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A Comprehensive Analysis of Neutrosophic Bonferroni Mean Operator

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Abstract

The Neutrosophic Bonferroni operator is a novel operator that we provide in this paper. Then the arithmetic operations for Neutrosophic Bonferroni operator is developed which tells the existence of Neutrosophic Bonferroni operator. Then its properties were discussed with special cases. To group decision-making issues with several attributes, arithmetic ranking operations and the Neutrosophic approach are used. The result is compared with the existing methodology. The suggested approach will more accurately give the decision maker the ideal attribute than the existing system does. Neutrophic multicriteria is a method of decision-making that makes use of ambiguity to integrate various criteria or factors—often with imprecise or ambiguous data—to reach a result. The neutrosophic multicriteria analysis enables the assessment of subjective and qualitative factors, which can assist in resolving conflicting goals and preferences. In Neutrosophic Multi-Attribute Group Decision Making (NMAGDM) problems, all the data supplied by the decision makers (DMs) is expressed in single-value Neutrosophic triangular and trapezoidal numbers, which are studied in this work and can improve the flexibility and precision of capturing uncertainty and aggregating preferences. Studying this operator is crucial because it can be utilised to resolve multi-attribute

Keywords: Group decision making in multi-attributes using Neutrosophic(NMAGDM),Neutrosophic Bonferroni operator, weighted Neutrosophic Bonferroni operator, Neutrosophic operator.

1.Introduction

[1] was first introduced the fuzzy set theory. This theory was used in many areas which is explained in [2]as the essential ideas in fuzzy set theory are covered in Fundamentals of Fuzzy Sets. Its four-part structure makes it simple

to reference both more recent and earlier findings in the subject, In [3] the definitions of the axioms pertaining to the fundamental relationships between the entropy, distance, and similarity metrics of fuzzy collections are discussed, [4] as Using probabilistic data, we created a novel decision-making model and aggregated the data using the instantaneous probability idea. This kind of probability introduces the decision maker's attitude, which changes the objective probability and in [5] as the theory and procedure of decision making are provided by the grey relational degree-based decision making approach. The above all can deviate in various situation which was simplified by various fuzzy members like [6] used interval valued fuzzy members produced by fuzzy disjunctive and conjunctive normal forms, serve as a type II fuzzy set model to depict the second order semantic uncertainty achieved by the linguistic connectives that combine two or more fuzzy, ambiguous ideas, [7] used vague sets, [8] used intuitionistic fuzzy sets, [9] used interval type 2 fuzzy sets, [9] used fuzzy multisets. This application was clearly explained in [10] as a method for handling several qualities The suggested aggregation operators are used to make decisions in an intuitionistic fuzzy environment, and an illustration is given to show the practicality and accuracy of the recommended approach, [11] and [12]as generalization of a fuzzy set is a membership function and a non-membership function define an intuitionistic fuzzy set. In this study, we first present a technique based on the accuracy and score functions for comparing two intuitionistic fuzzy values. In [13], Xia et al. recently presented an intuitionistic multiplicative preference relation to characterize the preference information provided by a decision maker over a set of objects. Next, we develop some aggregation operators for aggregating intuitionistic fuzzy values, such as the intuitionistic fuzzy ordered weighted averaging operator, intuitionistic fuzzy hybrid aggregation operator, and intuitionistic fuzzy weighted averaging operator, and establish various properties of these operators. The intuitionistic multiplicative preference relation is made up of all the 2-tuples, which can simultaneously express how much one thing is prior to another and how much it is not. Compared to the conventional multiplicative preference relation, the 2-tuples can more fully reflect the decision maker's preferences over objects because each component derives its value from the closed interval [1/9, 9]. Finding a way to extract the object's priority weights from an intuitionistic multiplicative preference relation is a key topic of research for decision making with such information.

The intricacy of the problem has increased along with the introduction of various sorts of fuzzy members. Consequently, the Bonferroni operator was introduced as a new operator. [14] introduced the aggregation operator for mean for the first time. With the aid of the OWA operator, this was made more generic, and [20] provides the Choquet integral. The above-mentioned generalised approach is also provided by [21]. The Bonferroni mean(BM) operator of interval type-2 is defined in [15]. Additionally, [16] applies this Bonferroni mean as the Bonferroni geometric mean, which is a generalisation of the Bonferroni mean and geometric mean and can reflect the correlations of the combined arguments. To more correctly define the uncertainty and fuzziness, membership, non-membership, and uncertainty information could be taken into consideration using an intuitionistic fuzzy set. We go on building the intuitionistic fuzzy geometric Atanassov To collect the intuitionistic fuzzy information of Atanassov, define the interdependence between arguments using the Bonferroni mean. A few characteristics and unique circumstances of this mean are also looked at [17], since it is a desired feature if the BM can capture the correlations between the input arguments. It seems, nevertheless, that the existing literature only discusses using the BM to aggregate crisp numbers—it does not handle other types of reasoning. In this work, we investigate the BM in intuitionistic fuzzy environments. We construct an intuitionistic fuzzy BM (IFBM) and discuss possible specific cases for it. Next, using fuzzy multi-attribute group decision making (FMAGDM) scenarios in which the decision makers' (DMs') input is represented as trapezoidal interval type-2 fuzzy sets (IT2 FS), the weighted IFBM is used to multicriteria decision making. This is done in [18]. We introduce the idea of interval possibility mean value and provide a new method for calculating the possibility degree of two trapezoidal IT2 FS. The type-2 fuzzy geometric Bonferroni mean operator for trapezoidal intervals and the type-2 fuzzy weighted geometric Bonferroni mean operator for trapezoidal intervals (TIT2FWGBM) are the two aggregation techniques that we then develop and the Bonferroni mean (BM) is a crucial aggregation operator in decision-making, as stated in [19]. A useful aspect of the BM is its capacity to record the relationship between the individual attributes or the aggregation arguments. Proposed by Jin et al. in 2016, the extensions of the BM consist of the optimum weighted geometric Bonferroni mean (OWGBM) and the generalised optimised weighted geometric Bonferroni mean (GOWGBM). However, the OWGBM and GOWGBM lack both reducibility and boundedness, which may lead to unsuitable and irrational aggregation outputs as well as poor decision-making. To overcome these existing limitations, we propose two new measures: the generalised normalised weighted geometric Bonferroni mean (GNWGBM) and the normalised weighted geometric Bonferroni mean (NWGBM), which are based on the GOWGBM and the normalised weighted Bonferroni mean (NWBM).

Now, this can be expanded upon in this paper. The aggregating operations of a suggested Neutrosophic Bonferroni operator are defined. [22] using Bonferroni power aggregation operator but the evaluation process is limited in satisfying sum squares of non-membership and membership value. By using the above operators there will be flaws in final calculation and that can be overcome by a proposed operator Neutrosophic Bonferroni operator which satisfying some required properties and theorems and it is extended to weighted Neutrosophic Bonferroni operator with its properties and theorems. Determining the concepts of neutrosophic possibility mean value and the degree of neutrosophic possibility of two and three trapezoidal and triangular neutrosophic sets is the aim of this work. The neutrosophic Bonferroni mean operator in triangular and trapezoidal arrangements [23].This essay attempted to give a summary of every method that may be used to address the traffic issue [24]. It also applies the given approach to a profit analysis decision-making problem in [25]

Thus, the paper is formulated as follows in Section 2, the basic definitions and theorems with proof of Neutrosophic Bonferroni mean operator and theorem is given. In section 3, the properties of Neutrosophic Bonferroni operator will be explained. In section 4, the weighted Neutrosophic Bonferroni operator is given with properties and theorems are given. In section 5, the conclusion is given.

