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NeutroAlgebra of Neutrosophic Triplets using $\{Z_n, \times\}$

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Abstract. Smarandache in 2019 has generalized the algebraic structures to NeutroAlgebraic structures and AntiAlgebraic structures. In this paper, authors, for the first time, define the NeutroAlgebra of neutrosophic triplets group under usual $+$ and $\times$, built using $\{Z_n, \times\}$, n a composite number, $5 < n < \infty$, which are not partial algebras. As idempotents in Z_n alone are neutrals that contribute to neutrosophic triplets groups, we analyze them and build NeutroAlgebra of idempotents under usual $+$ and $\times$, which are not partial algebras. We prove in this paper the existence theorem for NeutroAlgebra of neutrosophic triplet groups. This proves the neutrals assocaited with neutrosophic triplet groups in $\{Z_n, \times\}$ under product is a NeutroAlgebra of triplets. We also prove the non-existence theorem of NeutroAlgebra for neutrosophic triplets in case of Z_n when $n = 2p, 3p$ and $4p$ (for some primes p). Several open problems are proposed. Further, the NeutroAlgebras of extended neutrosophic triplet groups have been obtained.

Keywords: neutrosophic triplets; neutrosophic extended triplets; neutrosophic triplet group; neutrosophic extended triplet group; NeutroAlgebra; partial algebra; NeutroAlgebra of neutrosophic triplets; NeutroAlgebra of neutrosophic extended triplets; AntiAlgebra

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1. **Introduction**

The neutrosophic theory proposed by Smarandache in [1] has become a powerful tool in the study/analysis of real-world data as they are dominated by uncertainty, inconsistency, and indeterminacy. Neutrosophy deals with the neutralities and indeterminacies of real-world problems. The innovative concept of neutrosophic triplet groups was introduced by [2], which gives for any element a in $(G, *)$, the $anti(a)$ and $neut(a)$ satisfying conditions

$$a * neut(a) = neut(a) * a = a$$

and

$$a * anti(a) = anti(a) * a = neut(a)$$

where $neut(a)$ is not the identity element or the classical identity of the group. They call $(a, neut(a), anti(a))$ as the neutrosophic triplet group. These neutrosophic triplets built using Z_n are always symmetric about the neutral elements. For if $(a,\ neut(a), anti(a))$ is neutrosophic triplet then $(anti(a), neut(a), a)$ there by giving a perfect symmetry of a and $anti(a)$ about the $neut(a)$. The study of neutralities have been carried out by several researchers in neutrosophic algebraic structures like neutrosophic triplet rings, groups, neutrosophic quadruple vector spaces, neutrosophic semi idempotents, duplets and triplets in neutrosophic rings, neutrosophic triplet in biaglebras, neutrosophic triplet classical group and their applications, triplet loops, subgroups, cancellable semigroups and Abel-Grassman groupoids [2–24].

[13] has defined a classical group structure on these neutrosophic triplet groups and has obtained several interesting properties and given open conjectures. Smarandache [2] defined the Neutrosophic Extended Triplet, when the neutral element is allowed to be the classical unit element. Zhang et al has defined neutrosophic extended triplet group and have obtained several results in [25]. Later [26] have obtained some results on neutrosophic extended triplet groups with partial order defined on it. More results about neutrosophic triplet groups and neutrosophic extended triplet groups can be found in [25–32] .

We in this paper study the very new notion of NeutroAlgebra introduced by [33]. Several interesting results are obtained in [12, 34–36], and they introduced Neutro BC Algebra and sub Neutro BI Algebra and so on. NeutroAlgebras and AntiAlgebras in the classical number systems were studied in [37].

