



New Family of Neutrosophic Soft Sets

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Abstract: The goal of this paper is to study and discuss the neutrosophic soft set theory by introducing, new family of neutrosophic soft sets and because the concept of topological spaces is one of the most powerful concepts in system analysis, we introduced the concept of neutrosophic soft topological spaces depending on this the new family. Furthermore, we introduced new definitions, properties, concerning the neutrosophic soft closuer, the neutrosophic soft interior, the neutrosophic soft exterior and the neutrosophic soft boundary in details.

Keywords: neutrosophic soft set theory, of neutrosophic soft topological spaces, new operations for neutrosophic soft sets. families of neutrosophic soft sets.

1. Introduction

D Moloasov[1] introduced the notion of soft set in 1999 . In the same year F Smarndache firstly introduced the neutrosophic set theory [2]. Which is the generalization of the class set conventional fuzz set [3] and intuitionistic set fuzz [4]. The soft set theory and the neutrosophic set theory have been applied to many different fields, (see for example [5-64]) .

In 2012 Maji[65] combined the concept of soft set and neutrosophic set together by introducing the current mathematical framework called neutrosophic soft set and later this concept has been modified by S.Bromi[66]. Faruk[67] redefied neutrosophic soft set, and their operations, also presented an application of neutrosohpic soft set, in decision making. In 2017 Bera[68] introduced neutrosohpic soft topological spaces using different subsets of the parameters set for each soft set. In 2019 and In 2020 Taha[69] and Evanzalin[70] introduced the neutrosohpic soft topological spaces differently from the study[58]. More works on the concept, of neutrosohpic soft set can be found in [71, 72, 73, 74, 75, 76, 77, 78, 79, 80,81,82].

In this research, we studied and discussed the neutrosophic soft set theory by introducing, new family of neutrosophic soft sets, new operations for neutrosophic soft sets and we also we introduce the theory of neutrosophic soft topological spaces depending on this the new family.

The research is organized as follows: In section2, we first recall the necessary definitions needed in this work we then recall two families of neutrosohpic soft sets with explaining the properties of each family. In section3, the neutrosophic soft set theory is studied and discussed by introducing, new family of neutrosophic soft sets [namely third family], new operations for neutrosophic soft sets, comparison between the new family and other families, new definitions and examples. In section4, the theory of neutrosophic soft topological spaces is investigated depending on the new family and also, new definitions, characterization, the neutrosophic soft closure, the neutrosophic soft interior, the neutrosophic soft exterior and the neutrosophic soft boundary are introduced in details.

2. Preliminaries

In this section, we will recall the necessary definitions needed in this work, we then recall two families of neutrosohpic soft sets with explaining the properties of each family.

2.1. Definition [83]

If K is the initial universe then the neutrosohpic set A is defined as follows:

$$A = \{ \langle k, T_A(k), I_A(k), V_A(k) \rangle, k \in K \}$$

where, the functions $T_rI_rV: K \rightarrow]-0,+1[$ and

$$-0 \le T_A(k) + I_A(k) + V_A(k) \le +3$$

For any two neutrosohpic sets:

$$A = \{ \langle k, T_A(k), I_A(k), V_A(k) \rangle, k \in K \}$$

$$B = \{ < k, T_B(k), I_B(k), V_B(k) >, k \in K \}$$

- ❖ $A \subseteq B \leftrightarrow T_A(k) \le T_B(k)$, $I_A(k) \le I_B(k)$, $V_A(k) \ge V_B(k)$, for all, $k \in K$.
- **♦** $A \cup B = \{ \langle k, T_A(k) \lor T_B(k), I_A(k) \lor I_B(k), V_A(k) \land V_B(k) \}$, $k \in K \}$.
- **♦** A ∩ B= {< k, $T_A(k) \land T_B(k)$, $I_A(k) \land I_B(k)$, $V_A(k) \lor V_B(k) > , k ∈ K }.$
- $A = B \leftrightarrow A \subseteq B \text{ and } B \subseteq A$.
- \bullet The complement of A denoted by A^C is defined as:

$$(A)^c = \{ \langle k, 1 - T_A(k), 1 - I_A(k), 1 - V_A(k) \rangle, k \in K \}.$$

2.2. Definition [1]

Let K be an initial universe set and E be a set of parameters. Consider a set $A \neq \emptyset$, $A \subseteq E$. A pair (F,A) is called a soft set (over K) if and only if F is a mapping from A into the set of all the subsets of K.

First family [65]

Let K be an initial universe set and E_K be a set of parameters . Consider a set $D \neq \emptyset$, $D \subseteq E_K$. A pair (F,D) is called a neutrosohpic soft set (over K) if and only if F is a mapping from A into the set of all the neutrosohpic sets over K.

Note that , we will denote simply by F_D of the pair (F,D) and the set of all the neutrosohpic sets over K with respect to this family will be denoted by $N_1(K)$.

