NSS
Neutrosophic Sets and Systems,Vol. 38,2020
University of New Mexico

On Deneutrosophication

Kanika Mandal[1]
[1]Nalanda Mahila College, Patliputra University, Bihar, India; boson89@yahoo.com
*boson89@yahoo.com

Abstract. Deneutrosophication is a process to evaluate real output from neutrosophic information. The paper presents on a novel deneutrosophication algorithm. The process is developed with similarity measure and probability density function (PDF). This similarity measure is newly defined to prepare a correct transformation from neutrosophic set (NS) to fuzzy set (FS). Then an approach to find PDF is formulated which relates with fuzzy set. Finally, the algorithm has been implemented in solving a critical path problem to find out the completion time of a certain project.

Keywords: Deneutrosophication; Similarity measure; Probability density function

1. Introduction

Modern technology and science can not be evolved without dealing with uncertainty. Before the era of fuzzy, problems of uncertainty would solved by theory of probability only. Fuzzy set (FS) [1] can easily cope up with uncertainty by its membership grade (Zadah, 1965). But the necessity of handling incomplete information addressed to the introduction of intuitionistic fuzzy set (IFS) [2] (Atanasov, 1985). IFS can not link indeterminacy. Neutrosohic set [3] (NS) can overcome all the limitations of FS, IFS due to its easy relationship between mathematical and formal language (Smarandache, 1998).

One of the most challenging issue to the neutrosohic researchers is to innovate better solution approach or method to reach to a better decision or conclusion from several pieces of neutrosophic informations. Similarity measure, a popular information measure method, is used in different researches to solve real life problems. Ye [4] in 2014 solved decision making (DM) problem by introducing Jaccard, Dice and cosine similarity measures for single valued neutrosohic sets (SVNSs). Further, the author (Ye, 2015) eliminated limitations of the cosine similarity measure in [5] and used in a medical diagnosis problem. In [6], using the similarity function entropy measure was also developed (Aydogdu, 2015). Some more similarity functions [7] were introduced for recovery of some disadvantages of Jaccard, Dice and cosine similarity measures and applied to a minimum spanning tree problem (Mandal et al., 2016). It is also effectively used in medical diagnosis by using the euclidean distance based similarity measure [8] (Donghai Liu et al., 2018), hybrid distance based similarity measure for refined neutrosophic sets [9] (Vakkas Ulucay et al., 2019). Further similarity measure is applied in smart port development [10] (Jihong chen et al., 2019), selection of cricket players [11] (Muhammad et al., 2020), medical diagnosis as well as lecturer selection for universities [12] (Saeed et al., 2020).

In recent years different effective methods have been implemented to deal with several existing problems. Abdel-Basset et al. [13] developed a model on the basis of neutrosophic AHP which was succesfully used in Egypt steel industry. In [14] Plithogenic aggregation operator was proposed to aggregate the opinions of decision makers and was used in best worst method in solving supply chain, ware house location and plant evaluation problem (Abdel-Basset et al., 2020). In order to monitoring the spread of epidemic covid 19, a novel approach using best worst and Topsis method [15] is introduced (Abdel-Basset et al., 2020). Another technique health fog method was discussed in [16] to assist covid patient (Yasser et al., 2020). In [17] Carmen et al. analyzed emotional intelligence of some randomly selected university students. Further, contributions in neutrosophic researches are on medical diagnosis [18–21], smart product service systems [23], decision making in personnel selection [24], recommending in museum room [25], predicting tax time series [26], Sustainability of goat and sheep production systems [27], also in [28–32] etc.

The methods used in the above discussed liturature correspond to real data, not the actual real output to solve problems. But sometimes it becomes necessary to evaluate the actual real output of the respective event from the neutrosohic information. Suppose a decision maker put his decision on some activity time of a certain project by a SVNS $\langle 15, (0.3, 0.5, 0.6)\rangle$. Now a common thought arises upon us about the expected activity completion time. Deneutrosophication gives the answer, it is the process which can evaluate the crisp value from the neutrosophic data. Smarandache et al. [33] first discussed a deneutrosophication method in 2005 by using synthesization and center of gravity method. The synthesization process according to [33] corresponds a NS to different FSs resulting different crisp values. Using removal area method, a deneutrosophication technique [34] was studied on pentagonal neutrosophic number and utilized in MST problem (Chakraborty et al.,2019). A deneutrosophication equation computed in [35] gives the neat truth grade not the real output of the respective event (Azzah Awang et al., 2019). Deneutrosophicated value for a trapezoidal fuzzy number (TFN) was found in [36] using the score function involving center of gravity of TFN (Said Broumi et al., 2019).