Here we introduce NeutroAlgebra under the usual product and sum in case of idempotents in the semigroups $\{Z_n, \times\}$, n a composite number, $5 < n < \infty$. This study is very important for all the neutrosophic triplets in $\{Z_n, \times\}$, happen to be contributed only by the idempotents, which are the only neutrals in $\{Z_n, \times\}$. We obtain NeutroAlgebras under usual + and × in the case of neutrosophic triplet groups and neutrosophic extended triplet groups. It is pertinent to keep on record we define classical product on neutrosophic triplets, and they are classical groups under product of these triplets. This paper has six sections. Section one is introductory in nature, and basic concepts are recalled in section two. Section three obtains the existence and non-existence theorem on NeutroAlgebras under usual + or × using neutrosophic triplet groups. In section four, a similar study is carried out in the case of neutrosophic extended triplet groups. The fifth section provides a discussion on this topic, and the final section gives the conclusions based on our study and some open conjectures which will be taken for future research by the authors.

2. **Basic Concepts**

Here we recall some basic definitions which is important to make this paper a self contained one.

Definition 2.1. Let us assume that N is an empty set and with binary operation * defined on it. N is called a neutrosophic triplet set (NTS) if for any $a \in N$, there exists a neutral of "a" (denoted by neut(a)), and an opposite of "a" (denoted by anti(a)) satisfying the following conditions:

$$a * neut(a) = neut(a) * a = a$$

$$a * anti(a) = anti(a) * a = neut(a).$$

And, the neutrsophic triple is given by (a, neut(a), anti(a)).

In a neutrosophic triplet set (N, *), $a \in N$, neut(a) and anti(a) may not be unique.

In the definition given in [2], the neutral element cannot be an unit element in the usual sense, and then this restriction is removed, using the concept of a neutrosophic extended triplet in [26].

The classical unit element can be regarded as a special neutral element. The notion of neutrosophic triplet groups and that of neutrosophic extended triplet groups are distinctly dealt with in this paper.

Definition 2.2. Let us assume that (N, *) is a neutrosophic triplet set. Then, N is called a neutrosophic triplet group, if it satisfies:

(1) Closure Law, i.e., $a * b \in N, \forall a, b \in N$;

(2) Associativity, i.e., $(a * b) * c = a * (b * c), \forall a, b, c \in N$

A neutrosophic triplet group $(N, *)$ is said to be commutative, if $a * b = b * a, \forall a, b \in N$.

Let $\langle A \rangle$ be a concept (as in terms of attribute, idea, proposition, or theory). By the neutrosphication process, we split the non-empty space into three regions two opposite ones corresponding to $\langle A \rangle$ and $\langle anti\ A \rangle$, and one neutral (indeterminate) $\langle neut\ A \rangle$ (also denoted $\langle neutro\ A \rangle$) between the opposites, which may or may not be disjoint; depending on the application, but their union equals the whole space.

A NeutroAlgebra is an algebra that has at least one neutro operation or one neutro axiom (axiom that is true for some elements, indeterminate or false for the other elements) [33]. A partial algebra has at the minimum one partial operation, and all its axioms are classical. Through a theorem in [34], proved that NeutroAlgebra is a generalization of partial algebra, and also give illustrations of NeutroAlgebras that are not partial algebras. Boole has defined the Partial Algebra (based on Partial Function) as an algebra whole operation is partially well-defined, and partially undefined (this undefined goes under Indeterminacy with respect to NeutroAlgebra). Therefore, a Partial Algebra (Partial Function) has some elements for which the operation is undefined (not outer-defined). Similarly an AntiAlgebra is a nonempty set that is endowed with at least one anti-operation (or anti-function) or at least one anti-axiom.

3. NeutroAlgebras of neutrosophic triplets using $\{Z_n, \times\}$

Here for the first time authors build NeutroAlgebras using neutrosophic triplets group built using the modulo integers Z_n; n a composite number. Neutrosophic triplet groups and extended neutrosophic triplet groups were studied by [25, 26]. First we define NeutroAlgebra using the non-trivial idempotents of Z_n, n a composite number. This study is mandatory as all the neutral elements of neutrosophic triplets build using Z_n are only the non-trivial idempotents of Z_n. Next we give the existence and non existence theorems in case of NeutroAlgebras for these neutrosophic triplet sets. We give some interesting properties about them. Further it is important to note unless several open conjectures about idempotents in Z_n given in [13], are solved or some progress is made in that direction it will not be possible to completely characterize NeutroAlgebras of the neutrosophic triplet groups or extended neutrosophic triplet groups. We will be using [13] to get NeutroAlgebras of idempotents and NeutroAlgebra of neutrosophic triplet sets. First we provide examples of NeutroAlgebra using subsets of the semigroup $\{Z_n, \times\}$ and then NeutroAlgebra of idempotents in $\{Z_n, \times\}$.