2. Neutrosophic Bonferroni operators:

Definition 2.1:

Let

$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$

$= (\left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$

represent the collection of Neutrosophic members, and we define the Neutrosophic Bonferroni mean for s,t≥0 as

$$NBM^{(s,t)}\left((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\right) = \left(\frac{1}{s+t}\left(\bigotimes_{\substack{i,j=1\\ i\neq j}}^{m} \left(sTN_i \oplus tTN_j\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\bigotimes_{\substack{i,j=1\\ i\neq j}}^{m} \left(sIN_i \oplus tIN_j\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\bigotimes_{\substack{i,j=1\\ i\neq j}}^{m} \left(sFN_i \oplus tFN_j\right)\right)^{\frac{1}{m(m-1)}}\right) \quad (1)$$

<u>Theorem 2.1:</u>

Let

$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$

$= (\left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$

represent the set of Neutrosophic members, and in the case where s,t≥0, the aggregation operation on (1) is likewise a Neutrosophic member, as shown by

$NBM^{(s,t)}\left((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\right) = (TN, IN, FN) =$

$\left((TN^U, IN^U, FN^U), (TN^L, IN^L, FN^L)\right)$where

$(\mathrm{TN}^{\mathrm{U}}, \mathrm{IN}^{\mathrm{U}}, \mathrm{FN}^{\mathrm{U}})$

$$= \left(\begin{array}{c} \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i1}^{U} \oplus tTN_{j1}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i1}^{U} \oplus tIN_{j1}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i1}^{U} \oplus tFN_{j1}^{U} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i2}^{U} \oplus tTN_{j2}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i2}^{U} \oplus tIN_{j2}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i2}^{U} \oplus tFN_{j2}^{U} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i3}^{U} \oplus tTN_{j3}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i3}^{U} \oplus tIN_{j3}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i3}^{U} \oplus tFN_{j3}^{U} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i4}^{U} \oplus tTN_{j4}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i4}^{U} \oplus tIN_{j4}^{U} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i4}^{U} \oplus tFN_{j4}^{U} \right) \right)^{\frac{1}{m(m-1)}} \right) \\ \min_{i=1,2,3,\ldots,m\left(Th_i^U, Ih_i^U, Fh_i^U\right)} \end{array} \right) \tag{2}$$

And

(TN^{L}, IN^{L}, FN^{L})

$$= \left(\begin{array}{c} \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i1}^{L} \oplus tTN_{j1}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{n} \left(sIN_{i1}^{L} \oplus tIN_{j1}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{n} \left(sFN_{i1}^{L} \oplus tFN_{j1}^{L} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i2}^{L} \oplus tTN_{j2}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i2}^{L} \oplus tIN_{j2}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i2}^{L} \oplus tFN_{j2}^{L} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i3}^{L} \oplus tTN_{j3}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i3}^{L} \oplus tIN_{j3}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i3}^{L} \oplus tFN_{j3}^{L} \right) \right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sTN_{i4}^{L} \oplus tTN_{j4}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sIN_{i4}^{L} \oplus tIN_{j4}^{L} \right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes_{\substack{i,j=1 \\ i\neq j}}^{m} \left(sFN_{i4}^{L} \oplus tFN_{j4}^{L} \right) \right)^{\frac{1}{m(m-1)}} \right) \\ min_{i=1,2,3,\ldots,m\left(Th_i^L, Ih_i^L, Fh_i^L\right)} \end{array} \right) \tag{3}$$

The proof of the above theorem is dome by mathematical induction,

Proof:

We start the proof by proving

$$\left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN_i\oplus tTN_j\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN_i\oplus tIN_j\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN_i\oplus tFN_j\right)\right)\right)$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^U_{i1}\oplus tTN^U_{j1}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^U_{i1}\oplus tIN^U_{j1}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^U_{i1}\oplus tFN^U_{j1}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^U_{i2}\oplus tTN^U_{j2}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^U_{i2}\oplus tIN^U_{j2}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^U_{i2}\oplus tFN^U_{j2}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^U_{i3}\oplus tTN^U_{j3}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^U_{i3}\oplus tIN^U_{j3}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^U_{i3}\oplus tFN^U_{j3}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^U_{i4}\oplus tTN^U_{j4}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^U_{i4}\oplus tIN^U_{j4}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^U_{i4}\oplus tFN^U_{j4}\right)\right)\right)\\ min_{i=1,2,3,\dots,m\left(Th^U_i,Ih^U_i,Fh^U_i\right)}\end{array}\right),\\ \left(\begin{array}{c}\left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^L_{i1}\oplus tTN^L_{j1}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^L_{i1}\oplus tIN^L_{j1}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^L_{i1}\oplus tFN^L_{j1}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^L_{i2}\oplus tTN^L_{j2}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^L_{i2}\oplus tIN^L_{j2}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^L_{i2}\oplus tFN^L_{j2}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^L_{i3}\oplus tTN^L_{j3}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^L_{i3}\oplus tIN^L_{j3}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^L_{i3}\oplus tFN^L_{j3}\right)\right)\right),\\ \left(\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sTN^L_{i4}\oplus tTN^L_{j4}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sIN^L_{i4}\oplus tIN^L_{j4}\right)\right),\left(\bigotimes^{m}_{\substack{i,j=1\\ i,j\text{ is not}\\ \text{same}}}\left(sFN^L_{i4}\oplus tFN^L_{j4}\right)\right)\right)\\ min_{i=1,2,3,\dots,m\left(Th^L_i,Ih^L_i,Fh^L_i\right)}\end{array}\right)\end{array}\right)$$

(4)

