathbb{N}}_B}(\check{v}_i)\right|\Big). \end{aligned}$$

The similarity measure follows the four conditions mentioned above.

Definition 2.8. Let $\underset{\sim}{\mathbb{N}}$ be neutrosophic cubic fuzzy set in $\overset{*}{V}$, then the support of neutrosophic cubic fuzzy set $\underset{\sim}{\mathbb{N}}^*$ is defined by

$$\underset{\sim}{\mathbb{N}}^* = \Big\{ \Big[T^L_{\underset{\sim}{\mathbb{N}}}(\check{v}), T^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\Big] \supset [0,0], \Big[I^L_{\underset{\sim}{\mathbb{N}}}(\check{v}), I^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\Big] \supset [0,0], \Big[F^L_{\underset{\sim}{\mathbb{N}}}(\check{v}), F^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\Big] \subset [1,1]; \\ \Big\langle T_{\underset{\sim}{\mathbb{N}}}(\check{v}) > 0, I_{\underset{\sim}{\mathbb{N}}}(\check{v}) > 0, F_{\underset{\sim}{\mathbb{N}}}(\check{v}) < 1 \Big\rangle \,|\check{v} \in \overset{*}{V} \Big\}$$

Definition 2.9. Let $\underset{\sim}{\mathbb{N}}$ be a non empty neutrosophic cubic fuzzy number given by $\underset{\sim}{\mathbb{N}} = \Big\langle \widetilde{T}_{\underset{\sim}{\mathbb{N}}}, \widetilde{I}_{\underset{\sim}{\mathbb{N}}}, \widetilde{F}_{\underset{\sim}{\mathbb{N}}}, T_{\underset{\sim}{\mathbb{N}}}, I_{\underset{\sim}{\mathbb{N}}}, F_{\underset{\sim}{\mathbb{N}}} \Big\rangle$, where $\widetilde{T}_{\underset{\sim}{\mathbb{N}}} = [T^L_{\underset{\sim}{\mathbb{N}}}, T^U_{\underset{\sim}{\mathbb{N}}}], \widetilde{I}_{\underset{\sim}{\mathbb{N}}} = [I^L_{\underset{\sim}{\mathbb{N}}}, I^U_{\underset{\sim}{\mathbb{N}}}], \widetilde{F}_{\underset{\sim}{\mathbb{N}}} = [F^L_{\underset{\sim}{\mathbb{N}}}, F^U_{\underset{\sim}{\mathbb{N}}}]$, then its functions for ranking, accuracy and certainty can be stated as follows:

$$s(\underset{\sim}{\mathbb{N}}) = \frac{\frac{\left[4+T^L_{\underset{\sim}{\mathbb{N}}}(\check{v})-I^L_{\underset{\sim}{\mathbb{N}}}(\check{v})-F^L_{\underset{\sim}{\mathbb{N}}}(\check{v})+T^U_{\underset{\sim}{\mathbb{N}}}(\check{v})-I^U_{\underset{\sim}{\mathbb{N}}}(\check{v})-F^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\right]}{6} + \frac{\left[2+T_{\underset{\sim}{\mathbb{N}}}(\check{v})-I_{\underset{\sim}{\mathbb{N}}}(\check{v})-F_{\underset{\sim}{\mathbb{N}}}(\check{v})\right]}{3}}{2}, \tag{6}$$

$$a(\underset{\sim}{\mathbb{N}}) = \frac{\left[\left(T^L_{\underset{\sim}{\mathbb{N}}}(\check{v}) - F^L_{\underset{\sim}{\mathbb{N}}}(\check{v}) + T^U_{\underset{\sim}{\mathbb{N}}}(\check{v}) - F^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\right)/2 + T_{\underset{\sim}{\mathbb{N}}}(\check{v}) - F_{\underset{\sim}{\mathbb{N}}}(\check{v})\right]}{2}, \tag{7}$$

$$c(\underset{\sim}{\mathbb{N}}) = \frac{\left[\left(T^L_{\underset{\sim}{\mathbb{N}}}(\check{v}) + T^U_{\underset{\sim}{\mathbb{N}}}(\check{v})\right)/2 + T_{\underset{\sim}{\mathbb{N}}}(\check{v})\right]}{2}; \qquad s(\underset{\sim}{\mathbb{N}}), a(\underset{\sim}{\mathbb{N}}), c(\underset{\sim}{\mathbb{N}}) \in [0,1] \tag{8}$$

Definition 2.10. Let $\underset{\sim}{\mathbb{N}}_1$ and $\underset{\sim}{\mathbb{N}}_2$ be two non empty neutrosophic cubic values, where $S_{\underset{\sim}{\mathbb{N}}_1}$ and $S_{\underset{\sim}{\mathbb{N}}_2}$ are score values and $H_{\underset{\sim}{\mathbb{N}}_1}$ and $H_{\underset{\sim}{\mathbb{N}}_2}$ are accuracy functions of $\underset{\sim}{\mathbb{N}}_1$ and $\underset{\sim}{\mathbb{N}}_2$ respectively.