Let
$$F_{1D}$$
, $F_{2D} \in N_1(K)$. Then:

1) The union between them($F_D \sqcup G_B$) is defined by $H = F_D \sqcup G_B$ as follows:

$$\begin{split} T_{H(p)}(\,k\,\,) &= \begin{cases} &T_{F(p)}(\,k\,\,) & \text{if } p \in D \setminus B \\ &T_{G(p)}(\,k\,\,) & \text{if } p \in B \setminus D \\ & \max{\{T_{F(p)}(\,k\,\,), T_{G(p)}(\,k\,\,)\}}, & \text{if } p \in D \cap B \end{cases} \\ I_{H(p)}(\,k\,\,) &= \begin{cases} &I_{F(p)}(\,k\,\,) & \text{if } p \in D \setminus B \\ &I_{G(p)}(\,k\,\,) & \text{if } p \in B \setminus D \\ &\frac{(I_{F(p)}(\,k) + I_{G(p)}(\,k\,))}{2} & \text{if } p \in D \cap B \end{cases} \end{split}$$

$$V_{H(p)}(\;k\;)\; = \begin{cases} V_{F(p)}(\;k\;) & \text{if} \;\; p \in D \setminus B \\ V_{G(p)}(\;k\;) & \text{if} \;\; p \in B \setminus D \\ \min\{V_{F(p)}(\;k\;), V_{G(p)}(\;k\;)\}_{.} & \text{if} \;\; p \in D \cap B \;\; . \end{cases}$$

2) The interstation between them($F_D \sqcap G_B$) is defined by $H = F_D \sqcap G_B$ as follows:

$$T_{H(p)}(k) = \min \{T_{F(p)}(k), T_{G(p)}(k)\}$$

$$I_{H(p)}(k) = \frac{\left(I_{F(p)}(k) + I_{G(p)}(k)\right)}{2}$$

$$V_{H(p)}(k) = \max \{V_{F(p)}(k), V_{G(p)}(k)\}.$$

- 3) $F_D \subseteq G_B$ if and only if
- 1) D⊆B
- $2) \ T_{F(p)}(\,k\,\,) \, \leq \, T_{G(p)}(\,k\,\,) \ , \\ I_{F(p)}(\,k\,\,) \, \leq \, I_{G(p)}(\,k\,\,) \ , \\ V_{F(p)}(\,k\,\,) \, \geq \, V_{G(p)}(\,k\,\,) \ , \\ \text{for all } p \in D \ , k \in K \,. \\$
 - 4) The complement of F_D is defined as:

$$(F_D)^c = \{(p, \{ < k, T_{F(p)}(k), I_{F(p)}(k), V_{F(p)}(k) >, k \in K \}), p \in D\}.$$

2.3. Definition [70]

A neutrosohpic soft set F_D over the universe K is called a null neutrosohpic soft set and denoted by \emptyset_N if $T_{F(p)}(k) = 0$, $I_{F(p)}(k) = 0$, $I_{F(p)}(k$

2.4. Definition [70]

A neutrosohpic soft set F_D over the universe K is called an absolute is called an absolute neutrosohpics soft set and denoted by K_N if $T_{F(p)}(k)=1$, $I_{F(p)}(k)=1$, $V_{F(p)}(k)=0$, for all $p\in D$, $k\in K$.

Second family [66]

Let K be an initial universe set and E be a set of parameters , P(Y) be the set of all the subsets of K and V be a neutosohpic set over E. Then a neutrosohpic parameterized soft sets .

$$\Omega_{V} = \{(< p, T_{V}(p), I_{V}(p), W_{V}(p), f_{V}(p) >), p \in E\}$$

where , the functions T_V , I_V , W_V : $E \rightarrow [0,1]$ and f_V : $E \rightarrow P(K)$

and
$$f_V(p) = \emptyset$$
 if $T_V(p) = 0$, $I_V(p) = 1$ and $W_V(p) = 1$.

Here, the functions T_V , I_V , W_V are called membership function, indeterminacy function and non-membership function of parameterized soft set (for short , Np_soft set), respectively .

Let
$$\Omega_V$$
 , $\mho_L \in Np_soft set$.

Now : If $f_V(p) = K$, $T_V(p) = 0$, $I_V(p) = 0$ and $W_V(p) = 1$, $\forall p \in E$, then Ω_V is called a V_empty Np_soft set (for short Ω_{\emptyset_V}). If $V = \emptyset$, then the V_empty Np_soft set is called an empty Np_soft set (for short Ω_{\emptyset}). If $f_V(p) = K$, $T_V(p) = 1$, $I_V(p) = 0$ and $W_V(p) = 0$, $\forall p \in E$, then, Ω_V is called a V_universal Np_soft set (for short $\Omega_{\widetilde{V}}$,), if V = E, then the V_universal Np_soft set is called an V_universal Np_soft set (for short $\Omega_{\widetilde{E}}$).

$$\Omega_V \sqsubseteq \mho_L \, \leftrightarrow T_V(p) \le T_L(p), I_V(p) \ge I_L(p), W_V(p) \ge W_L(p), f_V(p) \le f_L(p) \ , p \in E \ .$$

$$\Omega_V \sqcup \mho_L \ = \ \left\{ \left(< p, \max\{T_V(p), T_L(p)\}, \min \ \{I_V(p), I_L(p)\}, \min \ \{W_V(p), W_L(p)\}, f_V(p) \cup f_L(p) > \right), p \in E \right\} \ .$$

$$\Omega_{V} \sqcap \mho_{L} \ = \ \left\{ \left(< p, \min \ \{ T_{V}(p), T_{L}(p) \}, \max \ \{ I_{V}(p), I_{L}(p) \}, \max \ \{ W_{V}(p), W_{L}(p) \}, f_{V}(p) \cap f_{L}(p) > \right), p \in E \right\} \ .$$