Then by arithmetic operations on Neutrosophic we get the following equations

$$\left(\left((sTN_i \oplus tTN_j)\right),\left((sIN_i \oplus tIN_j)\right),\left((sFN_i \oplus tFN_j)\right)\right)$$

$$= \left(\left(\begin{array}{c} \left(\left((sTN_{i1}^U \oplus tTN_{j1}^U)\right),\left((sIN_{i1}^U \oplus tIN_{j1}^U)\right),\left((sFN_{i1}^U \oplus tFN_{j1}^U)\right)\right), \\ \left(\left((sTN_{i2}^U \oplus tTN_{j2}^U)\right),\left((sIN_{i2}^U \oplus tIN_{j2}^U)\right),\left((sFN_{i2}^U \oplus tFN_{j2}^U)\right)\right), \\ \left(\left((sTN_{i3}^U \oplus tTN_{j3}^U)\right),\left((sIN_{i3}^U \oplus tIN_{j3}^U)\right),\left((sFN_{i3}^U \oplus tFN_{j3}^U)\right)\right), \\ \left(\left((sTN_{i4}^U \oplus tTN_{j4}^U)\right),\left((sIN_{i4}^U \oplus tIN_{j4}^U)\right),\left((sFN_{i4}^U \oplus tFN_{j4}^U)\right)\right) \\ min_{i=1,2,3,\ldots,m\left(Th_i^U,Ih_i^U,Fh_i^U\right)} \end{array} \right), \left(\begin{array}{c} \left(\left((sTN_{i1}^L \oplus tTN_{j1}^L)\right),\left((sIN_{i1}^L \oplus tIN_{j1}^L)\right),\left((sFN_{i1}^L \oplus tFN_{j1}^L)\right)\right), \\ \left(\left((sTN_{i2}^L \oplus tTN_{j2}^L)\right),\left((sIN_{i2}^L \oplus tIN_{j2}^L)\right),\left((sFN_{i2}^L \oplus tFN_{j2}^L)\right)\right), \\ \left(\left((sTN_{i3}^L \oplus tTN_{j3}^L)\right),\left((sIN_{i3}^L \oplus tIN_{j3}^L)\right),\left((sFN_{i3}^L \oplus tFN_{j3}^L)\right)\right), \\ \left(\left((sTN_{i4}^L \oplus tTN_{j4}^L)\right),\left((sIN_{i4}^L \oplus tIN_{j4}^L)\right),\left((sFN_{i4}^L \oplus tFN_{j4}^L)\right)\right) \\ min_{i=1,2,3,\ldots,m\left(Th_i^L,Ih_i^L,Fh_i^L\right)} \end{array} \right) \right)$$

(a) for $m = 2$,

$$\left(\left(\bigotimes^2{}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}} (sTN_i \oplus tTN_j)\right),\left(\bigotimes^2{}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}} (sIN_i \oplus tIN_j)\right),\left(\bigotimes^2{}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}} (sFN_i \oplus tFN_j)\right)\right) =$$

$$\left(\left((sTN_1 \oplus tTN_2) \otimes (sTN_2 \oplus tTN_1)\right),\left((sIN_1 \oplus tIN_2)\otimes(sIN_2 \oplus tIN_1)\right),\left((sFN_1 \oplus tFN_2) \otimes (sFN_2 \oplus tFN_1)\right)\right)$$

$$= \left(\begin{array}{c}\left(\begin{array}{c}\left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i1}^{U} \oplus tTN_{j1}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i1}^{U} \oplus tIN_{j1}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i1}^{U} \oplus tFN_{j1}^{U}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i2}^{U} \oplus tTN_{j2}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i2}^{U} \oplus tIN_{j2}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i2}^{U} \oplus tFN_{j2}^{U}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i3}^{U} \oplus tTN_{j3}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i3}^{U} \oplus tIN_{j3}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i3}^{U} \oplus tFN_{j3}^{U}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i4}^{U} \oplus tTN_{j4}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i4}^{U} \oplus tIN_{j4}^{U}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i4}^{U} \oplus tFN_{j4}^{U}\right)\right)\right)\\ min\left(\left(Th_1^U, Ih_1^U, Fh_1^U\right), \left(Th_2^U, Ih_2^U, Fh_2^U\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i1}^{L} \oplus tTN_{j1}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i1}^{L} \oplus tIN_{j1}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i1}^{L} \oplus tFN_{j1}^{L}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i2}^{L} \oplus tTN_{j2}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i2}^{L} \oplus tIN_{j2}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i2}^{L} \oplus tFN_{j2}^{L}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i3}^{L} \oplus tTN_{j3}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i3}^{L} \oplus tIN_{j3}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i3}^{L} \oplus tFN_{j3}^{L}\right)\right)\right),\\ \left(\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sTN_{i4}^{L} \oplus tTN_{j4}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sIN_{i4}^{L} \oplus tIN_{j4}^{L}\right)\right),\left(\bigotimes^{2}_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}\left(sFN_{i4}^{L} \oplus tFN_{j4}^{L}\right)\right)\right)\\ min\left(\left(Th_1^L, Ih_1^L, Fh_1^L\right), \left(Th_2^L, Ih_2^L, Fh_2^L\right)\right)\end{array}\right)\end{array}\right)$$

Therefore, for $m = 2$, (4) is right

Suppose we assume that (4) is true for $m = k$, which is given by the following equations

$$\left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_i\oplus tTN_j\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_i\oplus tIN_j\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_i\oplus tFN_j\right)\right)\right)$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i1}^{U}\oplus tTN_{j1}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i1}^{U}\oplus tIN_{j1}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i1}^{U}\oplus tFN_{j1}^{U}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i2}^{U}\oplus tTN_{j2}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i2}^{U}\oplus tIN_{j2}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i2}^{U}\oplus tFN_{j2}^{U}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i3}^{U}\oplus tTN_{j3}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i3}^{U}\oplus tIN_{j3}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i3}^{U}\oplus tFN_{j3}^{U}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i4}^{U}\oplus tTN_{j4}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i4}^{U}\oplus tIN_{j4}^{U}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i4}^{U}\oplus tFN_{j4}^{U}\right)\right)\right)\\ min_{i=1,2,3,\dots,k\left(Th_i^{U},Ih_i^{U},Fh_i^{U}\right)}\end{array}\right),\\ \left(\begin{array}{c}\left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i1}^{L}\oplus tTN_{j1}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i1}^{L}\oplus tIN_{j1}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i1}^{L}\oplus tFN_{j1}^{L}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i2}^{L}\oplus tTN_{j2}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i2}^{L}\oplus tIN_{j2}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i2}^{L}\oplus tFN_{j2}^{L}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i3}^{L}\oplus tTN_{j3}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i3}^{L}\oplus tIN_{j3}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i3}^{L}\oplus tFN_{j3}^{L}\right)\right)\right),\\ \left(\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sTN_{i4}^{L}\oplus tTN_{j4}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sIN_{i4}^{L}\oplus tIN_{j4}^{L}\right)\right),\left(\otimes^{k}{}_{\substack{i,j=1\\ i,j\ is\ not\\ same}}\left(sFN_{i4}^{L}\oplus tFN_{j4}^{L}\right)\right)\right)\\ min_{i=1,2,3,\dots,k\left(Th_i^{L},Ih_i^{L},Fh_i^{L}\right)}\end{array}\right)\end{array}\right) \quad (5)$$

Now we have to prove for $m = k + 1$

$$\left(\left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k+1} \left(sTN_i \oplus tTN_j\right)\right), \left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k+1} \left(sIN_i \oplus tIN_j\right)\right), \left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k+1} \left(sFN_i \oplus tFN_j\right)\right)\right)$$

$$= \left(\left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k} \left(sTN_i \oplus tTN_j\right)\right), \left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k} \left(sIN_i \oplus tIN_j\right)\right), \left(\bigotimes_{\substack{i,j=1\\ i,j \text{ is not}\\ \text{same}}}^{k} \left(sFN_i \oplus tFN_j\right)\right)\right)$$
$$\otimes \left(\left(\otimes_{i=1}^{k} \left(sTN_i \oplus tTN_{k+1}\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_i \oplus tIN_{k+1}\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_i \oplus tFN_{k+1}\right)\right)\right)$$
$$\otimes \left(\left(\otimes_{j=1}^{k} \left(sTN_{k+1} \oplus tTN_j\right)\right), \left(\otimes_{j=1}^{k} \left(sIN_{k+1} \oplus tIN_j\right)\right), \left(\otimes_{j=1}^{k} \left(sFN_{k+1} \oplus tFN_j\right)\right)\right) \quad (6)$$

