



A TreeSoft Set with Interval Valued Neutrosophic Set in the era of Industry 4.0

Ali Alqazzaz^{*1} and Karam M. Sallam²

¹Department of Information Systems and Cybersecurity, College of Computing and Information Technology, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia.

Email: aqzaz@ub.edu.sa

²School of IT and Systems, Faculty of Science and Technology, University of Canberra, Canberra, Australia

Email: karam.sallam@canberra.edu.au

*Correspondence: aqzaz@ub.edu.sa

Abstract The introduction of Industry 4.0 has brought about a significant shift in the manufacturing and supply chain management sectors, requiring supplier selection procedures to be adjusted to this rapidly changing technical environment. This study aims to improve supplier selection in Industry 4.0. This selection contains various criteria, so multi-criteria decision-making (MCDM) is used to deal with these criteria. The interval-valued neutrosophic sets (IVNSs) are used to deal with uncertainty in the evaluation process. The IVNSs are integrated with the TreeSoft Set. The TOPSIS method is an MCDM method used to rank the alternatives. The results show that the economic criterion is the most important, and supplier 7 is the best.

Keywords: TreeSoft Set; Interval Valued Neutrosophic Set; Industry 4.0; Supplier Selection; Multi-Criteria Decision Making.

1. Introduction

Known as the fourth industrial revolution, or industry 4.0, supply networks have been significantly and extensively impacted. This change has profoundly impacted how companies plan, run, and maximize their supply chain processes. Fundamentally, Industry 4.0 denotes a paradigm change in how businesses use digital technology to improve supply chain responsiveness, productivity, and visibility. Integrating cutting-edge technology is one of Industry 4.0's most significant effects on supply chains. They include automation, big data analytics, machine learning, artificial intelligence (AI), and the Internet of Things (IoT)[1].

These advances make it possible to monitor supply chain operations in real-time, which makes data-driven decision-making easier. Massive volumes of data are gathered by Internet of Things (IoT) sensors installed on assets like machinery, vehicles, and products. These sensors provide essential insights into the whereabouts and condition of items at every stage of the supply chain. AI and machine learning algorithms may process this data to estimate demand, optimize routes, and even carry out anticipatory equipment repairs, lowering expenses and downtime[2], [3].

The idea of a "smart" and networked supply chain ecosystem is another critical component of Industry 4.0's effects on supply chains. Businesses may use digital twins to build virtual versions of their supply chains, enabling detailed modeling and simulation. This helps companies to find bottlenecks, test different scenarios, and create more effective plans. These digital twins may also be utilized for real-time process monitoring and control, allowing quick modifications to minimize interruptions or take advantage of opportunities. An essential component of supply chains enabled by Industry 4.0 is automation and robots. The physical transportation of items is more accurate and efficient when autonomous robots, drones, and automated material handling systems are used[4], [5].

Warehouses and distribution centers increasingly use automated solutions to expedite order fulfillment and lower mistake rates. Furthermore, localized and on-demand production made possible by 3D printing in manufacturing might substantially change the supply chain by lowering the need for large stocks and long-distance shipping[6], [7].

Industry 4.0 encourages openness and cooperation through the supply chain. Blockchain technology builds secure, unchangeable ledgers of goods movements and transactions. This improves transparency and confidence, especially in sectors like food and pharmaceuticals with intricate, multi-tiered supply chains. By providing a common source of truth, stakeholders can guarantee the safety and legitimacy of the product. Moreover, Industry 4.0 makes it possible to transition from a linear supply chain model to a more sustainable and circular one. Businesses may minimize waste, maximize resource utilization, and lessen their environmental impact using data-driven insights[8], [9]. Supply chain strategies are starting to place a greater emphasis on sustainability and corporate social responsibility, and Industry 4.0 technologies facilitate the achievement of these goals.

This study used the concept of multi-criteria decision-making[10]–[12] for supplier selection in Industry 4.0 under an interval-valued neutrosophic set, and TreeSoft set. The MCDM has various applications[13]–[16].

It may be inferred from the literature that most studies have focused on quantitative, quantifiable attributes when examining supplier selection in Industry 4.0. Although fuzzy set theories (FSs) are useful for handling uncertain issues, they are only suitable for handling inconsistent or ambiguous data. As a result, in the context of supplier selection in Industry 4.0, sophisticated computational methods like Neutrosophic Sets (NSs) [17], a generalization of FSs, and Intuitionistic Fuzzy Sets (IFSs) have gained significance[18], [19].

Up to this point, no research has yet addressed the Industry 4.0 challenge of supplier selection utilizing the IVNS subset of NSs[20]–[22]. One key feature of Zhang et al.'s IVNS theory is that the membership, non-membership, and indeterminacy functions are considered as intervals rather than a single precise value. Furthermore, resolving intricate scientific and technical issues in IVNS takes into account the viewpoints of several experts with varying degrees of training, expertise, and interest. As a result, to evaluate performance in a group MCDM issue[23]–[26], the views of many experts must be combined.

2. Materials and Methods

This section introduces two parts: the interval-valued neutrosophic sets (IVNSs)[27] and the TreeSoft (TS) with IVNSs (TSIVNS).

Definition 1.

