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Optimization of triangular neutrosophic based economic order quantity model under preservation technology and power demand with shortages

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Abstract: The primary objective of this article is to develop a mathematical model and determine the optimal policies of an inventory system involving power demand and controlled deterioration through preservation technology. This model comes in handy in a power demand-oriented inventory system with demand high at the end of the period. The model incorporates backlogged shortages and linear holding cost. The triangular neutrosophic numbers (TNN's) are used for a nuanced representation of uncertain and imprecise inventory-related expenses. An efficient algorithm is constructed to minimize the total cost, and obtain optimal positive inventory time, optimum cycle time and minimum preservation technology investment. Few numerical examples are used to illustrate and validate the model. The comparative study conducted between models with and without preservation technology investment reveals a significant reduction in total inventory costs facilitated by the preservation facility. Also, the numerical results obtained in crisp and neutrosophic environment are compared. Specific previously obtained results are discussed to illustrate the theoretical findings. Sensitivity analysis of the model provides managerial insights replicating reality.

Keywords: economic order quantity; power demand; deteriorating items; complete backlogging; preservation technology; triangular neutrosophic number

1. Introduction

Inventory systems in the modern days face a huge challenge due to uncertain conditions and stiff competition in the product markets. This calls for new strategies or technologies to sustain in the global scenario. Deterioration of items is a common phenomenon in any inventory system. Many consequences arise due to unforeseen deterioration or breakage of items in stock. Unplanned deterioration often leads to unexpected costs for replacements, repairs, or disposal of obsolete items. Hence, stock preserving policies could be considered to reduce these costs and also avoid upscaling of shortages due to unexpected deterioration. The impact of deterioration can be significantly reduced by implementing preservation methods. A strategy is designed to minimize the cost of deterioration while making preservation investments. Moreover, stock preserving policies play a crucial role in reducing the financial burden associated with inventory maintenance.

Demand is one of the factors that influences the working mechanism of inventory management. The demand for most products is inherently time-dependent, influenced by critical factors such as freshness, seasonality, and the introduction of new products. Power demand represents a specific form of time-dependent demand. The power pattern of demand can be visualized in situations where the demand is high either at the beginning or end of a cycle. Inventory systems with variable demand like power demand are likely to face shortages in supply. In such cases, to retain the goodwill the demand is backlogged. This approach helps businesses retain customer goodwill by acknowledging demand during shortages, preventing immediate customer dissatisfaction. It allows organizations to manage variable demand effectively while maintaining positive customer relationships.

In general, the costs associated with the inventory are uncertain. Neutrosophic numbers are utilized to account for the imprecision in systems. Due to the uncertainty of many parameters in real-life scenarios, neutrosophic numbers are invaluable for mitigating uncertainty.

This article attempts to define new strategies by considering the implementation of preservation technology in an inventory system with power demand. The proposed model is particularly suited for items that align with the power pattern of demand. Consequently, the inventory model used in this study can be beneficial for products like: (i) Fresh vegetables, Bakery items, Milk-based products, or Seafoods, which experience higher demand at the beginning of the inventory period compared to the end, since fresh or new food products are preferred by the customers. (ii) Marginally discounted products including frozen meat and Ice creams which experience higher demand towards the end of the expiry period and also, newly introduced products which are assessed for their performance at the beginning and purchased towards the end of the period when the performance is promising. (iii) Clothing and apparel, Consumer electronics, Exercise equipment, Stationery and office supplies, which maintain a nearly constant demand throughout the inventory cycle. With the implementation of a well-suited preservation strategy, these items experience a significantly prolonged shelf life.

The remaining part of the article is structured in the following manner: Section 2 addresses the literature relevant to the current investigation and the study's contributions. As for Section 3, it presents the concept of neutrosophic numbers and their de-neutrosophication technique. Section 4 contains nomenclature and assumptions. The model is formulated and developed for more discussion under crisp and neutrosophic environment in Section 5. Section 6 presents iterative algorithms for determining the optimal solution. In Section 7, some particular inventory models derived from proposed model. In Section 8, a sensitivity analysis of a few system parameters is offered along with a numerical demonstration for testing the model. In Section 9, conclusions are provided.

2. Literature Review

In literature, researchers have examined various inventory models focusing on the deterioration of items. Whitin [1] was the first to introduce the idea of deterioration while modelling the inventory system. Ghare & Schrader [2] developed a mathematical model considering constant deterioration of items in stock. Sachan [3] revised these models to include shortages in a deteriorating inventory environment. Shah et al. [4] later incorporated object degradation after a certain period. Recently, Hatibaruah and Saha [5] suggested a model for managing items that deteriorate with a two-parameter Weibull distribution.