In this article first time similarity measure is used to have a meaningful relation between NS and FS. To establish this, a similarity function is posed by redefining the axiomatic definition. It also overcome the limitation of existing ones [5–7]. Then, an approach to relate FS with PDF is proposed. Finally, a deneutrosophication algorithm is established which is more generalized and robust in comparison with [33–36] as it can provide a single crisp value, deal with both continuous and discrete neutrosohic data and find the real output of the respective event. A comparative study to show its consistency and effectiveness has been delivered. At last a critical path method is solved as an application of the deneutrosohication approach.

2. **Preliminaries**

2.1. *Neutrosophic set [3]*

Let U be an universe of discourse, then the neutrosophic set A is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$, where the functions T, I, F: $U \to]^{-}0, 1^{+}[$ define respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or falsehood) of the element $x \in U$ to the set A with the condition $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

2.2. *Single Valued Neutrosophic Sets(SVNS) [38]*

Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS, A, in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. Therefore, a SVNS A can be written as $A_{SVNS} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$.

2.3. *Single valued triangular neutrosophic number [40]*

A triangular neutrosophic number $A = \langle (a_1, b_1, c_1); w_a, u_a, y_a\rangle$ is a special neutrosophic set on the real number set R, whose truth-membership indeterminacy-membership and falsity-membership functions are defined as follows:

$$T_A(x) = \begin{cases} \frac{(x-a_1)w_a}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ w_a & , x = b_1 \\ \frac{(c_1-x)w_a}{c_1-b_1} & , b_1 \leq x \leq c_1 \\ 0 & , otherwise. \end{cases}$$

$$I_A(x) = \begin{cases} \frac{b_1-x+u_a(x-a_1)}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ u_a & , x = b_1 \\ \frac{(x-b_1)+u_a(c_1-x)}{c_1-b_1} & , 0.5 \leq x \leq 0.7 \\ 1 & , otherwise. \end{cases}$$

$$F_A(x) = \begin{cases} \frac{b_1-x+y_a(x-a_1)}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ y_a & , x = b_1 \\ \frac{(x-b_1)+y_a(c_1-x)}{c_1-b_1} & , b_1 \leq x \leq c_1 \\ 1 & , otherwise. \end{cases}$$

2.4. *Single valued trapezoidal neutrosophic number [40]*

A single valued trapezoidal neutrosophic number $\langle A = (a_1, b_1, c_1, d_1); w_a, u_a, y_a \rangle$ is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy-membership, and a falsity-membership are given as follows:

$$T_A(x) = \begin{cases} \frac{(x-a_1)w_a}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ w_a & , b_1 \leq x \leq c_1 \\ \frac{(d_1-x)w_a}{d_1-c_1} & , c_1 \leq x \leq d_1 \\ 0 & , otherwise. \end{cases}$$

$$I_A(x) = \begin{cases} \frac{b_1-x+u_a(x-a_1)}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ u_a & , b_1 \leq x \leq c_1 \\ \frac{(x-c_1)+u_a(d_1-x)}{d_1-c_1} & , c_1 \leq x \leq d_1 \\ 1 & , otherwise. \end{cases}$$

$$F_A(x) = \begin{cases} \frac{b_1-x+u_a(x-a_1)}{b_1-a_1} & , a_1 \leq x \leq b_1 \\ y_a & , b_1 \leq x \leq c_1 \\ \frac{(x-c_1)+y_a(d_1-x)}{d_1-c_1} & , c_1 \leq x \leq d_1 \\ 1 & , otherwise. \end{cases}$$

2.5. *similarity measure [5, 6]*

Similarity measure s for SVNS(X) is a real function on universe X such that $S : SVNS(X) \times SVNS(X) \rightarrow [0, 1]$ and satisfies the following properties: (i) $0 \leq s(A, B) \leq 1, \forall A, B \in SVNS(X)$,
(ii) $s(A, B) = s(B, A), \forall A, B \in SVNS(X)$,
(iii) $s(A, B) = 1$, if and only if $A = B, \forall A, B \in SVNS(X)$.
(iv) If $A \subset B \subset C$, $s(A, B) \geq s(A, C)$ and $s(B, C) \geq s(A, C)$ $\forall A, B \in SVNS(X)$.

3. Limitations of existing similarity functions and redefinition

According to the definition of similarity measure function as defined in subsec 2.5, $s(A,B)$ should be 1 if and only if $A = B$. But there is no such necessary and sufficient condition for which $s(A,B)$ is zero.

$(1,0,0)$ and $(0,0,1)$ represent respectively the total affirmation and the total denial of the belongingness of an element to a given NS. Obviously, the similarity between $\{(1,0,0)\}$ and $\{(0,0,1)\}$ is zero. The existing formulee in [5–7], violate this argument. So, a new similarity measure is proposed by redefining the definition.