(1) If $S_{\underset{\sim}{\mathbb{N}}_1} > S_{\underset{\sim}{\mathbb{N}}_2} \Rightarrow \underset{\sim}{\mathbb{N}}_1 > \underset{\sim}{\mathbb{N}}_2$

(2) $S_{\underset{\sim}{\mathbb{N}}_1} = S_{\underset{\sim}{\mathbb{N}}_2}$ and $H_{\underset{\sim}{\mathbb{N}}_1} > H_{\underset{\sim}{\mathbb{N}}_2} \Rightarrow \underset{\sim}{\mathbb{N}}_1 > \underset{\sim}{\mathbb{N}}_2$; $H_{\underset{\sim}{\mathbb{N}}_1} = H_{\underset{\sim}{\mathbb{N}}_2} \Rightarrow \underset{\sim}{\mathbb{N}}_1 = \underset{\sim}{\mathbb{N}}_2$

3. **Neutrosophic Cubic Dombi Hamy Mean Operation**

Definition 3.1. The operator Hamy Mean (HM) is stated as follows:

$$HM^{(\mu)} = (\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) = \frac{\sum\limits_{1\leq \ldots < k_{(\mu)} < \ldots \leq k} \left(\prod\limits_{j=1}^{\mu} \underset{\sim}{\mathbb{N}}_{ij}\right)^{1/\mu}}{C^{\mu}_k}$$

where μ is a parameter, $\mu = 1, 2, \ldots, k$ and $k_1, k_2, \ldots, k_\mu$ are μ integer values taken from the set $(1, 2, \ldots, k)$ of k, C^μ_k is the binomial co-efficient, $C^\mu_k = \frac{k!}{\mu!(k-\mu)!}$

Definition 3.2. Dombi has formulated a generator for the development of Dombi T-norm and T-conorm that is shown as follows:

$$D(p,q) = \frac{1}{1 + \left(\left(\frac{1-p}{p}\right)^\varrho + \left(\frac{1-q}{q}\right)^\varrho\right)^{1/\varrho}}, \quad D^c(p,q) = 1 - \frac{1}{1 + \left(\left(\frac{p}{1-p}\right)^\varrho + \left(\frac{q}{1-q}\right)^\varrho\right)^{1/\varrho}}$$

where $\varrho > 0, (p,q) \in [0,1]$.

Definition 3.3. For two neutrosophic cubic sets (NCs), $\underset{\sim}{\mathbb{N}}_1 = \left\langle \widetilde{T}_1, \widetilde{I}_1, \widetilde{F}_1, T_1, I_1, F_1 \right\rangle$, where $\widetilde{T}_1 = [T_1^L, T_1^U]$, $\widetilde{I}_1 = [I_1^L, I_1^U]$, $\widetilde{F}_1 = [F_1^L, F_1^U]$, and $\underset{\sim}{\mathbb{N}}_2 = \left\langle \widetilde{T}_2, \widetilde{I}_2, \widetilde{F}_2, T_2, I_2, F_2 \right\rangle$, where $\widetilde{T}_2 = [T_2^L, T_2^U]$, $\widetilde{I}_2 = [I_2^L, I_2^U]$, $\widetilde{F}_2 = [F_2^L, F_2^U]$. The basic Dombi Hamy operations are defined as follows:

$$\begin{aligned}
\underset{\sim}{\mathbb{N}}_1 \oplus \underset{\sim}{\mathbb{N}}_2 = \Bigg\{ & \left[1 - \frac{1}{1 + \left[\left(\frac{T_1^L}{1-T_1^L}\right)^{\varrho} + \left(\frac{T_2^L}{1-T_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\left(\frac{T_1^U}{1-T_1^U}\right)^{\varrho} + \left(\frac{T_2^U}{1-T_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right], \\
& \left[1 - \frac{1}{1 + \left[\left(\frac{I_1^L}{1-I_1^L}\right)^{\varrho} + \left(\frac{I_2^L}{1-I_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\left(\frac{I_1^U}{1-I_1^U}\right)^{\varrho} + \left(\frac{I_2^U}{1-I_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right], \\
& \left[\frac{1}{1 + \left[\left(\frac{1-F_1^L}{F_1^L}\right)^{\varrho} + \left(\frac{1-F_2^L}{F_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1 + \left[\left(\frac{1-F_1^U}{F_1^U}\right)^{\varrho} + \left(\frac{1-F_2^U}{F_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right]; \\
& \left\langle \frac{1}{1 + \left[\left(\frac{1-T_1}{T_1}\right)^{\varrho} + \left(\frac{1-T_2}{T_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1 + \left[\left(\frac{1-I_1}{I_1}\right)^{\varrho} + \left(\frac{1-I_2}{I_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \right. \\
& \left. 1 - \frac{1}{1 + \left[\left(\frac{F_1}{1-F_1}\right)^{\varrho} + \left(\frac{F_2}{1-F_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle . \Bigg\}
\end{aligned}$$