The effect of investing in preservation technologies on an inventory system for deteriorating items has been investigated by several researchers. Selected works of preservation-related inventory models shown in table. 1. Hsu et al. [6] initially proposed the preservation technology principle. With the preservation technologies, they established an inventory model based on constant demand. Later, Dye and Hsieh [7] developed a model of economic production quantity (EPQ) based on preservation-based approach, considering time-reliant demand. He and Huang [8] suggested an inventory policy for degrading items accounting for the rate of deterioration which is negatively exponentially proportional to the amount spent on preservation technologies.

Table 1: Selected preservation-related inventory models from 2010

Authors	Demand type	Deterioration	Preservation Technology	Permitted Shortages	Backlog's nature	Cost environment	Objective function
Hsu et al. (2010) [6]	Constant	✓	✓	✓	Full	Crisp	Maximize profit
Dye & Hsieh (2012) [7]	Constant	✓	✓	✓	Partial	Crisp	Maximize profit
He & Huang (2013) [8]	Price- reliant	✓	✓	×	-	Crisp	Maximize profit
Singh & Sharma (2013) [9]	Ramp-Type	✓	✓	✓	Partial	Crisp	Minimize cost
Zhang et al. (2015) [10]	Price-reliant	✓	✓	×	-	Crisp	Maximize profit
Mishra et al. (2017) [11]	Price and stock reliant	✓	✓	✓	Partial & Full	Crisp	Maximize profit
Li et al. (2019) [12]	Price-reliant	✓	✓	✓	Partial	Crisp	Maximize profit
Das et al. (2020) [13]	Price- reliant	✓	✓	✓	Partial	Crisp	Maximize profit
Khanna et al. (2020) [14]	Stock reliant	✓	✓	×	-	Crisp	Minimize cost
Bhawaria and Rathore (2021) [15]	Price and stock reliant	✓	✓	✓	Partial & Full	Crisp	Minimize cost
Mahapatra et al. (2022) [27]	Uncertain with promotional effort	✓	✓	✓	Full	Crisp & fuzzy	Minimize cost
Mohanta et al. (2023) [35]	Selling price, promotional effort, downstream trade credit	✓	✓	×	-	Triangular neutrosophic numbers	Maximize profit
This Paper	Time-reliant power demand pattern	✓	✓	✓	Full	crisp & Triangular neutrosophic numbers	Minimize cost

Singh and Sharma [9] introduced a preservation technology based two stage trade credit-financing model. Zhang et al. [10] suggested a supply chain model incorporating preservation technologies where demand is reliant on stock. Mishra et al. [11] created a preservation inventory model under shortages, considering demand to be dependent on both cost and stock levels. Price reliant inventory models with preservation strategy have been proposed by Li et al. [12] & Das et al. [13]. A stock-dependent demand inventory model based on preservation technologies was developed by Khanna et al. [14]. Bhawaria and Rathore [15] presented a controllable deteriorating inventory model with Hybrid-Type demand.

In the literature, few inventory models have been studied by considering deteriorating items with power pattern of demand. Naddor [16] was the first to identify and formulate this pattern of demand. Datta and Pal [17] investigated the power demand inventory model with a varying deterioration. Lee and Wu [18] accounted for power demand and shortages in their model. Dye [19] expanded the model to incorporate backlogging in relation to time spent. Later, Rajeswari and Vanjikkodi [20] analyzed an inventory model with power demand when deterioration is constant. San-José et al. [21] introduced an inventory system that focuses on maximizing the return on inventory investment in the context of time-reliant power demand. San-José et al. [22] recently devised a sustainable inventory system for a product with demand exhibiting a power pattern over time, wherein shortages are entirely backlogged.

Several researchers have developed their inventory models by considering the complete backlog of shortages. Posner and Yansouni [23] drew insight from the impatience of customers and associated it with backorders. Abad [24] initiated the thought that the part of backlogged demand can be expressed as a function of waiting time. Valliathal and Uthayakumar [25] presented a deteriorating stocking model with two-warehouse and partial, fully backlog of shortages. Mashud [26] introduced an economic order quantity (EOQ) model that considers deterioration, price and stock depend demand, incorporating a complete backlog of shortages. Mahapatra et al. [27] have recently put forward an inventory model dealing with uncertain demand in the presence of complete backlog for shortages.