3.1. *Proposed similarity measure*

Along with the four properties in 2.5, the similarity measure is redefined with an extra fifth property:

(v) $s(A,B) = 0$, if $A = \{(1,0,0)\}$ and $B = \{(0,0,1)\}$,

which gives the sufficient condition for which the similarity between A and B will be zero.

Following the proposed function which satisfies all the property of similarity measure along with the fifth property.

$$s(A,B) = 1 - \frac{1}{2n}\sum_{i=1}^{n}[|T_A(x_i) - T_B(x_i)| + \max\{|I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}] \tag{1}$$

Clearly, $s(A,B)$ satisfies all the required properties of similarity measure function as defined in 3.1.

3.2. *Remark*

Consider $A = \{(1,0,0)\}$ and $B = \{(0,1,1)\}$. Then, using equation 1, we get $s(A,B) = 0$. Thus, the condition (stated as property (v)) for which similarity is zero is sufficient but not necessary.

3.2.1. *Example 1 [7]*

Let $A = \{(x_1,(0.2,0.5,0.6)),(x_2,(0.2,0.4,0.4))\}$ and
$B = \{(x_1,(0.2,0.4,0.4)),(x_2,(0.4,0.2,0.3))\}$ be two SVNSs.

TABLE 1. Similarity values ($s(A,B)$) in different methods

	Using [5]	*Using* [6]	*using equ*(3) *in* [7]	*using equ*(4) *in* [7]	*By proposed formula*
$s(A,B)$	0.977	0.867	0.815	0.975	0.85

3.2.2. *Example 2 [7]*

Consider $C = \{(x_1,(0.3,0.2,0.3)),(x_2,(0.5,0.2,0.3)),(x_3,(0.5,0.3,0.2))\}$ and $D = \{(x_1,(0.7,0,0.1)),(x_2,(0.6,0.1,0.2)),(x_3,(0.6,0.1,0.2))\}$.

TABLE 2. $s(C,D)$ using different methods

	Using [5]	*Using* [6]	*using equ*(3) *in* [7]	*using equ*(4) *in* [7]	*By proposed formula*
$s(C,D)$	0.9629	0.844	0.79	0.964	0.8167

From the above examples, it may be observed that the proposed formula gives consistent result.

Using the similarity measure function, we can convert neutrosophic fuzzy data to fuzzy data and hence PDF also which is discussed in the following section.

4. The proposed approach to formulate PDF from neutrosophic fuzzy data

We segregate the approach in two intermediate steps:

Step 1:

Synthesization is the process to convert a NS (here we consider SVNS) into a FS. The process evaluates the overall truth over the truth, indeterminacy and falsity membership function. In the neutrosophic set theory, $I_N = \{(1,0,0)\}$ can be considered as the reference NS, as it signifies the full membership of an element to a given set. Full membership in FS is indicated by the membership value 1. So, $(1,0,0)$ (in NS) is equivalent to the maximum membership 1 (in fuzzy) since both implies total belongingness to the respective set. The more the similarity between each $(T_A(x),I_A(x),F_A(x))$ and I_N is, the more the belongingness of the element to the set i.e., I_N stands for the reference NS. Thus, we get the following proposition to convert a NS into a FS:

Proposition:

Let $A_N = \{\langle x,(T_A(x),I_A(x),F_A(x))\rangle, x \in X\}$ be a NS. Its equivalent fuzzy membership set is defined as $A_F = \{(x,\mu_A(x)) : x \in X\}$, where $\mu_A(x) = s((T_A(x),I_A(x),F_A(x)),(1,0,0))$. So, using equation 1,

$$\mu_A(x) = 1 - \frac{1}{2}[(1-T_A(x))+ \max\{I_A(x),F_A(x)\}] \tag{2}$$

As the range of the similarity measure function is the unit interval $[0,1]$, $\mu_A(x) \in [0,1]$ $\forall x \in X$. Hence, the membership function of the derived fuzzy set belongs to $[0,1]$ and thus it satisfies the property of membership function of a FS.

The larger is the similarity measure value between $(T_A(x),I_A(x),F_A(x))$ and $(1,0,0)$, the larger is the belonginess of x to the respective set, A, the more is the membership value.

Moreover, $(1, 0, 0)$, the neutrosohic number which denotes the full belonginess to a set, is transformed into the membership value 1 which confirms to be the full belonginess to the set. Again $(0, 0, 1)$, the neutrosohic number which denotes the zero belonginess to a set, is transformed into the membership value 0 which confirms to be the zero belonginess to the set. So, the results of the transformation are desirable and meaningful.

