



## T-MBJ NEUTROSOPHIC SET UNDER M-SUBALGEBRA

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**Abstract.** In this paper, the idea of T-MBJ neutrosophic set is introduced in which MBJ-neutrosophic set is used to present this new set called T-MBJ neutrosophic set. furthermore the notion of T-MBJ neutrosophic M-subalgebra on  $G$ -algebra is also introduced and provide the conditions for T-MBJ neutrosophic M-subalgebra. The word M in the term M-subalgebra, represents the initial of author's first name Mohsin. We study the T-MBJ neutrosophic set through different characteristics and also prove some results for better understanding of newly define T-MBJ neutrosophic set.

**Keywords:**  $G$ -algebra; T-MBJ neutrosophic set; T-MBJ neutrosophic M-subalgebra.

### 1. Introduction

Smarandacha [3,4] extended the intuitionistic fuzzy set to neutrosophic set. Barbhuiya [22] wrote in detailed about t-intuitionistic fuzzy using the concept of subalgebra and different other characteristics. Takallo et al. [5] extensively explained the BMBJ-neutrosophic subalgebra. Senapati et al. [27] used cubic set and applied it to subalgebras, ideals and closed ideals of  $B$ -algebra. Imai and Iseki [7, 28] defined the  $BCK$ -algebra and  $BCI$ -algebra. Bandaru et al. [21] first time introduced the  $G$ -algebra. Zadeh [8,9] introduced unique sets called fuzzy set and interval-valued fuzzy set. Saeid [1] studied interval-valued fuzzy subalgebra of  $B$ -algebra. Khalid et al. [10] defined T-neutrosophic cubic set and explained this set with important results. Senapati et al. [25] studied  $L$ -fuzzy  $G$ -subalgebra of  $G$ -algebra. Lots of work on  $BG$ -algebras [2] have been done by the researchers [26]. Khalid et al. [11] investigated neutrosophic soft cubic subalgebra. Khalid et al. [12] studied translation and multiplication of intuitionistic fuzzy set through some theorems. Khalid et al. [13] first time done the magnification of translation of set MBJ-neutrosophic and proved the results to explain the magnification. Khalid et al. [14]

studied MBJ-neutrosophic T-ideal on B-algebra. Khalid et al. [15] wrote the extensively important results for multiplicative interpretation of neutrosophic cubic set. Takallo et al. [16] discussed the application of MBJ-neutrosophic set. Neggers et al. [6] studied the fundamental theorem of  $B$ -homomorphism for  $B$ -algebra. Biswas [23] investigated the membership function of interval valued fuzzy set. Ahn [24] proved different results for fuzzy subalgebra. Jun et al. [29] deeply studied neutrosophic cubic set with different characteristics. Basset et. al [17] done the detailed study on hybrid neutrosophic multiple criteria group decision making approach for project selection. Hemavati et. al [20] investigated the  $\beta$ -subalgebra using the interval valed fuzzy set. Surya et. al [18] worked on MBJ neutrsophic  $\beta$ -subalgebra. Basset et. al [19] worked on novel group decision making model based on neutrsophic set for the heart disease.

This paper is presented to define the T-MBJ neutrosophic set and provide the condition for T-MBJ neutrosophic set [T-MBJ NS] to be a T-MBJ neutrosophic M-subalgebra on  $G$ -algebra. We also investigate some properties and proved some results for T-MBJ neutrosophic M-subalgebra [T-MBJ NMSU].

## 2. Preliminaries

Here, some basic definitions are written that are helpful to present this paper.

**Definition 2.1.** A nonempty set  $Y$  with a constant  $0$  and a binary operation  $*$  is said to be  $G$ -algebra [21] if it fulfills these axioms:

$$G1: t_1 * t_1 = 0$$

$$G2: t_1 * (t_1 * t_2) = t_2, \text{ for all } t_1, t_2 \in Y.$$

A  $G$ -algebra is denoted by  $(Y, *, 0)$ .

**Definition 2.2.** A nonempty set  $Y$  with a constant  $0$  and a binary operation  $*$  is said to be  $B$ -algebra [6] if it fulfills these axioms:

$$B1: t_1 * t_1 = 0$$

$$B2: t_1 * 0 = t_1 \quad B2: (t_1 * t_2) * t_3 = t_1 * (t_3(0 * t_2)), \text{ for all } t_1, t_2, t_3 \in Y.$$

**Definition 2.3.** Let  $S$  be a subset of  $G$ -algebra is called a subalgebra [21] of  $Y$  if  $t_1 * t_2 \in S$   $\forall t_1, t_2 \in S$ .

**Definition 2.4.** Function  $f | Y \rightarrow X$  of  $B$ -algebra is called homomorphic [6] if  $f(t_1 * t_2) = f(t_1) * f(t_2) \forall t_1, t_2 \in Y$ . If  $f | Y \rightarrow X$  is a  $B$ -homomorphic, then  $f(0) = 0$ .

**Definition 2.5.** Let  $C$  be a fuzzy set [8] in  $Y$  is defined as  $C = \{ \langle t_1, \vartheta_C(t_1) \rangle \mid t_1 \in Y \}$ , where  $\vartheta_C(t_1)$  is called the existence ship value of  $t_1$  in  $C$  and  $\vartheta_C(t_1) \in [0, 1]$ .

For a fuzzy set's family  $C_i = \{ \langle t_1, \vartheta_{C_i}(t_1) \rangle \mid t_1 \in Y \}$  in  $Y$ , where  $i \in H$  and  $H$  is index set, Join ( $\vee$ ) and meet ( $\wedge$ ) are defined as follow:

$$\vee_{i \in H} C_i = (\vee_{i \in H} \vartheta_{C_i})(t_1) = \sup\{\vartheta_{C_i} \mid i \in H\},$$

and

$$\wedge_{i \in H} C_i = (\wedge_{i \in H} \vartheta_{C_i})(t_1) = \inf\{\vartheta_{C_i} \mid i \in H\}$$

respectively,  $\forall t_1 \in Y$ .

**Definition 2.6.** [23] Let two elements  $D_1, D_2 \in D[0, 1]$ . If  $D_1 = [(t_1)_1^-, (t_1)_1^+]$  and  $D_2 = [(t_1)_2^-, (t_1)_2^+]$ , then  $rmax(D_1, D_2) = [max((t_1)_1^-, (t_1)_2^-), max((t_1)_1^+, (t_1)_2^+)]$  which is denoted by  $D_1 \vee^r D_2$  and  $rmin(D_1, D_2) = [min((t_1)_1^-, (t_1)_2^-), min((t_1)_1^+, (t_1)_2^+)]$  which is denoted by  $D_1 \wedge^r D_2$ . Thus, if  $D_i = [((t_1)_1)_i^-, ((t_1)_2)_i^+] \in D[0, 1]$  for  $i = 1, 2, 3, \dots$ , then they defined  $rsup_i(D_i) = [sup_i(((t_1)_1)_i^-), sup_i(((t_1)_1)_i^+)]$ , i.e.,  $\vee_i^r D_i = [\vee_i(((t_1)_1)_i^-), \vee_i(((t_1)_1)_i^+)]$ . In the same way they defined  $rinf_i(D_i) = [inf_i(((t_1)_1)_i^-), inf_i(((t_1)_1)_i^+)]$ , i.e.,  $\wedge_i^r D_i = [\wedge_i(((t_1)_1)_i^-), \wedge_i(((t_1)_1)_i^+)]$ . Now they called  $D_1 \geq D_2 \iff (t_1)_1^- \geq (t_1)_2^-$  and  $(t_1)_1^+ \geq (t_1)_2^+$ . Similarly they defined the relations  $D_1 \leq D_2$  and  $D_1 = D_2$ .

