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The neutrosophic quaternions numbers

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Abstract: This article aims to study the neutrosophic quaternion numbers, where we defined the neutrosophic quaternions numbers and the two equal neutrosophic quaternions numbers, also, the neutrosophic quaternions numbers algebra were introduced by studying addition, multiplication, division and conjugate of a neutrosophic quaternions number. In addition, we have discussed how to calculate the absolute value of a neutrosophic quaternions number and its inverted.

Keywords: neutrosophic; quaternion numbers; division; multiplication; the absolute value of a neutrosophic quaternions number.

1. Introduction and Preliminaries

In an attempt to replace the current logics, Smarandache introduced the neutrosophic logic to illustrate a mathematical model of redundancy, uncertainty, contradiction, unknown, ambiguity, undefined, inconsistency, vagueness, imprecision, and incompleteness. Smarandache defined neutrosophic real number [2-4], probabilities according to neutrosophic logic [3-5-13], the neutrosophic statistics [4][6], he has also introduced the concept of integration and differentiation in neutrosophic [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Chakraborty utilized pentagonal neutrosophic number in networking problems, and Shortest Path Problems [11-12]. Yaser Alhasan probed the concepts of neutrosophic in the complex numbers [7-14-10].

Paper consists of 3 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, frames the neutrosophic quaternion numbers. In 3th section, a conclusion to the paper is given.

2. Main Discussion

The neutrosophic quaternions numbers

Definition1

We call the numbers that take the form:

$$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

the neutrosophic quaternions numbers, denoted by symbol H_N; where $\acute{c}, \acute{d}, \acute{c}_1, \acute{d}_1, \acute{c}_2, \acute{d}_2, \acute{c}_3, \acute{d}_3$ are real numbers, while $I =$ indeterminacy and $\breve{\imath}, \breve{\jmath}, \breve{k}$ are units such that:

$$\breve{\imath}^2 = \breve{\jmath}^2 = k^2 = \breve{\imath}\breve{\jmath}\breve{k} = -1$$
$$\breve{\imath}\breve{\jmath} = \breve{k} = -\breve{\jmath}\breve{\imath}$$
$$\breve{\jmath}\breve{k} = \breve{\imath} = -\breve{k}\breve{\jmath}$$
$$\breve{k}\breve{\imath} = \breve{\jmath} = -\breve{\imath}\breve{k}$$

We can noted that every neutrosophic quaternions number has two parts, a neutrosophic real (scalar) part and a neutrosophic vector part, where:

$\acute{c} + \acute{d}I$ is the neutrosophic real (scalar) part and $(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$ is the neutrosophic vector part

Example 1

1) $q_I = 3 + 7I + (-4 + 8I)\breve{\imath} + (7 - 3I)\breve{\jmath} - (5 + 9I)\breve{k}$
2) $q_I = 4I\breve{\imath} + (5 + I)\breve{\jmath} + (-1 + 2I)\breve{k}$
3) $q_I = 3I + (2 + 3I)\breve{\imath} + (-1 + 2I)\breve{k}$
4) $q_I = 6 + I + (9 - 4I)\breve{\imath}$

Note:

✓ $0_{H_N} = 0 + 0I + (0 + 0I)\breve{\imath} + (0 + 0I)\breve{\jmath} + (0 + 0I)\breve{k}$

✓ $0_{H_N} = 1 + 0I + (0 + 0I)\breve{\imath} + (0 + 0I)\breve{\jmath} + (0 + 0I)\breve{k}$

Definition2

Let $q_I,\ p_I \in H_N$ where:

$$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

$$p_I = \acute{\alpha} + \acute{b}I + \acute{u}_I = \acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}$$

then: $q_I = p_I$ if and only if:

$$\acute{c} = \acute{\alpha} \ \ and\ \acute{v}_I = \acute{u}_I$$

hence:

$$\acute{c}_1 + \acute{d}_1 I = \acute{\alpha}_1 + \acute{b}_1 I \quad \Rightarrow \quad \acute{c}_1 = \acute{\alpha}_1 \ and\ \acute{d}_1 = \acute{b}_1$$

$$\acute{c}_2 + \acute{d}_2 I = \acute{\alpha}_2 + \acute{b}_2 I \quad \Rightarrow \quad \acute{c}_2 = \acute{\alpha}_2 \ and\ \acute{d}_2 = \acute{b}_2$$

$$\acute{c}_3 + \acute{d}_3 I = \acute{\alpha}_3 + \acute{b}_3 I \quad \Rightarrow \quad \acute{c}_3 = \acute{\alpha}_3 \ and\ \acute{d}_3 = \acute{b}_3$$

2.1 The neutrosophic quaternions numbers algebra

2.1.1 Addition of the neutrosophic quaternions numbers

Let $q_I,\ p_I \in H_N$ where:

$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$
$p_I = \acute{\alpha} + \acute{b}I + \acute{u}_I = \acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}$

then:

$$q_I + p_I = (\acute{c} + \acute{d}I + \acute{v}_I) + (\acute{\alpha} + \acute{b}I + \acute{u}_I)$$

$$= \left((\acute{c} + \acute{\alpha}) + (\acute{d} + \acute{b})I\right) + \left((\acute{c}_1 + \acute{\alpha}_1) + (\acute{b}_1 + \acute{d}_1)I\right)\breve{\imath} + \left((\acute{c}_2 + \acute{\alpha}_2) + (\acute{b}_2 + \acute{d}_2)I\right)\breve{\jmath} + \left((\acute{c}_3 + \acute{\alpha}_3) + (\acute{b}_3 + \acute{d}_3)I\right)\breve{k}$$

Example 2

Let $q_I = 8 + 7I + (-5 + 8I)\breve{\imath} + (7 - 4I)\breve{\jmath} - (5 + 9I)\breve{k}$ *and* $p_I = 2I + (2 - 3I)\breve{\imath} + (3 - I)\breve{\jmath} + (-1 + 2I)\breve{k}$

then:

$$q_I + p_I = \left(8 + 7I + (-5 + 8I)\breve{\imath} + (7 - 4I)\breve{\jmath} - (5 + 9I)\breve{k}\right) + \left(2I + (2 - 3I)\breve{\imath} + (3 - I)\breve{\jmath} + (-1 + 2I)\breve{k}\right)$$
$$= (8 + 9I) + (-3 + 5I)\breve{\imath} + (10 - 5I)\breve{\jmath} + (-6 - 7I)\breve{k}$$

Note:

- ✓ Clearly, zero is neutral for addition.
- ✓ For every number $q_I \in H_N$, its additive counterpart is:

$$-q_I = -\acute{c} - \acute{d}I - \acute{v}_I = -\acute{c} - \acute{d}I - (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

2.1.2 Multiplication of the neutrosophic quaternions numbers

Let $q_I,\ p_I \in H_N$ where:

$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$
$p_I = \acute{\alpha} + \acute{b}I + \acute{u}_I = \acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$

then:

$$q_I.p_I = (\acute{c} + \acute{d}I + \acute{v}_I)(\acute{\alpha} + \acute{b}I + \acute{u}_I)$$

$$= \left[\acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}\right]\left[\acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}\right]$$

$$\begin{aligned} = {} & (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) + (\acute{c} + \acute{d}I)(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{c} + \acute{d}I)(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{c} + \acute{d}I)(\acute{\alpha}_3 + \acute{b}_3 I)\breve{k} \\ & + (\acute{\alpha} + \acute{b}I)(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} \\ & + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}\,(\acute{\alpha}_3 + \acute{b}_3 I)\breve{k} + (\acute{\alpha} + \acute{b}I)(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath}(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} \\ & + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath}(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{\alpha} + \acute{b}I)(\acute{c}_3 + \acute{d}_3 I)\breve{k} \\ & + (\acute{c}_3 + \acute{d}_3 I)\breve{k}(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}(\acute{\alpha}_3 + \acute{b}_3 I)\breve{k} \end{aligned}$$

$$\begin{aligned} = {} & (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) + (\acute{c} + \acute{d}I)(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{c} + \acute{d}I)(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{c} + \acute{d}I)(\acute{\alpha}_3 + \acute{b}_3 I)\breve{k} \\ & + (\acute{\alpha} + \acute{b}I)(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_1 + \acute{b}_1 I) + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} \\ & + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}\,(\acute{\alpha}_3 + \acute{b}_3 I)\breve{k} + (\acute{\alpha} + \acute{b}I)(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath}(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} \\ & - (\acute{\alpha}_2 + \acute{b}_2 I)(\acute{c}_2 + \acute{d}_2 I) + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{\alpha} + \acute{b}I)(\acute{c}_3 + \acute{d}_3 I)\breve{k} \\ & + (\acute{c}_3 + \acute{d}_3 I)\breve{k}(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)(\acute{\alpha}_3 + \acute{b}_3 I) \end{aligned}$$

$$= (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - [(\acute{c_1} + \acute{d_1}I)(\acute{\alpha_1} + \acute{b_1}I) + (\acute{\alpha_2} + \acute{b_2}I)(\acute{c_2} + \acute{d_2}I) + (\acute{c_3} + \acute{d_3}I)(\acute{\alpha_3} + \acute{b_3}I)] + (\acute{c} + \acute{d}I)[(\acute{\alpha_1} + \acute{b_1}I)\breve{\imath} + (\acute{\alpha_2} + \acute{b_2}I)\breve{\jmath} + (\acute{\alpha_3} + \acute{b_3}I)\breve{k}] + (\acute{\alpha} + \acute{b}I)[(\acute{c_1} + \acute{d_1}I)\breve{\imath} + (\acute{c_2} + \acute{d_2}I)\breve{\jmath} + (\acute{c_3} + \acute{d_3}I)\breve{k}] + (\acute{c_1} + \acute{d_1}I)(\acute{\alpha_2} + \acute{b_2}I)\breve{k} - (\acute{c_1} + \acute{d_1}I)(\acute{\alpha_3} + \acute{b_3}I)\breve{\jmath} - (\acute{c_2} + \acute{d_2}I)(\acute{\alpha_1} + \acute{b_1}I)\breve{k} + (\acute{c_2} + \acute{d_2}I)(\acute{\alpha_3} + \acute{b_3}I)\breve{\imath} + (\acute{c_3} + \acute{d_3}I)(\acute{\alpha_1} + \acute{b_1}I)\breve{\jmath} - (\acute{c_3} + \acute{d_3}I)(\acute{\alpha_2} + \acute{b_2}I)\breve{\imath}$$