Step 2:

Formulate PDF from fuzzy membership function:

[Case 1.] The variable is discrete in (S):

Let $S = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse and
$A = \{x_i, \mu_A(x_i) : x_i \in S\}$ be a fuzzy set with discrete membership function. Consider X to be the random variable corresponding to the event space S. The density function of the random variable X is defined as

$$P(X = x_i) = f(x_i) = \frac{\mu_A(x_i)}{\Delta} \tag{3}$$

where $\Delta = \sum_{-\infty}^{\infty} \mu_A(x_i)$. The proposed function $f(x_i)$ satisfies the required properties of a density function:

(1) $f(x_i) \geq 0$, as membership function $\mu_A(x_i) \geq 0$.

(2) $\sum_{-\infty}^{\infty} f(x_i) = \frac{\sum_{-\infty}^{\infty} \mu_A(x_i)}{\sum_{-\infty}^{\infty} \mu_A(x_i)} = 1$.

[Case 2.] The variable is continuous in (S):

Let
$A = \{(x, \mu_A(x)) : x \in S\}$ be a fuzzy set. Consider X to be the random variable corresponding to the event space S. The density function of the random variable X is defined as

$$f(x) = \frac{\mu_A(x)}{\Delta} \tag{4}$$

where $\Delta = \int_{-\infty}^{\infty} \mu_A(x)dx$. Clearly $f(x)$ also satisfies the desired properties of a density function.

4.1. *Example*

Discrete case: Let $S = \{x_1, x_2, x_3\}$ be the universal set and the neutrosophic set is defined as
$A_N = \{(x_1, (0.7, 0.5, 0.2)), (x_2, (0.7, 0.8, 0.9)), (x_3, (0.3, 0.8, 0.9))\}$
Using equation (2), equivalent fuzzy set $A_F = \{(x_1, 0.6), (x_2, 0.4), (x_3, 0.2)\}$. So, the corresponding density function (from equation (3)) at x_i, $(i = 1, 2, 3)$ are as follows:
$f(x_1) = \frac{0.6}{1.2} = 0.5$, $f(x_2) = 0.33$, $f(x_3) = 0.17$.

Continuous case: When the universal set S is a continuous, the degree of membership can be represented by a function which can take various shapes and forms like triangular membership function, trapezoidal membership function etc.

Consider the NS, A, defined on the interval $S = [0, 10]$ of real numbers by the truth, indeterminacy and falsity membership functions: $T_A(x) = \frac{1}{1+x}$, $I_A(x) = \frac{1}{1+x^2}$, $F_A(x) = \frac{1}{1+x^3}$. Then its equivalent fuzzy membership function is

$$\mu_A(x) = \begin{cases} 1 - \frac{1}{2}\left[(1 - \frac{1}{1+x}) + \frac{1}{1+x^3}\right] & ,0 \leq x \leq 1 \\ 1 - \frac{1}{2}\left[(1 - \frac{1}{1+x}) + \frac{1}{1+x^2}\right] & ,1 \leq x \leq 10 \end{cases}$$

Also the corresponding density function is $f_A(x) = \frac{\mu_A(x)}{\Delta}$, where $\Delta = \int_0^{10} \mu_A(x)dx = 5.4382$.

5. **Proposed deneutrosophication method**

Deneutrosophication is the process to convert neutrosophic data to crisp data corresponding universe of discourse. In this section, a deneutrosophication method is presented through the algorithm given below:

Step 1. Convert NS to FS using proposition given in section 4.

Step 2. Formulate PDF $f(x_i)$ or $f(x)$ of the random variable X according to discrete case or continuous case respectively from the fuzzy membership function using step 2 of section 4.

Step 3. Find the expectation of X. i.e., $E(X) = \sum_{x_i \in S} x_i f(x_i)$ (discrete case) $= \int_{x \in S} x f(x)dx$ (for continuous case).

Step 4. Deneutrosophicated value $= E(X)$.

5.1. *Examples*

5.1.1. *Example 1*

Let us consider the example (4.1). According to the proposed steps of the algorithm 5, the deneutrosophicated value is $\int_0^{10} x f(x)dx = 5.0816$.

5.1.2. *Example 2*

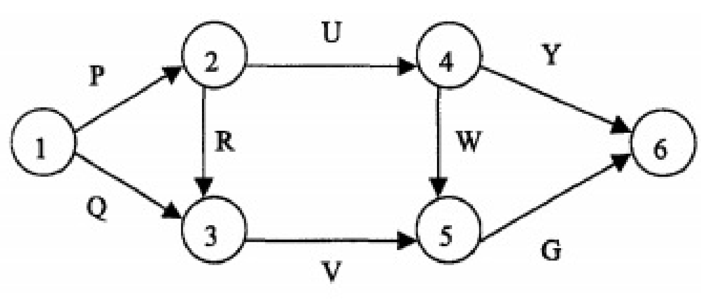

FIGURE 1. Capacitated network

We focus only on the arc $1 - 2$ of the network (fig 1). Suppose flow through the arc is represented by a trapezoidal neutrosophic number P, where $P = ((0.3, 0.4, 0.5, 0.7); 0.5, 0.4, 0.3)$. To find the mean flow through the arc, deneutrosohication is useful.