Using the arithmetic operations defined for Neutrosophic member, we get

$$\left(\left(\otimes_{i=1}^{k} \left(sTN_i \oplus tTN_{k+1}\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_i \oplus tIN_{k+1}\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_i \oplus tFN_{k+1}\right)\right)\right)$$
$$= \left(\begin{pmatrix} \left(\left(\otimes_{i=1}^{k} \left(sTN_{i1}^U \oplus tTN_{(k+1)1}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i1}^U \oplus tIN_{(k+1)1}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i1}^U \oplus tFN_{(k+1)1}^U\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i2}^U \oplus tTN_{(k+1)2}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i2}^U \oplus tIN_{(k+1)2}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i2}^U \oplus tFN_{(k+1)2}^U\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i3}^U \oplus tTN_{(k+1)3}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i3}^U \oplus tIN_{(k+1)3}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i3}^U \oplus tFN_{(k+1)3}^U\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i4}^U \oplus tTN_{(k+1)4}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i4}^U \oplus tIN_{(k+1)4}^U\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i4}^U \oplus tFN_{(k+1)4}^U\right)\right)\right) \\ \min_{i=1,2,\dots,k} \left((Th_i^U, Ih_i^U, Fh_i^U), (Th_{k+1}^U, Ih_{k+1}^U, Fh_{k+1}^U)\right) \end{pmatrix}, \begin{pmatrix} \left(\left(\otimes_{i=1}^{k} \left(sTN_{i1}^L \oplus tTN_{(k+1)1}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i1}^L \oplus tIN_{(k+1)1}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i1}^L \oplus tFN_{(k+1)1}^L\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i2}^L \oplus tTN_{(k+1)2}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i2}^L \oplus tIN_{(k+1)2}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i2}^L \oplus tFN_{(k+1)2}^L\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i3}^L \oplus tTN_{(k+1)3}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i3}^L \oplus tIN_{(k+1)3}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i3}^L \oplus tFN_{(k+1)3}^L\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k} \left(sTN_{i4}^L \oplus tTN_{(k+1)4}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sIN_{i4}^L \oplus tIN_{(k+1)4}^L\right)\right), \left(\otimes_{i=1}^{k} \left(sFN_{i4}^L \oplus tFN_{(k+1)4}^L\right)\right)\right) \\ \min_{i=1,2,\dots,k} \left((Th_i^L, Ih_i^L, Fh_i^L), (Th_{k+1}^L, Ih_{k+1}^L, Fh_{k+1}^L)\right) \end{pmatrix}\right)$$

And

$$\left(\left(\otimes_{j=1}^{k} \left(sTN_{k+1} \oplus tTN_j\right)\right), \left(\otimes_{j=1}^{k} \left(sIN_{k+1} \oplus tIN_j\right)\right), \left(\otimes_{j=1}^{k} \left(sFN_{k+1} \oplus tFN_j\right)\right)\right)$$

$$= \begin{pmatrix} \begin{pmatrix} \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)1}^{U} \oplus tTN_{j1}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)1}^{U} \oplus tIN_{j1}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)1}^{U} \oplus tFN_{j1}^{U}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)2}^{U} \oplus tTN_{j2}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)2}^{U} \oplus tIN_{j2}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)2}^{U} \oplus tFN_{j2}^{U}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)3}^{U} \oplus tTN_{j3}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)3}^{U} \oplus tIN_{j3}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)3}^{U} \oplus tFN_{j3}^{U}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)4}^{U} \oplus tTN_{j4}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)4}^{U} \oplus tIN_{j4}^{U}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)4}^{U} \oplus tFN_{j4}^{U}\right)\right)\right) \\ \min_{j=1,2,\ldots,k}\left(\left(Th_{(k+1)}^{U}, Ih_{(k+1)}^{U}, Fh_{(k+1)}^{U}\right), \left(Th_{j}^{U}, Ih_{j}^{U}, Fh_{j}^{U}\right)\right) \end{pmatrix}, \\ \begin{pmatrix} \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)1}^{L} \oplus tTN_{j1}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)1}^{L} \oplus tIN_{j1}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)1}^{L} \oplus tFN_{j1}^{L}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)2}^{L} \oplus tTN_{j2}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)2}^{L} \oplus tIN_{j2}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)2}^{L} \oplus tFN_{j2}^{L}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)3}^{L} \oplus tTN_{j3}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)3}^{L} \oplus tIN_{j3}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)3}^{L} \oplus tFN_{j3}^{L}\right)\right)\right), \\ \left(\left(\otimes_{j=1}^{k}\left(sTN_{(k+1)4}^{L} \oplus tTN_{j4}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sIN_{(k+1)4}^{L} \oplus tIN_{j4}^{L}\right)\right), \left(\otimes_{j=1}^{k}\left(sFN_{(k+1)4}^{L} \oplus tFN_{j4}^{L}\right)\right)\right) \\ \min_{j=1,2,\ldots,k}\left(\left(Th_{(k+1)}^{L}, Ih_{(k+1)}^{L}, Fh_{(k+1)}^{L}\right), \left(Th_{j}^{L}, Ih_{j}^{L}, Fh_{j}^{L}\right)\right) \end{pmatrix} \end{pmatrix}$$

The above two equations and equation (5) will applied in (6). The resulting equation will gives

$$\left(\left(\otimes_{\substack{i,j=1\\ i\neq j}}^{k+1}\left(sTN_i \oplus tTN_j\right)\right), \left(\otimes_{\substack{i,j=1\\ i\neq j}}^{k+1}\left(sIN_i \oplus tIN_j\right)\right), \left(\otimes_{\substack{i,j=1\\ i\neq j}}^{k+1}\left(sFN_i \oplus tFN_j\right)\right)\right)$$