we can write it by the form:

$$q_I.p_I = (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - \acute{v}_I.\acute{u}_I + (\acute{c} + \acute{d}I)\acute{u}_I + (\acute{\alpha} + \acute{b}I)\acute{v}_I + \acute{v}_I \times \acute{u}_I$$

where:

$$\acute{v}_I.\acute{u}_I = (\acute{c_1} + \acute{d_1}I)(\acute{\alpha_1} + \acute{b_1}I) + (\acute{c_2} + \acute{d_2}I)(\acute{\alpha_2} + \acute{b_2}I) + (\acute{c_3} + \acute{d_3}I)(\acute{\alpha_3} + \acute{b_3}I)$$

$$\acute{v}_I \times \acute{u}_I = \begin{vmatrix} \breve{\imath} & \breve{\jmath} & \breve{k} \\ \acute{c_1} + \acute{d_1}I & \acute{c_2} + \acute{d_2}I & \acute{c_3} + \acute{d_3}I \\ \acute{\alpha_1} + \acute{b_1}I & \acute{\alpha_2} + \acute{b_2}I & \acute{\alpha_3} + \acute{b_3}I \end{vmatrix}$$

Result1:

Multiplication of the neutrosophic quaternions numbers is not commutative because:

$$\acute{v}_I \times \acute{u}_I \neq \acute{u}_I \times \acute{v}_I$$

Example 3

Let $q_I = 2 + I + (1 - 4I)\breve{\imath} + (8 - 3I)\breve{\jmath} + (6 + 4I)\breve{k}$ *and* $p_I = 7I + (3 - 3I)\breve{\imath} + (2 - 5I)\breve{\jmath} + (-4 + 2I)\breve{k}$

then:

$$q_I.p_I = (2 + I + (1 - 4I)\breve{\imath} + (8 - 3I)\breve{\jmath} + (6 + 4I)\breve{k})(7I + (3 - 3I)\breve{\imath} + (2 - 5I)\breve{\jmath} + (-4 + 2I)\breve{k})$$

$$= 14I + 7I - [3 - 3I - 12I + 12I + 16 - 40I - 6I + 15I - 24 + 12I - 16I + 8I] + [(6 - 6I + 3I - 3I)\breve{\imath} + (4 - 10I + 2I - 5I)\breve{\jmath} + (-8 + 4I - 4I + 2I)\breve{k}] + [(7I - 28I)\breve{\imath} + (56I - 2I)\breve{\jmath} + (42I + 28I)\breve{k}] + \begin{vmatrix} \breve{\imath} & \breve{\jmath} & \breve{k} \\ 1 - 4I & 8 - 3I & 6 + 4I \\ 3 - 3I & 2 - 5I & -4 + 2I \end{vmatrix}$$

$$= 21I - (-5 - 30I) + (6 - 6I)\breve{\imath} + (4 - 13I)\breve{\jmath} + (-8 + 2I)\breve{k} + 21I\breve{\imath} + 54I\breve{\jmath} + 70I\breve{k} + (-45 + 64I)\breve{\imath} + (22 - 28I)\breve{\jmath} + (22 + 34I)\breve{k}$$

$$= 5 + 57I + (39 + 79I)\breve{\imath} + (26 - 13I)\breve{\jmath} + (12 + 106I)\breve{k}$$

Result2:

1) The neutrosophic quaternions numbers H_N is closed in relation to the addition operation, as the product of adding two neutrosophic quaternions numbers is a neutrosophic quaternions numbers, its real part is $(\acute{c} + \acute{\alpha}) + (\acute{d} + \acute{b})I$, and its vector part is:

   $$((\acute{c_1} + \acute{\alpha_1}) + (\acute{b_1} + \acute{d_1})I)\breve{\imath} + ((\acute{c_2} + \acute{\alpha_2}) + (\acute{b_2} + \acute{d_2})I)\breve{\jmath} + ((\acute{c_3} + \acute{\alpha_3}) + (\acute{b_3} + \acute{d_3})I)\breve{k}.$$