A few inventory models have been developed in the literature by treating the cost parameters as triangular neutrosophic numbers (TNNs). The neutrosophic theory was first developed by Smarandache [28]. It efficiently expresses uncertain, contradictory, and incomplete information. Classical inventory models rely on crisp values, which cannot accurately reflect the inherent uncertainties and inaccuracies connected to actual inventory systems. Mullai and Broumi [29] proposed an inventory model that treats demand and ordering cost as TNNs to address this. Mullai and Surya [30] proposed a price break EOQ model under a neutrosophic environment. Pal and Chakraborty [31] created a time-discounted triangular neutrosophic-based degrading inventory model. Mondal et al. [32] created Logistic-growth demand-dependent EOQ model with neutrosophic coefficients under trade credit. Sugapriya et al. [33] presented power demand dependent two-warehouse deteriorating inventory model under trapezoidal bipolar neutrosophic environment. Recently, numerous researchers (Bhavani et al. [34], and Mohanta et al. [35]) established inventory models under a triangular neutrosophic environment.

To the best of our knowledge, no researcher has explored the combined impact of preservation technology on a deteriorating inventory model where demand follows a power pattern over time, while considering linear holding costs and neutrosophic cost parameters. The present study aims to fill this research gap by investigating the application of preservation technology to EOQ models with demand following a power pattern under neutrosophic environment.

Unique Contribution of this Study:

The subsequent contributions emphasize the novelty of this study:

1. Customers' demand size during the entire cycle follows power pattern of time.

2. Preservation technology is employed to mitigate the deterioration rate to fulfil customer demand, especially when it peaks towards the end of the scheduled period.
3. Shortages are allowed and that are completely backlogged. The nature of the holding cost is linear function of time.
4. The proposed model calculates the total cost within a neutrosophic environment, considering cost parameters as triangular neutrosophic numbers.
5. We have determined the optimal preservation technology cost, cycle length, and the time of shortage that minimize the total cost per unit time in the proposed model.

3. Preliminaries

Definition 3.1 [28]

Suppose X is the universal set. A "neutrosophic set (NS)" Ũ in X is defined by a truth, hesitation, false membership functions $\psi_{\tilde{U}}, \zeta_{\tilde{U}}, \nu_{\tilde{U}}$ respectively. Here, $\psi_{\tilde{U}}, \zeta_{\tilde{U}}$ and $\nu_{\tilde{U}}$ are real-valued parameters in the interval [0,1]. The NS Ũ can be expressed as $\tilde{U} = \left\{ \left\langle x; \left[\psi_{\tilde{U}}(x), \zeta_{\tilde{U}}(x), \nu_{\tilde{U}}(x) \right] \right\rangle : x \in X \,\&\, \psi_{\tilde{U}}(x), \zeta_{\tilde{U}}(x), \nu_{\tilde{U}}(x) \in]0^{-}, 1^{+}[\right\}$. The sum of the three membership functions is not constrained, allowing for flexibility in the membership functions $0^{-} \le \psi_{\tilde{U}}(x) + \zeta_{\tilde{U}}(x) + \nu_{\tilde{U}}(x) \le 3^{+}$.

Definition 3.2 [36]

If a set Ũ in the universal discourse X satisfies the condition $\tilde{A} = \left\{ \left\langle x; \left[\psi_{\tilde{U}}(x), \zeta_{\tilde{U}}(x), \nu_{\tilde{U}}(x) \right] \right\rangle : x \in X \right\}$, then it is considered to be a "Single-Valued neutrosophic set" of a Single-Valued independent variable x. Here, truth, hesitation, false membership functions are $\psi_{\tilde{U}}(x) : X \to [0,1]$, $\zeta_{\tilde{U}}(x) : X \to [0,1]$, $\nu_{\tilde{U}}(x) : X \to [0,1]$ respectively. These functions are used by the decision maker to represent their degree of belief in the variable. Additionally, $\psi_{\tilde{U}}(x) + \zeta_{\tilde{U}}(x) + \nu_{\tilde{U}}(x)$ is constrained to lie within the interval [0,3].

Definition 3.3:

Suppose we have three variables, ʊ, ʋ, and ʌ, such that $\psi_{\tilde{U}}(\square) = 1, \zeta_{\tilde{U}}(\upsilon) = 1, \nu_{\tilde{U}}(\Lambda) = 1$. In this case, the set Ũ is classified as "neutro-normal".