The complement of Ω_V is defined as:

$$\Omega_V^C = \{(\langle p, W_V(p), I_V(p), I_V(p), f_{VC}(p) \rangle | p \in E \}, \text{ where, } f_{VC}(p) = K - f_V(p) .$$

3. Third family (New family)

In this section, we will study and discuss the neutrosophic soft set theory giving new definitions, example, new family of neutrosophic soft sets, new operations for neutrosophic soft sets and comparison between the new family and first family.

3.1. Definition

A neutrosohpic soft set F_D on the universe K is denoted by the set of ordered pairs

$$F_D = \{(p, f_D(p)), p \in E_K, \}$$
.

It can be written as: $F_D = \{ (p, \{ < q^{(T_{f_D(p)}(q), I_{f_D(p)}(q), V_{f_D(p)}(q))} >), p \in E_K \}.$

Where,

 $f_{\rm D}$ is a mapping such that

$$f_{\mathrm{D}}: \left\{ \begin{matrix} \mathrm{D} \longrightarrow \mathrm{P}(\mathrm{K}) \\ \mathrm{D}^{\mathrm{C}} \longrightarrow <\mathrm{q}^{(0\,\prime\,0\,\prime\,1)} > \end{matrix} \right. , \mathrm{q} \in \mathrm{K} \, .$$

 E_K is the set of all possible parameters under consideration with respect to K , $D \subseteq E_K$.

P(K) is the set of all the neutrosohpic sets over K.

Form now on , the set of all the neutrosohpic sets over K with respect to this family (Third family) will be denoted by $N_3(K)$.

3.2. Example

Let K=
$$\{q_1,q_2,q_3,q_4\}$$
 and D \subseteq E_K = $\{p_1,p_2,p_3,p_4\}$, such that D = $\{p_1,p_2\}$.

Suppose that

$$\begin{split} &f_{1_{\mathrm{D}}}\left(\mathbf{p}_{1}\right) = \, \{ \, <\mathbf{q}_{1}\,^{(\,0,6'\,0,3'\,0,7)},\mathbf{q}_{2}\,^{(\,0,5\,\prime\,0,4'\,0,5)}, \,\, \mathbf{q}_{3}\,^{(\,0,7'\,0,4'\,0,3)},\mathbf{q}_{4}\,^{(\,0,8'\,0,4'\,0,3)} \,> \, \}. \\ &f_{1_{\mathrm{D}}}\left(\mathbf{p}_{2}\right) = \{ \, <\mathbf{q}_{1}\,^{(\,0,7'\,0,3'\,0,5)}, \,\, \mathbf{q}_{2}\,^{(\,0,6'\,0,7'\,0,3)}, \,\, \mathbf{q}_{3}\,^{(\,0,7'\,0,3'\,0,5)}, \,\, \mathbf{q}_{4}\,^{(\,0,6'\,0,3'\,0,6)} \,> \, \} \,\, . \\ &f_{2_{\mathrm{D}}}\left(\mathbf{p}_{1}\right) = \{ \, <\mathbf{q}_{1}\,^{(\,0,6'\,0,4'\,0,5)}, \,\, \mathbf{q}_{2}\,^{(\,0,6'\,0,5'\,0,4)} \,\, , \,\, \mathbf{q}_{3}\,^{(\,0,7'\,0,4'\,0,5)}, \,\, \mathbf{q}_{4}\,^{(\,0,7'\,0,5'\,0,6)} \,> \, \}. \\ &f_{2_{\mathrm{D}}}\left(\mathbf{p}_{1}\right) = \{ \, <\mathbf{q}_{1}\,^{(\,0,7'\,0,6'\,0,6)}, \,\, \mathbf{q}_{2}\,^{(\,0,8'\,0,4'\,0,5)}, \,\, \mathbf{q}_{3}\,^{(\,0,7'\,0,4'\,0,6)}, \,\, \mathbf{q}_{4}\,^{(\,0,6'\,0,3'\,0,5)} \,> \, \} \,. \end{split}$$