2) The neutrosophic quaternions numbers H_N is closed in relation to the multiplication operation, as the product of multipl two neutrosophic quaternions numbers is a neutrosophic quaternions numbers, its real part is $(\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - \acute{v}_I.\acute{u}_I$, and its vector part is $(\acute{c} + \acute{d}I)\acute{v}_I + (\acute{\alpha} + \acute{b}I)\acute{u}_I + \acute{v}_I \times \acute{u}_I$.
3) Multiplication accepts distribution on addition from the right and the left, so if we have three neutrosophic quaternions numbers $q_I, p_I, r_I \in H_N$, then:

$$q_I(p_I + r_I) = q_I.p_I + q_I.r_I$$

$$(p_I + r_I)q_I = p_I.q_I + r_I.q_I$$

4) The neutrality of multiplying numbers is $1 + 0I$

2.2 The neutrosophic quaternions numbers conjugate

Definition3

Let $q_I \in H_N$, where $q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$. The neutrosophic quaternions number conjugate define by the following form:

$$\overline{q_I} = \acute{c} + \acute{d}I - \acute{v}_I = \acute{c} + \acute{d}I - (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)\breve{k}.$$

Example 4

i. $q_I = 28 + 4I + (14 - 17I)\breve{\imath} + (17 - 3I)\breve{\jmath} - (77 - 45I)\breve{k}$

$\Rightarrow \overline{q_I} = 28 + 4I - (14 - 17I)\breve{\imath} - (17 - 3I)\breve{\jmath} + (77 - 45I)\breve{k}$

ii. $q_I = (1 - 13I)\breve{\jmath} + (9 - I)\breve{k} \quad \Rightarrow \quad \overline{q_I} = -(1 - 13I)\breve{\jmath} - (9 - I)\breve{k}$

Result3:

1. The neutrosophic quaternions number conjugate of $\overline{q_I}$ is the same The neutrosophic quaternions number q_I.

$$\overline{(\overline{q_I})} = q_I$$

Proof:

Let $q_I \in H_N$, where $q_I = \acute{c} + \acute{d}I + \acute{v}_I$, then:

$$\overline{q_I} = \acute{c} + \acute{d}I - \acute{v}_I$$

$$\overline{(\overline{q_I})} = \overline{(\acute{c} + \acute{d}I - \acute{v}_I)} = \acute{c} + \acute{d}I + \acute{v}_I = q_I$$

2. If $q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$

then:

- $q_I + \overline{q_I} = 2(\acute{c} + \acute{d}I) = Re(q_I)$
- $q_I - \overline{q_I} = 2\acute{v}_I = 2(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + 2(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + 2(\acute{c}_3 + \acute{d}_3 I)\breve{k} = V(q_I)$

where $Re(q_I)$ is the neutrosophic real part (scalar) of the complex number and $V(q_I)$ is the neutrosophic vector part.

3. The neutrosophic quaternions number is real (scalar) if and only if $q_I = \overline{q_I}$, and it is vector if and only if $q_I = -\overline{q_I}$.

Remarks1:

$$\overline{q_{I_1} + q_{I_2}} = \overline{q_{I_1}} + \overline{q_{I_2}}$$

Proof:

Let $q_{I_1}, q_{I_2} \in H_N$, where

$q_{I_1} = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$

$q_{I_2} = \acute{\acute{c}} + \acute{\acute{d}}I + \acute{\acute{v}}_I = \acute{\acute{c}} + \acute{\acute{d}}I + (\acute{\acute{c}}_1 + \acute{\acute{d}}_1 I)\breve{\imath} + (\acute{\acute{c}}_2 + \acute{\acute{d}}_2 I)\breve{\jmath} + (\acute{\acute{c}}_3 + \acute{\acute{d}}_3 I)\breve{k}$

then:

$$q_{I_1} + q_{I_2} = \left(\acute{c} + \acute{d}I + \acute{\acute{c}} + \acute{\acute{d}}I\right) + \left(\acute{c}_1 + \acute{d}_1 I + \acute{\acute{c}}_1 + \acute{\acute{d}}_1 I\right)\breve{\imath} + \left(\acute{c}_2 + \acute{d}_2 I + \acute{\acute{c}}_2 + \acute{\acute{d}}_2 I\right)\breve{\jmath} + \left(\acute{c}_3 + \acute{d}_3 I + \acute{\acute{c}}_3 + \acute{\acute{d}}_3 I\right)\breve{k}$$

$$\overline{q_{I_1} + q_{I_2}} = \left(\acute{c} + \acute{d}I + \acute{\acute{c}} + \acute{\acute{d}}I\right) - \left(\acute{c}_1 + \acute{d}_1 I + \acute{\acute{c}}_1 + \acute{\acute{d}}_1 I\right)\breve{\imath} - \left(\acute{c}_2 + \acute{d}_2 I + \acute{\acute{c}}_2 + \acute{\acute{d}}_2 I\right)\breve{\jmath} - \left(\acute{c}_3 + \acute{d}_3 I + \acute{\acute{c}}_3 + \acute{\acute{d}}_3 I\right)\breve{k}$$

$$= \acute{c} + \acute{d}I - (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)\breve{k} + \acute{\acute{c}} + \acute{\acute{d}}I + (\acute{\acute{c}}_1 + \acute{\acute{d}}_1 I)\breve{\imath} + (\acute{\acute{c}}_2 + \acute{\acute{d}}_2 I)\breve{\jmath} + (\acute{\acute{c}}_3 + \acute{\acute{d}}_3 I)\breve{k}$$

$$= \overline{q_{I_1}} + \overline{q_{I_2}}$$

Theorem1

The conjugate of multiplication two neutrosophic quaternions numbers is equal to the multiplication of their two conjugates.