Definition 3.4:

The set Ũ is considered "neutro-convex" if the following criteria are met

i. $\psi_{\tilde{U}}(\omega p + (1-\omega) q) \ge \min(\psi_{\tilde{U}}(p), \psi_{\tilde{U}}(q))$

ii. $\varsigma_{\tilde{U}}(\omega p + (1-\omega) q) \ge \min(\varsigma_{\tilde{U}}(p), \varsigma_{\tilde{U}}(q))$

iii. $\nu_{\tilde{U}}(\omega p + (1-\omega) q) \ge \min(\nu_{\tilde{U}}(p), \nu_{\tilde{U}}(q))$

Here, $\omega \in [0,1]$ while the variables p and q are real numbers.

Definition 3.5 [37]

A "triangular single valued neutrosophic number" (Ũ) can be expressed as Ũ=<(q_1,q_2,q_3:ψ),(r_1,r_2,r_3:ς),(s_1,s_2,s_3:ν)>. Here, $\psi_{\tilde{U}} : R \to [0,1]$, $\zeta_{\tilde{U}} : R \to [0,1]$, $\nu_{\tilde{U}} : R \to [0,1]$ are truth, hesitation, and false membership function, respectively, that are defined as follows:

$$\psi_{\tilde{U}}(x) = \begin{cases} \dfrac{x - q_1}{q_2 - q_1}, & \text{for } q_1 \le x < q_2 \\ 1, & \text{for } x = q \\ \dfrac{q_3 - x}{q_3 - q_2}, & \text{for } q_2 < x \le q_2 \\ 0, & \text{otherwise} \end{cases}$$

$$\varsigma_{\tilde{U}}(x)=\begin{cases}\dfrac{x-q_1}{q_2-q_1}, & \text{for } q_1 \le x < q_2\\ 0, & \text{for } x = q\\ \dfrac{q_3-x}{q_3-q_2}, & \text{for } q_2 < x \le q_2\\ 1, & \text{otherwise}\end{cases}$$

$$\nu_{\tilde{U}}(x)=\begin{cases}\dfrac{x-q_1}{q_2-q_1}, & \text{for } q_1 \le x < q_2\\ 0, & \text{for } x = q\\ \dfrac{q_3-x}{q_3-q_2}, & \text{for } q_2 < x \le q_2\\ 1, & \text{otherwise}\end{cases}$$

Definition 3.6: De-neutrosophic Technique

The removal of area approach [37] has been utilized in this model to compute the de-neutrosophic value for the triangular single-valued neutrosophic number $\tilde{U}$=<(q_1,q_2,q_3:ψ), (r_1,r_2,r_3:ς), (s_1,s_2,s_3:ν)>. The resulting de-neutrosophic value of $\tilde{U}$ is expressed as,

$$D(\tilde{U})=\frac{1}{12}(q_1+2q_2+q_3+r_1+2r_2+r_3+s_1+2s_2+s_3) \tag{1}$$

4. Nomenclature and assumptions

4.1 Nomenclature

Parameters

- π_o – The cost of each order placed.
- π_p – cost per unit of purchase.
- π_b – Backordered cost of each unit per short-time unit.
- π_d – Deteriorating cost of each unit.
- y_0 – The rate of deterioration.

Decision Variable

- t_1 – The instant that the level of inventory is zero, $t_1 \geq 0$.
- T –The duration of the cycle, ($T=t_1+t_2$).
- ξ – Investing in preservation technologies per unit of time.

Other variables

- Q – Order volume throughout a cycle of length T, (Q=MI+MB).

Functions

- t_2 – a period in the cycle time where shortages are permitted, $t_2 \geq 0$.
- h(t) – (h+bt), The cost per unit and per time unit for keeping inventory.
- MI – The highest amount of inventory during [0, T].
- MB – The highest number of units backordered during a stockout period.
- $I_1(t)$ – At time t, the amount of positive inventory, $0 \le t \le t_1$.
- $I_2(t)$ – At time t, the amount of negative inventory, $t_1 \le t \le T$.
- TC – The overall cost per unit of time.

4.2 Assumptions

- The demand rate is formulated as $D(t)=\dfrac{dt^{\frac{1}{\eta}-1}}{\eta T^{\frac{1}{\eta}-1}}$ at any time t, where T is the planning horizon, η can be any positive number, and d is a positive constant. In this expression, when η > 1, most of the demand occurs at the beginning; when η=1, demand is constant; when $\eta < 1$, most of the demand occurs at end shown in figure 1.