$$= \begin{pmatrix} \begin{pmatrix} \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i1}^{U} \oplus tTN_{j1}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i1}^{U} \oplus tIN_{j1}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i1}^{U} \oplus tFN_{j1}^{U}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i2}^{U} \oplus tTN_{j2}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i2}^{U} \oplus tIN_{j2}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i2}^{U} \oplus tFN_{j2}^{U}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i3}^{U} \oplus tTN_{j3}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i3}^{U} \oplus tIN_{j3}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i3}^{U} \oplus tFN_{j3}^{U}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i4}^{U} \oplus tTN_{j4}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i4}^{U} \oplus tIN_{j4}^{U}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i4}^{U} \oplus tFN_{j4}^{U}\right)\right)\right) \\ min_{i=1,2,3,\ldots,k+1\left(Th_i^U, Ih_i^U, Fh_i^U\right)} \end{pmatrix}, \\ \begin{pmatrix} \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i1}^{L} \oplus tTN_{j1}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i1}^{L} \oplus tIN_{j1}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i1}^{L} \oplus tFN_{j1}^{L}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i2}^{L} \oplus tTN_{j2}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i2}^{L} \oplus tIN_{j2}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i2}^{L} \oplus tFN_{j2}^{L}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i3}^{L} \oplus tTN_{j3}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i3}^{L} \oplus tIN_{j3}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i3}^{L} \oplus tFN_{j3}^{L}\right)\right)\right), \\ \left(\left(\otimes_{i=1}^{k+1}\left(sTN_{i4}^{L} \oplus tTN_{j4}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sIN_{i4}^{L} \oplus tIN_{j4}^{L}\right)\right), \left(\otimes_{i=1}^{k+1}\left(sFN_{i4}^{L} \oplus tFN_{j4}^{L}\right)\right)\right) \\ min_{i=1,2,3,\ldots,k+1\left(Th_i^L, Ih_i^L, Fh_i^L\right)} \end{pmatrix} \end{pmatrix}$$

Next we prove (1) is true,

By the arithmetic operations defined for Neutrosophic member and equation),

It is verified that the below equation (1) is true for any n.

$$NBM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_n, IN_n, FN_n)\big)$$
$$= \left(\frac{1}{s+t} \left(\otimes^{m}{}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(sTN_i \oplus tTN_j\right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t} \left(\otimes^{m}{}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(sIN_i \oplus tIN_j\right) \right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t} \left(\otimes^{m}{}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(sFN_i \oplus tFN_j\right) \right)^{\frac{1}{m(m-1)}} \right)$$

Now we prove some important property for Neutrosophic Bonferroni mean(NBM)

3.Neutrosophic Bonferroni properties:

Property 3. 1:

This property is also called as idempotency on NBM.

Let

$$(TN_i, IN_i, FN_i) = \big((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\big)$$
$$= (\big((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\big),$$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ be the and $s, t \geq 0$. If every $(TN_i, IN_i, FN_i) = \big((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\big)$ are equal for all i.

(i.e) $(TN_i, IN_i, FN_i) = \big((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\big) = (TN_0, IN_0, FN_0)$;

$((TN_0, IN_0, FN_0) = \big((TN_0^U, IN_0^U, FN_0^U), (TN_0^L, IN_0^L, FN_0^L)\big)$ then

$$NBM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_n, IN_n, FN_n)\big) = ((TN_0, IN_0, FN_0) = \big((TN_0^U, IN_0^U, FN_0^U), (TN_0^L, IN_0^L, FN_0^L)\big) \quad (7)$$

Property 3.2:

This property is also called as boundedness on NBM.

$$(TN_i, IN_i, FN_i) = \big((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\big)$$
$$= (\big((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\big),$$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ be the set of members of the Neutrosophic and for $s, t \geq 0$ and also we have $\left((TN_-^U, IN_-^U, FN_-^U), (TN_-^L, IN_-^L, FN_-^L)\right) =$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left(\min_i Ta_{i1}^U, \min_i Ia_{i1}^U, \min_i Fa_{i1}^U\right), \left(\min_i Ta_{i2}^U, \min_i Ia_{i2}^U, \min_i Fa_{i2}^U\right), \left(\min_i Ta_{i3}^U, \min_i Ia_{i3}^U, \min_i Fa_{i3}^U\right), \\ \left(\min_i Ta_{i4}^U, \min_i Ia_{i4}^U, \min_i Fa_{i4}^U\right), \left(\min_i Th_i^U, \min_i Ih_i^U, \min_i Fh_i^U\right)\end{array}\right), \\ \left(\begin{array}{c}\left(\min_i Ta_{i1}^L, \min_i Ia_{i1}^L, \min_i Fa_{i1}^L\right), \left(\min_i Ta_{i2}^L, \min_i Ia_{i2}^L, \min_i Fa_{i2}^L\right), \left(\min_i Ta_{i3}^L, \min_i Ia_{i3}^L, \min_i Fa_{i3}^L\right), \\ \left(\min_i Ta_{i4}^L, \min_i Ia_{i4}^L, \min_i Fa_{i4}^L\right), \left(\min_i Th_i^L, \min_i Ih_i^L, \min_i Fh_i^L\right)\end{array}\right)\end{array}\right)$$

And$\left((TN_+^U, IN_+^U, FN_+^U), (TN_+^L, IN_+^L, FN_+^L)\right) =$

$$\left(\begin{array}{c}\left(\begin{array}{c}\left(\max_i Ta_{i1}^U, \max_i Ia_{i1}^U, \max_i Fa_{i1}^U\right), \left(\max_i Ta_{i2}^U, \max_i Ia_{i2}^U, \max_i Fa_{i2}^U\right), \left(\max_i Ta_{i3}^U, \max_i Ia_{i3}^U, \max_i Fa_{i3}^U\right), \\ \left(\max_i Ta_{i4}^U, \max_i Ia_{i4}^U, \max_i Fa_{i4}^U\right), \left(\max_i Th_i^U, \max_i Ih_i^U, \max_i Fh_i^U\right)\end{array}\right), \\ \left(\begin{array}{c}\left(\max_i Ta_{i1}^L, \max_i Ia_{i1}^L, \max_i Fa_{i1}^L\right), \left(\max_i Ta_{i2}^L, \max_i Ia_{i2}^L, \max_i Fa_{i2}^L\right), \left(\max_i Ta_{i3}^L, \max_i Ia_{i3}^L, \max_i Fa_{i3}^L\right), \\ \left(\max_i Ta_{i4}^L, \max_i Ia_{i4}^L, \max_i Fa_{i4}^L\right), \left(\max_i Th_i^L, \max_i Ih_i^L, \max_i Fh_i^L\right)\end{array}\right)\end{array}\right)$$

Then we have,

$$(TN_-, IN_-, FN_-) \leq NBM^{(s,t)}\left((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\right) \leq (TN_+, IN_+, FN_+)$$
(8)

<u>Property 3.3:</u>

This property is also called as monotonicity on NBM.

$$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$$
$$= (\left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ and for $s, t \geq 0$ and

$$(TM_i, IM_i, FM_i) = \left((TM_i^U, IM_i^U, FM_i^U), (TM_i^L, IM_i^L, FM_i^L)\right)$$
$$= (\left((Tb_{i1}^U, Ib_{i1}^U, Fb_{i1}^U), (Tb_{i2}^U, Ib_{i2}^U, Fb_{i2}^U), (Tb_{i3}^U, Ib_{i3}^U, Fb_{i3}^U), (Tb_{i4}^U, Ib_{i4}^U, Fb_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$$

$((Tb_{i1}^L, Ib_{i1}^L, Fb_{i1}^L), (Tb_{i2}^L, Ib_{i2}^L, Fb_{i2}^L), (Tb_{i3}^L, Ib_{i3}^L, Fb_{i3}^L), (Tb_{i4}^L, Ib_{i4}^L, Fb_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ and for $s, t \geq 0$ and also $\left((Ta_{ik}^U \leq Tb_{ik}^U), (Ia_{ik}^U \leq Ib_{ik}^U), (Fa_{ik}^U \leq Fb_{ik}^U)\right)$ and $\left((Ta_{ik}^L \leq Tb_{ik}^L), (Ia_{ik}^L \leq Ib_{ik}^L), (Fa_{ik}^L \leq Fb_{ik}^L)\right)$