Ahn et al. [24] defined fuzzy subalgebra, which is defined below.

**Definition 2.7.** A nonempty set  $C = \{ \langle t_1, \vartheta_C(t_1) \rangle \mid t_1 \in Y \}$  is called a fuzzy subalgebra [24] of  $Y$  if  $\vartheta_C(t_1 * t_2) \geq \min\{\vartheta_C(t_1), \vartheta_C(t_2)\} \forall t_1, t_2 \in Y$ .

**Definition 2.8.** For any  $C_i = (\rho_i, \lambda_i)$  [29] where  $\rho_i = \{ \langle t_1; \rho_{iE}(t_1), \rho_{iI}(t_1), \rho_{iN}(t_1) \rangle \mid t_1 \in Y \}$ ,  $\lambda_i = \{ \langle t_1; \lambda_{iE}(t_1), \lambda_{iI}(t_1), \lambda_{iN}(t_1) \rangle \mid t_1 \in Y \}$  for  $i \in H$ , P-union, P-inersection, R-union and R-intersection is defined respectively by **P-union**  $\cup_P C_i = (\cup_{i \in H} \rho_i, \vee_{i \in H} \lambda_i)$ , **P-intersection**  $\cap_P C_i = (\cap_{i \in H} \rho_i, \wedge_{i \in H} \lambda_i)$ , **R-union**  $\cup_R C_i = (\cup_{i \in H} \rho_i, \wedge_{i \in H} \lambda_i)$ , **R-intersection:**  $\cap_R C_i = (\cap_{i \in H} \rho_i, \vee_{i \in H} \lambda_i)$ , where

$$\begin{aligned} \cup_{i \in H} \rho_i &= \{ \langle t_1; (\cup_{i \in H} \rho_{iE})(t_1), (\cup_{i \in H} \rho_{iI})(t_1), (\cup_{i \in H} \rho_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \vee_{i \in H} \lambda_i &= \{ \langle t_1; (\vee_{i \in H} \lambda_{iE})(t_1), (\vee_{i \in H} \lambda_{iI})(t_1), (\vee_{i \in H} \lambda_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \cap_{i \in H} \rho_i &= \{ \langle t_1; (\cap_{i \in H} \rho_{iE})(t_1), (\cap_{i \in H} \rho_{iI})(t_1), (\cap_{i \in H} \rho_{iN})(t_1) \rangle \mid t_1 \in Y \}, \\ \wedge_{i \in H} \lambda_i &= \{ \langle t_1; (\wedge_{i \in H} \lambda_{iE})(t_1), (\wedge_{i \in H} \lambda_{iI})(t_1), (\wedge_{i \in H} \lambda_{iN})(t_1) \rangle \mid t_1 \in Y \}. \end{aligned}$$

**Definition 2.9.** Let  $B = (\vartheta_B, \nu_B)$  be an IFS of BG-algebra  $Y$  and  $t \in [0, 1]$ , then the IFS  $B^t$  is said to be t-intuitionistic fuzzy subset [1] of  $Y$  w.r.t  $B$  and is defined as  $B^t = \{ \langle t_1, \vartheta_{B^t}(t_1), \nu_{B^t}(t_1) \rangle \mid t_1 \in Y \} = \langle \vartheta_{B^t}, \nu_{B^t} \rangle$ , where  $\vartheta_{B^t}(t_1) = \min\{\vartheta_B(t_1), t\}$  and  $\nu_{B^t}(t_1) = \max\{\nu_B(t_1), 1 - t\} \forall t_1 \in Y$ .

**Definition 2.10.** Let  $B^t = (\vartheta_{B^t}, \nu_{B^t})$  be a t-IFS of BG-algebra  $Y$  and  $t \in [0, 1]$  then  $B^t$  is said to be t-IFSU [23] of  $Y$  if it fulfills these axioms.

- (i)  $\vartheta_{B^t}(t_1 * t_2) \geq \min\{\vartheta_{B^t}(t_1), \vartheta_{B^t}(t_2)\}$  and
- (ii)  $\nu_{B^t}(t_1 * t_2) \leq \max\{\nu_{B^t}(t_1), \nu_{B^t}(t_2)\} \forall t_1, t_2 \in Y$ .

**Definition 2.11.** A MBJ-neutrosophic set [16] in  $Y$  is a structure of the form  $C = \{\langle M_C t_1, \hat{B}_C t_1, J_C t_1 \rangle \mid t_1 \in Y\}$ , where  $M_C$  and  $J_C$  are fuzzy sets in  $Y$  and  $M_C$  is a truth membership function,  $J_C$  is a false membership function and  $\hat{B}$  is an indeterminate interval valued membership function.

### 3. T-MBJ Neutrosophic M-Subalgebras

**Definition 3.1.** A nonempty set of the form  $C^t = (M^t, \hat{B}^t, J^t)$  is called a T-MBJ neutrosophic set (**T-MBJ NS**) of  $Y$ , where  $C^t = \{\langle t_1, M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle \mid t_1 \in Y\} = \langle M^t, \hat{B}^t, J^t \rangle$  with two independent components is defined as  $M^t(t_1) = \{\min(M, t)(t_1)\}$ ,  $\hat{B}^t = \{rmin(\hat{B}, t')(t_1)\}$  and  $J^t(t_1) = \{\max(J, 2 - t - t')(t_1)\} \forall t, t', 2 - t - t' \in [0, 1]$ , where  $M^t$  is truth membership function,  $\hat{B}$  is an indeterminate interval valued membership function and  $J^t$  is a false membership function.

**Definition 3.2.** Let  $C^t = (M^t, \hat{B}, J^t)$  be a T-MBJ neutrosophic set. Then  $C^t$  is T-MBJ NMSU under binary operation  $*$ , where  $t_1, t_2, t, t', 2 - t - t', \aleph, \Re \in [0, 1]$  if it satisfies the following three conditions: N1:

$$\begin{aligned} \min(M((t_1 * \aleph) * (t_2 * \Re)), t) &= M^t((t_1 * \aleph) * (t_2 * \Re)) \succeq \min\{M^t(t_1 * \aleph), M^t(t_2 * \Re)\} \\ \min(\hat{B}((t_1 * \aleph) * (t_2 * \Re)), t') &= \hat{B}^t((t_1 * \aleph) * (t_2 * \Re)) \succeq rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \Re)\} \\ \min(J((t_1 * \aleph) * (t_2 * \Re)), 2 - t - t') &= J^t((t_1 * \aleph) * (t_2 * \Re)) \preceq \max\{J^t(t_1 * \aleph), J^t(t_2 * \Re)\}. \end{aligned}$$

For our simplicity we replace the  $2 - t - t'$  with  $\Im$ .