Example 3.1. Let $S = \{Z_{15}, \times\}$ be a semigroup under product modulo 15. Now consider the subset $A = \{5, 10, 14\} \in S$. The Cayley table for A is given in Table 1, where outer-defined elements are denoted by *od*.

TABLE 1. Cayley Table for A

$\times$	5	10	14
5	10	5	10
10	5	10	5
14	10	5	od

We see the table has outer-defined elements denoted by *od*. So A is a NeutroAlgebra which is not a partial algebra, since the operation 14×14 is outer-defined. $14 \times 14 \equiv 1 \pmod{15}$, but $1 \notin \{5, 10, 14\}$. Therefore Table 1 is only a NeutroAlgebra. Every subset of S need not be a NeutroAlgebra. For take $B = \{3, 6, 9, 12\}$ a subset in S. Consider the Cayley table for B is given in Table 2.

B is not a NeutroAlgebra as every term in the cell is defined and associativity axiom is totally true..

TABLE 2. Cayley Table for B

$\times$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

Clearly B is a subsemigroup of S, in fact a group under $\times$ modulo 15 with 6 as its multiplicative identity, so S is a Smarandache semigroup [10].

Consider $C = \{2, 7, 8\}$ a subset of S. The Cayley table for C is given in Table 3, this has every cell to be outer-defined.

TABLE 3. Cayley Table for C

$\times$	2	7	8
2	od	od	od
7	od	od	od
8	od	od	od

So C is not a NeutroAlgebra or a subsemigroup but an AntiAlgebra since the operation $\times$ is totally outer-defined under $\times$ modulo 15.

Thus we can categorically put forth the following facts.

Every classical algebraic structure A with binary operations defined on it is such that any proper subset B of A with inherited operation of A falls under the three categories;

(1) B can be a proper substructure of a stronger structure of A with the inherited operations of A.
(2) B can only be a NeutroAlgebra, which may be a Partial Algebra, when some operation is undefined, and all other operations are well-defined and all axioms are true.
(3) B can be an AntiAlgebra when at least one operation is totally outer-defined. or at least one axiom is totally false.

Under these circumstances if one wants to get a NeutroAlgebra which is not a partial algebra for a proper subset of a classical algebraic structure one should exploit the special axioms satisfied by them, to this end we study the property of idempotents in the semigroup $\{Z_n, \times\}$.

We also in case of neutrosophic triplet group obtain a NeutroAlgebra which is not a partial algebra.

First we give examples of NeutroAlgebra which are not partial algebras using idempotents of the semigroup S $= \{Z_n, \times\}$.

Example 3.2. Let S $= \{Z_6, \times\}$ be the semigroup under product modulo 6. The nontrivial idempotents of S are $V = \{3, 4\}$. The Cayley table for V is given in Table 4,

TABLE 4. Cayley Table for V

$\times$	3	4
3	3	od
4	od	4

So V is a NeutroAlgebra under $\times$ but not a partial algebra. For the same V define operation $+$ modulo 6, the Cayley table for V is given in Table 5 and V is AntiAlgebra and not a partial algebra either.

TABLE 5. Cayley Table for V

$+$	3	4
3	od	od
4	od	od

Suppose we take $W = \{0, 1, 3, 4\}$ the collection of trivial and non trivial idempotents of S, and if we take S as a whole set but study the idempotent axiom in W we see from Table 6.

TABLE 6. Cayley Table for W

$\times$	0	1	3	4
0	0	0	0	0
1	0	1	3	4
3	0	3	3	0
4	0	4	3	4

Suppose we find the Cayley table for W under $+$ we get the Cayley table given in the following Table 7.