$$\overline{q_I . p_I} = \overline{p_I} . \overline{q_I}$$

where $q_I,\ p_I \in H_N$

Proof:

Let $q_I,\ p_I \in H_N$ where:

$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$

$p_I = \acute{\alpha} + \acute{b}I + \acute{u}_I = \acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$

then:

$$q_I . p_I = (\acute{c} + \acute{d}I + \acute{v}_I)(\acute{\alpha} + \acute{b}I + \acute{u}_I)$$
$$= [\acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}][\acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}]$$

$$= (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - [(\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_1 + \acute{b}_1 I) + (\acute{\alpha}_2 + \acute{b}_2 I)(\acute{c}_2 + \acute{d}_2 I) + (\acute{c}_3 + \acute{d}_3 I)(\acute{\alpha}_3 + \acute{b}_3 I)] + (\acute{c} + \acute{d}I)[(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\imath} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{\jmath} + (\acute{\alpha}_3 + \acute{b}_3 I)\breve{k}] + (\acute{\alpha} + \acute{b}I)[(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}] + (\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_2 + \acute{b}_2 I)\breve{k} - (\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_3 + \acute{b}_3 I)\breve{\jmath} - (\acute{c}_2 + \acute{d}_2 I)(\acute{\alpha}_1 + \acute{b}_1 I)\breve{k} + (\acute{c}_2 + \acute{d}_2 I)(\acute{\alpha}_3 + \acute{b}_3 I)\breve{\imath} + (\acute{c}_3 + \acute{d}_3 I)(\acute{\alpha}_1 + \acute{b}_1 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)(\acute{\alpha}_2 + \acute{b}_2 I)\breve{\imath}$$

we can write it by the form:

$$q_I . p_I = (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - \acute{v}_I . \acute{u}_I + (\acute{c} + \acute{d}I)\acute{u}_I + (\acute{\alpha} + \acute{b}I)\acute{v}_I + \acute{v}_I \times \acute{u}_I$$

where:

$$\acute{v}_I . \acute{u}_I = (\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_1 + \acute{b}_1 I) + (\acute{c}_2 + \acute{d}_2 I)(\acute{\alpha}_2 + \acute{b}_2 I) + (\acute{c}_3 + \acute{d}_3 I)(\acute{\alpha}_3 + \acute{b}_3 I)$$

$$\acute{v}_I \times \acute{u}_I = \begin{vmatrix} \breve{\imath} & \breve{\jmath} & \breve{k} \\ \acute{c}_1 + \acute{d}_1 I & \acute{c}_2 + \acute{d}_2 I & \acute{c}_3 + \acute{d}_3 I \\ \acute{\alpha}_1 + \acute{b}_1 I & \acute{\alpha}_2 + \acute{b}_2 I & \acute{\alpha}_3 + \acute{b}_3 I \end{vmatrix}$$

then:

$\overline{q_I . p_I} = (\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - \acute{v}_I . \acute{u}_I - (\acute{c} + \acute{d}I)\acute{u}_I - (\acute{\alpha} + \acute{b}I)\acute{v}_I - \acute{v}_I \times \acute{u}_I$

$\overline{p_I} . \overline{q_I} = (\acute{\alpha} + \acute{b}I - \acute{u}_I)(\acute{c} + \acute{d}I - \acute{v}_I)$

$$= [\acute{a} + \acute{b}I - (\acute{a}_1 + \acute{b}_1 I)\breve{\imath} - (\acute{a}_2 + \acute{b}_2 I)\breve{\jmath} - (\acute{a}_3 + \acute{b}_3 I)\breve{k}][\acute{c} + \acute{d}I - (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{c}_3 + \acute{d}_3 I)\breve{k}]$$

$$= (\acute{a} + \acute{b}I)(\acute{c} + \acute{d}I) - (\acute{a} + \acute{b}I)(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{a} + \acute{b}I)(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{a} + \acute{b}I)(\acute{c}_3 + \acute{d}_3 I)\breve{k} - (\acute{c} + \acute{d}I)(\acute{a}_1 + \acute{b}_1 I)\breve{\imath} - (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_1 + \acute{d}_1 I) + (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_2 + \acute{d}_2 I)\breve{k} - (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_3 + \acute{d}_3 I)\breve{\jmath} - (\acute{c} + \acute{d}I)(\acute{a}_2 + \acute{b}_2 I)\breve{\jmath} - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_1 + \acute{d}_1 I)\breve{k} - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_2 + \acute{d}_2 I) - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_3 + \acute{d}_3 I)\breve{\imath} - (\acute{c} + \acute{d}I)(\acute{a}_3 + \acute{b}_3 I)\breve{k} + (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_1 + \acute{d}_1 I)\breve{\jmath} - (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_2 + \acute{d}_2 I)\breve{\imath} - (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_3 + \acute{d}_3 I)$$