$$\begin{aligned}
\underset{\sim}{\mathbb{N}}_1 \otimes \underset{\sim}{\mathbb{N}}_2 = \Bigg\{ & \left[\frac{1}{1 + \left[\left(\frac{1-T_1^L}{T_1^L}\right)^{\varrho} + \left(\frac{1-T_2^L}{T_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1 + \left[\left(\frac{1-T_1^U}{T_1^U}\right)^{\varrho} + \left(\frac{1-T_2^U}{T_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right], \\
& \left[\frac{1}{1 + \left[\left(\frac{1-I_1^L}{I_1^L}\right)^{\varrho} + \left(\frac{1-I_2^L}{I_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1 + \left[\left(\frac{1-I_1^U}{I_1^U}\right)^{\varrho} + \left(\frac{1-I_2^U}{I_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right], \\
& \left[1 - \frac{1}{1 + \left[\left(\frac{F_1^L}{1-F_1^L}\right)^{\varrho} + \left(\frac{F_2^L}{1-F_2^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\left(\frac{F_1^U}{1-F_1^U}\right)^{\varrho} + \left(\frac{F_2^U}{1-F_2^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right]; \\
& \left\langle 1 - \frac{1}{1 + \left[\left(\frac{T_1}{1-T_1}\right)^{\varrho} + \left(\frac{T_2}{1-T_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\left(\frac{I_1}{1-I_1}\right)^{\varrho} + \left(\frac{I_2}{1-I_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \right. \\
& \left. \frac{1}{1 + \left[\left(\frac{1-F_1}{F_1}\right)^{\varrho} + \left(\frac{1-F_2}{F_2}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle . \Bigg\}
\end{aligned}$$

$$\vartheta\underset{\sim}{\mathbb{N}}_1=\left\{\left[1-\frac{1}{1+\left[\vartheta\left(\frac{T_1^L}{1-T_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\vartheta\left(\frac{T_1^U}{1-T_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],\right.$$
$$\left[1-\frac{1}{1+\left[\vartheta\left(\frac{I_1^L}{1-I_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\vartheta\left(\frac{I_1^U}{1-I_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],$$
$$\left[\frac{1}{1+\left[\vartheta\left(\frac{1-F_1^L}{F_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\vartheta\left(\frac{1-F_1^U}{F_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right];$$
$$\left.\left\langle\frac{1}{1+\left[\vartheta\left(\frac{1-T_1}{T_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\vartheta\left(\frac{1-I_1}{I_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\vartheta\left(\frac{F_1}{1-F_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right\rangle.\right\}$$

$$[\underset{\sim}{\mathbb{N}}_1]^{\vartheta}=\left\{\left[\frac{1}{1+\left[\vartheta\left(\frac{1-T_1^L}{T_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\vartheta\left(\frac{1-T_1^U}{T_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],\right.$$
$$\left[\frac{1}{1+\left[\vartheta\left(\frac{1-I_1^L}{I_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\vartheta\left(\frac{1-I_1^U}{I_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],$$
$$\left[1-\frac{1}{1+\left[\vartheta\left(\frac{F_1^L}{1-F_1^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\vartheta\left(\frac{F_1^U}{1-F_1^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right];$$
$$\left.\left\langle 1-\frac{1}{1+\left[\vartheta\left(\frac{T_1}{1-T_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\vartheta\left(\frac{I_1}{1-I_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\vartheta\left(\frac{1-F_1}{F_1}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right\rangle.\right\}$$

4. Dombi Hamy Mean Aggregation Operators to Neutrosophic Cubic Numbers

The NCFDHM Operator: The NCFDHM Operator is defined as follows, based on the Dombi and Hamy mean operations

Theorem 4.1. *Let* $\underset{\sim}{\mathbb{N}}_j=(\widetilde{T}_j,\widetilde{I}_j,\widetilde{F}_j,T_j,I_j,F_j)$*, where* $\widetilde{T}_j=[T_j^L,T_j^U]$*,* $\widetilde{I}_j=[I_j^L,I_j^U],\widetilde{F}_j=[F_j^L,F_j^U]$ $(j=1,2,\ldots,k)$ *be a non empty collection of NCFNs. The compressed value by the NCFDHM operators is also an NCFNs where*

$$NCFDHM^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1,\underset{\sim}{\mathbb{N}}_2,\ldots,\underset{\sim}{\mathbb{N}}_k)=\bigoplus_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\left(\bigotimes_{j=1}^{\mu}\underset{\sim}{\mathbb{N}}_{ij}\right)^{1/\mu}}{C_k^{\mu}}$$

$$= \left\{ \left[1 - \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{1 - T_{ij}^L}{T_{ij}^L} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{1 - T_{ij}^U}{T_{ij}^U} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}} \right], \right.$$

$$\left[1 - \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{1 - I_{ij}^L}{I_{ij}^L} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{1 - I_{ij}^U}{I_{ij}^U} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}} \right],$$

$$\left[\frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{F_{ij}^L}{1 - F_{ij}^L} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}}, \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{F_{ij}^U}{1 - F_{ij}^U} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}} \right];$$

$$\left\langle \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{T_{ij}}{1 - T_{ij}} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}}, \right.$$

$$\frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{I_{ij}}{1 - I_{ij}} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}},$$

$$\left. \left. 1 - \frac{1}{1 + \left[\frac{\mu}{C_k^{\mu}} \sum_{1 \le \ldots < k_{(\mu)} < \ldots \le k} \frac{1}{\sum_{j=1}^{\mu} \left(\frac{1 - F_{ij}}{F_{ij}} \right)^{\varrho}} \right]^{\frac{1}{\varrho}}} \right\rangle . \right\}$$

Proof.