W itself is a NeutroAlgebra under usual $+$ with several undefined terms. W under usual product is a subsemigroup of idempotents of S; where as S under sum of idempotents is a NeutroAlgebra which is not a partial algebra under the axiom of the property of idempotency.

Now if we take for any subset of S the axiom of idempotent property we get NeutroAlgebras which are not partial algebras.

To this effect we provide an example.

TABLE 7. Cayley Table for W

+	0	1	3	4
0	0	1	3	4
1	1	od	4	od
3	3	4	0	1
4	4	od	1	od

Example 3.3. Let S = $\{Z_{42}, \times\}$ be the semigroup under product modulo 42. The trivial and non trivial idempotents of S are $B = \{0, 1, 7, 15, 21, 22, 28, 36\}$. We define + modulo 42 on this set of idempotents keeping the resultant what we need is the axiom of idempotency. The Cayley table for B is given in Table 8.

TABLE 8. Cayley Table for B

+	0	1	7	15	21	22	28	36
0	0	1	7	15	21	22	28	36
1	1	od	od	od	22	od	od	od
7	7	od	od	22	28	od	od	1
15	15	od	22	od	36	od	1	od
21	21	22	28	36	0	1	7	15
22	22	od	od	od	1	od	od	od
28	28	od	od	1	7	od	od	22
36	36	od	1	od	15	od	22	od

Thus B is a NeutroAlgebra which is not a partial algebra under the axiom of idempotency. Thus we have a large class of NeutroAlgebras which are not partial algebras.

As the main theme of this paper is study of neutrosophic triplets using modulo integers $\{Z_n, \times\}$ and prove the existence theorem and non-existence theorem of NeutroAlgebra of neutrosophic triplet groups.

In view of all these we have the following existence theorem of NeutroAlgebra of neutrosophic triplets.

Theorem 3.4. *Let $S = \{Z_n, \times\}$, n not a prime, $5 < n < \infty$. Let V be the collection of all non trivial idempotents that is all neutrals of S, where 0 and 1 are not in S. Then V under product is a NeutroAlgebra of triplets.*

Proof. Let $W = \{w_1, w_2, \ldots, w_t\}$ be the non trivial idempotents of S. It is proved in [13] that if W_i is the set of all neutrosophic triplets of a non trivial idempotent w_i in S which serves as the neutral for the collection W_i then $\{W_i, \times\}$ is a neutrosophic triplet classical group under usual product and i varies over all neutrals; $1 \leq i \leq t$. If V is the collection of all neutrosophic triplets (this V will include all W_i for different neutrals or non trivial idempotents in S), associated with $S = \{Z_n, \times\}$; then V is not closed under usual product [13] and there are many undefined elements under usual product so V is a NeutroAlgebra of neutrosophic triplets. Hence the claim. □

In view of this we have the following partial non existence theorem of NeutroAlgebra of neutrosophic triplets under + for Z_{np} where $n = 2, 3$ and 4 for some values of P provided in the Tables 9, 10 and 11. We have for Z_n, n a product of more than two primes can have NeutroAlgebra of neutrosophic triplets under +.

Theorem 3.5. *Let $S = \{Z_{np}, \times\}$; where n = 2, 3 and 4, (p a specific prime and np is not a square of a prime, prime values refer Tables 9, 10, and 11) be a semigroup under product modulo np. If V denotes the collection of all idempotents associated with the non trivial idempotents of Z_{np} then $\{V, +\}$ is never a NeutroAlgebra of triplets for $n = 2$, 3 and 4.*

Proof. Recall from [13] that there are two idempotents in all the three cases when $n = 2p$ or $3p$ or $4p$ given in Tables 9, 10 and 11. □

Table 9. Idempotent table for Z_{2p}

S.no	Z_{2p}	p	p+1
1	Z_6	3	4
2	Z_{10}	5	6
3	Z_{14}	7	8
4	Z_{22}	11	12
5	Z_{26}	13	14
6	Z_{34}	17	18
7	Z_{38}	19	20
8	Z_{46}	23	24
9	Z_{58}	29	30

We see any sum of the idempotents is 1 and product is 0.