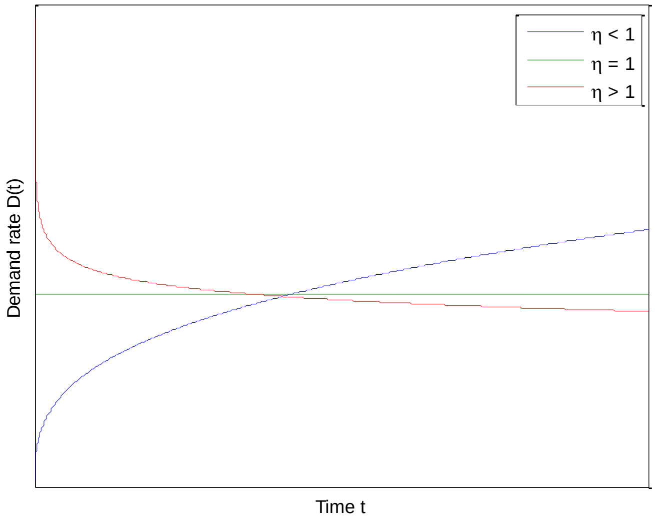

Figure 1. Demand depiction

- The deterioration rate y_0 is constant, $0 < y_0 << 1$, this can be controlled by investing in preservation strategy $y(\xi) = y_0 e^{-m\xi}$ which satisfies the condition $\frac{dy(\xi)}{d\xi} < 0$, $\frac{d^2 y(\xi)}{d\xi^2} > 0$ and $y(0)=y_0$, where m is the investment's sensitivity parameter. $0 < m < 1$.
- It is considered that the time-dependent holding cost, $h(t)=h+bt$.
- A single kind of item makes up the inventory.
- The replenishment rate is considered to be infinite.
- There is no limit to the planning horizon.
- Lead time for delivery is zero.
- The shortage is permitted, and it is fully backordered.

5. Mathematical Model

Figure 2 shows the level of available inventory at any given time.

Inventory level prior to the shortage

Inventory level between [0, t_1] is depends on demand and deterioration. The differential equation can be used to depict the inventory amount during [0, t_1] is

$$\frac{dI_1(t)}{dt} + y(\xi)I_1(t) = -\frac{dt^{\frac{1}{\eta}-1}}{\eta T^{\frac{1}{\eta}-1}}, \qquad 0 \le t \le t_1 \qquad (2)$$

with $I_1(t_1) = 0$ as the boundary condition.

Equation (1)'s solution is provided by

$$I_1(t) = \frac{d}{T^{\frac{1}{\eta}-1}}\left[(1 - y(\xi)t)\left(t_1^{\frac{1}{\eta}} - t^{\frac{1}{\eta}}\right) + \frac{y(\xi)}{1+\eta}\left(t_1^{\frac{1+\eta}{\eta}} - t^{\frac{1+\eta}{\eta}}\right)\right], \; 0 \le t \le t_1 \qquad (3)$$

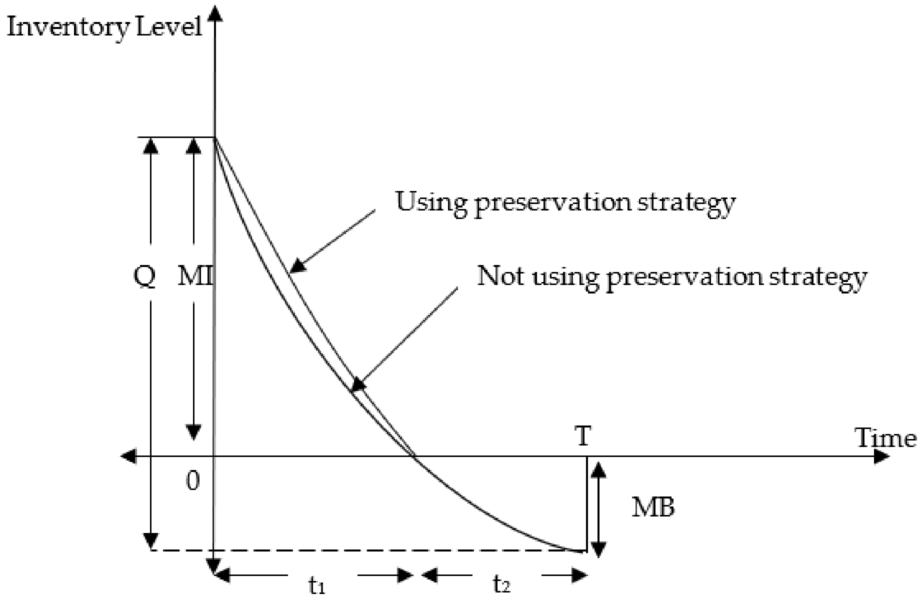

Figure 2. Inventory system depiction

Inventory level throughout stockout period

Inventory level between [t_1, T] is depends on demand. The differential equation can be used to depict the inventory amount during [t_1, T] is