$$\text{Let} \bigotimes_{j=1}^{\mu} \underset{\sim}{\mathbb{N}}_{ij} = \left\{ \left[\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \right.$$

$$\left[\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right],$$

$$\left[1-\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right];$$

$$\left. \left\langle 1-\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle . \right\}$$

$$\text{Thus,} \left(\bigotimes_{j=1}^{\mu} \underset{\sim}{\mathbb{N}}_{ij}\right)^{1/\mu} = \left\{ \left[\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \right.$$

$$\left[\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right],$$

$$\left[1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right];$$

$$\left. \left\langle 1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle . \right\}$$

Thereafter, $\bigoplus_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\left(\bigotimes_{j=1}^{\mu}\underset{\sim}{\mathbb{N}}_{ij}\right)^{1/\mu}$

$$=\left\{\left[1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],\right.$$

$$\left[1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],$$

$$\left[1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-F_{ij}^{L}}{F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-F_{ij}^{U}}{F_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right];$$

$$\left\langle\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-T_{ij}}{T_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-I_{ij}}{I_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\right.$$

$$\left.\left.1-\frac{1}{1+\left[\sum\limits_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum\limits_{j=1}^{\mu}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right\rangle.\right\}$$

Therefore, $NCFDHM^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) = \dfrac{\bigoplus_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\left(\bigotimes_{j=1}^{\mu}\underset{\sim}{\mathbb{N}}_{ij}\right)^{1/\mu}}{C_k^{\mu}}$

$$= \left\{ \left[1-\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{x}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}} \right], \right.$$

$$\left[1-\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}} \right],$$

$$\left[\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}} \right];$$

$$\left\langle \frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}, \right.$$

$$\frac{1}{1+\left[\frac{\mu}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},$$

$$\left. \left. 1-\frac{1}{1+\left[\frac{y}{C_k^{\mu}}\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{1}{\sum_{j=1}^{\mu}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}} \right\rangle . \right\}$$

hence the proof. □

Then we discuss some properties of NCFDHM Operator

1. Idempotency: If for all $\underset{\sim}{\mathbb{N}}_j = \left\langle \widetilde{T}_j, \widetilde{I}_j, \widetilde{F}_j, T_j, I_j, F_j \right\rangle$, where $\widetilde{T}_j = [T_j^L, T_j^U]$, $\widetilde{I}_j = [I_j^L, I_j^U]$, $\widetilde{F}_j = [F_j^L, F_j^U]$ $(j = 1, 2, \ldots, k)$ are equal, that is, $\underset{\sim}{\mathbb{N}}_j = \underset{\sim}{\mathbb{N}}\ \forall j$, then $NCFDHM^{\check{v}}(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) = \underset{\sim}{\mathbb{N}}$

2. Commuatativity: Let $C_j = (\widetilde{T}_j, \widetilde{I}_j, \widetilde{F}_j, T_j, I_j, F_j)$, where $\widetilde{T}_j = [T_j^L, T_j^U]$, $\widetilde{I}_j = [I_j^L, I_j^U]$, $\widetilde{F}_j = [F_j^L, F_j^U]$ $(j = 1, 2, \ldots, k)$ is the collection of neutrosophic cubic numbers

$NCFDHM^{\check{v}}\left(\ddot{\underset{\sim}{\mathbb{N}}}_1, \ddot{\underset{\sim}{\mathbb{N}}}_2, \ldots, \ddot{\underset{\sim}{\mathbb{N}}}_k\right)$ be any permuation of $NCFDHM^{(\check{v})}\left(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k\right)$

$$NCFDHM^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1), (\underset{\sim}{\mathbb{N}}_2), \ldots, (\underset{\sim}{\mathbb{N}}_k) = NCFDHM^{(\check{v})}(\ddot{\underset{\sim}{\mathbb{N}}}_1), (\ddot{\underset{\sim}{\mathbb{N}}}_2), \ldots, (\ddot{\underset{\sim}{\mathbb{N}}}_k)$$

3. Monotonicity: Let $\underset{\sim}{C}_j = \left\langle \widetilde{T}_j, \widetilde{I}_j, \widetilde{F}_j, T_j, I_j, F_j \right\rangle$, where $\widetilde{T}_j = [T_j^L, T_j^U]$, $\widetilde{I}_j = [I_j^L, I_j^U]$, $\widetilde{F}_j = [F_j^L, F_j^U]$ $(j = 1, 2, \ldots, k)$ be the set of neutrosophic cubic numbers

If $S_{C_j}(\check{v}) \geq S_{\mathbb{N}_j}(\check{v})$ and $C_j(\check{v}) \geq \underset{\sim}{\mathbb{N}}_j(\check{v})$ then

$$NCFDHM^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) \leq NCFDHM^{\check{v}}(\underset{\sim}{C}_1, \underset{\sim}{C}_2, \ldots, \underset{\sim}{C}_k)$$

4. Boundary: $\underset{\sim}{\mathbb{N}}_i^- \leq NCFDHM^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1), (\underset{\sim}{\mathbb{N}}_2), \ldots, (\underset{\sim}{\mathbb{N}}_n) \leq \underset{\sim}{\mathbb{N}}_i^+$, where

$\tilde{\underset{\sim}{\mathbb{N}}}_i^- = \left\{inf\left(\left[T_i^-, T_i^+\right]\right), sup\left(\left[I_i^-, I_i^+\right]\right), sup\left(\left[F_i^-, F_i^+\right]\right); min\left(T_i\right), max\left(I_i\right), max\left(F_i\right)\right\},$