Then using equation (2) the equivalent fuzzy membership function defined on the interval $[0.3, 0.7]$ is $\mu_P(x)$, where

$$\mu_P(x) = \begin{cases} 1 - \frac{1}{2}[1 - \frac{(x-0.3)0.5}{0.1} + \frac{0.4-x+0.3(x-0.3)}{0.1} & , 0.3 \leq x \leq 0.4 \\ 1 - \frac{1}{2}[1 - 0.5 + 0.4] & , 0.4 \leq x \leq 0.5 \\ 1 - \frac{1}{2}[1 - \frac{(0.7-x)0.5}{0.2} + \frac{(x-0.5)+0.4(0.7-x)}{0.2} & , 0.5 \leq x \leq 0.7 \\ 0 & , otherwise. \end{cases}$$

the corresponding PDF $f_P(x) = \frac{\mu_P(x)}{\Delta}$, where $\Delta = \int_{0.3}^{0.7} \mu_P(x)dx = 0.1375$.
So, the deneutrosophicated value i.e., mean flow through the arc is $\int_{0.3}^{0.7} x f_P(x)dx = 0.48$.

5.1.3. *Some more examples and comparative study*

Following are some more examples shown in Table 3.

TABLE 3. Comparison study on deneutrosohicated values

Different Single valued neutrosophic numbers	*Deneutrosophi-cated value using* [33] *considering* $(a = 0.4, b = 0.3, c = 0.2, d = 0.1)$	*Using* [33] *considering* $(a = 0.6, b = 0.2, c = 0.15, d = 0.05)$	*Using* [33] *considering* $(a = 0.45, b = 0.4, c = 0.05, d = 0.1)$	*Using proposed method*
Triangular				
$\langle(2, 4, 5); 0.3, 0.4, 0.5\rangle$	3.5909	3.6042	3.6458	3.6667
$\langle(1, 2, 4); 0.7, 0.8, 0.9\rangle$	2.4	2.38	2.35	2.33
Trapezoidal				
$\langle(0.3, 0.5, 0.8, 0.9); 0.2, 0.5, 0.8\rangle$	0.6230	0.6220	0.5887	0.6424
$\langle(0.4, 0.6, 0.7, 0.8); 0.5, 0.4, 0.2\rangle$	0.6169	0.6521	0.6222	0.6406
$\langle(1, 2, 5, 6); 0.8, 0.6, 0.4\rangle$	3.5	3.5	3.5	3.5
SVNN with discrete membership				
$\langle 5, (0.6, 0.4, 0.3)\rangle$, $\langle 5.5, (0.8, 0.3, 0.2)\rangle$, $\langle 6, (0.5, 0.3, 0.4)\rangle$	5.51	5.516	5.507	5.519
$\langle 3, (0.7, 0.4, 0.3)\rangle$, $\langle 3.5, (0.6, 0.3, 0.3)\rangle$, $\langle 3.4, (0.7, 0.4, 0.4)\rangle$	3.292	3.291	3.29	3.3

5.1.4. *Results discussion and comparison analysis*

From the Table 3, it is shown that the transformed crisp value of each SVNN using proposed deneutrosophication method is almost similar to the different deneutrosophicated values using existing method [33] for different choices of a, b, c, d. So, the values evaluated by the proposed deneutrosophication are consistent. On the other hand, according to [33], the membership values of the FS obtained from the NS depend on the choice of the parameters a, b, c, d which leads to the fact that for a given NS, there can be different membership values of the fuzzy set and so the different crisp values which is not desirable. Again, in the table, it is seen that the proposed technique is applicable on neutrosohic number with both continuous (Triangular, trapezoidal) and discrete membership grade. In this sense, the proposed deneutrosophication is more robust than the existing [34, 36].