(9)

Property 4:

This property is also called as commutivity on NBM.

$$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$$
$$= (\left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ and for $s, t \geq 0$ and

$$(TN'_i, IN'_i, FN'_i) = \left((TN_i'^U, IN_i'^U, FN_i'^U), (TN_i'^L, IN_i'^L, FN_i'^L)\right)$$
$$= (\left((Ta_{i1}'^U, Ia_{i1}'^U, Fa_{i1}'^U), (Ta_{i2}'^U, Ia_{i2}'^U, Fa_{i2}'^U), (Ta_{i3}'^U, Ia_{i3}'^U, Fa_{i3}'^U), (Ta_{i4}'^U, Ia_{i4}'^U, Fa_{i4}'^U), (Th_i'^U, Ih_i'^U, Fh_i'^U)\right),$$

$((Ta_{i1}'^L, Ia_{i1}'^L, Fa_{i1}'^L), (Ta_{i2}'^L, Ia_{i2}'^L, Fa_{i2}'^L), (Ta_{i3}'^L, Ia_{i3}'^L, Fa_{i3}'^L), (Ta_{i4}'^L, Ia_{i4}'^L, Fa_{i4}'^L), (Th_i'^L, Ih_i'^L, Fh_i'^L)))(i = 1,2, \dots, m)$ be the permutation number of above Neutrosophic member and for $s, t \geq 0$. Then,

$$NBM^{(s,t)}\left((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\right) =$$
$$NBM^{(s,t)}\left((TN'_1, IN'_1, FN'_1), (TN'_2, IN'_2, FN'_2), \dots, (TN'_m, IN'_m, FN'_m)\right) \quad (10)$$

By giving parameters s, t different values, we will get different values.

4.Neutrosophic weighted Bonferroni operator:

Definition 4.1:

Let

$$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$$
$$= (\left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right),$$

$((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)))(i = 1,2, \dots, m)$ and for $s, t \geq 0$ and $(Tw, Iw, Fw) = \left((Tw_1, Iw_1, Fw_1), (Tw_2, Iw_2, Fw_2) \dots (Tw_m, Iw_m, Fw_m)\right)$ be the weight vector for $(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$, where $(Tw_i \geq 0, Iw_i \geq 0, Fw_i \geq 0)$ and $\sum_{i=0}^{n} Tw_i + \sum_{i=0}^{m} Iw_i + \sum_{i=0}^{m} Fw_i = 1$, then the Neutrosophic weighted Bonferroni operator is defined as

$$NWBM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\big) = \left(\frac{1}{s+t}\left(\bigotimes^{m}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(s(TN_i)^{w_i} \oplus t\left(TN_j\right)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\bigotimes^{m}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(s(IN_i)^{w_i} \oplus t\left(IN_j\right)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\bigotimes^{m}_{\substack{i,j=1 \\ i,j \text{ is not} \\ \text{same}}} \left(s(FN_i)^{w_i} \oplus t\left(FN_j\right)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right) \quad (14)$$

Theorem 4.1:

Let

$$(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$$
$$= \begin{pmatrix} \left((Ta_{i1}^U, Ia_{i1}^U, Fa_{i1}^U), (Ta_{i2}^U, Ia_{i2}^U, Fa_{i2}^U), (Ta_{i3}^U, Ia_{i3}^U, Fa_{i3}^U), (Ta_{i4}^U, Ia_{i4}^U, Fa_{i4}^U), (Th_i^U, Ih_i^U, Fh_i^U)\right), \\ \left((Ta_{i1}^L, Ia_{i1}^L, Fa_{i1}^L), (Ta_{i2}^L, Ia_{i2}^L, Fa_{i2}^L), (Ta_{i3}^L, Ia_{i3}^L, Fa_{i3}^L), (Ta_{i4}^L, Ia_{i4}^L, Fa_{i4}^L), (Th_i^L, Ih_i^L, Fh_i^L)\right) \end{pmatrix}$$

$(i = 1,2, \dots, n)$ and for s,t≥0 are $(Tw, Iw, Fw) = \left((Tw_1, Iw_1, Fw_1), (Tw_2, Iw_2, Fw_2) \dots (Tw_m, Iw_m, Fw_m)\right)$ be the weight vector for $(TN_i, IN_i, FN_i) = \left((TN_i^U, IN_i^U, FN_i^U), (TN_i^L, IN_i^L, FN_i^L)\right)$, where $(Tw_i \geq 0, Iw_i \geq 0, Fw_i \geq 0)$ and $\sum_{i=0}^{m} Tw_i + \sum_{i=0}^{m} Iw_i + \sum_{i=0}^{m} Fw_i = 1$. Additionally, a Neutrosophic member, so we have

$$NWBM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_n, IN_n, FN_n)\big) = (TN_w, IN_w, FN_w) = \left((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\right) \quad (15)$$

Where

$$(TN_w^U, IN_w^U, FN_w^U) = \left(\begin{array}{l} \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i1}^U)^{w_i} \oplus t(TN_{j1}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i1}^U)^{w_i} \oplus t(IN_{j1}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i1}^U)^{w_i} \oplus t(FN_{j1}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i2}^U)^{w_i} \oplus t(TN_{j2}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i2}^U)^{w_i} \oplus t(IN_{j2}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i2}^U)^{w_i} \oplus t(FN_{j2}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i3}^U)^{w_i} \oplus t(TN_{j3}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i3}^U)^{w_i} \oplus t(IN_{j3}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i3}^U)^{w_i} \oplus t(FN_{j3}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i4}^U)^{w_i} \oplus t(TN_{j4}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i4}^U)^{w_i} \oplus t(IN_{j4}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i4}^U)^{w_i} \oplus t(FN_{j4}^U)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right) \\ \qquad min_{i=1,2,3,\dots,m(Th_i^U, Ih_i^U, Fh_i^U)} \end{array} \right) \tag{16}$$

And

$$(TN_w^L, IN_w^L, FN_w^L) = \left(\begin{array}{l} \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i1}^L)^{w_i} \oplus t(TN_{j1}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i1}^L)^{w_i} \oplus t(IN_{j1}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i1}^L)^{w_i} \oplus t(FN_{j1}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i2}^L)^{w_i} \oplus t(TN_{j2}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i2}^L)^{w_i} \oplus t(IN_{j2}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i2}^L)^{w_i} \oplus t(FN_{j2}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i3}^L)^{w_i} \oplus t(TN_{j3}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i3}^L)^{w_i} \oplus t(IN_{j3}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i3}^L)^{w_i} \oplus t(FN_{j3}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right), \\ \left(\frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(TN_{i4}^L)^{w_i} \oplus t(TN_{j4}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(IN_{i4}^L)^{w_i} \oplus t(IN_{j4}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}}, \frac{1}{s+t}\left(\otimes^m_{\substack{i,j=1 \\ i,j \text{ is not} \\ same}} \left(s(FN_{i4}^L)^{w_i} \oplus t(FN_{j4}^L)^{w_j}\right)\right)^{\frac{1}{m(m-1)}} \right) \\ \qquad min_{i=1,2,3,\dots,n(Th_i^L, Ih_i^L, Fh_i^L)} \end{array} \right) \tag{17}$$

Now we define the property for Neutrosophic weighted Bonferroni operator

<u>Property 4.1:</u>

This property is also called as idempotency on NBWM.