The neutrosophic set k can be defined with three membership functions as truth, indeterminacy, and falsity membership degrees $T_k(u_i)$, $I_k(u_i)$, and $F_k(u_i)$,

$$k = \{(T_k(u_i), I_k(u_i), F_k(u_i)) | u_i \in U\} \tag{1}$$

Where U refers to the universal numbers including components u_i .

$$0^- \leq \sup T_k(u_i) + \sup I_k(u_i) + \sup F_k(u_i) \leq 3^+ \tag{2}$$

Definition 2.

We can define an IVNS as:

$$T_k(u_i) = [\inf T_k(u), \sup T_k(u)] \tag{3}$$

$$I_k(u_i) = [\inf I_k(u), \sup I_k(u)] \tag{4}$$

$$F_k(u_i) = [\inf F_k(u), \sup F_k(u)] \tag{5}$$

$$k = \{[\inf T_k(u), \sup T_k(u)], [\inf I_k(u), \sup I_k(u)], [\inf F_k(u), \sup F_k(u)] | u \in U\} \tag{6}$$

Definition 3.

We can present some IVNS operations as:

Let interval-valued neutrosophic numbers as: $k_1 =$

$$[\inf T_{k_1}(u), \sup T_{k_1}(u)], [\inf I_{k_1}(u), \sup I_{k_1}(u)], [\inf F_{k_1}(u), \sup F_{k_1}(u)] \text{ and } k_2 =$$

$$[\inf T_{k_2}(u), \sup T_{k_2}(u)], [\inf I_{k_2}(u), \sup I_{k_2}(u)], [\inf F_{k_2}(u), \sup F_{k_2}(u)]$$

$$k_1 + k_2 = \left[\begin{array}{l} [\inf T_{k_1}(u) + \inf T_{k_2}(u) - \inf T_{k_1}(u) \cdot \inf T_{k_2}(u), \\ \sup T_{k_1}(u) + \sup T_{k_2}(u) - \sup T_{k_1}(u) \cdot \sup T_{k_2}(u)] \\ [\inf I_{k_1}(u) \cdot \inf I_{k_2}(u), \sup I_{k_1}(u) \cdot \sup I_{k_2}(u)], \\ [\inf F_{k_1}(u) \cdot \inf F_{k_2}(u), \sup F_{k_1}(u) \cdot \sup F_{k_2}(u)] \end{array} \right] \tag{7}$$

$$k_1 \cdot k_2 = \left[\begin{array}{l} [\inf T_{k_1}(u) \cdot \inf T_{k_2}(u), \sup T_{k_1}(u) \cdot \sup T_{k_2}(u)] \\ [\inf I_{k_1}(u) + \inf I_{k_2}(u) - \inf I_{k_1}(u) \cdot \inf I_{k_2}(u), \\ \sup I_{k_1}(u) + \sup I_{k_2}(u) - \sup I_{k_1}(u) \cdot \sup I_{k_2}(u)] \\ [\inf F_{k_1}(u) + \inf F_{k_2}(u) - \inf F_{k_1}(u) \cdot \inf F_{k_2}(u), \\ \sup F_{k_1}(u) + \sup F_{k_2}(u) - \sup F_{k_1}(u) \cdot \sup F_{k_2}(u)] \end{array} \right] \tag{8}$$

$$s \cdot k_1 = \left[\begin{array}{l} [1 - (1 - \inf T_{k_1}(u))^s, 1 - (1 - \sup T_{k_1}(u))^s], \\ [(\inf I_{k_1}(u))^s, (\sup I_{k_1}(u))^s], \\ [(\inf F_{k_1}(u))^s, (\sup F_{k_1}(u))^s] \end{array} \right] \tag{9}$$

$$k_1^s = \left[\begin{array}{l} [(\inf T_{k_1}(u))^s, (\sup T_{k_1}(u))^s], \\ [1 - (1 - \inf I_{k_1}(u))^s, 1 - (1 - \sup I_{k_1}(u))^s], \\ [1 - (1 - \inf F_{k_1}(u))^s, 1 - (1 - \sup F_{k_1}(u))^s] \end{array} \right] \tag{10}$$

2.1 TreeSoft [28]

Let U be a universe disclosure and H a non-empty subset of U , with $P(H)$ be a powerset of H .

Let TSR be a set of attributes of the problem (criteria),

$$TSR = \{TSR_1, TSR_2, \dots, TSR_n\}, n \geq 1 \quad (11)$$

Where $TSR_1, TSR_2, \dots, TSR_n$ are criteria of the first level of the tree.

Each attribute $TSR_i, 1 \leq i \leq n$, is formed by sub – attributes:

$$TSR_1 = \{TSR_{1,1}, TSR_{1,2}, \dots, \}$$

$$TSR_2 = \{TSR_{2,1}, TSR_{2,2}, \dots, \}$$

.

.

$$TSR_n = \{TSR_{n,1}, TSR_{n,2}, \dots, \}$$

Where $TSR_{i,j}$ are sub-attributes.