6. Numerical Example

6.1. *Example 1*

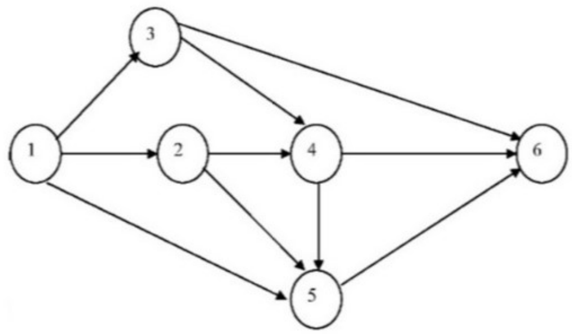

FIGURE 2. Poject Network

Fig. 2 shows a construction project network. The duration of each activity are estimated by three estimators. To find the critical path and expected project completion time, a decision organization is assigned to evaluate the degrees of each estimated activity time. The estimated times and the corresponding degrees given by the organization in single valued neutrosophic forms are given in the table 4

TABLE 4. Estimated activity times

Activity	*Time duration*	*and corresonding*	*degrees*
	Expert I	*Expert II*	*Expert III*
$1-2$	$\langle 8,(0.4,0.3,0.5)\rangle$	$\langle 8.5,(0.7,0.3,0.4)\rangle$	$\langle 9,(0.6,0.4,0.3)\rangle$
$1-3$	$\langle 10,(0.4,0.3,0.2)\rangle$	$\langle 9.7,(0.4,0.2,0.3)\rangle$	$\langle 8.5,(0.2,0.2,0.5)\rangle$
$1-5$	$\langle 13.5,(0.6,0.3,0.2)\rangle$	$\langle 13.33,(0.6,0.1,0.3)\rangle$	$\langle 14,(0.5,0.3,0.2)\rangle$
$2-4$	$\langle 4,(0.3,0.5,0.4)\rangle$	$\langle 5.5,(0.5,0.3,0.2)\rangle$	$\langle 6,(0.5,0.55,0.3)\rangle$
$2-5$	$\langle 5,(0.7,0.3,0.2)\rangle$	$\langle 4.4,(0.5,0.3,0.3)\rangle$	$\langle 4,(0.6,0.3,0.2)\rangle$
$3-4$	$\langle 6,(0.73,0.3,0.3)\rangle$	$\langle 5.65,(0.8,0.3,0.2)\rangle$	$\langle 5,(0.85,0.4,0.3)\rangle$
$4-5$	$\langle 4,(0.26,0.37,0.29)\rangle$	$\langle 4.45,(0.32,0.21,0.46)\rangle$	$\langle 4.8,(0.39,0.34,0.27)\rangle$
$3-6$	$\langle 8,(0.69,0.49,0.20)\rangle$	$\langle 8.6,(0.65,0.40,0.3)\rangle$	$\langle 7.77,(0.75,0.20,0.30)\rangle$
$4-6$	$\langle 7,(0.6,0.4,0.3)\rangle$	$\langle 6.5,(0.8,0.3,0.2)\rangle$	$\langle 5.8,(0.5,0.3,0.4)\rangle$
$5-6$	$\langle 3.7,(0.6,0.45,0.33)\rangle$	$\langle 3.44,(0.55,0.39,0.4)\rangle$	$\langle 4,(0.7,0.3,0.2)\rangle$

Let X be the random variable representing the time duration for the activity $1-2$. Then X can be written as

$X = \{\langle 10,(0.4,0.3,0.5)\rangle, \langle 9.7,(0.7,0.3,0.4)\rangle, \langle 8.5,(0.6,0.4,0.3)\rangle\}$. So, to complete the activity $1-2$, the expected time duration is the deneutrosophicated value $= 8.544$. (Using 5). All the expected time durations for activities are given in the table 5.

Table 5. Expected activity times

Activity	*deneutrosohicated values*
$1-2$	8.54
$1-3$	9.52
$1-5$	13.6
$2-4$	5.25
$2-5$	4.48
$3-4$	5.54
$4-5$	4.44
$3-6$	8.11
$4-6$	6.45
$5-6$	3.73

Now we reach to a project network with crisp duration time for each activity and apply forward pass method as well as backward pass method to find the critical path. Let E_i and L_j denote the earliest occurrence time and latest allowable time corresponding to the i^{th} event.

Forward pass method:

starting time $E_1 = 0$, $E_2 = 0 + t_{12} = 8.54$, $E_3 = 0 + t_{13} = 9.52$, $E_4 = \max\{E_2 + t_{2,4}, E_3 + t_{34}\} = 15.06$.

Similarly, $E_5 = 19.5$, $E_6 = \max\{E_3 + t_{3,6}, E_4 + t_{4,6}, E_5 + t_{5,6}\} = 23.23$.

Backward pass method:

$L_6 = E_6 = 23.23$, $L_5 = L_6 - t_{5,6} = 19.5$, $L_4 = \min\{L_6 - t_{4,6}, L_5 - t_{4,5}\} = 15.06$, similarly, $L_3 = 9.52$, $L_2 = 9.81$, $L_1 = 0$.