$\tilde{\underset{\sim}{\mathbb{N}}}_i^+ = \left\{sup\left(\left[T_i^-, T_i^+\right]\right), inf\left(\left[I_i^-, I_i^+\right]\right), inf\left(\left[F_i^-, F_i^+\right]\right); max\left(T_i\right), min\left(I_i\right), min\left(F_i\right)\right\}.$

4.1. *The Weighted NCFWDHM Aggregation Operator*

Definition 4.2. Let $\underset{\sim}{\mathbb{N}}_j = \left\langle \widetilde{T}_j, \widetilde{I}_j, \widetilde{F}_j, T_j, I_j, F_j \right\rangle$, where $\widetilde{T}_j = [T_j^L, T_j^U]$, $\widetilde{I}_j = [I_j^L, I_j^U]$, $\widetilde{F}_j = [F_j^L, F_j^U]$ $(j = 1, 2, \ldots, k)$, be a set of NCFNs. The NCFWDHM Operator is

$$NCFWDHM_w^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) = \bigoplus_{1 \leq \ldots < k_{(\mu)} < \ldots \leq k} \frac{\left(\bigotimes_{j=1}^{\mu} (\underset{\sim}{\mathbb{N}}_{ij})^{w_{ij}}\right)^{1/\mu}}{C_k^{\mu}}$$

Theorem 4.3. *Let* $\underset{\sim}{\mathbb{N}}_j = \left\langle \widetilde{T}_j, \widetilde{I}_j, \widetilde{F}_j, T_j, I_j, F_j \right\rangle$, *where* $\widetilde{T}_j = [T_j^L, T_j^U]$, $\widetilde{I}_j = [I_j^L, I_j^U]$, $\widetilde{F}_j = [F_j^L, F_j^U]$ $(j = 1, 2, \ldots, k)$ *be a collection of non empty NCFNs. The compressed value by the NCFWDHM operators is also an NCFN where*

$$NCFWDHM_w^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k) = \bigoplus_{1 \leq \ldots < k_{(\mu)} < \ldots \leq k} \frac{\left(\bigotimes_{j=1}^{\mu} \left(\underset{\sim}{\mathbb{N}}_{ij}\right)^{w_{ij}}\right)^{1/\mu}}{C_k^{\mu}} =$$

$$\left\{\left[1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],\right.$$

$$\left[1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],$$

$$\left[\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right];$$

$$\left\langle\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{T_{ij}^{L}}{1-T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\right.$$

$$\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{I_{ij}^{L}}{1-I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},$$

$$\left.\left.1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-F_{ij}^{L}}{F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right\rangle.\right\}$$

Proof.

$$\text{Let}\quad \underset{\sim}{\mathbb{N}}_{ij}^{w_{ij}} = \Bigg\{ \left[\frac{1}{1+\left[w_{ij}\left(\frac{1-T_{ij}^L}{T_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[w_{ij}\left(\frac{1-T_{ij}^U}{T_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \left[\frac{1}{1+\left[w_{ij}\left(\frac{1-I_{ij}^L}{I_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[w_{ij}\left(\frac{1-I_{ij}^U}{I_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \left[1-\frac{1}{1+\left[w_{ij}\left(\frac{F_{ij}^L}{1-F_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[w_{ij}\left(\frac{F_{ij}^U}{1-F_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right]; \left\langle 1-\frac{1}{1+\left[w_{ij}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(w_{ij}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}\right)^{\frac{1}{\varrho}}}, \frac{1}{1+\left[w_{ij}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle. \Bigg\}$$

$$\bigotimes_{j=1}^{\mu} (\underset{\sim}{\mathbb{N}}_{ij})^{w_{ij}} = \Bigg\{ \left[\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{1-T_{ij}^L}{T_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{1-T_{ij}^U}{T_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \left[\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{1-I_{ij}^L}{I_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{1-I_{ij}^U}{I_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right], \left[1-\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{F_{ij}^L}{1-F_{ij}^L}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{F_{ij}^U}{1-F_{ij}^U}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right]; \left\langle 1-\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}, \frac{1}{1+\left[\sum_{j=1}^{\mu} w_{ij}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}} \right\rangle. \Bigg\}$$

Therefore,

$$\left(\bigotimes_{j=1}^{\mu}(\underset{\sim}{\mathbb{N}}_{ij})^{w_{ij}}\right)^{1/\mu}=\left\{\left[\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],\right.$$

$$\left[\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right],$$

$$\left[1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right];$$

$$\left\langle 1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{T_{ij}}{1-T_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},\right.$$

$$1-\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{I_{ij}}{1-I_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}},$$

$$\left.\left.\frac{1}{1+\left[\frac{1}{\mu}\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-F_{ij}}{F_{ij}}\right)^{\varrho}\right]^{\frac{1}{\varrho}}}\right\rangle\right\}$$

Thereafter, $\bigoplus_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\left(\bigotimes_{j=1}^{\mu}(\underset{\sim}{\mathbb{N}}_{ij})^{w_{ij}}\right)^{1/\mu}$

$$=\left\{\left[1-\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],\right.$$

$$\left[1-\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],$$

$$\left[\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right];$$

$$\left\langle\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{T_{ij}^{L}}{1-T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\right.$$

$$\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{I_{ij}^{L}}{1-I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},$$

$$\left.1-\frac{1}{1+\left[\sum_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-F_{ij}^{L}}{F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right\rangle.\Bigg\}$$