**Example 3.3.** Let  $Y = \{0, t_1 * \aleph, t_2 * \Re\}$  be a  $G$ -algebra with the following Cayley table.

$*$	0	$t_1 * \aleph$	$t_2 * \Re$
0	0	$t_1 * \aleph$	$t_2 * \Re$
$t_1 * \aleph$	$t_1 * \aleph$	0	$t_2 * \Re$
$t_2 * \Re$	$t_2 * \Re$	$t_1 * \aleph$	0

A T-MBJ neutrosophic set  $C^t = (M^t, \hat{B}, J^t)$  of  $X$  is defined by

$M^t_T$	0	$t_1 * \aleph$	$t_2 * \Re$	$\hat{B}^t$	[0.1,0.2]	[0.3,0.5]	[0.6,0.9]	$J^t$	0.2	0.4	0.8
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It is the routine work to check that set is T-MBJ neutrosophic M-subalgebra.

**Proposition 3.4.** Let  $C^t = \{\langle t_1, M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle\}$  be a T-MBJ NMSU of  $Y$ , then  $\forall t_1 \in Y, M^t(0 * \aleph) \succeq M^t(t_1 * \aleph), \hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph)$  and  $J^t(0 * \aleph) \preceq J^t(t_1 * \aleph)$ . Thus,  $M^t(0 * \aleph), \hat{B}^t(0 * \aleph)$  and  $J^t(0 * \aleph)$  are the upper bounds and lower bounds of  $M^t(t_1 * \aleph), \hat{B}^t(t_1 * \aleph)$  and  $J^t(t_1 * \aleph)$  respectively.

*Proof.*  $\forall t_1 \in Y$ , we have  $M^t((0 * \aleph)) = \min(M((0 * \aleph)), t) = \min(M((t_1 * \aleph) * (t_1 * \aleph)), t) \succeq \min\{\min(M((t_1 * \aleph)), t), \min(M(t_1 * \aleph), t)\} = \min(M(t_1 * \aleph), t) = M^t((t_1 * \aleph)) \Rightarrow M^t((0 * \aleph)) \succeq M^t((t_1 * \aleph))$ ,  $\hat{B}^t(0 * \aleph) = r\min(\hat{B}(0 * \aleph), t) = r\min(\hat{B}((t_1 * \aleph) * (t_1 * \aleph)), t) \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(t_1 * \aleph), t)\} = r\min(\hat{B}(t_1 * \aleph), t) = \hat{B}^t(t_1 * \aleph) \Rightarrow \hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph)$  and  $\max(J(0 * \aleph), \Im) = \max(J((t_1 * \aleph) * (t_1 * \aleph)), \Im) \succeq \max\{\max(J(t_1 * \aleph), \Im), \max(J(t_1 * \aleph), \Im)\} = \max(J(t_1 * \aleph), t) = J^t(t_1 * \aleph) \Rightarrow J^t(0 * \aleph) \preceq J^t(t_1 * \aleph)$ .  $\square$

**Theorem 3.5.** Let  $\mathcal{C}^t = \{\langle (t_1), M^t(t_1), \hat{B}^t(t_1), J^t(t_1) \rangle\}$  be a T-MBJ NMSU of  $Y$ . If there exists a sequence  $\{(t_1 * \aleph)_n\}$  of  $Y$  such that  $\lim_{n \rightarrow \infty} M^t((t_1 * \aleph)_n) = 0$ ,  $\lim_{n \rightarrow \infty} \hat{B}^t((t_1 * \aleph)_n) = [1, 1]$  and  $\lim_{n \rightarrow \infty} J^t((t_1 * \aleph)_n) = 0$ . Then  $M^t(0) = 0$ ,  $\hat{B}^t(0) = [1, 1]$  and  $J^t(0) = 0$ .

*Proof.* Using Proposition 3.4,  $M^t(0 * \aleph) \succeq M^t(t_1 * \aleph) \forall t_1 \in Y$ , then  $M^t(0 * \aleph) \succeq M^t((t_1 * \aleph)_n)$  for  $n \in \mathbf{Z}^+$ . Consider,  $0 \succeq M^t(0 * \aleph) \succeq \lim_{n \rightarrow \infty} M^t((t_1 * \aleph)_n) = 0$ . Hence,  $M^t(0 * \aleph) = 0$ . Using Proposition 3.4,  $\hat{B}^t(0 * \aleph) \succeq \hat{B}^t(t_1 * \aleph) \forall t_1 \in Y$ , so therefore  $\hat{B}^t(0 * \aleph) \succeq \hat{B}^t((t_1 * \aleph)_n)$  for  $n \in \mathbf{Z}^+$ . Consider,  $[1, 1] \succeq \hat{B}^t(0 * \aleph) \succeq \lim_{n \rightarrow \infty} \hat{B}^t((t_1 * \aleph)_n) = [1, 1]$ . Hence,  $\hat{B}^t(0 * \aleph) = [1, 1]$ . Again, using Proposition 3.4,  $J^t(0 * \aleph) \preceq J^t(t_1 * \aleph) \forall t_1 \in Y$ , so therefore  $J^t(0 * \aleph) \preceq J^t((t_1 * \aleph)_n)$  for  $n \in \mathbf{Z}^+$ . Consider,  $0 \preceq J^t(0 * \aleph) \preceq \lim_{n \rightarrow \infty} J^t((t_1 * \aleph)_n) = 0$ . Hence,  $J^t(0 * \aleph) = 0$ .  $\square$

**Theorem 3.6.** The R-intersection of any set of T-MBJ NMSU of  $Y$  is also a T-MBJ NMSU of  $Y$ .

*Proof.* Let  $\mathcal{C}_i^t = \{\langle t_1, M_i^t, \hat{B}_i^t, J_i^t \rangle \mid t_1 \in Y\}$  where  $i \in k$ , be a set of T-MBJ NMSU of  $Y$  and  $t_1, t_2 \in Y$  and  $t, \aleph, \mathfrak{R} \in [0, 1]$ . Then

$$\begin{aligned} (\vee(M_i^t)_i)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= \vee(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \sup(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &\succeq \sup\{\min\{\min(M_i, t)(t_1 * \aleph), \min(M_i, t)(t_2 * \mathfrak{R})\}\} \\ &= \min\{\sup(\min(M_i, t)(t_1 * \aleph)), \sup(\min(M_i, t)(t_2 * \mathfrak{R}))\} \\ &= \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_2 * \mathfrak{R})\} \\ &= \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_2 * \mathfrak{R})\} \\ \Rightarrow \vee(M_i^t)_i((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_2 * \mathfrak{R})\} \end{aligned}$$

and

$$\begin{aligned}
 (\cap(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \cap(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\
 &= \text{rinf}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\
 &\succeq \text{rinf}\{\text{rmin}\{\text{rmin}(\hat{B}_i, t')(t_1 * \aleph), (\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\}\} \\
 &= \text{rmin}\{\text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\} \\
 &= \text{rmin}\{\text{rinf}(\hat{B}_i^t)(t_1 * \aleph), \text{rinf}(\hat{B}_i^t)(t_1 * \aleph)\} \\
 &= \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_1 * \aleph)\} \\
 \Rightarrow \cap(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_1 * \aleph)\},
 \end{aligned}$$

and

$$\begin{aligned}
 (\vee(J^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \vee(\text{max}(J_i, \mathfrak{S})((t_1 * \aleph) * (t_2 * \aleph))) \\
 &= \text{sup}(\text{max}(J_i, \mathfrak{S})((t_1 * \aleph) * (t_2 * \aleph))) \\
 &\preceq \text{sup}\{\text{max}\{(\text{max}(J_i, \mathfrak{S})(t_1 * \aleph)), (\text{max}(J_i, \mathfrak{S})(t_1 * \aleph))\}\} \\
 &= \text{max}\{\text{sup}(\text{max}(J_i, \mathfrak{S})(t_1 * \aleph)), \text{sup}(\text{max}(J_i, \mathfrak{S})(t_1 * \aleph))\} \\
 &= \text{max}\{\text{sup}(J_i^t)(t_1 * \aleph), \text{sup}(J_i^t)(t_1 * \aleph)\} \\
 &= \text{max}\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\} \\
 \Rightarrow \vee(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \text{max}\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\},
 \end{aligned}$$

which show that  $R$ -intersection of  $\mathcal{C}_i^t$  is a T-MBJ NMSU of  $Y$ .  $\square$