$$\frac{dI_2(t)}{dt} = -\left(\frac{dt^{\frac{1}{\eta}-1}}{\eta T^{\frac{1}{\eta}-1}}\right), \qquad t_1 \le t \le T \tag{4}$$

with $I_2(t_1) = 0$ as a boundary condition.

$$I_2(t) = -\frac{d}{T^{\frac{1}{\eta}-1}}\left(t^{\frac{1}{\eta}} - t_1^{\frac{1}{\eta}}\right), \qquad t_1 \le t \le T \tag{5}$$

The highest amount of inventory during [0, T] is

$$MI = I_1(0) = \frac{d}{T^{\frac{1}{\eta}-1}}\left[t_1^{\frac{1}{\eta}} + \frac{y(\xi)t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right] \tag{6}$$

The number of backordered units are

$$MB = -I_2(T) = \frac{d}{T^{\frac{1}{\eta}-1}}\left(T^{\frac{1}{\eta}} - t_1^{\frac{1}{\eta}}\right) \tag{7}$$

Hence, the purchase volume during the time span [0, T] is Q = MI + MB.

$$Q = \frac{d}{T^{\frac{1}{\eta}-1}}\left[T^{\frac{1}{\eta}} + \frac{y(\xi)t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right] \tag{8}$$

Cost components:

The following cost elements make up each replenishment cycle's overall cost.

Ordering cost

$$OC = \pi_o \tag{9}$$

Holding cost

$$HC=\int_0^{t_1} h(t)I_1(t)dt$$

$$HC=\frac{hd}{T^{\frac{1}{\eta}-1}}\left[\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{y(\xi)t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}\right]+\frac{bd}{2T^{\frac{1}{\eta}-1}}\left[\frac{t_1^{\frac{1+2\eta}{\eta}}}{1+2\eta}+\frac{y(\xi)t_1^{\frac{1+3\eta}{\eta}}}{3(1+3\eta)}\right] \tag{10}$$

Backordered cost

$$BC=\pi_b\int_{t_1}^{T}\left(-I_2(t)\right)dt$$

$$BC=\frac{\pi_b d}{T^{\frac{1}{\eta}-1}}\left[-Tt_1^{\frac{1}{\eta}}+\frac{\eta T^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right] \tag{11}$$

Deterioration Cost

$$DC=\pi_d\left\{Q-\int_0^{t_1}\left(\frac{dt^{\frac{(1-\eta)}{\eta}}}{\eta T^{\frac{1}{\eta}-1}}\right)dt-\int_{t_1}^{T}\left(\frac{dt^{\frac{1-\eta}{\eta}}}{\eta T^{\frac{1}{\eta}-1}}\right)dt\right\}$$

$$DC=\pi_d\frac{dy(\xi)t_1^{\frac{1+\eta}{\eta}}}{(1+\eta)T^{\frac{1}{\eta}-1}} \tag{12}$$

Purchase cost

$$PC=\pi_p\times Q$$

$$PC=\frac{\pi_p d}{T^{\frac{1}{\eta}-1}}\left[T^{\frac{1}{\eta}}+\frac{y(\xi)t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right] \tag{13}$$

Preservation technology cost

$$PTC=\xi T$$

Therefore, the overall cost per unit of time is

$$TC\left(t_1,T,\xi\right)=\frac{1}{T}\left[OC+HC+BC+DC+PC+PTC\right]$$

$$\begin{aligned}TC\left(t_1,T,\xi\right)=\frac{1}{T}\Bigg\{&\pi_o+\frac{hd}{T^{\frac{1}{\eta}-1}}\left[\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{y(\xi)t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}\right]+\frac{bd}{2T^{\frac{1}{\eta}-1}}\left[\frac{t_1^{\frac{1+2\eta}{\eta}}}{1+2\eta}+\frac{y(\xi)t_1^{\frac{1+3\eta}{\eta}}}{3(1+3\eta)}\right]+\frac{\pi_d dy(\xi)t_1^{\frac{1+\eta}{\eta}}}{(1+\eta)T^{\frac{1}{\eta}-1}}\\&+\frac{\pi_b d}{T^{\frac{1}{\eta}-1}}\left[-Tt_1^{\frac{1}{\eta}}+\frac{\eta T^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\frac{\pi_p d}{T^{\frac{1}{\eta}-1}}\left[T^{\frac{1}{\eta}}+\frac{y(\xi)t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\xi T\Bigg\}\end{aligned} \tag{14}$$