Therefore,

$$NCFWDHM_{w}^{(\check{v})}(\underset{\sim}{\mathbb{N}}_1,\underset{\sim}{\mathbb{N}}_2,\ldots,\underset{\sim}{\mathbb{N}}_k)=\frac{\bigoplus_{1\leq\ldots<k_{(\mu)}<\ldots\leq k}\left(\bigotimes_{j=1}^{\mu}(\underset{\sim}{\mathbb{N}}_{ij})^{w_{ij}}\right)^{1/\mu}}{C_k^{\mu}}=$$

$$\left\{\left[1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-T_{ij}^{L}}{T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{y}w_{ij}\left(\frac{1-T_{ij}^{U}}{T_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],\right.$$

$$\left[1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{L}}{I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-I_{ij}^{U}}{I_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right],$$

$$\left[\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{F_{ij}^{L}}{1-F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{y}w_{ij}\left(\frac{F_{ij}^{U}}{1-F_{ij}^{U}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right];$$

$$\left\langle\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{T_{ij}^{L}}{1-T_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},\right.$$

$$\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{I_{ij}^{L}}{1-I_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}},$$

$$\left.\left.1-\frac{1}{1+\left[\frac{1}{C_k^{\mu}}\sum_{1\le\ldots<k_{(\mu)}<\ldots\le k}\frac{\mu}{\sum_{j=1}^{\mu}w_{ij}\left(\frac{1-F_{ij}^{L}}{F_{ij}^{L}}\right)^{\varrho}}\right]^{\frac{1}{\varrho}}}\right\rangle.\right\}$$

hence proved. □

5. Algorithms and Illustration of the Proposed MCDM Method

5.1. *Algorithm 1*

Step 1. For an MCDM problem, a neutrosophic cubic decision matrix $A = (a_{ij})_{n\times k}$ is constructed

Step 2. Fix corresponding relative ideal point over attributes of neutrosophic cubic sets

$$\underset{\sim}{\mathbb{N}}^*_{C_i} = \left\{ \left[\max\left\{ \underset{\sim}{T}^L_{\underset{\sim}{\mathbb{N}}^*} \right\}, \max\left\{ \underset{\sim}{T}^U_{\underset{\sim}{\mathbb{N}}^*} \right\} \right], \left[\min\left\{ \underset{\sim}{I}^L_{\underset{\sim}{\mathbb{N}}^*} \right\}, \min\left\{ \underset{\sim}{I}^U_{\underset{\sim}{\mathbb{N}}^*} \right\} \right], \left[\min\left\{ \underset{\sim}{F}^L_{\underset{\sim}{\mathbb{N}}^*} \right\}, \min\left\{ \underset{\sim}{F}^U_{\underset{\sim}{\mathbb{N}}^*} \right\} \right]; \left\langle \max\left\{ \underset{\sim}{T}_{\underset{\sim}{\mathbb{N}}^*} \right\}, \min\left\{ \underset{\sim}{I}_{\underset{\sim}{\mathbb{N}}^*} \right\}, \min\left\{ \underset{\sim}{F}_{\underset{\sim}{\mathbb{N}}^*} \right\} \right\rangle . \right\} \forall j = 1, 2, 3, \ldots, k$$

Step 3. Calculate similarity measure between corresponding alternatives $A_1, A_2, \ldots, A_n$ and relative ideal point of neutrosophic cubic sets $\underset{\sim}{\mathbb{N}}^*_C = \underset{\sim}{\mathbb{N}}^*_{C_i}(j = 1, 2, 3, \ldots, k)$ using equation.5

$$S\left(A_i, \underset{\sim}{\mathbb{N}}^*_C\right) \quad \forall i = 1, 2, 3, \ldots, n$$

Step 4. Choose the best alternatives A_i according to similarity values of $S\left(A_i, \underset{\sim}{\mathbb{N}}^*_C\right)$, for all $i = 1, 2, \ldots, n$

5.2. ***Algorithm 2***

Step 1. For an MCDM problem, a neutrosophic cubic decision matrix $A = (a_{ij})_{n\times k}$ is constructed

Step 2. Compute neutrosophic cubic aggregated vaules for each alternative over attributes by $NCFWDHM^{(\check{v})}_w(\underset{\sim}{\mathbb{N}}_1, \underset{\sim}{\mathbb{N}}_2, \ldots, \underset{\sim}{\mathbb{N}}_k)$

Step 3. Utilize the score formula (Eq.6) to obtain the score values of the alternatives.

Step 4. Rank the alternatives A_i according to score values.

5.3. *Illustration of the Models*

In this section, an illustration has been chosen on the basis of finding the poor from among the target group based on education level, employment and income. We employ the proposed multi criteria decision making algorithms to the chosen problem. Here we have taken the households of target group of people as alternatives $A_i(i = 1, 2, 3, \ldots, n)$ and we consider the attributes employment (C_1) , education level (C_2) and income (C_3). The values are represented by neutrosophic cubic numbers which covers both interval and individual poverty information.