**Theorem 3.7.** Let  $\mathcal{C}_i^t = \{\langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$  be a set of T-MBJ NMSU of  $Y$ , where  $i \in k$  and  $t \in [0, 1]$ . If  $\text{inf}\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \aleph)\}\} = \min\{\text{inf}(M_i^t)(t_1 * \aleph), \text{inf}(M_i^t)(t_1 * \aleph)\}$  and  $\text{inf}\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\text{inf}(J_i^t)(t_1 * \aleph), \text{inf}(J_i^t)(t_1 * \aleph)\} \forall t_1 \in Y$ , then  $P$ -intersection of  $\mathcal{C}_i^t$  is also a T-MBJ NMSU of  $Y$ .

*Proof.* Suppose that  $\mathcal{C}_i^t = \{\langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$  where  $i \in k$ , is a family of sets of T-MBJ NMSU of  $Y$  such that  $\text{inf}\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \aleph)\}\} = \min\{\text{inf}(M_i^t)(t_1 * \aleph), \text{inf}(M_i^t)(t_1 * \aleph)\}$  and  $\text{inf}\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\text{inf}(J_i^t)(t_1 * \aleph), \text{inf}(J_i^t)(t_1 * \aleph)\}$

$\aleph\}} \forall t_1, t_2 \in Y$  and  $t \in [0, 1]$ . Then

$$\begin{aligned} (\wedge(M^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \wedge(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \inf(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \inf\{\min\{(\min(M_i, t)(t_1 * \aleph)), (\min(M_i, t)(t_2 * \aleph))\}\} \\ &= \min\{\inf(\min(M_i, t)(t_1 * \aleph)), \inf(\min(M_i, t)(t_2 * \aleph))\} \\ &= \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_2 * \aleph)\} \\ &= \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_2 * \aleph)\} \\ \Rightarrow \wedge(M_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_2 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\cap(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \cap(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \text{rinf}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \text{rinf}\{\text{rmin}\{(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), (\text{rmin}(\hat{B}_i, t')(t_2 * \aleph))\}\} \\ &= \text{rmin}\{\text{rinf}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rinf}(\text{rmin}(\hat{B}_i, t')(t_2 * \aleph))\} \\ &= \text{rmin}\{\text{rinf}(\hat{B}_i^t)(t_1 * \aleph), \text{rinf}(\hat{B}_i^t)(t_2 * \aleph)\} \\ &= \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_2 * \aleph)\} \\ \Rightarrow \cap(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \text{rmin}\{\cap(\hat{B}_i^t)(t_1 * \aleph), \cap(\hat{B}_i^t)(t_2 * \aleph)\}, \end{aligned}$$

and

$$\begin{aligned} (\wedge(J^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \wedge(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \inf(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\preceq \inf\{\max\{(\max(J_i, \Im)(t_1 * \aleph)), (\max(J_i, \Im)(t_2 * \aleph))\}\} \\ &= \max\{\inf(\max(J_i, \Im)(t_1 * \aleph)), \inf(\max(J_i, \Im)(t_2 * \aleph))\} \\ &= \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_2 * \aleph)\} \\ &= \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\} \\ \Rightarrow \wedge(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \aleph)\}, \end{aligned}$$

which show that  $P$ -intersection of  $C_i^t$  is a T-MBJ NMSU of  $Y$ .  $\square$

**Theorem 3.8.** Let  $C_i^t = \{(t_1, (M_i^t), (\hat{B}_i^t), (J_i^t)) \mid t_1 \in Y\}$  where  $i \in k$ , be a family of sets of T-MBJ NMSU of  $Y$ . If  $\sup\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_2 * \aleph)\}\} = \min\{\sup(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \aleph)\}$  and  $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_2 * \aleph)\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_2 * \aleph)\}$

and  $\sup\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \forall t_1, t_2 \in Y$ , then  $P$ -union of  $\mathcal{C}_i^t$  is also a  $T$ -MBJ NMSU of  $Y$ .

*Proof.* Let  $\mathcal{C}_i^t = \{\langle t_1, (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y\}$  where  $i \in k$ , be a family of sets of  $T$ -MBJ NMSU of  $Y$  such that  $\sup\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \aleph)\}\} = \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_1 * \aleph)\}$  and  $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \aleph)\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \aleph)\}$  and  $\sup\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \aleph)\}\} = \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \forall t_1, t_2 \in Y$ . Then for  $t_1, t_2 \in Y$ , and  $t \in [0, 1]$ .

$$\begin{aligned} (\vee(M^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \vee(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \sup(\min(M_i, t)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \sup\{\min\{(\min(M_i, t)(t_1 * \aleph)), (\min(M_i, t)(t_1 * \aleph))\}\} \\ &= \min\{\sup(\min(M_i, t)(t_1 * \aleph)), \sup(\min(M_i, t)(t_1 * \aleph))\} \\ &= \min\{\sup(M_i^t)(t_1 * \aleph), \sup(M_i^t)(t_1 * \aleph)\} \\ &= \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_1 * \aleph)\} \\ \Rightarrow \vee(M_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \min\{\vee(M_i^t)(t_1 * \aleph), \vee(M_i^t)(t_1 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\cup(\hat{B}^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \cup(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \text{rsup}(\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \aleph))) \\ &\succeq \text{rsup}\{\text{rmin}\{(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), (\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\}\} \\ &= \text{rmin}\{\text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph))\} \\ &= \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \aleph)\} \\ &= \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \aleph)\} \\ \Rightarrow \cup(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\succeq \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \aleph)\} \end{aligned}$$

and

$$\begin{aligned} (\vee(J^t)_i)((t_1 * \aleph) * (t_2 * \aleph)) &= \vee(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &= \sup(\max(J_i, \Im)((t_1 * \aleph) * (t_2 * \aleph))) \\ &\preceq \sup\{\max\{(\max(J_i, \Im)(t_1 * \aleph)), (\max(J_i, \Im)(t_1 * \aleph))\}\} \\ &= \max\{\sup(\max(J_i, \Im)(t_1 * \aleph)), \sup(\max(J_i, \Im)(t_1 * \aleph))\} \\ &= \max\{\sup(J_i^t)(t_1 * \aleph), \sup(J_i^t)(t_1 * \aleph)\} \\ &= \max\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\} \\ \Rightarrow \vee(J_i^t)((t_1 * \aleph) * (t_2 * \aleph)) &\preceq \max\{\vee(J_i^t)(t_1 * \aleph), \vee(J_i^t)(t_1 * \aleph)\}, \end{aligned}$$



which show that  $P$ -union of  $\mathcal{C}_i^t$  is a T-MBJ NMSU of  $Y$ .  $\square$

**Theorem 3.9.** Let  $\mathcal{C}_i^t = \{ \langle t_1, (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y \}$  where  $i \in k$ , be a family of sets of T-MBJ NMSU of  $Y$ . If  $\inf\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \mathfrak{R})\}\} = \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \mathfrak{R})\}$  and  $\inf\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \mathfrak{R})\}\} = \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_1 * \mathfrak{R})\}$  and  $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \mathfrak{R})\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \forall t_1 \in Y$ , and  $t \in [0, 1]$  then  $R$ -union of  $\mathcal{C}_i^t$  is also a T-MBJ NMSU of  $Y$ .