Here in Z_{3p} and Z_{4p} also sum of idempotents is 1 and that product is 0. Tables are provided for them [13]. In case of $2p$ the nontrivial idempotents are p and $p+1$, clearly under sum this is a set. Thus we have proved the non-existence of NeutroAlgebra of idempotents under '+'.

To this effect first provide an example.

TABLE 10. Idempotent table for Z_{3p}

S. No.	Z_{3p}	p	p + 1	2p	2p + 1
1	Z_{15}	-	6	10	-
2	Z_{21}	7	-	-	15
3	Z_{33}	-	12	22	-
4	Z_{39}	13	-	-	27
5	Z_{51}	-	18	34	-
7	Z_{57}	19	-	-	39
8	Z_{69}	-	24	46	-
9	Z_{159}	-	54	106	-

TABLE 11. Idempotent table for Z_{4p}

S. No.	Z_{4p}	p	$p+1$	$3p$	$3p+1$
1	Z_{12}	-	4	9	-
2	Z_{20}	5	-	-	16
3	Z_{28}	-	8	21	-
4	Z_{44}	-	12	33	-
5	Z_{52}	13	-	-	40
6	Z_{76}	-	20	57	-
7	Z_{212}	53	-	-	160
8	Z_{388}	97	-	-	292
9	Z_{332}	-	84	249	-

Example 3.6. Consider the semigroup S = $\{Z_{10}, \times\}$. The nontrivial idempotents of S which contribute to the neutrosophic triplet set are; $\{6, 5\}$ in Z_{10}. Consider the neutrosophic triplet set $V = \{(5,5,5), (6,6,6), (8,6,2), (2,6,8), (4,6,4)\}$. It is proved $V \backslash \{(5,5,5)\}$ is a neutrosophic triplet classical group under × [13]. Now the Cayley table of V under usual product × is given in Table 12.

TABLE 12. Cayley Table for V

×	(5,5,5)	(6,6,6)	(8,6,2)	(2,6,8)	(4,6,4)
(5,5,5)	(5,5,5)	od	od	od	od
(6,6,6)	od	(6,6,6)	(8,6,2)	(2,6,8)	(4,6,4)
(8,6,2)	od	(8,6,2)	(4,6,4)	(6,6,6)	(2,6,8)
(2,6,8)	od	(2,6,8)	(6,6,6)	(4,6,4)	(8,6,2)
(4,6,4)	od	(4,6,4)	(2,6,8)	(8,6,2)	(6,6,6)

Clearly V is a NeutroAlgebra under usual product and not a partial algebra. Since we have not included the neutrals that is non trivial idempotents like 0 and 1 we have this to be only a NeutroAlgebra of triplets.

Table 13. Cayley Table for V

+	(5,5,5)	(6,6,6)	(8,6,2)	(2,6,8)	(4,6,4)
(5,5,5)	od	od	od	od	od
(6,6,6)	od	od	od	od	od
(8,6,2)	od	od	od	od	od
(2,6,8)	od	od	od	od	od
(4,6,4)	od	od	od	od	od

Thus the neutrosophic triplets collection yields only a set under addition where no pair of neutrosophic triplets gives under sum a neutrosophic triplet. Hence our claim no NeutroAlgebra neutrosophic triplets under addition. So V in Table 13 is an AntiAlgebra. Likewise the cases $3p$ and $4p$ from tables.

So if we include the non trivial idempotents 0 and 1 then we can get NeutroAlgebra of idempotents under + which is carried out in the following section.

Example 3.7. Consider the semigroup $S = \{Z_{105}, \times\}$ under $\times$ modulo 105. The non trivial idempotents are $V = \{15, 21, 36, 70, 85, 91\}$. Let M be the collection of all neutrosophic triplets using the idempotents in V. M contains elements say { (15, 15, 15), (21, 21, 21), (36, 36, 36), (30, 15, 60), (51, 36, 81)}, from the Cayley table of M under + we see there are some undefined terms also given in Table 14.