The TreeSoft set can be formed by:

$$F: P(Tree(TSR)) \rightarrow P(H) \quad (12)$$

$Tree(TSR)$ is the set of all nodes and leaves from level 1 to level m and $P(Tree(TSR))$ is the power set of the $Tree(TSR)$.

$$Tree(TSR) = \{TSR_i | i_1 = 1, 2, 3, \dots\} \cup \{TSR_i | i_1, i_2 = 1, 2, 3, \dots\} \cup \{TSR_i | i_1, i_2, i_3 = 1, 2, 3, \dots\} \cup \dots \\ \cup \{TSR_i | i_1, i_2, \dots, i_m = 1, 2, 3, \dots\} \quad (13)$$

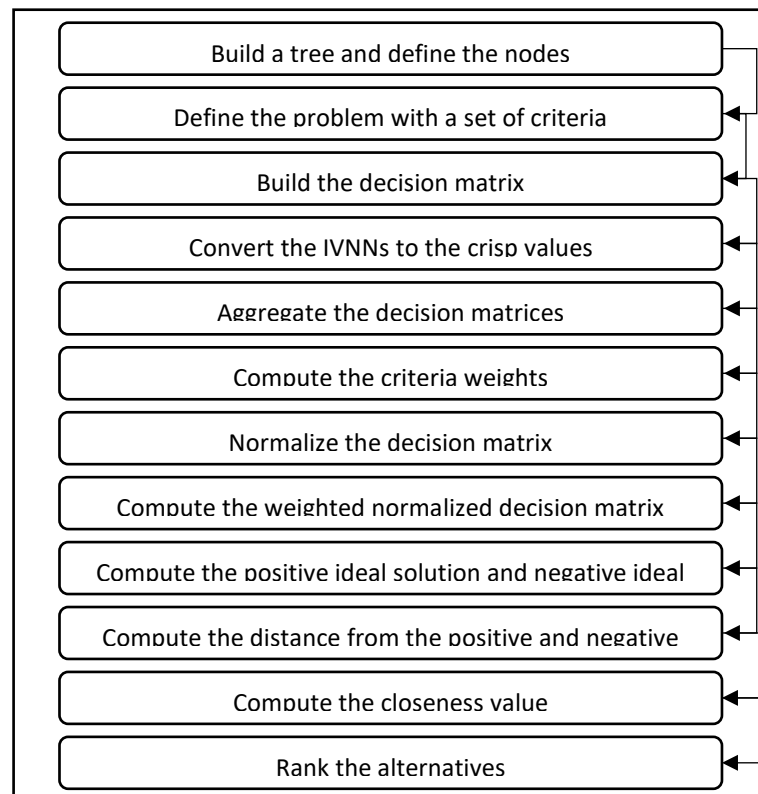


Figure 1. The steps of the TSIVNS TOPSIS method.

2.2 TSIVNS TOPSIS Method

This part introduces the TOPSIS method[29]–[31] with the TSIVNS. Figure 1 shows the research framework:

Step 1. Build a tree and define the nodes.

The tree has more than one level, in the first level, the main criteria and introduced as $TSR_1, TSR_2, \dots, TSR_n$

In the second level, the sub-criteria are introduced as $TSR_{1,1}, TSR_{1,2}, \dots$. And $TSR_{2,1}, TSR_{2,2}, \dots$

Step 2. Define the problem with a set of criteria.

The main, and sub-criteria are defined in this step by problem definition.

Step 3. Build the decision matrix[27].

The decision matrix is built based on the IVNSs

$$A = \begin{pmatrix} \left[\begin{array}{c} [\inf T_{k_{11}}(u), \sup T_{k_{11}}(u)], \\ [[\inf I_{k_{11}}(u), \sup I_{k_{11}}(u)]], \\ [[\inf F_{k_{11}}(u), \sup F_{k_{11}}(u)]], \\ \vdots \\ [\inf T_{k_{m1}}(u), \sup T_{k_{m1}}(u)], \\ [[\inf I_{k_{m1}}(u), \sup I_{k_{m1}}(u)]], \\ [[\inf F_{k_{m1}}(u), \sup F_{k_{m1}}(u)]], \end{array} \right] & \dots & \left[\begin{array}{c} [\inf T_{k_{1n}}(u), \sup T_{k_{1n}}(u)], \\ [[\inf I_{k_{1n}}(u), \sup I_{k_{1n}}(u)]], \\ [[\inf F_{k_{1n}}(u), \sup F_{k_{1n}}(u)]], \\ \vdots \\ [\inf T_{k_{mn}}(u), \sup T_{k_{mn}}(u)], \\ [[\inf I_{k_{mn}}(u), \sup I_{k_{mn}}(u)]], \\ [[\inf F_{k_{mn}}(u), \sup F_{k_{mn}}(u)]], \end{array} \right] \end{pmatrix} \quad (13)$$

Step 4. Convert the IVNNs to the crisp values[27].

The IVNNs are converted by the score function to crisp values, then the decision matrix as:

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \quad (14)$$

Step 5. Aggregate the decision matrices

The decision matrices are combined using the average method.

Step 6. Compute the criteria weights.

The criteria weights are computed by the average method.