*Proof.* Let  $\mathcal{C}_i^t = \{ \langle t_1, (M_i^t), (\hat{B}_i^t), (J_i^t) \rangle \mid t_1 \in Y \}$  where  $i \in k$ , and  $t \in [0, 1]$  be a family of sets of T-MBJ NMSU of  $Y$  such that  $\inf\{\min\{(M_i^t)(t_1 * \aleph), (M_i^t)(t_1 * \mathfrak{R})\}\} = \min\{\inf(M_i^t)(t_1 * \aleph), \inf(M_i^t)(t_1 * \mathfrak{R})\}$  and  $\inf\{\max\{(J_i^t)(t_1 * \aleph), (J_i^t)(t_1 * \mathfrak{R})\}\} = \max\{\inf(J_i^t)(t_1 * \aleph), \inf(J_i^t)(t_1 * \mathfrak{R})\}$  and  $\text{rsup}\{\text{rmin}\{(\hat{B}_i^t)(t_1 * \aleph), (\hat{B}_i^t)(t_1 * \mathfrak{R})\}\} = \text{rmin}\{\text{rsup}(\hat{B}_i^t)(t_1 * \aleph), \text{rsup}(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \forall t_1 \in Y$ , and  $t \in [0, 1]$ . Then for  $t_1, t_2 \in Y$  and  $t \in [0, 1]$ .

$$\begin{aligned} (\wedge(M_i^t))((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= (\wedge(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \inf\{(\min(M_i, t)((t_1 * \aleph) * (t_2 * \mathfrak{R})))\} \\ &\succeq \inf\{\min\{\min(M_i, t)(t_1 * \aleph), \min(M_i, t)(t_1 * \mathfrak{R})\}\} \\ &= \min\{\inf(\min(M_i, t)(t_1 * \aleph)), \inf(\min(M_i, t)(t_1 * \mathfrak{R}))\} \\ &= \min\{\inf((M_i^t)(t_1 * \aleph)), \inf((M_i^t)(t_1 * \mathfrak{R}))\} \\ &= \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_1 * \mathfrak{R})\} \\ \Rightarrow \wedge(M_i^t)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \min\{\wedge(M_i^t)(t_1 * \aleph), \wedge(M_i^t)(t_1 * \mathfrak{R})\}, \end{aligned}$$

and

$$\begin{aligned} (\cup(\hat{B}_i^t))((t_1 * \aleph) * (t_2 * \mathfrak{R})) &= (\cup(\text{rmin}(\hat{B}_i, t'))((t_1 * \aleph) * (t_2 * \mathfrak{R}))) \\ &= \text{rsup}\{\text{rmin}(\hat{B}_i, t')((t_1 * \aleph) * (t_2 * \mathfrak{R}))\} \\ &\succeq \text{rsup}\{\text{rmin}\{\text{rmin}(\hat{B}_i, t')(t_1 * \aleph), \text{rmin}(\hat{B}_i, t')(t_1 * \mathfrak{R})\}\} \\ &= \text{rmin}\{\text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \aleph)), \text{rsup}(\text{rmin}(\hat{B}_i, t')(t_1 * \mathfrak{R}))\} \\ &= \text{rmin}\{\text{rsup}((\hat{B}_i^t)(t_1 * \aleph)), \text{rsup}((\hat{B}_i^t)(t_1 * \mathfrak{R}))\} \\ &= \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \mathfrak{R})\} \\ \Rightarrow \cup(\hat{B}_i^t)((t_1 * \aleph) * (t_2 * \mathfrak{R})) &\succeq \text{rmin}\{\cup(\hat{B}_i^t)(t_1 * \aleph), \cup(\hat{B}_i^t)(t_1 * \mathfrak{R})\}, \end{aligned}$$

and

$$\begin{aligned}
 (\wedge(J_i^t))((t_1 * \aleph) * (t_2 * \ale�)) &= (\wedge(\max(J_i, \mathfrak{S}))((t_1 * \aleph) * (t_2 * \ale�))) \\
 &= \inf\{\max(J_i, \mathfrak{S})((t_1 * \aleph) * (t_2 * \ale�))\} \\
 &\preceq \inf\{\max\{\max(J_i, \mathfrak{S})(t_1 * \aleph), \max(J_i, \mathfrak{S})(t_2 * \ale�)\}\} \\
 &= \max\{\inf(\max(J_i, \mathfrak{S})(t_1 * \aleph)), \inf(\max(J_i, \mathfrak{S})(t_2 * \ale�))\} \\
 &= \max\{\inf((J_i^t)(t_1 * \aleph)), \inf((J_i^t)(t_2 * \ale�))\} \\
 &= \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \ale�)\} \\
 \Rightarrow \wedge(J_i^t)((t_1 * \aleph) * (t_2 * \ale�)) &\preceq \max\{\wedge(J_i^t)(t_1 * \aleph), \wedge(J_i^t)(t_2 * \ale�)\},
 \end{aligned}$$

which show that  $R$ -union of  $\mathcal{C}_i^t$  is a T-MBJ NMSU of  $Y$ .  $\square$

**Proposition 3.10.** *If a T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  is a TMBJ-neutrosophic  $M$ -subalgebra, then  $\forall t_1 \in Y, M^t(0 * t_1) \succeq M^t(t_1 * \aleph)$  and  $\hat{B}^t(0 * t_1) \succeq \hat{B}^t(t_1 * \aleph)$  and  $J^t(0 * t_1) \preceq J^t(t_1 * \aleph)$ .*

*Proof.*  $\forall t_1 \in Y, M^t(0 * t_1) = \min(M(0 * t_1), t) \succeq \min\{\min(M(0), t), \min(M(t_1 * \aleph), t)\} = \min\{\min(M(t_1 * \aleph), t), \min(M(t_1 * \aleph), t)\} \succeq \min\{\min\{\min(M(t_1 * \aleph), t), \min(M(t_1 * \aleph), t)\}, \min(M(t_1 * \aleph), t)\} = \min(M(t_1 * \aleph), t) = M^t(t_1 * \aleph)$  and  $\hat{B}^t(0 * t_1) = \text{rmin}(\hat{B}(0 * t_1), t') \succeq \text{rmin}\{\text{rmin}(\hat{B}(0), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\} = \text{rmin}\{\text{rmin}(\hat{B}(t_1 * \aleph), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\} \succeq \text{rmin}\{\text{rmin}\{\text{rmin}(\hat{B}(t_1 * \aleph), t'), \text{rmin}(\hat{B}(t_1 * \aleph), t')\}, \text{rmin}(\hat{B}(t_1 * \aleph), t')\} = \text{rmin}(\hat{B}(t_1 * \aleph), t') = \hat{B}^t(t_1 * \aleph)$  and  $J^t(0 * t_1) = \max(J(0 * t_1), \mathfrak{S}) \preceq \max\{\max(J(0), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\} = \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\} \preceq \max\{\max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_1 * \aleph), \mathfrak{S})\}, \max(J(t_1 * \aleph), \mathfrak{S})\} = \max(J(t_1 * \aleph), \mathfrak{S}) = J^t(t_1 * \aleph)$   $\square$