The decision matrix is given as follows:

$$DM = \left\{ \begin{array}{cccc} & C_1 & C_2 & C_3 \\ A_1 & \begin{bmatrix} [0.5,0.6], & [0.1,0.3], \\ [0.2,0.4]; & \langle 0.6,0.2,0.3\rangle \end{bmatrix} & \begin{bmatrix} [0.5,0.6], & [0.1,0.3], \\ [0.2,0.4]; & \langle 0.6,0.2,0.3\rangle \end{bmatrix} & \begin{bmatrix} [0.2,0.4], & [0.7,0.8], \\ [0.8,0.9]; & \langle 0.3,0.8,0.9\rangle \end{bmatrix} \\ A_2 & \begin{bmatrix} [0.6,0.8], & [0.1,0.2], \\ [0.2,0.3]; & \langle 0.7,0.1,0.2\rangle \end{bmatrix} & \begin{bmatrix} [0.6,0.7], & [0.1,0.2], \\ [0.2,0.3]; & \langle 0.6,0.1,0.2\rangle \end{bmatrix} & \begin{bmatrix} [0.3,0.4], & [0.6,0.7], \\ [0.8,0.9]; & \langle 0.3,0.6,0.9\rangle \end{bmatrix} \\ A_3 & \begin{bmatrix} [0.4,0.6], & [0.2,0.3], \\ [0.1,0.3]; & \langle 0.6,0.2,0.2\rangle \end{bmatrix} & \begin{bmatrix} [0.5,0.6], & [0.2,0.3], \\ [0.3,0.4]; & \langle 0.6,0.3,0.4\rangle \end{bmatrix} & \begin{bmatrix} [0.3,0.5], & [0.7,0.8], \\ [0.6,0.7]; & \langle 0.4,0.8,0.7\rangle \end{bmatrix} \\ A_4 & \begin{bmatrix} [0.7,0.8], & [0.1,0.2], \\ [0.1,0.2]; & \langle 0.8,0.1,0.2\rangle \end{bmatrix} & \begin{bmatrix} [0.6,0.7], & [0.1,0.2], \\ [0.1,0.3]; & \langle 0.7,0.1,0.2\rangle \end{bmatrix} & \begin{bmatrix} [0.3,0.4], & [0.6,0.7], \\ [0.7,0.8]; & \langle 0.3,0.7,0.8\rangle \end{bmatrix} \end{array} \right\}$$

5.4. *Algorithm 1*

Step 2. Corresponding relative ideal point over attributes of neutrosophic cubic sets

$$\underset{\sim}{\mathbb{N}}_C^* : \left\langle \underset{\sim}{\mathbb{N}}_{C_1}^*, \underset{\sim}{\mathbb{N}}_{C_2}^*, \underset{\sim}{\mathbb{N}}_{C_3}^* \right\rangle$$

$$\underset{\sim}{\mathbb{N}}_{C_1}^* = \{[0.7, 0.8], [0.1, 0.2], [0.1, 0.2]; \langle 0.8, 0.1, 0.2\rangle\}$$

$$\underset{\sim}{\mathbb{N}}_{C_2}^* = \{[0.6, 0.7], [0.1, 0.2], [0.1, 0.3]; \langle 0.7, 0.1, 0.2\rangle\}$$

$$\underset{\sim}{\mathbb{N}}_{C_3}^* = \{[0.3, 0.5], [0.6, 0.7], [0.6, 0.7]; \langle 0.4, 0.6, 0.7\rangle\}$$

Step 3. Similarity measure between alternatives $A_1, A_2, \ldots, A_m$ and relative ideal point of neutrosophic cubic sets $\underset{\sim}{N}_C^*$.

$$S\left(A_1, \underset{\sim}{\mathbb{N}}_C^*\right) = 0.8778, \quad S\left(A_2, \underset{\sim}{\mathbb{N}}_C^*\right) = 0.9370, \quad S\left(A_3, \underset{\sim}{\mathbb{N}}_C^*\right) = 0.9000, \quad S\left(A_4, \underset{\sim}{\mathbb{N}}_C^*\right) = 0.9778$$

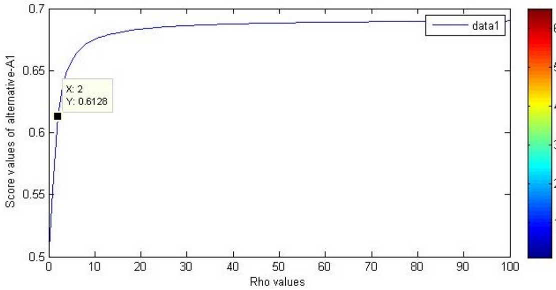
Step 4. The arrangement of alternatives according to similarity values of $S\left(A_i, \underset{\sim}{\mathbb{N}}_C^*\right)$, $i = 1, 2, \ldots, n$. $A_4 > A_2 > A_3 > A_1$

5.5. *Algorithm 2*

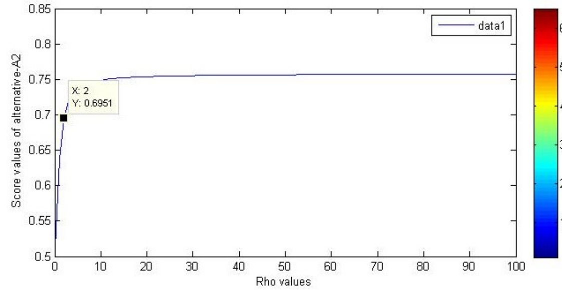
Step 2. Let the weighted values of the attributes be $W = (0.32, 0.38, 0.3)$, respectively. Take $\mu = 2$, $\varrho = 2$ then using NCFWDHM operators on alternatives A_i $(i = 1, 2, 3, 4)$ we get the aggregated values as shown in table.2