Step 7. Normalize the decision matrix.

$$T_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^n a_{ij}^2}} \quad (15)$$

Step 8. Compute the weighted normalized decision matrix

$$R_{ij} = T_{ij} \cdot w_j \quad (16)$$

Step 9. Compute the positive ideal solution and negative ideal solution.

$$P_j^+ = \left\{ \begin{array}{l} \max_{i=1, \dots, n} r_{ij} \text{ positive criteria} \\ \min_{i=1, \dots, n} r_{ij} \text{ negative criteria} \end{array} \right\} \quad (17)$$

$$P_j^- = \left\{ \begin{array}{l} \min_{i=1, \dots, n} r_{ij} \text{ positive criteria} \\ \max_{i=1, \dots, n} r_{ij} \text{ negative criteria} \end{array} \right\} \quad (18)$$

Step 10. Compute the distance between the positive and negative ideal solutions.

$$d^+(a_i) = \sqrt{\sum_{j=1}^m (r_{ik} - p_j^+)^2} \quad (19)$$

$$d^-(a_i) = \sqrt{\sum_{j=1}^m (r_{ik} - p_j^-)^2} \tag{20}$$

Step 11. Compute the closeness value.

$$S_i = \frac{d^-(a_i)}{d^-(a_i) + d^+(a_i)} \tag{21}$$

Step 12. Rank the alternatives.

3. Application

This section introduces the results of TRIVNS with the TOPSIS method to select the best supplier in industry 4.0. There are three experts are invited to evaluate the criteria and alternatives in this study.

Step 1. Build a tree and define the nodes.

We built the tree nodes with one level with ten nodes. The ten nodes present the main criteria in this study as shown in Figure 2.



Figure 2. Level 1 of nodes.

Step 2. Define the problem with a set of criteria.

There are ten criteria used in this study.

Step 3. Build the decision matrix.

We built three decision matrices by the opinions of experts in Eq. (13) as shown in Table 1.

Table 1. Three decision matrices by IVNNs.

	TRS ₁	TRS ₂	TRS ₃	TRS ₄	TRS ₅	TRS ₆	TRS ₇	TRS ₈	TRS ₉	TRS ₁₀
TRA₁	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])
TRA₂	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])