Then, we can view the neutrosophic soft sets F_{1D} , F_{2D} as :

$$F_{1_{D}} = \begin{cases} \left(p_{1}, \{ < q_{1}^{(0,6'0,3'0,7)}, \ q_{2}^{(0,5'0,4'0,5)}, \ q_{3}^{(0,7'0,4'0,3)}, \ q_{4}^{(0,8'0,4'0,3)} > \} \right) \\ \left(p_{2}, \{ < q_{1}^{(0,7'0,3'0,5)}, \ q_{2}^{(0,6'0,7'0,3)}, q_{3}^{(0,7'0,3'0,5)}, q_{4}^{(0,6'0,3'0,6)} > \} \right) \end{cases}$$

$$F_{2_{\mathrm{D}}} = \left. \left\{ \left(p_{1}, \{ < q_{1}^{\;\;(0,6\,\prime\,0,4\,\prime\,0,5)}, \; q_{2}^{\;\;(0,6\,\prime\,0,5\,\prime\,0,4)}, \; q_{3}^{\;\;(0,7\,\prime\,0,4\,\prime\,0,5)}, \; q_{4}^{\;\;(0,7\,\prime\,0,5\,\prime\,0,6)} > \} \right) \right\} \\ \left(p_{2}, \{ < q_{1}^{\;\;(0,7\,\prime\,0,6\,\prime\,0,6)}, \; q_{2}^{\;\;(0,8\,\prime\,0,4\,\prime\,0,5)}, \; q_{3}^{\;\;(0,7\,\prime\,0,4\,\prime\,0,6)}, \; q_{4}^{\;\;(0,6\,\prime\,0,3\,\prime\,0,5)} > \} \right) \right\}$$

Note that, if $f_D(p) = \langle q^{(0'0'1)} \rangle$, for all $p \in E$, $q \in K$, the element $(p, f_D(p))$ is not appeared in neutrosohpic soft set F_D .

3.3. Definition

The neutrosohpic soft complement $F^c{}_D$ of F_D is defined by the mapping $f_{D^c}(p) = f^c{}_D(p)$, where $f^c{}_D(p)$ is the complement of the set $f_D(p)$.

That is:

$$F^c{}_D = \ \{ \left(p, \{ < q^{(1-T_{f_D(p)}(q)) \, \prime (1-I_{f_D(p)}(q)) \, \prime \, (1-V_{f_D(p)}(q))} > \right), p \in E_K \}.$$

3.4. Definition

Let $F_D \in N_3(K)$, if $f_D(p) = \langle q^{(0'0'1)} \rangle$, $\forall p \in E_K$, $q \in K$, then F_D is called the null neutrosohpic soft set and denoted by $\widetilde{\emptyset}_D$.

3.5. Definition

Let $F_D \in N_3(K)$ if $f_D(p) = \langle q^{(1'1'0)} \rangle$, $\forall p \in D$, $q \in K$, then F_D is called the absolute neutrosohpic soft set and denoted by \widetilde{K}_D .

3.6. Definition

Let $F_{1_D} \in N_3(K)$, then F_{1_D} is called a neutrosohpic soft subset of F_{2_D} and denoted by $F_{1_D} \sqsubseteq F_{2_D}$ if $f_{1_D}(p) \sqsubseteq f_{2_D}(p)$, $\forall p \in E_K$.

3.7. Definition

Let F_{1_D} , $F_{2_D} \in N_3(K)$, then, the neutrosohpic soft intersection $(F_{1_D} \sqcap F_{2_D})$ and the neutrosohpic soft union $(F_{1_D} \sqcup F_{2_D})$ are defined by the mappings .

$$f_{1_{\mathrm{D}}}(\mathbf{p}) \sqcap f_{2_{\mathrm{D}}}(\mathbf{p})$$

 $f_{1_{\mathrm{D}}}(\mathbf{p}) \sqcup f_{2_{\mathrm{D}}}(\mathbf{p})$

3.8. Example

Let us consider neutrosohpic soft sets $\mathbf{F_{1}_{D}}, \ \mathbf{F_{2}_{D}}$ in example .

Then,

$$1) \quad F_{1_{D}} \sqcup F_{2_{D}} = \\ \begin{cases} \left(p_{1}, \{ < q_{1}^{\ (0,6'\ 0,4'\ 0,5)}, q_{2}^{\ (0,6'\ 0,5'\ 0,4)}, q_{3}^{\ (0,7'\ 0,4'\ 0,3)}, q_{4}^{\ (0,8'\ 0,5'\ 0,3)} > \} \right) \\ \left(p_{2}, \{ < q_{1}^{\ (0,7'\ 0,6'\ 0,5)}, q_{2}^{\ (0,8'\ 0,7'\ 0,3)}, q_{3}^{\ (0,7'\ 0,4'\ 0,5)}, q_{4}^{\ (0,6'\ 0,3'\ 0,5)} > \} \right) \\ \end{cases}$$

$$2) \quad F_{1_{\mathrm{D}}} \sqcap F_{2_{\mathrm{D}}} = \quad \left\{ \begin{pmatrix} \left(p_{1}, \{ < q_{1} \stackrel{(0,6 \vee 0,3 \vee 0,7)}{,} q_{2} \stackrel{(0,5 \vee 0,4 \vee 0,5)}{,} q_{3} \stackrel{(0,7 \vee 0,4 \vee 0,5)}{,} q_{4} \stackrel{(0,7 \vee 0,4 \vee 0,6)}{,} > \} \right) \\ \left(p_{2}, \{ < q_{1} \stackrel{(0,7 \vee 0,3 \vee 0,5)}{,} q_{2} \stackrel{(0,6 \vee 0,5 \vee 0,4)}{,} q_{3} \stackrel{(0,7 \vee 0,3 \vee 0,6)}{,} q_{4} \stackrel{(0,6 \vee 0,3 \vee 0,6)}{,} > \} \right) \\ \right\}$$