Let

$(TN_w, IN_w, FN_w) = \left((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\right)$

$$= \begin{pmatrix} \left(\left((Ta_{i1}^U)^w, (Ia_{i1}^U)^w, (Fa_{i1}^U)^w\right), \left((Ta_{i2}^U)^w, (Ia_{i2}^U)^w, (Fa_{i2}^U)^w\right), \left((Ta_{i3}^U)^w, (Ia_{i3}^U)^w, (Fa_{i3}^U)^w\right), \left((Ta_{i4}^U)^w, (Ia_{i4}^U)^w, (Fa_{i4}^U)^w\right), (Th_i^U, Ih_i^U, Fh_i^U)\right), \\ \left(\left((Ta_{i1}^L)^w, (Ia_{i1}^L)^w, (Fa_{i1}^L)^w\right), \left((Ta_{i2}^L)^w, (Ia_{i2}^L)^w, (Fa_{i2}^L)^w\right), \left((Ta_{i3}^L)^w, (Ia_{i3}^L)^w, (Fa_{i3}^L)^w\right), \left((Ta_{i4}^L)^w, (Ia_{i4}^L)^w, (Fa_{i4}^L)^w\right), (Th_i^L, Ih_i^L, Fh_i^L)\right) \end{pmatrix},$$

$(i = 1,2, \dots, m)$ and if every $(TN_w, IN_w, FN_w) = \left((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\right)$ are equal for all.

(i.e) $(TN_w, IN_w, FN_w) = \left((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\right) = (TN_0, IN_0, FN_0)$;

$((TN_0, IN_0, FN_0) = \left((TN_0^U, IN_0^U, FN_0^U), (TN_0^L, IN_0^L, FN_0^L)\right)$ then

$$NBWM^{(s,t)}\left((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \dots, (TN_m, IN_m, FN_m)\right) = ((TN_0, IN_0, FN_0)$$
$$= \left((TN_0^U, IN_0^U, FN_0^U), (TN_0^L, IN_0^L, FN_0^L)\right)$$

Property4.2:

This property is also called as boundedness on NBWM.

$(TN_w, IN_w, FN_w) = \left((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\right) =$

$$\begin{pmatrix} \left(\left((Ta_{i1}^U)^w, (Ia_{i1}^U)^w, (Fa_{i1}^U)^w\right), \left((Ta_{i2}^U)^w, (Ia_{i2}^U)^w, (Fa_{i2}^U)^w\right), \left((Ta_{i3}^U)^w, (Ia_{i3}^U)^w, (Fa_{i3}^U)^w\right), \left((Ta_{i4}^U)^w, (Ia_{i4}^U)^w, (Fa_{i4}^U)^w\right), (Th_i^U, Ih_i^U, Fh_i^U)\right), \\ \left(\left((Ta_{i1}^L)^w, (Ia_{i1}^L)^w, (Fa_{i1}^L)^w\right), \left((Ta_{i2}^L)^w, (Ia_{i2}^L)^w, (Fa_{i2}^L)^w\right), \left((Ta_{i3}^L)^w, (Ia_{i3}^L)^w, (Fa_{i3}^L)^w\right), \left((Ta_{i4}^L)^w, (Ia_{i4}^L)^w, (Fa_{i4}^L)^w\right), (Th_i^L, Ih_i^L, Fh_i^L)\right) \end{pmatrix} (i = 1,2, \dots, m)$$ and

for $s, t \geq 0$ and also we have $(TN_-, IN_-, FN_-) = \left((TN_-^U, IN_-^U, FN_-^U), (TN_-^L, IN_-^L, FN_-^L)\right) =$

$$\left(\begin{pmatrix} \left(\min_i (Ta_{i1}^U)^{w_i}, \min_i (Ia_{i1}^U)^{w_i}, \min_i (Fa_{i1}^U)^{w_i}\right), \left(\min_i (Ta_{i2}^U)^{w_i}, \min_i (Ia_{i2}^U)^{w_i}, \min_i (Fa_{i2}^U)^{w_i}\right), \left(\min_i (Ta_{i3}^U)^{w_i}, \min_i (Ia_{i3}^U)^{w_i}, \min_i (Fa_{i3}^U)^{w_i}\right), \\ \left(\min_i (Ta_{i4}^U)^{w_i}, \min_i (Ia_{i4}^U)^{w_i}, \min_i (Fa_{i4}^U)^{w_i}\right), \left(\min_i (Th_i^U)^{w_i}, \min_i (Ih_i^U)^{w_i}, \min_i (Fh_i^U)^{w_i}\right) \end{pmatrix}, \begin{pmatrix} \left(\min_i (Ta_{i1}^L)^{w_i}, \min_i (Ia_{i1}^L)^{w_i}, \min_i (Fa_{i1}^L)^{w_i}\right), \left(\min_i (Ta_{i2}^L)^{w_i}, \min_i (Ia_{i2}^L)^{w_i}, \min_i (Fa_{i2}^L)^{w_i}\right), \left(\min_i (Ta_{i3}^L)^{w_i}, \min_i (Ia_{i3}^L)^{w_i}, \min_i (Fa_{i3}^L)^{w_i}\right), \\ \left(\min_i (Ta_{i4}^L)^{w_i}, \min_i (Ia_{i4}^L)^{w_i}, \min_i (Fa_{i4}^L)^{w_i}\right), \left(\min_i Th_i^L, \min_i Ih_i^L, \min_i Fh_i^L\right) \end{pmatrix}\right)$$

And

$(TN_+, IN_+, FN_+) = \left((TN_+^U, IN_+^U, FN_+^U), (TN_+^L, IN_+^L, FN_+^L)\right) =$

$$\left(\begin{pmatrix} \left(\max_i (Ta_{i1}^U)^{w_i}, \max_i (Ia_{i1}^U)^{w_i}, \max_i (Fa_{i1}^U)^{w_i}\right), \left(\max_i (Ta_{i2}^U)^{w_i}, \max_i (Ia_{i2}^U)^{w_i}, \max_i (Fa_{i2}^U)^{w_i}\right), \left(\max_i (Ta_{i3}^U)^{w_i}, \max_i (Ia_{i3}^U)^{w_i}, \max_i (Fa_{i3}^U)^{w_i}\right), \\ \left(\max_i (Ta_{i4}^U)^{w_i}, \max_i (Ia_{i4}^U)^{w_i}, \max_i (Fa_{i4}^U)^{w_i}\right), \left(\max_i (Th_i^U)^{w_i}, \max_i (Ih_i^U)^{w_i}, \max_i (Fh_i^U)^{w_i}\right) \end{pmatrix}, \begin{pmatrix} \left(\max_i (Ta_{i1}^L)^{w_i}, \max_i (Ia_{i1}^L)^{w_i}, \max_i (Fa_{i1}^L)^{w_i}\right), \left(\max_i (Ta_{i2}^L)^{w_i}, \max_i (Ia_{i2}^L)^{w_i}, \max_i (Fa_{i2}^L)^{w_i}\right), \left(\max_i (Ta_{i3}^L)^{w_i}, \max_i (Ia_{i3}^L)^{w_i}, \max_i (Fa_{i3}^L)^{w_i}\right), \\ \left(\max_i (Ta_{i4}^L)^{w_i}, \max_i (Ia_{i4}^L)^{w_i}, \max_i (Fa_{i4}^L)^{w_i}\right), \left(\max_i Th_i^L, \max_i Ih_i^L, \max_i Fh_i^L\right) \end{pmatrix}\right)$$