5.1 Inventory model under triangular neutrosophic domain

In real markets, cost parameters are often uncertain, to overcome this neutrosophic numbers are used to represent various costs because they have truth, hesitation, and falsity membership functions that can address all types of parameter uncertainties. Specifically, this proposed inventory model utilizes TNNs to represent holding cost (h), purchase cost (π_p), ordering cost (π_o), deterioration cost (π_d), backordered cost (π_b). The format of the TNNs $\tilde{\pi}_{oN},\tilde{\pi}_{pN},\tilde{h}_N,\tilde{\pi}_{dN},\tilde{\pi}_{bN}$ is as follows:

$$\tilde{\pi}_{oN}=\langle(o_{11},o_{12},o_{13}),(o_{21},o_{22},o_{23}),(o_{31},o_{32},o_{33})\rangle$$
$$\tilde{\pi}_{pN}=\langle(p_{11},p_{12},p_{13}),(p_{21},p_{22},p_{23}),(p_{31},p_{32},p_{33})\rangle$$
$$\tilde{h}_{N}=\langle(h_{11},h_{12},h_{13}),(h_{21},h_{22},h_{21}),(h_{31},h_{32},h_{33})\rangle$$
$$\tilde{\pi}_{dN}=\langle(d_{11},d_{12},d_{13}),(d_{21},d_{22},d_{23}),(d_{31},d_{32},d_{33})\rangle$$
$$\tilde{\pi}_{bN}=\langle(b_{11},b_{12},b_{13}),(b_{21},b_{22},b_{23}),(b_{31},b_{32},b_{33})\rangle$$

By applying the removal area technique (1) to above neutrosophic costs, the resulting de-neutrosophize costs are $D(\tilde{\pi}_{oN}),D(\tilde{\pi}_{pN}),D(\tilde{h}_N),D(\tilde{\pi}_{dN}),D(\tilde{\pi}_{bN})$.

To calculate the total cost within the neutrosophic domain $\tilde{T}C_N$, we can substitute the de-neutrosophic values into equation (14), resulting in:

$$\tilde{T}C_N=\frac{1}{T}\left\{D(\tilde{\pi}_{oN})+\frac{D(\tilde{h}_N)d}{T^{\frac{1}{\eta}-1}}\left[\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{y(\xi)t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}\right]+\frac{bd}{2T^{\frac{1}{n}-1}}\left[\frac{t_1^{\frac{1+2\eta}{\eta}}}{1+2\eta}+\frac{y(\xi)t_1^{\frac{1+3\eta}{\eta}}}{3(1+3\eta)}\right]+\frac{D(\tilde{\pi}_{dN})dy(\xi)t_1^{\frac{1+\eta}{\eta}}}{(1+\eta)T^{\frac{1}{\eta}-1}}\right.$$
$$\left.+\frac{D(\tilde{\pi}_{bN})d}{T^{\frac{1}{\eta}-1}}\left[-Tt_1^{\frac{1}{\eta}}+\frac{\eta T^{\frac{1+\eta}{\eta}}}{1+n}+\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\frac{D(\tilde{\pi}_{pN})d}{T^{\frac{1}{\eta}-1}}\left[T^{\frac{1}{\eta}}+\frac{y(\xi)t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\xi T\right\} \quad (15)$$

6. Optimal Solution

In this section the necessary and sufficiency conditions for total cost's optimality are derived.

Necessary conditions:

By solving the equations $\frac{\partial TC}{\partial t_1}=0$, $\frac{\partial TC}{\partial T}=0$ and $\frac{\partial TC}{\partial \xi}=0$, the optimum value of t_1^*, T^*, ξ^*, and thereby the minimum average total cost per unit of time (TC^*), Q^* can be determined.