$$(F_{2D})^{C} = \begin{cases} \left(p_{1},\{< q_{1}^{(0,4'0,6'0,5)},q_{2}^{(0,4'0,5'0,6)},q_{3}^{(0,3'0,6'0,5)},q_{4}^{(0,3'0,5'0,4)}>\}\right) \\ \left(p_{2},\{< q_{1}^{(0,3'0,4'0,4)},q_{2}^{(0,2'0,6'0,5)},q_{3}^{(0,3'0,6'0,4)},q_{4}^{(0,4'0,7'0,5)}>\}\right) \\ \left(p_{3},\{< q_{1}^{(1'1'0)},q_{2}^{(1'1'0)},q_{3}^{(1'1'0)},q_{4}^{(1'1'0)},q_{4}^{(1'1'0)}>\}\right) \\ \left(p_{4},\{< q_{1}^{(1'1'0)},q_{2}^{(1'1'0)},q_{3}^{(1'1'0)},q_{4}^{(1'1'0)},q_{4}^{(1'1'0)}>\}\right) \end{cases}$$

3.9. Proposition

Let $F_{1D} \in N_3(K)$, then:

- $\bullet \quad F_{1_{\mathrm{D}}} \sqcup F_{1_{\mathrm{D}}} = F_{1_{\mathrm{D}}} \ .$
- $\bullet \quad F_{1_{\mathrm{D}}} \sqcap F_{1_{\mathrm{D}}} = F_{1_{\mathrm{D}}}.$
- $F_{1D} \sqcup \widetilde{\emptyset}_D = F_{1D}$.
- $\bullet \quad \ F_{1_{\rm D}} \sqsubseteq F_{1_{\rm D}} \ .$
- $\widetilde{\emptyset}_D \sqsubseteq F_{1_D}$.
- $\bullet \quad \ F_{1_D} \sqsubseteq \widetilde{K}_D \ .$
- $F_{1D} \sqcap \widetilde{\emptyset}_D = \widetilde{\emptyset}_D$.
- $\bullet \quad \ F_{1_{\rm D}} \sqcup \widetilde{K}_{\rm D} = \widetilde{K}_{\rm D} \ .$
- $F_{1D} \cap \widetilde{K}_D = F_{1D}$.

Proof: The proof of the remark is direct from the definition

3.10. Proposition

Let $F_{1_D} \in N_3(K)$, then:

- $(\widetilde{\emptyset}_D)^c = \widetilde{K}_D$
- $(\widetilde{K}_D)^c = \widetilde{\emptyset}_D$
- $\bullet \quad ((F_{1D})^c)^c = F_{1D}$

Proof: Straightforward.

3.11. Proposition

Let F_{1D} , F_{2D} and $F_{3D} \in N_3(K)$, then:

- $F_{1D} \sqcup F_{2D} = F_{2D} \sqcup F_{1D}$.
- $\bullet \quad \operatorname{F}_{1_{\operatorname{D}}} \sqcap \operatorname{F}_{2_{\operatorname{D}}} = \operatorname{F}_{2_{\operatorname{D}}} \sqcap \operatorname{F}_{1_{\operatorname{D}}} \ .$
- $\bullet \quad (F_{1_{\mathrm{D}}} \sqcup F_{2_{\mathrm{D}}})^{\mathsf{C}} = (F_{1_{\mathrm{D}}})^{\mathsf{C}} \sqcap (F_{2_{\mathrm{D}}})^{\mathsf{C}} \, .$
- $(F_{1_D} \sqcap F_{2_D})^C = (F_{1_D})^C \sqcup (F_{2_D})^C$.
- $\bullet \quad \left({F_1}_D \sqcap {F_2}_D \right) \mathrel{\sqcup} {F_3}_D = \left({F_1}_D \mathrel{\sqcup} {F_3}_D \right) \sqcap \left({F_2}_D \mathrel{\sqcup} {F_3}_D \right) \; .$
- $(F_{1_D} \sqcup F_{2_D}) \sqcap F_{3_D} = (F_{1_D} \sqcap F_{3_D}) \sqcup (F_{2_D} \sqcap F_{3_D})$.

Proof: Straightforward.

Comparison

Next, we will compare (new family) with the first family.

1- Dentition of neutrosohpic soft sets	
First family	Third family(new family)
$F_{D} = \{ (p, f_{D}(p)), p \in D, D \subseteq E_{k} \}$	$F_{D} = \{ (p, f_{D}(p)), p \in E_{K}, D \subseteq E_{k} \}$
Where, $F: D \rightarrow P(K)$	Where, $F: \begin{cases} D \rightarrow P(K) \\ D^C \rightarrow < q^{(0,0,1)} > \end{cases}$, $q \in K$