TABLE 2. Aggregated Values by NCFWDHM operator

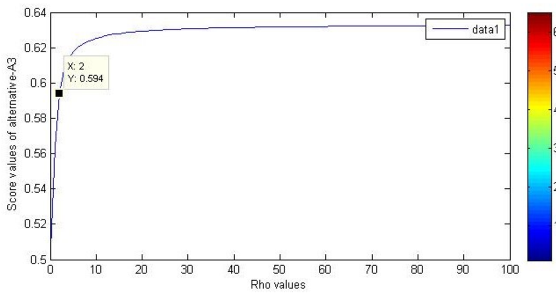
Atlernatives	Aggregated Values (NCFWDHM)
A_1	[0.5239, 0.6594], [0.1955, 0.4830], [0.2025, 0.4028]; $\langle 0.4133, 0.2025, 0.4838\rangle$
A_2	[0.6296, 0.7528], [0.1953, 0.3527], [0.2025, 0.3040]; $\langle 0.4616, 0.1011, 0.3536\rangle$
A_3	[0.5249, 0.6925], [0.3527, 0.4830], [0.2323, 0.3423]; $\langle 0.4218, 0.2649, 0.4966\rangle$
A_4	[0.6577, 0.7528], [0.1953, 0.3527], [0.1017, 0.2649]; $\langle 0.5775, 0.1017, 0.3533\rangle$



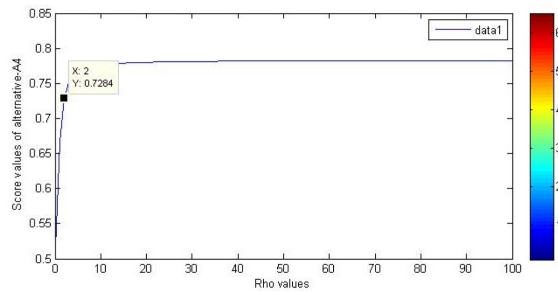
(A) Scores of alternative A_1



(B) Scores of alternative A_2



(C) Scores of alternative A_3



(D) Scores of alternative A_4

FIGURE 1. Scores of alternative A_i obtained by $NCFWDHM$ operator

Step 3. Score function of the alternatives are

$$S(A_1) = 0.6128, S(A_2) = 0.6951, S(A_3) = 0.5940, S(A_4) = 0.7284$$

Step 4. The alternatives are ranked based on their score values $A_4 > A_2 > A_1 > A_3$

5.6. *Comparative Analysis*

For comparison analysis, the proposed weighted dombi hamy neutrosophic cubic mean aggregation operator ($NCFWDHM$) is compared with an existing multi-criteria decision making method based on neutrosophic cubic aggregation operator ($WNCFGBM_w^{u,v}$) [9] and with the proposed decision making technique over similarity measure. The findings are shown in table.3

TABLE 3. Rank of the proposed methods

Proposed Methods	Ranking order
Algorithm 1 (Similarity Measure)	$A_4 > A_2 > A_3 > A_1$
Algorithm 2 ($NCFWDHM$)	$A_4 > A_2 > A_1 > A_3$
$WNCFGBM_w^{u,v}$ Operator [9]	$A_4 > A_2 > A_3 > A_1$

Also for a detailed comparision, we represent the score values of each alternative in Fig.1 by only changing the values of ϱ between 0 and 100 and we can find that the score values of each alternative are the same after certain values of ϱ. The values of the parameter μ have been chosen based on the total number of combinations of k data sets, that is, the total number of neutrosophic cubic numbers. When we have large data sets the values of μ will have more combinations to deal with. Therefore the WDHNCM operators are more applicable to large data sets for making decision. From table.3 we can see that the most optimistic results are A_4 and A_2. The ranking order of the alternatives are almost the same and slight changes exist between alternatives A_1 and A_3. The reason is that the proposed methods with the WDHNCM and ($WNCFGBM_w^{u,v}$) [9] aggregation operators have been applied for minimum data sets or, in other words, the given illustration exist for initial parameter, that is, for $\mu = 2$.

6. Conclusions

The advantage of neutrosophic cubic sets is presented in the article with a newly developed weighted dombi hamy neutrosophic cubic mean aggregation operators. Some of the basic neutrosophic cubic operations and properties are proved with respect to domi and hamy mean operations. Further the similarity measure and relative neutrosophic cubic ideal points are introduced and with the help of these ideas a new decision making method is developed. The comparative analysis has been made to prove the efficiency and validity of the proposed operators through numerical illustration. The important finding is that the proposed operators give more efficient results while having to deal with huge set of uncertain data or information due to their nature. Also the results are stable and more consistent with existing measures. The proposed aggregation operators can be extended to be used in existing decision making models like CODAS, AHP, MULTIMOORA, TOPSIS, VIKOR, etc. which would result in more new models. This study can further be extended to the field of artificial intelligence, medical and fault diagnosis with real time applications.

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