$$= (\acute{a} + \acute{b}I)(\acute{c} + \acute{d}I) - (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_1 + \acute{d}_1 I) - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_2 + \acute{d}_2 I) - (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_3 + \acute{d}_3 I) - (\acute{c} + \acute{d}I)(\acute{a}_1 + \acute{b}_1 I)\breve{\imath} - (\acute{c} + \acute{d}I)(\acute{a}_2 + \acute{b}_2 I)\breve{\jmath} - (\acute{c} + \acute{d}I)(\acute{a}_3 + \acute{b}_3 I)\breve{k} - (\acute{a} + \acute{b}I)(\acute{c}_1 + \acute{d}_1 I)\breve{\imath} - (\acute{a} + \acute{b}I)(\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} - (\acute{a} + \acute{b}I)(\acute{c}_3 + \acute{d}_3 I)\breve{k} + (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_2 + \acute{d}_2 I)\breve{k} - (\acute{a}_1 + \acute{b}_1 I)(\acute{c}_3 + \acute{d}_3 I)\breve{\jmath} - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_1 + \acute{d}_1 I)\breve{k} - (\acute{a}_2 + \acute{b}_2 I)(\acute{c}_3 + \acute{d}_3 I)\breve{\imath} + (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_1 + \acute{d}_1 I)\breve{\jmath} - (\acute{a}_3 + \acute{b}_3 I)(\acute{c}_2 + \acute{d}_2 I)\breve{\imath}$$

$$= (\acute{c} + \acute{d}I)(\acute{a} + \acute{b}I) - \acute{v}_I . \acute{u}_I - (\acute{c} + \acute{d}I)\acute{u}_I - (\acute{a} + \acute{b}I)\acute{v}_I - \acute{v}_I \times \acute{u}_I$$

$$\Rightarrow \quad \overline{q_I . p_I} = \overline{p_I} . \overline{q_I}$$

2.3 The absolute value of a neutrosophic quaternions number

Let $q_I \in H_N$ where: $q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$, the absolute value of a neutrosophic quaternions numbers defined by the following form:

$$|q_I| = \sqrt{(\acute{c} + \acute{d}I)^2 + (\acute{c}_1 + \acute{d}_1 I)^2 + (\acute{c}_2 + \acute{d}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2}$$

Example 5

Let $q_I = 1 - 4I + I\breve{\imath} + 2I\breve{\jmath} - I\breve{k}$, then:

$$|q_I| = \sqrt{(\acute{c} + \acute{d}I)^2 + (\acute{c}_1 + \acute{d}_1 I)^2 + (\acute{c}_2 + \acute{d}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2}$$

$$= \sqrt{(1 - 4I)^2 + (I)^2 + (2I)^2 + (I)^2}$$

$$= \sqrt{1 - 8I + 16I + I + 4I + I}$$

$$= \sqrt{1 + 14I}$$

$$\sqrt{1 + 14I} \equiv x + yI$$

$$1 + 14I \equiv x^2 + 2xyI + y^2$$

by identifying we get:

$$\begin{cases} x^2 = 1 \\ y^2 + 2xy = 14 \end{cases}$$

Since the absolute value is positive, we take: $x = 1$

then:

$$y^2 + 2y = 14 \implies y^2 + 2y - 14 = 0$$

$$y = \frac{-2 + 2\sqrt{15}}{2} = -1 + \sqrt{15} \approx 2.9$$

Therefore,

$$|q_I| = \sqrt{(1-4I)^2 + (I)^2 + (2I)^2 + (I)^2} = 1 + 2.9I$$

Theorem2

Let $q_I \in H_N$ where: $q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{i} + (\acute{c}_2 + \acute{d}_2 I)\breve{j} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$, multiplication the absolute value of q_I by its conjugate equals to square of the absolute value of q_I .

$$q_I . \overline{q_I} = |q_I|^2$$

Proof:

$$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{i} + (\acute{c}_2 + \acute{d}_2 I)\breve{j} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

$$\Longrightarrow \quad q_I = \acute{c} + \acute{d}I - \acute{v}_I = \acute{c} + \acute{d}I - (\acute{c}_1 + \acute{d}_1 I)\breve{i} - (\acute{c}_2 + \acute{d}_2 I)\breve{j} - (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

$$q_I . \overline{q_I} = (\acute{c} + \acute{d}I)^2(\acute{c} + \acute{d}I - \acute{v}_I)$$

$$= (\acute{c} + \acute{d}I)^2 - (\acute{c} + \acute{d}I)\acute{v}_I + (\acute{c} + \acute{d}I)\acute{v}_I - \acute{v}_I . \acute{v}_I$$

$$= (\acute{c} + \acute{d}I)^2 - \acute{v}_I . \acute{v}_I$$

$$= (\acute{c} + \acute{d}I)^2 + (\acute{c}_1 + \acute{d}_1 I)^2 + (\acute{c}_2 + \acute{d}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2 = |q_I|^2$$

$$\Longrightarrow \quad q_I . \overline{q_I} = |q_I|^2$$

Example 6

Let $q_I = 2 - 6\mathrm{I} + 3I\breve{i} + (1 + 2I)\breve{j} - 5\breve{k}$, then:

$$q_I . \overline{q_I} = |q_I|^2$$

$$= (2 - 6\mathrm{I})^2 + 9\mathrm{I} + (1 + 2I)^2$$

$$= 4 - 24I + 36I + 9I + 1 + 4I + 4I$$

$$= 5 + 29I$$

Remarks2:

Let $q_I \in H_N$, then:

1) $|q_I| = |\overline{q}_I| = |-q_I|$
2) $|q_I . p_I| = |q_I| . |p_I|$

Proof (2):

$$|q_I . p_I|^2 = q_I . p_I \overline{(q_I . p_I)} = q_I . p_I . \overline{p_I} . \overline{q}_I = q_I . |p_I|^2 . \overline{q}_I = q_I . \overline{q}_I . |p_I|^2 = |q_I|^2 . |p_I|^2$$

2.4 Division of neutrosophic quaternions numbers

Let $q_I,\ p_I \in H_N$ where:

$$q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{i} + (\acute{c}_2 + \acute{d}_2 I)\breve{j} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$
$$p_I = \acute{\alpha} + \acute{b}I + \acute{u}_I = \acute{\alpha} + \acute{b}I + (\acute{\alpha}_1 + \acute{b}_1 I)\breve{i} + (\acute{\alpha}_2 + \acute{b}_2 I)\breve{j} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$$

then:

$$\frac{q_I}{p_I} = \frac{\acute{c} + \acute{d}I + \acute{v}_I}{\acute{\alpha} + \acute{b}I + \acute{u}_I}$$

multiply the numerator and denominator by conjugate of p_I we get:

$$\frac{q_I}{p_I} = \frac{(\acute{c} + \acute{d}I + \acute{v}_I)(\acute{\alpha} + \acute{b}I - \acute{u}_I)}{(\acute{\alpha} + \acute{b}I + \acute{u}_I)(\acute{\alpha} + \acute{b}I - \acute{u}_I)}$$

$$= \frac{(\acute{c} + \acute{d}I + \acute{v}_I)(\acute{\alpha} + \acute{b}I - \acute{u}_I)}{(\acute{\alpha} + \acute{b}I)^2 - (\acute{u}_I)^2}$$

$$= \frac{(\acute{c} + \acute{d}I)(\acute{\alpha} + \acute{b}I) - \acute{v}_I.\acute{u}_I + (\acute{c} + \acute{d}I)\acute{v}_I + (\acute{\alpha} + \acute{b}I)\acute{u}_I + \acute{v}_I \times \acute{u}_I}{(\acute{\alpha} + \acute{b}I)^2 - (\acute{u}_I)^2}$$

where:

$$\acute{v}_I.\acute{u}_I = (\acute{c}_1 + \acute{d}_1 I)(\acute{\alpha}_1 + \acute{b}_1 I) + (\acute{c}_2 + \acute{d}_2 I)(\acute{\alpha}_2 + \acute{b}_2 I) + (\acute{c}_3 + \acute{d}_3 I)(\acute{c}_3 + \acute{d}_3 I)$$

$$\acute{v}_I \times \acute{u}_I = \begin{vmatrix} \breve{\imath} & \breve{\jmath} & \breve{k} \\ \acute{c}_1 + \acute{d}_1 I & \acute{c}_2 + \acute{d}_2 I & \acute{c}_3 + \acute{d}_3 I \\ \acute{\alpha}_1 + \acute{b}_1 I & \acute{\alpha}_2 + \acute{b}_2 I & \acute{\alpha}_3 + \acute{b}_3 I \end{vmatrix}$$

and $(\acute{\alpha} + \acute{b}I)^2 - (\acute{u}_I)^2 = (\acute{\alpha} + \acute{b}I)^2 + (\acute{\alpha}_1 + \acute{b}_1 I)^2 + (\acute{\alpha}_2 + \acute{b}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2$

Example 7

Let $q_I = 2 + (1 - 4I)\breve{\imath} - 3I\breve{\jmath} + (6 + 4I)\breve{k}$ *and* $p_I = 7I - 2I\breve{\imath} + (2 - 5I)\breve{\jmath} + 4\breve{k}$

then:

$$\frac{q_I}{p_I} = \frac{2 + (1 - 4I)\breve{\imath} - 3I\breve{\jmath} + (6 + 4I)\breve{k}}{7I - 2I\breve{\imath} + (2 - 5I)\breve{\jmath} + 4\breve{k}}$$

$$= \frac{(2 + (1 - 4I)\breve{\imath} - 3I\breve{\jmath} + (6 + 4I)\breve{k})(7I + 2I\breve{\imath} - (2 - 5I)\breve{\jmath} - 4\breve{k})}{(7I - 2I\breve{\imath} + (2 - 5I)\breve{\jmath} + 4\breve{k})(7I + 2I\breve{\imath} - (2 - 5I)\breve{\jmath} - 4\breve{k})}$$