Then we have,

$$(TN_-, IN_-, FN_-) \leq NBWM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \ldots, (TN_n, IN_n, FN_n)\big) \leq (TN_+, IN_+, FN_+)$$

Property 4.3:

This property is also called as monotonicity on NBWM.

$(TN_w, IN_w, FN_w) = \big((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\big) =$
$\begin{pmatrix} (((Ta_{i1}^U)^w, (Ia_{i1}^U)^w, (Fa_{i1}^U)^w), ((Ta_{i2}^U)^w, (Ia_{i2}^U)^w, (Fa_{i2}^U)^w), ((Ta_{i3}^U)^w, (Ia_{i3}^U)^w, (Fa_{i3}^U)^w), ((Ta_{i4}^U)^w, (Ia_{i4}^U)^w, (Fa_{i4}^U)^w), (Th_i^U, Ih_i^U, Fh_i^U)), \\ (((Ta_{i1}^L)^w, (Ia_{i1}^L)^w, (Fa_{i1}^L)^w), ((Ta_{i2}^L)^w, (Ia_{i2}^L)^w, (Fa_{i2}^L)^w), ((Ta_{i3}^L)^w, (Ia_{i3}^L)^w, (Fa_{i3}^L)^w), ((Ta_{i4}^L)^w, (Ia_{i4}^L)^w, (Fa_{i4}^L)^w), (Th_i^L, Ih_i^L, Fh_i^L)) \end{pmatrix} (i = 1,2, \ldots, m)$ and for $s, t \geq 0$

and
$(TM_w, IM_w, FM_w) = \big((TM_w^U, IM_w^U, FM_w^U), (TM_w^L, IM_w^L, FM_w^L)\big) =$
$\begin{pmatrix} (((Tb_{i1}^U)^w, (Ib_{i1}^U)^w, (Fb_{i1}^U)^w), ((Tb_{i2}^U)^w, (Ib_{i2}^U)^w, (Fb_{i2}^U)^w), ((Tb_{i3}^U)^w, (Ib_{i3}^U)^w, (Fb_{i3}^U)^w), ((Tb_{i4}^U)^w, (Ib_{i4}^U)^w, (Fb_{i4}^U)^w), (Th_i^U, Ih_i^U, Fh_i^U)), \\ (((Tb_{i1}^L)^w, (Ib_{i1}^L)^w, (Fb_{i1}^L)^w), ((Tb_{i2}^L)^w, (Ib_{i2}^L)^w, (Fb_{i2}^L)^w), ((Tb_{i3}^L)^w, (Ib_{i3}^L)^w, (Fb_{i3}^L)^w), ((Tb_{i4}^L)^w, (Ib_{i4}^L)^w, (Fb_{i4}^L)^w), (Th_i^L, Ih_i^L, Fh_i^L)) \end{pmatrix} (i = 1,2, \ldots, m)$ and for $s, t \geq 0$ and also $\big(((Ta_{ik}^U)^w \leq (Tb_{ik}^U)^w), ((Ia_{ik}^U)^w \leq (Ib_{ik}^U)^w), ((Fa_{ik}^U)^w \leq (Fb_{ik}^U)^w)\big)$ and
$\big(((Ta_{ik}^L)^w \leq (Tb_{ik}^L)^w), ((Ia_{ik}^L)^w \leq (Ib_{ik}^L)^w), ((Fa_{ik}^L)^w \leq (Fb_{ik}^L)^w)\big)$

Then we have

$$NBWM^{(s,t)}\big((TN_1, IN_1, FN_1), (TN_2, IN_2, FN_2), \ldots, (TN_m, IN_m, FN_m)\big)$$
$$\leq NBWM^{(s,t)}\big((TM_1, IM_1, FM_1), (TM_2, IM_2, FM_2), \ldots, (TM_m, IM_m, FM_m)\big)$$

Property 4.4:

This property is also called as commutivity on NBWM.

$(TN_w, IN_w, FN_w) = \big((TN_w^U, IN_w^U, FN_w^U), (TN_w^L, IN_w^L, FN_w^L)\big) =$
$\begin{pmatrix} (((Ta_{i1}^U)^w, (Ia_{i1}^U)^w, (Fa_{i1}^U)^w), ((Ta_{i2}^U)^w, (Ia_{i2}^U)^w, (Fa_{i2}^U)^w), ((Ta_{i3}^U)^w, (Ia_{i3}^U)^w, (Fa_{i3}^U)^w), ((Ta_{i4}^U)^w, (Ia_{i4}^U)^w, (Fa_{i4}^U)^w), (Th_i^U, Ih_i^U, Fh_i^U)), \\ (((Ta_{i1}^L)^w, (Ia_{i1}^L)^w, (Fa_{i1}^L)^w), ((Ta_{i2}^L)^w, (Ia_{i2}^L)^w, (Fa_{i2}^L)^w), ((Ta_{i3}^L)^w, (Ia_{i3}^L)^w, (Fa_{i3}^L)^w), ((Ta_{i4}^L)^w, (Ia_{i4}^L)^w, (Fa_{i4}^L)^w), (Th_i^L, Ih_i^L, Fh_i^L)) \end{pmatrix} (i = 1,2, \ldots, n)$ and for $s, t \geq 0$

By giving parameters s, t different values, we have some different result.

5. Conclusion:

The classical Bonferroni mean operator and possibility degree have been extended in the trapezoidal and triangular neutrosophic environment to better organise and model the uncertainties and indeterminacy inside multi-attribute decision analysis. In FMAGDM, the neutrosophic Bonferroni operator can combine several decisions or evaluations from multiple decision-makers. Neutropphic surroundings, as opposed to trapezoidal and triangular contexts, are able to convey the decision-makers' ambiguity, indecision, and uncertainty. Based on the neutrosophic possibility degree and the TITRNWBM operator, we have introduced a novel approach for NMAGDM. Numerous difficult multiple-attribute decision-making issues can be resolved with the help of the suggested Neutrosophic Bonferroni operator and weighted Neutrosophic Bonferroni operator, both of which meet the necessary criteria and theorems. Therefore, we see this as a starting point for future research using this operator for solving multiple attributes decision making problems.

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