TRA₃	([0.1,0.2],	([0.2,0.4],	([0.2,0.4],	([0.7,0.9],	([0.7,0.9],	([0.6,0.8],	([0.6,0.8],	([0.4,0.6],	([0.4,0.6],	([0.4,0.6],
	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.2,0.3],	[0.2,0.3],	[0.3,0.4],	[0.3,0.4],	[0.4,0.5],	[0.4,0.5],	[0.4,0.5],
	[0.7,0.8])	[0.5,0.6])	[0.5,0.6])	[0.1,0.2])	[0.1,0.2])	[0.2,0.4])	[0.2,0.4])	[0.3,0.4])	[0.3,0.4])	[0.3,0.4])
TRA₄	([0.6,0.8],	([0.1,0.2],	([0.2,0.4],	([0.6,0.8],	([0.7,0.9],	([0.6,0.8],	([0.7,0.9],	([0.2,0.4],	([0.6,0.8],	([0.6,0.8],
	[0.3,0.4],	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.2,0.3],	[0.3,0.4],	[0.2,0.3],	[0.5,0.6],	[0.3,0.4],	[0.3,0.4],
	[0.2,0.4])	[0.7,0.8])	[0.5,0.6])	[0.2,0.4])	[0.1,0.2])	[0.2,0.4])	[0.1,0.2])	[0.5,0.6])	[0.2,0.4])	[0.2,0.4])
TRA₅	([0.2,0.4],	([0.6,0.8],	([0.2,0.4],	([0.2,0.4],	([0.6,0.8],	([0.7,0.9],	([0.2,0.4],	([0.2,0.4],	([0.7,0.9],	([0.7,0.9],
	[0.5,0.6],	[0.3,0.4],	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.2,0.3],	[0.5,0.6],	[0.5,0.6],	[0.2,0.3],	[0.2,0.3],
	[0.5,0.6])	[0.2,0.4])	[0.5,0.6])	[0.5,0.6])	[0.2,0.4])	[0.1,0.2])	[0.5,0.6])	[0.5,0.6])	[0.1,0.2])	[0.1,0.2])
TRA₆	([0.7,0.9],	([0.7,0.9],	([0.2,0.4],	([0.7,0.9],	([0.6,0.8],	([0.4,0.6],	([0.2,0.4],	([0.6,0.8],	([0.6,0.8],	([0.1,0.2],
	[0.2,0.3],	[0.2,0.3],	[0.5,0.6],	[0.2,0.3],	[0.3,0.4],	[0.4,0.5],	[0.5,0.6],	[0.3,0.4],	[0.3,0.4],	[0.5,0.6],
	[0.1,0.2])	[0.1,0.2])	[0.5,0.6])	[0.1,0.2])	[0.2,0.4])	[0.3,0.4])	[0.5,0.6])	[0.2,0.4])	[0.2,0.4])	[0.7,0.8])
TRA₇	([0.4,0.6],	([0.7,0.9],	([0.7,0.9],	([0.6,0.8],	([0.2,0.4],	([0.4,0.6],	([0.6,0.8],	([0.6,0.8],	([0.4,0.6],	([0.1,0.2],
	[0.4,0.5],	[0.2,0.3],	[0.2,0.3],	[0.3,0.4],	[0.5,0.6],	[0.4,0.5],	[0.3,0.4],	[0.3,0.4],	[0.4,0.5],	[0.5,0.6],
	[0.3,0.4])	[0.1,0.2])	[0.1,0.2])	[0.2,0.4])	[0.5,0.6])	[0.3,0.4])	[0.2,0.4])	[0.2,0.4])	[0.3,0.4])	[0.7,0.8])
TRA₈	([0.1,0.2],	([0.1,0.2],	([0.6,0.8],	([0.2,0.4],	([0.6,0.8],	([0.7,0.9],	([0.7,0.9],	([0.7,0.9],	([0.2,0.4],	([0.2,0.4],
	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.5,0.6],	[0.3,0.4],	[0.2,0.3],	[0.2,0.3],	[0.2,0.3],	[0.5,0.6],	[0.5,0.6],
	[0.7,0.8])	[0.7,0.8])	[0.2,0.4])	[0.5,0.6])	[0.2,0.4])	[0.1,0.2])	[0.1,0.2])	[0.1,0.2])	[0.5,0.6])	[0.5,0.6])
TRA₉	([0.1,0.2],	([0.1,0.2],	([0.2,0.4],	([0.2,0.4],	([0.6,0.8],	([0.6,0.8],	([0.4,0.6],	([0.4,0.6],	([0.1,0.2],	([0.4,0.6],
	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.3,0.4],	[0.4,0.5],	[0.4,0.5],	[0.5,0.6],	[0.4,0.5],
	[0.7,0.8])	[0.7,0.8])	[0.5,0.6])	[0.5,0.6])	[0.2,0.4])	[0.2,0.4])	[0.3,0.4])	[0.3,0.4])	[0.7,0.8])	[0.3,0.4])
TRA₁₀	([0.7,0.9],	([0.7,0.9],	([0.2,0.4],	([0.2,0.4],	([0.1,0.2],	([0.2,0.4],	([0.4,0.6],	([0.6,0.8],	([0.7,0.9],	([0.6,0.8],
	[0.2,0.3],	[0.2,0.3],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.4,0.5],	[0.3,0.4],	[0.2,0.3],	[0.3,0.4],
	[0.1,0.2])	[0.1,0.2])	[0.5,0.6])	[0.5,0.6])	[0.7,0.8])	[0.5,0.6])	[0.3,0.4])	[0.2,0.4])	[0.1,0.2])	[0.2,0.4])
	TRS₁	TRS₂	TRS₃	TRS₄	TRS₅	TRS₆	TRS₇	TRS₈	TRS₉	TRS₁₀
TRA₁	([0.1,0.2],	([0.2,0.4],	([0.6,0.8],	([0.1,0.2],	([0.2,0.4],	([0.6,0.8],	([0.6,0.8],	([0.7,0.9],	([0.1,0.2],	([0.2,0.4],
	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.5,0.6],	[0.5,0.6],	[0.3,0.4],	[0.3,0.4],	[0.2,0.3],	[0.5,0.6],	[0.5,0.6],
	[0.7,0.8])	[0.5,0.6])	[0.2,0.4])	[0.7,0.8])	[0.5,0.6])	[0.2,0.4])	[0.2,0.4])	[0.1,0.2])	[0.7,0.8])	[0.5,0.6])
TRA₂	([0.6,0.8],	([0.1,0.2],	([0.2,0.4],	([0.2,0.4],	([0.1,0.2],	([0.4,0.6],	([0.1,0.2],	([0.1,0.2],	([0.2,0.4],	([0.1,0.2],
	[0.3,0.4],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.4,0.5],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],	[0.5,0.6],
	[0.2,0.4])	[0.7,0.8])	[0.5,0.6])	[0.5,0.6])	[0.7,0.8])	[0.3,0.4])	[0.7,0.8])	[0.7,0.8])	[0.5,0.6])	[0.7,0.8])
TRA₃	([0.1,0.2],	([0.6,0.8],	([0.2,0.4],	([0.7,0.9],	([0.6,0.8],	([0.6,0.8],	([0.6,0.8],	([0.6,0.8],	([0.4,0.6],	([0.1,0.2],
	[0.5,0.6],	[0.3,0.4],	[0.5,0.6],	[0.2,0.3],	[0.3,0.4],	[0.3,0.4],	[0.3,0.4],	[0.3,0.4],	[0.4,0.5],	[0.5,0.6],
	[0.7,0.8])	[0.2,0.4])	[0.5,0.6])	[0.1,0.2])	[0.2,0.4])	[0.2,0.4])	[0.2,0.4])	[0.2,0.4])	[0.3,0.4])	[0.7,0.8])

TRA₄	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])
TRA₅	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₆	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])
TRA₇	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₈	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])
TRA₉	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₁₀	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.6,0.8], [0.3,0.4], [0.2,0.4])
	TRS ₁	TRS ₂	TRS ₃	TRS ₄	TRS ₅	TRS ₆	TRS ₇	TRS ₈	TRS ₉	TRS ₁₀
TRA₁	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])
TRA₂	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₃	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])
TRA₄	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])

TRA₅	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])
TRA₆	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₇	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])
TRA₈	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])
TRA₉	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.4,0.6], [0.4,0.5], [0.3,0.4])
TRA₁₀	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.5,0.6], [0.7,0.8])	([0.2,0.4], [0.5,0.6], [0.5,0.6])	([0.4,0.6], [0.4,0.5], [0.3,0.4])	([0.6,0.8], [0.3,0.4], [0.2,0.4])	([0.7,0.9], [0.2,0.3], [0.1,0.2])	([0.6,0.8], [0.3,0.4], [0.2,0.4])

Step 4. Convert the IVNNs to the crisp values.