So, the critical path is $1-3-4-5-6$ (as $E_1 = L_1$, $E_3 = L_3$, $E_4 = L_4$, $E_5 = L_5$, $E_6 = L_6$) and corresponding project completion time 23.23.

7. Conclusion and future scope

We discuss a deneutrosophication method for both the continuous and discrete cases, establishing a relation among neutrosophic fuzzy set, fuzzy set and probability density function. A well defined similarity measure function is formulated which meaningfully transform neutrosophic data to fuzzy data. The comparative study proves the consistency and effectiveness of the proposed algorithm. The method can be applied in any kinds of problem to deal with neutrosophic information. Various types of decision making methods to get the optimum solution is introduced in different researches but deneutrosophication not only measure the best among several informations but also can evaluate the crisp value of the universe of discourse.

In future our objective is to use this method on transformed neutrosophic covid 19 data of an area and comment on the infected and infective so that corrective measures can be taken to reduce the impact of the epidemic of the area.

Acknowledgment

The author is grateful to Prof. Abdel-Basset and reviewers for their suggestion and constructive comments which has become very helpful to elevate the paper.

References

[1] Zadeh, L.A.; Fuzzy Sets. Information and Control 1965. 8, 338-353.

[2] Atanassov, K.; Intuitionistic Fuzzy sets. Fuzzy Sets and Systems 1986. 87-96.

[3] Smarandache, F.; A unifying field in logics neutrosophy: neutrosophic probability, set and logic, Amer. Res. Press, Rehoboth, USA, 1998.

[4] Ye, J.; Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making. International Journal of Fuzzy Systems 2014. 16, 204-215.

[5] Ye, J.; Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial Intelligence in Medicine 2015. 63, 171-179.

[6] Aydogdu, A.; On Similarity and Entropy of Single Valued Neutrosophic Sets. Journal of Intelligent and Fuzzy Systems 2015. 29, 67-74.

[7] Mandal, K.; Basu, K.; Improved Similarity Measure in Neutrosophic Environment and its Application in Finding Minimum Spanning Tree. Journal of Intelligent and Fuzzy Systems 2016. 31, 1721-1730.

[8] Liu, D.; Liu, G.; Some Similarity Measures of Neutrosophic Sets Based on the Euclidean Distance and Their Application in Medical Diagnosis. Computational and Mathematical Methods in Medicine 2018. 2018, 7325938-7325946.

[9] Ulucay, V.; Kihc, A.; Sahin, M.; Deniz, H.; A New Hybrid Distance-Based Similarity Measure for Refined Neutrosophic Sets and its Application in Medical Diagnosis. Mathematica 2019. 35, 83–96.

[10] Chen, J.; Xue, K.; Ye, J.; Huang, T.; Tian Y.; Hua, C.; Simplified Neutrosophic Exponential Similarity Measures for Evaluation of Smart Port Development. Symmetry 2019. 11, 485-495.

[11] Saqlain, M.; Jafar, N.; Moin, S.; Saeed, M.; and Broumi, S.; Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets. Neutrosophic Sets and Systems 2020. 32, 317-329.

[12] Saeed, M.; Saqlain, M.; Mehmood, A.; Naseer, K.; and Yaqoob, S.; Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making. Neutrosophic Sets and Systems 2020. 33, 183-207.

[13] Abdel-Basset, M.; Ding, B.W.; Mohamed, R.; Metawa, N.; An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries. Risk Management 2020. 1-27. https://doi.org/10.1057/s41283-020-00061-4.

[14] Abdel-Basset, M.; Mohameda, R.; El-Nasser, A.; Zaieda, H.; Gamala, A.; Smarandache, F.; Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In Optimization Theory Based on Neutrosophic and Plithogenic Sets, Ist edition; Florentine Smarandache, Abdel Basset 2020; Academic press. pp. 1-19.

[15] Abdel-Basst, M.; Mohamed, R.; Elhoseny, M.; A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans. Health Informatics Journal 2020. DOI: 10.1177/1460458220952918.

[16] Yasser, I.; Twakol, A.; El-Khalek, A.A.A.; Samrah, A.; Salama, A.A.; COVID-X: Novel Health-Fog Framework Based on Neutrosophic Classifier for Confrontation Covid-19. Neutrosophic Sets and Systems 2020. 35, 1-21.

[17] Chicaiza, V.V.V.; Paspuel, O.G.A.; Yesenia, P.; Neutrosophic Psychology for Emotional Intelligence Analysis in Students of the Autonomous University of Los Andes, Ecuador. Neutrosophic Sets and Systems 2020. 34, 1-8.

[18] Broumi, S.; Deli, I.; Correlation Measure for Neutrosophic Refined Sets and Its Application in Medical Diagnosis. Palestine Journal of Mathematics 2014. 3, 11-19.