**Lemma 3.11.** *If a T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  is a T-MBJ neutrosophic  $M$ -subalgebra, then  $\mathcal{C}^t((t_1 * \aleph) * (t_2 * \ale�)) = \mathcal{C}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \ale�)))) \forall t_1, t_2 \in Y$ .*

*Proof.* Let  $Y$  be a  $G$ -algebra and  $t_1, t_2 \in Y$ . Then we know by lemma that  $t_2 * \ale� = 0 * (0 * (t_2 * \ale�))$ . Hence,  $M^t((t_1 * \aleph) * (t_2 * \ale�)) = M^t((t_1 * \aleph) * (0 * (0 * (t_2 * \ale�))))$  and  $\hat{B}^t((t_1 * \aleph) * (t_2 * \ale�)) = \hat{B}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \ale�))))$  and  $J^t((t_1 * \aleph) * (t_2 * \ale�)) = J^t((t_1 * \aleph) * (0 * (0 * (t_2 * \ale�))))$ . Therefore,  $\mathcal{C}^t((t_1 * \aleph) * (t_2 * \ale�)) = \mathcal{C}^t((t_1 * \aleph) * (0 * (0 * (t_2 * \ale�))))$   $\square$

**Proposition 3.12.** *If T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  is a T-MBJ NMSU, then  $\forall t_1, t_2 \in Y, M^t((t_1 * \aleph) * (0 * (t_2 * \ale�))) \succeq \min\{M^t(t_1 * \aleph), M^t(t_2 * \ale�)\}$  and  $\hat{B}^t((t_1 * \aleph) * (0 * (t_2 * \ale�))) \succeq \text{rmin}\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \ale�)\}$  and  $J^t((t_1 * \aleph) * (0 * (t_2 * \ale�))) \preceq \max\{J^t(t_1 * \aleph), J^t(t_2 * \ale�)\}$ .*

*Proof.* Let  $t_1, t_2 \in Y$ . Then we have  $M^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = \min(M((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), t) \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * (t_2 * \mathfrak{R})), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(t_2 * \mathfrak{R}), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_2 * \mathfrak{R})\}$  and  $\hat{B}^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = r\min(\hat{B}((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), t') \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t'), r\min(\hat{B}(0 * (t_2 * \mathfrak{R})), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t'), r\min(\hat{B}(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \mathfrak{R})\}$  and  $J^t((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))) = \max(J((t_1 * \aleph) * (0 * (t_2 * \mathfrak{R}))), \mathfrak{S}) \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(0 * (t_2 * \mathfrak{R})), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), t), \max(J(t_2 * \mathfrak{R}), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_2 * \mathfrak{R})\}$  by Definition and Proposition.  $\square$

**Proposition 3.13.** *If T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  fulfills the following statements, then  $\mathcal{C}^t$  refers to a T-MBJ NMSU of  $Y$ .*

- (1)  $M^t(0 * t_1) \succeq M^t(t_1 * \aleph)$  and  $\hat{B}^t(0 * t_1) \succeq \hat{B}^t(t_1 * \aleph)$  and  $J^t(0 * t_1) \preceq J^t(t_1 * \aleph) \forall t_1 \in Y$ .
- (2)  $M^t(t_1 * (0 * t_2)) \succeq \min\{M^t(t_1 * \aleph), M^t(t_1 * \mathfrak{R})\}$  and  $\hat{B}^t(t_1 * (0 * t_2)) \succeq r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \mathfrak{R})\}$  and  $J^t(t_1 * (0 * t_2)) \preceq \max\{J^t(t_1 * \aleph), J^t(t_1 * \mathfrak{R})\} \forall t_1, t_2 \in Y$  and  $t \in [0, 1]$ .

*Proof.* Let T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  fulfills the above statements (1 and 2). Then by Lemma 3.11, we have  $M^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\min(M((t_1 * \aleph) * (t_2 * \mathfrak{R})), t)\} = \{\min(M(t_1 * (0 * (0 * t_2))), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * t_2), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(0 * t_2), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_1 * \mathfrak{R})\}$  and  $\hat{B}^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} = \{r\min(\hat{B}(t_1 * (0 * (0 * t_2))), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(0 * t_2), t')\} \succeq r\min\{r\min(\hat{B}(t_1 * \aleph), t), r\min(\hat{B}(0 * t_2), t')\} = r\min\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \mathfrak{R})\}$  and  $J^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\max(J((t_1 * \aleph) * (t_2 * \mathfrak{R}))), \mathfrak{S}\} = \{\max(J(t_1 * (0 * (0 * t_2))), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), t), \max(J(0 * t_2), \mathfrak{S})\} \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(0 * t_2), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_1 * \mathfrak{R})\} \forall t_1, t_2 \in Y$ . Hence,  $\mathcal{C}^t$  is T-MBJ NMSU of  $Y$ .  $\square$

**Theorem 3.14.** *The T-MBJ neutrosophic set  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  of  $Y$  is a T-MBJ NMSU of  $Y \iff M^t$  and  $\hat{B}^{t-}, \hat{B}^{t+}$  and  $J^t$  are fuzzy subalgebra of  $Y$ .*

*Proof.* Suppose  $M^t, \hat{B}^{t-}, \hat{B}^{t+}$  and  $J^t$  are fuzzy subalgebra of  $Y$  and  $t_1, t_2 \in Y$  and  $t, t', \mathfrak{S} \in [0, 1]$ . Then  $M^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\min(M((t_1 * \aleph) * (t_2 * \mathfrak{R})), t)\} \succeq \min\{\min(M(t_1 * \aleph), t), \min(M(t_2 * \mathfrak{R}), t)\} = \min\{M^t(t_1 * \aleph), M^t(t_2 * \mathfrak{R})\}$  and  $\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}^-((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} \succeq r\min\{r\min(\hat{B}^-(t_1 * \aleph), t'), r\min(\hat{B}^-(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \mathfrak{R})\}$  and  $\hat{B}^{t+}((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{r\min(\hat{B}^+((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'\} \succeq r\min\{r\min(\hat{B}^+(t_1 * \aleph), t'), r\min(\hat{B}^+(t_2 * \mathfrak{R}), t')\} = r\min\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \mathfrak{R})\}$  and  $J^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = \{\max(J((t_1 * \aleph) * (t_2 * \mathfrak{R}))), \mathfrak{S}\} \preceq \max\{\max(J(t_1 * \aleph), \mathfrak{S}), \max(J(t_2 * \mathfrak{R}), \mathfrak{S})\} = \max\{J^t(t_1 * \aleph), J^t(t_2 * \mathfrak{R})\}$ . Now,  $\hat{B}^t((t_1 * \aleph) * (t_2 * \mathfrak{R})) = [\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \mathfrak{R})), \hat{B}^{t+}((t_1 * \aleph) * (t_2 * \mathfrak{R}))] = [r\min(\hat{B}^-((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t'), r\min(\hat{B}^+((t_1 * \aleph) * (t_2 * \mathfrak{R}))), t')] \succeq$

$[rmin\{rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^-(t_1 * \aleph), t')\}, rmin\{rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^+(t_1 * \aleph), t')\}] = [rmin\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_1 * \aleph)\}, rmin\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)\}] \succeq rmin\{[\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)], [\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t+}(t_1 * \aleph)]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_1 * \aleph)\}$ . Therefore,  $\mathcal{C}^t$  is T-MBJ NMSU of  $Y$ .