Table 14. Cayley Table for M

+	(15,15,15)	(21,21,21)	(36,36,36)	(30,15,60)	(51,36,81)
(15,15,15)	od	(36,36,36)	od	od	od
(21,21,21)	(36,36,36)	od	od	(51,36,81)	od
(36,36,36)	od	od	od	od	od
(30,15,60)	od	(51,36,81)	od	od	od
(51,36,81)	od	od	od	od	od

Hence we have a NeutroAlgebra of neutrosophic triplets under +.

We propose some open problems in this regard in the final section of this paper.

Now we find ways to get NeutroAlgebra of neutrosophic triplets under +. The possibility is by using extended neutrosophic triplets group we can have for all Z_n, n any composite number

NeutroAlgebra of neutrosophic triplets under +. Unless the conjectures proposed in [13] is solved complete characterization is not possible, only partial results and examples to that effect are possible.

In the following section we discuss NeutroAlgebra of extended neutrosophic triplet sets.

4. NeutroAlgebra of extended neutrosophic triplets using $\{Z_n, \times\}$

In this section we prove the existence of NeutroAlgebra of extended neutrosophic triplets using $\{Z_n, \times\}$, for more about extended neutrosophic triplets refer [2, 26] under both + and $\times$. Throughout this section we assume the collection of idempotents contains both the trivial idempotents 1 and 0. It is thus mandatory the neutrosophic triplet set collection contains (0, 0, 0) and (1, 1, 1) apart from the neutrosophic triplets of the form (a, 1, anti a = inverse of a), where a is in Z_n which has inverse in Z_n.

We first prove the collection of all trivial and non trivial idempotents in Z_n is a NeutroAlgebra under + and also under $\times$.

Theorem 4.1. *Let $S = \{Z_n, \times\}$ be the semigroup under product modulo $n, 5 < n < \infty$. Let $V = \{$Collection of all idempotents in Z_n including 0 and 1 $\}$.*

(1) *$V \setminus \{0, 1\}$ is a NeutroAlgebra of idempotents under $\times$ modulo n.*
(2) *V is a NeutroAlgebra of idempotents under + mod n.*

Proof. Consider $V \setminus \{0, 1\}$ for every x in $V \setminus \{1, 0\}$ is such that $x \times x = x$, so $V \setminus \{1, 0\}$ is a NeutroAlgebra under $\times$. Hence (1) is true.

Proof of (2): To show V is a NeutroAlgebra of idempotents under +. Since 0 is in V we have for every $x \in V$; $0 + x = x$ is in V, however we do not in general have the sum of two idempotents to be an idempotent. For instance 1 +1 = 2 is not an idempotent so $(V, +)$ has undefined elements, hence undefined. Thus (2) is proved. □

We provide an example to this effect.

Example 4.2. Let $S = \{Z_{10}, n, \times\}$ be the semigroup under $\times$ modulo 10. The trivial and non trivial idempotents are $V = \{0, 1, 5, 6\}$. It is easily verified V is a NeutroAlgebra under +, for 6 + 6 = 2 modulo 10. However V is not a NeutroAlgebra under $\times$, but $V \setminus \{0, 1\}$is a NeutroAlgebra under $\times$ modulo 10. For 6 + 5 = 1 modulo 10, so $V \setminus \{1, 0\}$ is a NeutroAlgebra.

Now the neutrosophic triplets of S associated with the idempotents V are $N = \{$ (0, 0, 0), (1, 1, 1), (5, 1, 5), (3, 1, 7), (7, 1, 3), (5, 5, 5), (6, 6, 6), (4 ,6, 4), (2, 6, 8) and (8, 6, 2) $\}$. We see N under + is a NeutroAlgebra, for (1, 1, 1) + (7, 1, 3) = (8, 2, 4) is not in N. N is not a NeutroAlgebra under +. But $N \setminus \{$ (0, 0, 0), (1, 1, 1), (5, 1, 5), (3, 1, 7), (7, 1, 3) $\} = W$ neutrosophic triplets formed by the non trivial idempotents 5 and 6 is a NeutroAlgebra as (5, 5, 5) × (2, 6, 8) = (0, 0, 0) which is not in W. Hence the claim. If $\{(0,0,0)\}$ is added, then the set V becomes a NeutroAlgebra under $+$.