2- Intersection of neutrosohpic soft sets	
First family	Third family(new family)
$F_{1A} \sqcap F_{2B} = F_{3(A \cap B)}$ $F_{3}: (A \cap B) \longrightarrow P(K)$ $F_{3}(p) = F_{1}(p) \text{ or } F_{2}(p)$	$F_{1D} \sqcap F_{2D} = F_{3D}$ $F_{3}: \begin{cases} D \longrightarrow P(K) \\ D^{c} \longrightarrow < q^{(0 \prime 0 \prime 1)} > \end{cases}, q \in K$ $F_{3}(p) = F_{1}(p) \sqcap F_{2}(p)$

3- Union of neutrosohpic soft sets	
First family	Third family(new family)
$F_{1_{A}} \sqcup F_{2_{B}} = F_{3_{(A \cup B)}}$ $F_{3} \colon (A \cap B) \longrightarrow P(K)$	$F_{1_{\mathbf{D}}} \sqcup F_{2_{\mathbf{D}}} = F_{3_{\mathbf{D}}}$
$F_{3}(p) = \begin{cases} F_{1_{A}} & \text{if } p \in A \setminus B \\ F_{2_{B}} & \text{if } p \in B \setminus A \\ F_{1}(p) \sqcup F_{2}(p), \text{if } p \in A \cap B \end{cases}$	$F_3: \begin{cases} D \to P(K) \\ D^C \to < k^{(0'0'1)} > \end{cases}, k \in K$ $F_3(p) = F_1(p) \sqcup F_2(p)$

4- Complement of neutrosohpic soft sets	
First family	Third family(new family)
$(F_{D})^{c} = F^{c}_{ D}$	$(F_D)^c = F^c_D$
Where $F^c:]D \rightarrow P(K)$ "	Where, $F^c: \begin{cases} D \to P(K) \\ D^C \to \end{cases}$, $q \in K$.
Where, $\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	

4. Neutrosohpic Soft Topology

In this section, we will investigate the theory of neutrosophic soft topological spaces depending on the new family $(N_3(K))$ and we present new definitions, characterization and properties concerning the neutrosophic soft closure, the neutrosophic soft interior, the neutrosophic soft exterior, the neutrosophic soft boundary .

4.1. Definition

Let K be the initial universe E_K be set of parameters $D \subseteq E_K$ and $\mu \subseteq N_3(K)$, we say that μ is a neutrosophic soft topology on K, if it satisfies the following conditions:

- 1) $\widetilde{\emptyset}_D$, $\widetilde{K}_D \in \mu$.
- 2) $F_{1_D} \sqcap F_{2_D} \in \mu$, $\forall F_{1_D}, F_{2_D} \in \mu$.
- 3) $\sqcup \{F_{i_D}, i \in I\} \in \mu, \forall F_{i_D} \in \mu$.

The pair (K, μ) (or simply K) is a neutrosophic soft topological spaces or $((N_3 - Top)$ for short).

- \diamond The elements of μ are called a neutrosohpic open the family sets .
- \diamond A neutrosohpic soft F_{1D} is called a neutrosohpic soft closed set, if its complement is a neutros ohpic soft open set .

4.2. Proposition

Let (K,μ) be (N_3-Top) , then the family of neutrosohpic soft closed sets $(C(\widetilde{K}_D)$ for short) has the following properties :

- 1) $\widetilde{\emptyset}_D$, $\widetilde{K}_D \in C(\widetilde{K}_D)$.
- 2) $F_{1_D} \sqcup F_{2_D} \in \mu$, $\forall F_{1_D}, F_{2_D} \in C(\widetilde{K}_D)$.
- 3) $\sqcap \{F_{i_D}, i \in I\} \in C(\widetilde{K}_D), \forall F_{i_D} \in C(\widetilde{K}_D)$.

Proof: Straightforward.

4.3. Example

Let K =
$$\{q_1,q_2\}$$
 , D \subseteq E_K , such that D = $\{p_1\}$ and $F_{1_D},F_{2_D} \in N_3(K)$,

such that:

$$\begin{split} &F_{1_{\mathrm{D}}} = \left\{ \left(p_{1}, \{ < q_{1}{}^{(0,5\,\prime 0,1\prime\,0,4)}, q_{2}{}^{(0,4\prime\,0,3\prime\,0,8)} > \} \right) \right\} \\ &F_{2_{\mathrm{D}}} = \left\{ \left(p_{1}, \{ < q_{1}{}^{(0,5\prime\,0,1\prime\,0,3)}, q_{2}{}^{(0,5\prime\,0,3\prime\,0,6)} > \} \right) \right\} \end{split}$$

Then , $\mu = \{ \widetilde{\emptyset}_D, \widetilde{K}_D, F_{1D}, F_{2D} \}$ is a neutrosohpic soft topology on K and (K, μ) is a neutrosohpic soft topological space .

4.4. Proposition

Let (K,μ_1) and (K,μ_2) be two neutrosohpic soft topological spaces on K, then $(K,\mu_1 \cap \mu_2)$ is a neutrosohpic soft topological spaces on K.