The IVNNs are converted by the score function to crisp values, then the decision matrix as:

Step 5. Aggregate the decision matrices

Step 6. Compute the criteria weights as shown in Figure 3. The economy has the highest importance and society has the lowest importance.

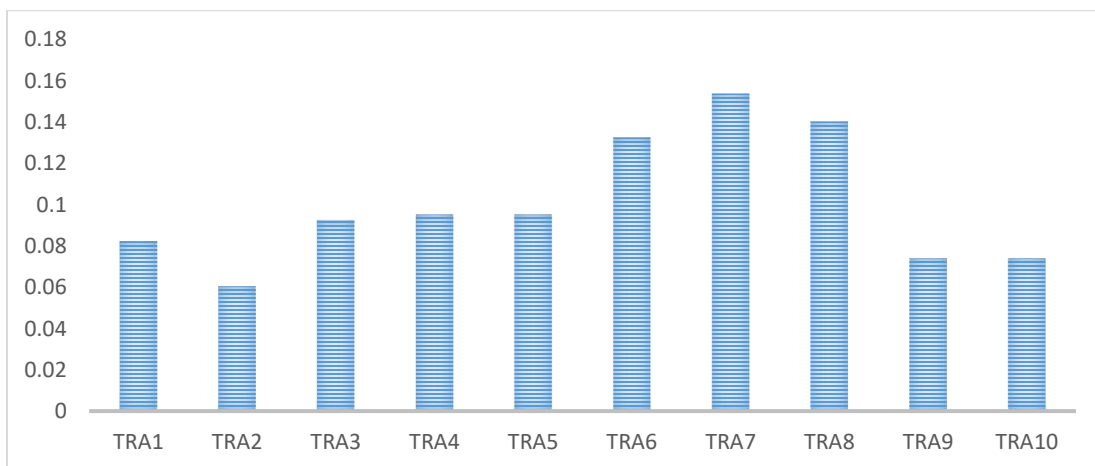


Figure 3. The criteria weights.

The criteria weights are computed by the average method.

Step 7. Normalize the decision matrix by using Eq. (15) as shown in Table 2.

Table 2. Normalization decision matrix.

	TRS ₁	TRS ₂	TRS ₃	TRS ₄	TRS ₅	TRS ₆	TRS ₇	TRS ₈	TRS ₉	TRS ₁₀
TRA ₁	0.165528	0.185954	0.444651	0.16747	0.182788	0.378925	0.382012	0.357618	0.143203	0.207747
TRA ₂	0.340355	0.143451	0.199523	0.182524	0.141008	0.274478	0.248961	0.218545	0.185633	0.160262
TRA ₃	0.205515	0.262107	0.199523	0.446902	0.450877	0.309294	0.298753	0.316227	0.247511	0.276996
TRA ₄	0.435208	0.143451	0.16912	0.440316	0.362095	0.296339	0.284061	0.23179	0.375687	0.46298
TRA ₅	0.195286	0.376336	0.322087	0.182524	0.3325	0.281765	0.311814	0.245035	0.457896	0.411538
TRA ₆	0.386852	0.458688	0.266031	0.487358	0.318574	0.34411	0.171416	0.302982	0.323532	0.261168
TRA ₇	0.315247	0.480825	0.492157	0.344349	0.220216	0.226707	0.382012	0.38742	0.337676	0.160262
TRA ₈	0.245502	0.143451	0.444651	0.27849	0.407356	0.419409	0.443232	0.344373	0.17149	0.292825
TRA ₉	0.150649	0.143451	0.199523	0.197578	0.407356	0.378925	0.276714	0.316227	0.233368	0.276996
TRA ₁₀	0.504953	0.480825	0.199523	0.197578	0.141008	0.170031	0.276714	0.38742	0.479995	0.46298

Step 8. Compute the weighted normalized decision matrix by using Eq. (16) as shown in Table 3.

Table 3. Weighted normalized decision matrix.

	TRS ₁	TRS ₂	TRS ₃	TRS ₄	TRS ₅	TRS ₆	TRS ₇	TRS ₈	TRS ₉	TRS ₁₀
TRA ₁	0.013631	0.011252	0.041053	0.015932	0.017389	0.050232	0.058744	0.050196	0.010586	0.015357
TRA ₂	0.028027	0.00868	0.018421	0.017364	0.013415	0.036386	0.038284	0.030675	0.013723	0.011847
TRA ₃	0.016923	0.015861	0.018421	0.042516	0.042894	0.041001	0.045941	0.044386	0.018297	0.020477
TRA ₄	0.035837	0.00868	0.015614	0.041889	0.034448	0.039284	0.043681	0.032534	0.027772	0.034225
TRA ₅	0.016081	0.022773	0.029737	0.017364	0.031632	0.037352	0.047949	0.034394	0.033849	0.030422
TRA ₆	0.031856	0.027756	0.024562	0.046364	0.030307	0.045617	0.026359	0.042527	0.023917	0.019307
TRA ₇	0.025959	0.029095	0.045439	0.032759	0.02095	0.030053	0.058744	0.054379	0.024962	0.011847
TRA ₈	0.020216	0.00868	0.041053	0.026494	0.038753	0.055598	0.068158	0.048337	0.012677	0.021647
TRA ₉	0.012405	0.00868	0.018421	0.018796	0.038753	0.050232	0.042551	0.044386	0.017251	0.020477
TRA ₁₀	0.041581	0.029095	0.018421	0.018796	0.013415	0.02254	0.042551	0.054379	0.035483	0.034225

Step 9. Compute the positive ideal solution and negative ideal solution by using Eqs. (17 and 18).