Conversely, assume that  $\mathcal{C}^t$  is a T-MBJ NMSU of  $Y$ . For any  $t_1, t_2 \in Y$ ,  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M(t_1 * \aleph), t), min(M(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$  and  $[\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \aleph)), \hat{B}^{t+}((t_1 * \aleph) * (t_2 * \aleph))] = \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}^t(t_1 * \aleph), t'), rmin(\hat{B}^t(t_2 * \aleph), t')\} = rmin\{[rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^-(t_2 * \aleph), t'), rmin(\hat{B}^+(t_2 * \aleph), t')]\} = [rmin\{rmin(\hat{B}^-(t_1 * \aleph), t'), rmin(\hat{B}^-(t_2 * \aleph), t')\}, rmin\{rmin(\hat{B}^+(t_1 * \aleph), t'), rmin(\hat{B}^+(t_2 * \aleph), t')\}] = [rmin\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \aleph)\}, rmin\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \aleph)\}]. Thus,  $\hat{B}^{t-}((t_1 * \aleph) * (t_2 * \aleph)) \succeq min\{\hat{B}^{t-}(t_1 * \aleph), \hat{B}^{t-}(t_2 * \aleph)\}$ ,  $\hat{B}^{t+}((t_1 * \aleph) * (t_2 * \aleph)) \succeq min\{\hat{B}^{t+}(t_1 * \aleph), \hat{B}^{t+}(t_2 * \aleph)\}$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), \mathfrak{S})\} \preceq max\{max(J(t_1 * \aleph), \mathfrak{S}), max(J(t_2 * \aleph), \mathfrak{S})\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$ . Hence  $M^t$  and  $\hat{B}^{t+}, \hat{B}^{t-}$  and  $J^t$  are fuzzy subalgebra of  $Y$ .  $\square$$

**Theorem 3.15.** Let  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  be a T-MBJ NMSU of  $Y$ . Then the sets  $I_{M^t}$ ,  $I_{\hat{B}^t}$  and  $I_{J^t}$  which are defined as  $I_{M^t} = \{t_1 \in Y \mid M^t(t_1 * \aleph) = M^t(0)\}$ ,  $I_{\hat{B}^t} = \{t_1 \in Y \mid \hat{B}^t(t_1 * \aleph) = \hat{B}^t(0)\}$  and  $I_{J^t} = \{t_1 \in Y \mid J^t(t_1 * \aleph) = J^t(0)\}$  are T-MBJ neutrosophic M-subalgebra of  $Y$ .

*Proof.* Let  $t_1, t_2 \in I_{M^t}$ . Then  $M^t(t_1 * \aleph) = M^t(0) = M^t(t_2 * \aleph)$  and so,  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{(min(M(t_1 * \aleph), t), (min(M(t_2 * \aleph), t))\} = M^t(0)$ . By using Proposition 3.4, as we know that  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = M^t(0)$  or equivalently  $(t_1 * \aleph) * (t_2 * \aleph) \in I_{M^t}$ .

Now we let  $t_1, t_2 \in I_{\hat{B}^t}$ . Then  $\hat{B}^t(t_1 * \aleph) = \hat{B}^t(0) = \hat{B}^t(t_2 * \aleph)$  and so,  $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{(rmin(\hat{B}^t(t_1 * \aleph), t'), (rmin(\hat{B}^t(t_2 * \aleph), t'))\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} = \hat{B}^t(0)$ . By using Proposition 3.4, as we know that  $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \hat{B}^t(0)$  or equivalently  $(t_1 * \aleph) * (t_2 * \aleph) \in I_{\hat{B}^t}$ .

Again we let  $t_1, t_2 \in I_{J^t}$ . Then  $J^t(t_1 * \aleph) = J^t(0) = J^t(t_2 * \aleph)$  and so,  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), t)\} \preceq max\{(max(J(t_1 * \aleph), t), (max(J(t_2 * \aleph), t))\} = J^t(0)$ . Again by using Proposition 3.4, as we know that  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = J^t(0)$  or equivalently  $(t_1 * \aleph) * (t_2 * \aleph) \in I_{J^t}$ . Hence the sets  $I_{M^t}$ ,  $I_{\hat{B}^t}$  and  $I_{J^t}$  are subalgebra of  $Y$ .  $\square$

**Definition 3.16.** Let  $\mathcal{C}^t = \{M^t, \hat{B}^t, J^t\}$  be a T-MBJ neutrosophic set of  $Y$ . For  $[s_1, s_2] \in D[0, 1]$  and  $t_1, t_2 \in [0, 1]$ , the set  $U(M^t \mid t) = \{t_1 \in Y \mid M^t(t_1 * \aleph) \succeq t\}$  is called upper  $t_1$ -level of  $\mathcal{C}^t$  and the set  $U(\hat{B}^t \mid [s_1, s_2]) = \{s_1, s_2 \in Y \mid \hat{B}^t(t_1 * \aleph) \succeq [s_1, s_2]\}$  is called upper  $[s_1, s_2]$ -level

of  $\mathcal{C}^t$  and  $L(J^t | \acute{t}) = \{t_1 \in Y \mid J^t(t_1 * \aleph) \preceq \acute{t} \text{ is called lower } (t_1 * \aleph)\text{-level of } \mathcal{C}^t.$

**Theorem 3.17.** *If  $\mathcal{C}^t = (M^t, \hat{B}^t, J^t)$  is T-MBJ NMSU of  $Y$ , then the upper  $\acute{t}$ -level, upper  $[s_1, s_2]$ -level and lower  $\acute{t}$ -level of  $\mathcal{C}^t$  are subalgebra of  $Y$ .*

*Proof.* Let  $t_1, t_2 \in U(M^t | \acute{t})$ . Then  $M^t(t_1 * \aleph) \succeq \acute{t}$  and  $M^t(t_2 * \aleph) \succeq \acute{t}$ . It follows that  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M(t_1 * \aleph), M(t_2 * \aleph)), t\} = min\{min(M(t_1 * \aleph), t), min(M(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} \succeq \acute{t} \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in U(M^t | \acute{t})$ . Hence  $U(M^t | \acute{t})$  is a subalgebra of  $Y$ . Let  $t_1, t_2 \in U(\hat{B}^t | [s_1, s_2])$ . Then  $\hat{B}^t(t_1 * \aleph) \succeq [s_1, s_2]$  and  $\hat{B}^t(t_2 * \aleph) \succeq [s_1, s_2]$ . It follows that  $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}(t_1 * \aleph), \hat{B}(t_2 * \aleph)), t'\} = rmin\{rmin(\hat{B}(t_1 * \aleph), t'), rmin(\hat{B}(t_2 * \aleph), t')\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} \succeq [s_1, s_2] \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in U(\hat{B}^t | [s_1, s_2])$ . Hence,  $U(\hat{B}^t | [s_1, s_2])$  is a subalgebra of  $Y$ . Let  $t_1, t_2 \in L(J^t | \acute{t})$ . Then  $J^t(t_1 * \aleph) \preceq \acute{t}$  and  $J^t(t_2 * \aleph) \preceq \acute{t}$ . It follows that  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(J((t_1 * \aleph) * (t_2 * \aleph)), \Im)\} \preceq max\{max(J(t_1 * \aleph), J(t_2 * \aleph)), \Im\} = max\{max(J(t_1 * \aleph), t), max(J(t_2 * \aleph), t)\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\} \preceq \acute{t} \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in L(J^t | \acute{t})$ . Hence  $L(J^t | \acute{t})$  is a subalgebra of  $Y$ .  $\square$