$$= \frac{(2 + (1 - 4I)\breve{\imath} - 3I\breve{\jmath} + (6 + 4I)\breve{k})(7I + 2I\breve{\imath} - (2 - 5I)\breve{\jmath} - 4\breve{k})}{(7I)^2 - (-2I\breve{\imath} + (2 - 5I)\breve{\jmath} + 4\breve{k})^2}$$

$$= \frac{24 + 45I - 18I\breve{\imath} + (-4 + 11I)\breve{\jmath} + (8 + 28I)\breve{k} + (12 - 30I)\breve{\imath} + (4 + 4I)\breve{\jmath} + (-2 - I)\breve{k}}{49I - (-4I - (2 - 5I)^2 - 16)}$$

$$= \frac{24 + 45I + (12 - 48I)\breve{\imath} + 15I\breve{\jmath} + (6 + 27I)\breve{k}}{20 + 9I}$$

$$= \frac{24 + 45I}{20 + 9I} + \frac{12 - 48I}{20 + 9I}\breve{\imath} + \frac{15I}{20 + 9I}\breve{\jmath} + \frac{6 + 27I}{20 + 9I}\breve{k}$$

$$= \frac{6}{5} + \frac{171}{145}I + \left(\frac{3}{5} + \frac{267}{145}I\right)\breve{\imath} + \left(\frac{15}{29}I\right)\breve{\jmath} + \left(\frac{3}{10} + \frac{243}{290}I\right)\breve{k}$$

2.5 Inverted Neutrosophic quaternions numbers

Definition4

We define Inverted $q_I \in H_N$ as $q_I^{-1} \in H_N$, whereas:

$$q_I.q_I^{-1} = q_I^{-1}.q_I = 1_{H_N}$$

whereas: $q_I \neq 0_{H_N}$

Remark3:

$$|q_I|^2 = q_I.\overline{q_I} \quad \Rightarrow \quad q_I = \frac{|q_I|^2}{\overline{q_I}} \quad \Rightarrow \quad {q_I}^{-1} = \frac{\overline{q_I}}{|q_I|^2}$$

Proof:

Let $q_I \in H_N$ where: $q_I = \acute{c} + \acute{d}I + \acute{v}_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}$, then:

$${q_I}^{-1} = \frac{1}{q_I} = \frac{1}{\acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + (\acute{c}_2 + \acute{d}_2 I)\breve{\jmath} + (\acute{c}_3 + \acute{d}_3 I)\breve{k}}$$

$$= \frac{\acute{c} + \acute{d}I}{(\acute{\alpha} + \acute{b}I)^2 + (\acute{\alpha}_1 + \acute{b}_1 I)^2 + (\acute{\alpha}_2 + \acute{b}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2} - \frac{(\acute{c}_1 + \acute{d}_1 I)}{(\acute{\alpha} + \acute{b}I)^2 + (\acute{\alpha}_1 + \acute{b}_1 I)^2 + (\acute{\alpha}_2 + \acute{b}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2}\breve{\imath} - \frac{(\acute{c}_2 + \acute{d}_2 I)}{(\acute{\alpha} + \acute{b}I)^2 + (\acute{\alpha}_1 + \acute{b}_1 I)^2 + (\acute{\alpha}_2 + \acute{b}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2}\breve{\jmath} - \frac{(\acute{c}_3 + \acute{d}_3 I)}{(\acute{\alpha} + \acute{b}I)^2 + (\acute{\alpha}_1 + \acute{b}_1 I)^2 + (\acute{\alpha}_2 + \acute{b}_2 I)^2 + (\acute{c}_3 + \acute{d}_3 I)^2}\breve{k}$$

Example 8

$$\frac{1}{2 + I + (1 - 4I)\breve{\imath} + (8 - 3I)\breve{\jmath} + (6 + 4I)\breve{k}} = \frac{2 + I}{105 + 38I} - \frac{(1 - 4I)}{105 + 38I}\breve{\imath} - \frac{(8 - 3I)}{105 + 38I}\breve{\jmath} - \frac{(6 + 4I)}{105 + 38I}\breve{k}$$

$$= \frac{2}{105} - \frac{29}{15015}I + \left(-\frac{1}{105} + \frac{458}{15015}I\right)\breve{\imath} + \left(-\frac{8}{105} + \frac{619}{15015}I\right)\breve{\jmath} + \left(-\frac{6}{105} - \frac{192}{15015}I\right)\breve{k}$$

Remark4:

$$(p_I q_I)^{-1} = {q_I}^{-1}.{p_I}^{-1} \qquad \text{, whereas: } p_I.q_I \neq 0_{H_N}$$

Remark5:

Since any neutrosophic complex number $q_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath}$ can be written in the form:

$$q_I = \acute{c} + \acute{d}I + (\acute{c}_1 + \acute{d}_1 I)\breve{\imath} + 0\breve{\jmath} + 0\breve{k}$$

then:

$$R_N \subseteq C_N \subseteq H_N$$

5. Conclusions

In this paper, we introduced the neutrosophic quaternions numbers, where all algebraic operations were studied on it. Also, we studied the absolute value of a neutrosophic quaternions number and its inverted.

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