TABLE 15. Cayley Table for V

+	(0, 0, 0)	(5,5,5)	(6,6,6)	(8,6,2)	(2,6,8)	(4,6,4)
(0, 0, 0)	(0, 0, 0)	(5,5,5)	(6,6,6)	(8,6,2)	(2,6,8)	(4,6,4)
(5,5,5)	(5,5,5)	od	od	od	od	od
(6,6,6)	(6,6,6)	od	od	od	od	od
(8,6,2)	(8,6,2)	od	od	od	od	od
(2,6,8)	(2,6,8)	od	od	od	od	od
(4,6,4)	(4,6,4)	od	od	od	od	od

Theorem 4.3. *Let $S = \{Z_n, \times\}$ be a semigroup under $\times$ modulo n, where n is not a prime and $5 < n < \infty$. Let $N = \{$collection of all extended neutrosophic triplet set including (0, 0, 0), and all neutrosophic triplets associated with the trivial idempotent 1$\}$.*

(1) *N is a NeutroAlgebra under $+$ of extended neutrosophic triplets set.*
(2) *$N \setminus \{(0,0,0)\}$ is a NeutroAlgebra of extended neutrosophic triplet set under product modulo n.*

Proof. Let N be the collection of all extended neutrosophic triplets including (0, 0, 0) and (1, 1, 1) and other triplets associated with the neutral 1.

Proof of (1): In the case extended triplet N we see sum of two idempotents need not be idempotent for (1, 1, 1) + (1, 1, 1) = (2, 2, 2) is not in N, hence N is the NeutroAlgebra of extended neutrosophic triplets which is not a partial algebra as the axiom of neutrosophic triplets is not satisfied.

Proof of (2): Consider $N \setminus \{(0,0,0)\}$. Clearly in general the product of any two idempotents is not an idempotent in Z_n, and several triplets are undefined and do not in general satisfy the triplet relation [13]. Hence the claim. □

5. Discussions

The study of NeutroAlgebra introduced by [33] is very new, here the authors built NeutroAlgebra using idempotents of $\{Z_n, \times\}$ a semigroup under $\times$ modulo n for appropriate n which are not partial algebras. Likewise NeutroAlgebra built using neutrosophic triplets set and extended neutrosophic triplets set. Some open problems based on our study is proposed in the section on conclusions.

6. **Conclusions**

For the first time authors have NeutroAlgebra using idempotents of a semigroup $S = \{Z_n, \times\}$; n a composite number $5 < n < \infty$, neutrosophic triplets and extended neutrosophic triplets. We have obtained NeutroAlgebras of idempotents which are not partial algebras under the classical operation of $+$ and $\times$ only using $S = \{Z_n, \times\}$, the semigroup under product for appropriate n. We have obtained both existence and non-existence theorem for NeutroAlgebras of idempotents in S. We suggest certain open problems for researchers as well as these problems will be taken by the authors for future study.

Problem 1: Does there exist a n (n a composite number) such that using $\{Z_n, \times\}$ there are no non trivial NeutroAlgebra of neutrosophic triplet set and NeutroAlgebra in extended neutrosophic triplet set?

Problem 2. Does there exist a n, n a composite number such that $\{Z_n, \times\}$ has its collection of trivial and non trivial idempotents denoted by N to be such that;

- $(N, +)$ is not NeutroAlgebra of idempotents ?
- $(N, \times)$ is not a NeutroAlgebra of idempotents?

Problem 3: Prove in case of $\{Z_{3p}, \times\}$ and $\{Z_{4p}, \times\}$, the idempotents are only of the form mentioned in Tables 10 and 11 respectively.

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