**Theorem 3.18.** *Any subalgebra of  $Y$  can be considered as upper  $\acute{t}$ -level, upper  $[s_1, s_2]$ -level and lower  $\acute{t}$ -level of some T-MBJ NMSU of  $Y$ .*

*Proof.* Let  $\mathcal{D}^t$  be a T-MBJ NMSU of  $Y$ , and  $\mathcal{C}^t$  be a T-MBJ neutrosophic set on  $Y$  defined by

$$M^t = \begin{cases} [\nu] & \text{if } t_1 \in \mathcal{D}^t \\ 1, & \text{otherwise.} \end{cases} \quad \hat{B}^t = \begin{cases} [\mu_1, \mu_2] & \text{if } t_1 \in \mathcal{D}^t \\ [0, 0] & \text{otherwise.} \end{cases}, \quad J^t = \begin{cases} [\nu] & \text{if } t_1 \in \mathcal{D}^t \\ 0, & \text{otherwise.} \end{cases}$$

$\forall [\mu_1, \mu_2] \in D[0, 1]$  and  $\nu \in [0, 1]$ . Now we discuss the following cases.

*Case 1.* If  $\forall t_1, t_2 \in \mathcal{D}^t$  then  $M^t(t_1 * \aleph) = \nu$ ,  $\hat{B}^t(t_1 * \aleph) = [\mu_1, \mu_2]$ ,  $J^t(t_1 * \aleph) = \nu$  and  $M^t(t_2 * \aleph) = \nu$ ,  $\hat{B}^t(t_2 * \aleph) = [\mu_1, \mu_2]$ ,  $J^t(t_2 * \aleph) = \nu$ . Thus  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \nu = min\{\nu, \nu\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$  and  $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = [\mu_1, \mu_2] = rmin\{[\mu_1, \mu_2], [\mu_1, \mu_2]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \nu = max\{\nu, \nu\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$ .

*Case 2.* If  $t_1 \in \mathcal{R}^t$  and  $t_2 \notin \mathcal{R}^t$ , then  $M^t(t_1 * \aleph) = \nu$ ,  $\hat{B}^t(t_1 * \aleph) = [\mu_1, \mu_2]$ ,  $J^t(t_1 * \aleph) = \nu$  and  $M^t(t_2 * \aleph) = 0$ ,  $\hat{B}^t(t_2 * \aleph) = [0, 0]$ ,  $J^t(t_2 * \aleph) = 1$ . Thus  $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 0 = min\{\nu, 0\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}$ ,  $\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[\mu_1, \mu_2], [0, 0]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{\nu, 1\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$ .

*Case 3.* If  $t_1 \notin \mathcal{R}^t$  and  $t_2 \in \mathcal{R}^t$ , then  $M^t(t_1 * \aleph) = 0$ ,  $\hat{B}^t(t_1 * \aleph) = [0, 0]$ ,  $J^t(t_1 * \aleph) = 1$  and  $M^t(t_2 * \aleph) = \nu$ ,  $\hat{B}^t(t_2 * \aleph) = [\mu_1, \mu_2]$ ,  $J^t(t_2 * \aleph) = \nu$ . Thus  $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 0 =$

$min\{0, \nu\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[0, 0], [\mu_1, \mu_2]\}$   
 $= rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{1, \nu\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$ .

Case 4. If  $t_1 \notin \aleph^t$  and  $t_2 \notin \aleph^t$ , then  $M^t(t_1 * \aleph) = 0, \hat{B}^t(t_1 * \aleph) = [0, 0], J^t(t_1 * \aleph) = 1$  and  $M^t(t_2 * \aleph) = 0, \hat{B}^t(t_2 * \aleph) = [0, 0], J^t(t_2 * \aleph) = 1$ . Thus  $M^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq 1 = min\{0, 0\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\}, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) \succeq [0, 0] = rmin\{[0, 0], [0, 0]\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\}$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) \preceq 1 = max\{1, 1\} = max\{J^t(t_1 * \aleph), J^t(t_2 * \aleph)\}$ . Therefore,  $\mathcal{C}^t$  is a T-MBJ NMSU of  $Y$ .  $\square$

**Theorem 3.19.** *Let  $\mathcal{C}^t$  be a subset of  $Y$  and  $\mathcal{C}^t$  be a T-MBJ neutrosophic set on  $Y$  which is given in the proof of above Theorem. If  $\mathcal{C}^t$  is considered as lower level subalgebra and upper level subalgebra of some T-MBJ NMSU of  $Y$ , then  $\mathcal{C}^t$  is a T-MBJ neutrosophic cubic one of  $Y$ .*

*Proof.* Let  $\mathcal{C}^t$  be a T-MBJ NMSU of  $Y$ , and  $t_1, t_2 \in \mathcal{C}^t$ . Then  $M^t(t_1 * \aleph) = M^t(t_2 * \aleph) = \gamma, \hat{B}^t(t_1 * \aleph) = \hat{B}^t(t_2 * \aleph) = [\alpha_1, \alpha_2]$  and  $J^t(t_1 * \aleph) = J^t(t_2 * \aleph) = \beta$ . Thus  $M^t((t_1 * \aleph) * (t_2 * \aleph)) = \{min(M^t((t_1 * \aleph) * (t_2 * \aleph)), t)\} \succeq min\{min(M^t(t_1 * \aleph), t), min(M^t(t_2 * \aleph), t)\} = min\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} = min\{\gamma, \gamma\} = \gamma, \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in \mathcal{C}^t, \hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)) = \{rmin(\hat{B}^t((t_1 * \aleph) * (t_2 * \aleph)), t')\} \succeq rmin\{rmin(\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)), t'\} = rmin\{rmin(\hat{B}^t(t_1 * \aleph), t'), rmin(\hat{B}^t(t_2 * \aleph), t')\} = rmin\{\hat{B}^t(t_1 * \aleph), \hat{B}^t(t_2 * \aleph)\} = rmin\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$  and  $J^t((t_1 * \aleph) * (t_2 * \aleph)) = \{max(M^t((t_1 * \aleph) * (t_2 * \aleph)), \aleph)\} \preceq max\{max(M^t(t_1 * \aleph), \aleph), max(M^t(t_2 * \aleph), \aleph)\} = max\{M^t(t_1 * \aleph), M^t(t_2 * \aleph)\} = max\{\beta, \beta\} = \beta, \Rightarrow (t_1 * \aleph) * (t_2 * \aleph) \in \mathcal{C}^t$ . Hence, proof is completed.  $\square$

**4. Conclusions**

In this paper, T-MBJ neutrosophic set is defined and notion of T-MBJ neutrosophic M-subalgebra is also introduced by set of conditions on G-algebra. T-MBJ neutrosophic M-subalgebra of G-algebra has investigated by p-union, P-intersection, R-union, R-intersection and some results. For future work this study will be use to discuss the normal ideals, multiplication, translation and magnification of T-MBJ neutrosophic set.

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