Poof:

Let (K,μ_1) and (K,μ_2) be two neutrosohpic soft topological spaces on K. It can be seen clearly that $\widetilde{\emptyset}_D$, $\widetilde{K}_D \in \mu_1 \cap \mu_2$. If $F_{1_D}, F_{2_D} \in \mu_1 \cap \mu_2$, then $F_{1_D}, F_{2_D} \in \mu_1$ and $F_{1_D}, F_{2_D} \in \mu_2$. It is given that

 $\begin{aligned} F_{1_D}\sqcap F_{2_D}\in \mu_1\,, & \ F_{1_D}\sqcap F_{2_D}\in \mu_2\,. \end{aligned} \text{ Thus } F_{1_D}\sqcap F_{2_D}\in \mu_1\cap \mu_2\,. \text{ Let } \{F_{i_D},i\in I\}\in \mu_1\cap \mu_2\,, \text{ then } F_{i_D}\in \mu_1,\forall\, i\in I \text{ and } F_{i_D}\in \mu_2\,,\forall\, i\in I\,. \end{aligned} \text{ Then } F_{i_D}\in \mu_1\cap \mu_2\,,\forall\, i\in I \text{ So, we have } \sqcup \{F_{i_D},i\in I\}\in \mu_1\cap \mu_2$

4.5. Remark

Let (K,μ_1) and (K,μ_2) be two neutrosohpic soft topological spaces on K, then $(K,\mu_1 \cup \mu_2)$ may not be correct. It can be seen from the following example.

4.6. Example

Let K =
$$\{q_1, q_2\}$$
 , D \subseteq E_K , such that D = $\{p_1\}$ and $F_{1D}, F_{2D} \in N_3(K)$,

such that:

$$\begin{split} &F_{1_{\mathrm{D}}} = \left\{ \left(p_{1}, \{ < q_{1}^{(0,2'0,4'0,6)}, q_{2}^{(0,1'0,3'0,5)} > \} \right) \right\}. \\ &F_{2_{\mathrm{D}}} = \left\{ \left(p_{1}, \{ < q_{1}^{(0,4'0,6'0,8)}, q_{2}^{(0,3'0,5'0,7)} > \} \right) \right\}. \end{split}$$

Then , $\mu_1 = \{ \ \widetilde{\varnothing}_D \ , \widetilde{K}_D \ , F_{1_D} \}$ and $\mu_2 = \{ \ \widetilde{\varnothing}_D \ , \widetilde{K}_D \ , F_{2_D} \}$ are two neutrosohpic soft topology on K. But $\mu_1 \cup \mu_2 = \{ \ \widetilde{\varnothing}_D \ , \widetilde{K}_D \ , F_{1_D} , F_{2_D} \} \}$ is not neutrosohpic soft topology on K .

4.7. Definition

Let $F_D \in N_3(K)$. The interior of F_D is union of all neutrosohpic soft open sets contained in F_D , denoted by $int(F_D)$. That is

$$int(F_D) = \sqcup \{ \ F_{1_D} \ : \ F_{1_D} \ is \ neutrosohpic \ soft \ open \ set, \ F_{1_D} \sqsubseteq (F_D) \ \} \ .$$

4.8. Definition

Let $F_D \in N_3(K)$. The interior of F_D is intersection of all neutrosohpic soft closed sets containing in F_D , denoted by $cl(F_D)$. That is

$$cl(F_D) = \sqcup \{ F_{1_D} : F_{1_D} \text{ is neutrosohpic soft closed set, } F_{1_D} \supseteq (F_D) \}$$
.

4.9. Proposition

Let (K,μ) be $(N_3 - Top)$, $F_D \in N_3(K)$. Then:

- 1) F_D is a neutrosohpic soft open (closed) set if and only if $F_D = int(F_D)$ ($F_D = cl(F_D)$).
- 2) $\operatorname{cl}((F_D)^C) = (\operatorname{int}(F_D))^C$.
- 3) $int((F_D)^C) = (cl(F_D))^C$.

4.10. Proposition

Let $F_{1_D}, F_{2_D} \in N_3(K)$, Then:

- 1. $int(F_{1D}) \subseteq F_{1D}$
- 2. $int(int(F_{1D})) = int(F_{1D})$.
- 3. $int(F_{1D}) \sqsubseteq int(F_{2D})$, whenever $F_{1D} \sqsubseteq F_{2D}$.
- 4. $\operatorname{int}(F_{1n} \sqcap F_{2n}) = \operatorname{int}(F_{1n}) \sqcap \operatorname{int}(F_{2n})$.
- 5. $\operatorname{int}(F_{1_D} \sqcup F_{2_D}) \supseteq \operatorname{int}(F_{1_D}) \sqcup \operatorname{int}(F_{2_D})$.
- 6. $F_{1D} \subseteq cl(F_{1D})$.
- 7. $cl(cl(F_{1D})) = cl(F_{1D})$.
- 8. $cl(F_{1D}) \sqsubseteq cl(F_{2D})$, whenever $F_{1D} \sqsubseteq F_{2D}$.

9.
$$\operatorname{cl}(F_{1_{\mathrm{D}}} \sqcap F_{2_{\mathrm{D}}}) \sqsubseteq \operatorname{cl}(F_{1_{\mathrm{D}}}) \sqcap \operatorname{cl}(F_{2_{\mathrm{D}}})$$
.

10.
$$cl(F_{1D} \sqcup F_{2D}) = cl(F_{1D}) \sqcup cl(F_{2D})$$
.