Step 10. Compute the distance from the positive and negative ideal solutions by using Eqs. (19 and 20).

Step 11. Compute the closeness value by using Eq. (21) as shown in Figure 4.

Step 12. Rank the alternatives. Alternative 7 is the best and alternative 2 is the worst.

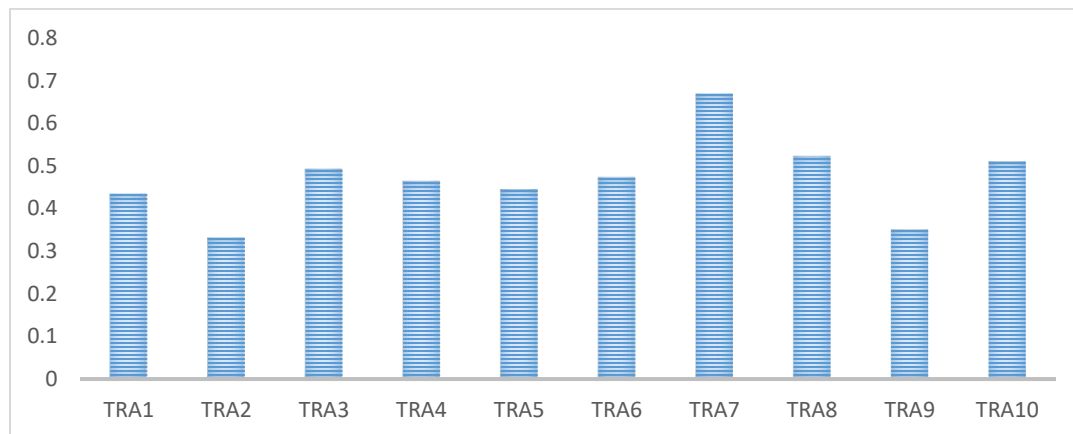


Figure 4. The closeness values.

4. Conclusions

This paper used the MCDM methodology for the supplier selection in Industry 4.0. This study used the TOPSIS method as an MCDM method for ranking alternatives and used the best one. The TOPSIS method is integrated with the IVNSs and TreeSoft Set. Three experts are invited to evaluate the criteria and options in this study. This study used ten criteria and ten suppliers. The decision matrices are built by the opinions of experts. Then, we use the IVNNs to evaluate the requirements and alternatives. The IVNNs are converted to the crisp values. The results show that the economic criterion has the highest weight, and the social criterion has the lowest. Alternative 7 is the best, and alternative 2 is the worst.

Acknowledgment

The authors are thankful to the Deanship of Graduate Studies and Scientific Research at the University of Bisha for supporting this work through the Fast-Track Research Support Program.

References

- [1] S. K. Sahoo, S. S. Goswami, and R. Halder, "Supplier Selection in the Age of Industry 4.0: A Review on MCDM Applications and Trends," *Decision Making Advances*, vol. 2, no. 1, pp. 32–47, 2024.
- [2] L. S. Dalenogare, G. B. Benitez, N. F. Ayala, and A. G. Frank, "The expected contribution of Industry 4.0 technologies for industrial performance," *International Journal of production economics*, vol. 204, pp. 383–394, 2018.
- [3] H. Lasi, P. Fettke, H.-G. Kemper, T. Feld, and M. Hoffmann, "Industry 4.0," *Business & information systems engineering*, vol. 6, pp. 239–242, 2014.
- [4] A. G. Frank, L. S. Dalenogare, and N. F. Ayala, "Industry 4.0 technologies: Implementation patterns in manufacturing companies," *International journal of production economics*, vol. 210, pp. 15–26, 2019.