[19] Deli, I.; Broumi, S.; Smarandache, F.; On Neutrosophic Refined Sets and their Application in Medical Diagnosis. Journal of new theory 2015. 3, 88-98.

[20] Martin, N.; Smarandache, F.; Pradeepa, I.; Gandhi, N.R.; Pandiamma P.; Exploration of the Factors Causing Autoimmune Diseases using Fuzzy Cognitive Maps with Concentric Neutrosophic Hypergraphic Approach. Neutrosophic Sets and Systems 2020. 35, 232-238.

[21] Samuel, A.E.; Narmadhagnanam, R.; Pi-Distance of Rough Neutrosophic Sets for Medical Diagnosis. Neutrosophic Sets and Systems 2019. 28, 51-57.

[22] Cruzaty, L.E.V.; Tomalá, M.R.; Gallo, C.M.C.; A Neutrosophic Statistic Method to Predict Tax Time Series in Ecuador. Neutrosophic Sets and Systems 2020. 34, 33-39.

[23] Abdel-Basst, M.; Mohamed, R.; Elhoseny, M.; A novel framework to evaluate innovation value proposition for smart product–service systems. Environmental Technology & Innovation 2020. 20, 101036.

[24] Abdel-Basset, M.; Gamal, A.; Son, L.H.; Smarandache, F.; A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. Applied Sciences 2020. 10, 1202.

[25] Ona, X.C.; Burgos, M.J.; Vallejo, G.A.C; Cepeda, M.O.; Neutrosophic Decision Map for critical success factors prioritization in a museum of religious Art. Neutrosophic Sets and Systems 2020. 34, 48-46.

[26] Cruzaty, L.E.V.; Tomalá, M.R.; Gallo, C.M.C.; A Neutrosophic Statistic Method to Predict Tax Time Series in Ecuador. Neutrosophic Sets and Systems 2020. 34, 33-39.

[27] Sudha, A.S.; Gomes, L.F.A.M.; Vijayalakshmi, K.R.; Assessment of MCDM problems by TODIM using aggregated weights. Neutrosophic Sets and Systems 2020. 35, 78-98.

[28] Mandal, K.; Basu, K.; Vector Aggregation Operator and Score Function to solve Multi Criteria Decision Making Method In Neutrosophic Environment. International journal of machine learning and cybernatics 2019. 10, 1373-1383.

[29] Zaied, A.N.H; Ismail, M.; Gamal, A.; AbdelAziz, N.M.; An Integrated Neutrosophic and MOORA for Selecting Machine Tool. Neutrosophic Sets and Systems 2019. 28, 23-33.

[30] Rana, S.; Qayyum, M.; Saeed, M.; Smarandache, F.; Khan, B.A; Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique . Neutrosophic Sets and Systems 2019. 28, 34-50.

[31] Mondal, K.; Pramanik, S.; Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 2014. 6, 28-34.

[32] Pramanik, S.; Dey, P.P.; Giri, B.C.; Smarandache, F.; An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. Neutrosophic Sets and Systems 2017. 17, 20-28.

[33] Wang; Smarandache; Zhang; Sunderraman; Interval neutrosophic sets and logic Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. Phoenix, Arizona:Hexis, 2008.

[34] Chakraborty, A.; Mondal, S.; Broumi, S., De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree. Neutrosophic Sets and Systems 2019. 29, 1-17.

[35] Awang, A.; Aizam, N.A.H; Abdullah, L.; An Integrated Decision-Making Method Based on Neutrosophic Numbers for Investigating Factors of Coastal Erosion. Symmetry 2019. 11, 328.

[36] Broumi, S.; Nagarajan, D.; Bakali, A.; Talea, M.; Smarandache, F.; Lathamaheswari, M.; The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment. Complex and Intelligent Systems 2019. 5, 391-402.

[37] Jun Ye, Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method. Journal of Intelligent Systems 2014. 23, 311-324.

[38] Wang H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R.; Single valued neutrosophic sets. Multispace and Multistructure 2010, 4, 410-413.

[39] Ye, J.; A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent and Fuzzy Systems 2014. 26, 2459-2466.

[40] Deli, I.; Subas, Y.; Single valued neutrosophic numbers and their applications to multicriteria decision making problem. International journal of machine learning and cybernatics 2014. 251, 406-432.

[41] Saqlain, M.; Jafar, N.; Moin, S.; Saeed, M.; Broumi, S.; Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets. Neutrosophic Sets and Systems 2020. 32, 317-329.

[42] Mohana, K.; Mohanasundari, M.; On Some Similarity Measures of Single Valued Neutrosophic Rough Sets. Neutrosophic Sets and Systems 2019. 24, 10-22.

Received: June 2, 2020. Accepted: Nov 25, 2020