4.11. Remark

The converse of (property (1,3,6,9)) in above theorem is not true in general. It can be seen from the following examples.

4.12.Example

Let K= $\{q_1,q_2\}$, D \subseteq E_K , such that D = $\{p_1\}$ and $\,F_{1_D},F_{2_D}\,\in\,N_3(K)\,$,

such that:

$$F_{1_{\mathrm{D}}} = \left\{ \left(p_{1}, \left\{ < q_{1}^{(0,5'0,5'0,5)}, q_{2}^{(0,4'0,4'0,4)} > \right\} \right) \right\},$$

$$F_{2D} = \left\{ \left(p_1, \left\{ < q_1^{(0,6'\,0,6'\,0,6)}, q_2^{(0,3'\,0,3'\,0,3)} > \right\} \right) \right\}$$

$$F_{3D} = \left\{ \left(p_1, \left\{ < q_1^{(0.5' \, 0.5' \, 0.6)}, q_2^{(0.3' \, 0.3' \, 0.4)} > \right\} \right) \right\},$$

$$F_{4_{\mathrm{D}}} = \left\{ \left(p_{1}, \left\{ < q_{1}^{(0,6'\,0,6'\,0,5)}, q_{2}^{(0,4'\,0,4'\,0,3)} > \right\} \right) \right\},$$

Then , μ = { $\widetilde{\emptyset}_D$, \widetilde{K}_D , F_{1_D} , F_{2_D} , F_{3_D} , F_{4_D} } is a neutrosohpic soft topology on K .

Note that:

- 1) $\operatorname{int}(F_{1_D} \cup F_{2_D}) \not\sqsubseteq \operatorname{int}(F_{1_D}) \sqcup \operatorname{int}(F_{2_D})$.
- 2) $F_{1D} \not\sqsubseteq int(F_{1D})$

4.13. Example

Let K= $\{q_1,q_2\}$, D \subseteq E_K , such that D = $\{p_1\}$ and $\,F_{1_D},F_{2_D}\,\in\,N_3(K)$,

such that:

$$F_{1_{\mathrm{D}}} = \left\{ \left(p_{1}, \left\{ < q_{1}^{(0,1'0,1'0,9)}, q_{2}^{(0,2'0,2'0,8)} > \right\} \right) \right\} \, ,$$

$$F_{2_{\mathrm{D}}} = \bigg\{ \Big(p_{1}, \{ < q_{1}^{(0,9'\,0,9'\,0,1)}, q_{2}^{(0,8'\,0,8'\,0,2)} > \} \Big) \bigg\}.$$

Then , μ = { $\widetilde{\emptyset}_D$, \widetilde{K}_D , F_{1_D} , F_{2_D} } is a neutrosohpic soft topology on K .

Note that:

- 1) $\operatorname{cl}(F_{1D}) \sqcup \operatorname{cl}(F_{2D}) \not\sqsubseteq \operatorname{cl}(F_{1D} \cup F_{2D})$.
- 2) $cl(F_{1D}) \not\sqsubseteq F_{1D}$.

4.14. Definition

Let $F_D \in N_3(K)$. The neutrosohpic soft exterior of F_D is denoted by $ext(F_D)$ and is defined as: $ext(F_D) = int((F_D)^C)$.

4.15. Definition

Let $F_D \in N_3(K)$. The neutrosohpic soft boundary of F_D is denoted by $br(F_D)$ and is defined as: $br(F_D) = cl((F_D)^c) \sqcap (cl(F_D))$. It must be notion that $br(F_D) = br((F_D)^c)$.

4.16. Proposition

Let (K,μ) be $(N_3 - Top)$, $F_D \in N_3(K)$. Then:

- 1) $br((F_D)^C) = ext(F_D) \sqcup int(F_D)$.
- 2) $cl(F_D) = br(F_D) \sqcup int(F_D)$.
- 3) $br(F_D) \sqcap int(F_D) = \widetilde{\emptyset}_D$.
- 4) $br(int(F_D)) \sqsubseteq br(F_D)$.

Proof: Straightforward.

4.17. Proposition

Let (K,μ) be $(N_3 - Top)$, $F_D \in N_3(K)$. Then:

- 1) F_D is a neutrosohpic soft open set \leftrightarrow br $(F_D) \sqcap (F_D) = \widetilde{\emptyset}_D$.
- 2) F_D is a neutrosohpic soft closed set \leftrightarrow br $(F_D) \sqsubseteq (F_D)$.

Proof: Straightforward.

Conclusion

- -The neutrosophic soft set theory is studied and discussed by introducing, new family of neutrosophic soft sets, new operations for neutrosophic soft set .
- -The neutrosophic soft set theory is studied and discussed by introducing, new family of neutrosophic soft sets [namely third family], new operations for neutrosophic soft sets, comparison between the new family and other families, new definitions and examples.
- New definitions, characterization, the neutrosophic soft closure, the neutrosophic soft interior, the neutrosophic soft exterior and the neutrosophic soft boundary are introduced in details .
- -We expect this research will promote the future study on theory of neutrosohpic soft sets, the theory of neutrosophic soft topological spaces and many other general frameworks .

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