- [5] D. Gorecky, M. Schmitt, M. Loskyll, and D. Zühlke, "Human-machine-interaction in the industry 4.0 era," in *2014 12th IEEE international conference on industrial informatics (INDIN)*, Ieee, 2014, pp. 289–294.
- [6] M. Ghobakhloo, "Industry 4.0, digitization, and opportunities for sustainability," *Journal of cleaner production*, vol. 252, p. 119869, 2020.
- [7] M. Sony and S. Naik, "Key ingredients for evaluating Industry 4.0 readiness for organizations: a literature review," *Benchmarking: An International Journal*, vol. 27, no. 7, pp. 2213–2232, 2020.
- [8] A. Raj, G. Dwivedi, A. Sharma, A. B. L. de Sousa Jabbour, and S. Rajak, "Barriers to the adoption of industry 4.0 technologies in the manufacturing sector: An inter-country comparative perspective," *International Journal of Production Economics*, vol. 224, p. 107546, 2020.
- [9] C. Bai, P. Dallasega, G. Orzes, and J. Sarkis, "Industry 4.0 technologies assessment: A sustainability perspective," *International journal of production economics*, vol. 229, p. 107776, 2020.
- [10] Mohamed Abouhawwash, Nitin Mittal, Asiye Yilmaz Adkinson, "Single Valued Neutrosophic Set with Multi-Criteria Decision Making Methodology for Wind Turbine Development," *Neutrosophic Optimization and Intelligent Systems (NOIS)*, vol. 1, pp. 31–38, 2024, DOI: <https://doi.org/10.61356/j.nois.2024.16189>.
- [11] I. Mahdavi, N. Mahdavi-Amiri, A. Heidarzade, and R. Nourifar, "Designing a model of fuzzy TOPSIS in multiple criteria decision making," *Applied Mathematics and Computation*, vol. 206, no. 2, pp. 607–617, 2008.
- [12] Nada A. Nabeeh, Karam M. Sallam, "A Combined Compromise Solution (CoCoSo) of MCDM Problems for Selection of Medical Best Bearing Ring," *Neutrosophic Optimization and Intelligent Systems (NOIS)*, vol. 1, pp. 1–13, 2024, DOI: <https://doi.org/10.61356/j.nois.2024.16089>.
- [13] M. Mohamed, "BHARAT Decision Making Model: Harness an Innovative MCDM Methodology for Recommending Beneficial E-Commerce Website," *Multicriteria Algorithms with Applications*, vol. 2, pp. 53–64, 2024.
- [14] S. N. S. Bathusha, S. Jayakumar, and S. A. K. Raj, "The Energy of Interval-Valued Complex Neutrosophic Graph Structures: Framework, Application and Future Research Directions," *Neutrosophic Systems with Applications*, vol. 13, pp. 67–101, 2024.
- [15] Z. Mohamed, M. M. Ismail, and A. Abd El-Gawad, "Sustainable supplier selection using neutrosophic multi-criteria decision making methodology," *Sustain. Mach. Intell. J.*, vol. 3, 2023.
- [16] M. Jameel and S. Tanwar, "A Deeper Monitoring and Evaluation of the Nature of Barriers to Climate Change Adaptation Planning under Fuzzy Multi-Criteria Decision Making Methodology,"

- Multicriteria Algorithms with Applications*, vol. 2, pp. 43–52, 2024.
- [17] F. Smarandache, "A unifying field in Logics: Neutrosophic Logic.," in *Philosophy*, American Research Press, 1999, pp. 1–141.
- [18] F. Smarandache, *Introduction to neutrosophic statistics*. Infinite Study, 2014.
- [19] F. Smarandache, *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability*. Infinite Study, 2013.
- [20] F. Smarandache, *Symbolic neutrosophic theory*. Infinite Study, 2015.
- [21] F. Smarandache *et al.*, "Introduction to neutrosophy and neutrosophic environment," in *Neutrosophic Set in Medical Image Analysis*, Elsevier, 2019, pp. 3–29.
- [22] F. Smarandache and M. Jdid, "An Overview of Neutrosophic and Plithogenic Theories and Applications," 2023.
- [23] Muhammad Saeed, Imrana Shafique, "Relation on Fermatean Neutrosophic Soft Set with Application to Sustainable Agriculture," *HyperSoft Set Methods in Engineering (HSSE)*, vol. 1, pp. 21–33, 2024, DOI: <https://doi.org/10.61356/j.hsse.2024.18250>.
- [24] J.-J. Wang, Y.-Y. Jing, C.-F. Zhang, and J.-H. Zhao, "Review on multi-criteria decision analysis aid in sustainable energy decision-making," *Renewable and sustainable energy reviews*, vol. 13, no. 9, pp. 2263–2278, 2009.
- [25] D. Jato-Espino, E. Castillo-Lopez, J. Rodriguez-Hernandez, and J. C. Canteras-Jordana, "A review of application of multi-criteria decision making methods in construction," *Automation in construction*, vol. 45, pp. 151–162, 2014.
- [26] Darvin Manuel Ramírez Guerra, Osmanys Pérez Peña, John Alex Torres Yanez, "Neutrosophic Evaluation of Ethical Factors in Remote Medical Care," *HyperSoft Set Methods in Engineering (HSSE)*, vol. 1, pp. 11–20, 2024, DOI: <https://doi.org/10.61356/j.hsse.2024.17950>.
- [27] R. S. U. Haq, M. Saeed, N. Mateen, F. Siddiqui, and S. Ahmed, "An interval-valued neutrosophic based MAIRCA method for sustainable material selection," *Engineering Applications of Artificial Intelligence*, vol. 123, p. 106177, 2023.
- [28] F. Smarandache, *Practical applications of IndetermSoft Set and IndetermHyperSoft Set and introduction to TreeSoft Set as an extension of the MultiSoft Set*. Infinite Study, 2022.
- [29] C. M. Brugha, "Structure of multi-criteria decision-making," *Journal of the Operational research Society*, vol. 55, no. 11, pp. 1156–1168, 2004.
- [30] M. Panda and A. K. Jagadev, "TOPSIS in multi-criteria decision making: a survey," in *2018 2nd International Conference on Data Science and Business Analytics (ICDSBA)*, IEEE, 2018, pp. 51–54.

- [31] E. Roszkowska, "Multi-criteria decision making models by applying the TOPSIS method to crisp and interval data," *Multiple Criteria Decision Making/University of Economics in Katowice*, vol. 6, no. 1, pp. 200–230, 2011.

Received: 09 Nov,2023 Accepted: 27 Feb,2024