where,

$$\frac{\partial TC}{\partial t_1}=\frac{1}{\eta T^{\frac{1}{\eta}}}\left\{hd\left[t_1^{\frac{1}{\eta}}+\frac{1}{2}y_0e^{-m\xi}t_1^{\frac{1+\eta}{\eta}}\right]+\frac{bd}{2}\left[t_1^{\frac{1+\eta}{\eta}}+\frac{1}{3}y_0e^{-m\xi}t_1^{\frac{1+2\eta}{\eta}}\right]+\pi_d dy_0e^{-m\xi}t_1^{\frac{1}{\eta}}\right.$$
$$\left.+\pi_b d\left[-Tt_1^{\frac{1-\eta}{\eta}}+t_1^{\frac{1}{\eta}}\right]+\pi_p d\left[y_0e^{-m\xi}t_1^{\frac{1}{\eta}}\right]\right\}=0 \quad (16)$$

$$\frac{\partial TC}{\partial T}=-\frac{1}{\eta T^{\frac{1}{\eta}+1}}\left\{\pi_o T^{\frac{1}{\eta}-1}+hd\left[\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{y(\xi)t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}\right]+\frac{bd}{2}\left[\frac{t_1^{\frac{1+2\eta}{\eta}}}{1+2\eta}+\frac{y(\xi)t_1^{\frac{1+3\eta}{\eta}}}{3(1+3\eta)}\right]+\frac{\pi_d dy(\xi)t_1^{\frac{1+\eta}{\eta}}}{(1+\eta)}\right.$$
$$\left.+\pi_b d\left[-Tt_1^{\frac{1}{\eta}}+\frac{\eta T^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\pi_p d\left[T^{\frac{1}{\eta}}+\frac{1}{1+\eta}\left(y(\xi)t_1^{\frac{1+\eta}{\eta}}\right)\right]+\xi T^{\frac{1}{\eta}}\right\}$$

$$+\frac{1}{T^{\frac{1}{\eta}}}\left\{\frac{(1-\eta)\pi_o T^{\frac{1}{\eta}-2}}{\eta}+\pi_b d\left[T^{\frac{1}{\eta}}-t_1^{\frac{1}{\eta}}\right]+\frac{\pi_p d T^{\frac{1}{\eta}-1}}{\eta}+\frac{\xi T^{\frac{1}{\eta}-1}}{\eta}\right\}=0 \quad (17)$$

$$\frac{\partial TC}{\partial \xi}=1-\frac{m y_0 e^{-m\xi}}{T^{\frac{1}{\eta}}}\left\{\frac{h d t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}+\frac{b d t_1^{\frac{1+3\eta}{\eta}}}{6(1+3\eta)}+\frac{\pi_d d t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{\pi_p d}{1+\eta} t_1^{\frac{1+\eta}{\eta}}\right\}=0 \quad (18)$$

Sufficiency conditions:

A sufficient condition for t_1^*, T^*, ξ^* to be minimum point of TC is that the Hessian matrix $H(t_1,T,\xi)=\begin{pmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\partial T} & \frac{\partial^2 TC}{\partial t_1\partial \xi}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T\partial \xi}\\ \frac{\partial^2 TC}{\partial \xi\partial t_1} & \frac{\partial^2 TC}{\partial \xi\partial T} & \frac{\partial^2 TC}{\partial \xi^2}\end{pmatrix}$ evaluated at t_1^*, T^* and ξ^*, is positive definite.

The Hessian matrix H (t_1, T, ξ) is said to be positive definite if the signs of the principal minor determinants of H (t_1, T, ξ) are positive. (i.e., D_1, D_2, $D_3>0$).
where

$$D_1=\frac{\partial^2 TC}{\partial t_1^2},\ D_2=\begin{vmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\partial T}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial T^2}\end{vmatrix} \text{ and } D_3=\begin{vmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\partial T} & \frac{\partial^2 TC}{\partial t_1\partial \xi}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T\partial \xi}\\ \frac{\partial^2 TC}{\partial \xi\partial t_1} & \frac{\partial^2 TC}{\partial \xi\partial T} & \frac{\partial^2 TC}{\partial \xi^2}\end{vmatrix}$$

$$\frac{\partial^2 TC}{\partial t_1^2}=\frac{1}{\eta^2 T^{\frac{1}{\eta}}}\left\{hd\left[t_1^{\frac{1-\eta}{\eta}}+(1+\eta)\frac{y_0}{2}e^{-m\xi}t_1^{\frac{1}{\eta}}\right]+\frac{bd}{2}\left[(1+\eta)t_1^{\frac{1}{\eta}}+\frac{(1+2\eta)}{3}y_0 e^{-m\xi}t_1^{\frac{1+\eta}{\eta}}\right]\right.$$
$$\left.+\pi_b d\left[-(1-\eta)T t_1^{\frac{1-2\eta}{\eta}}+t_1^{\frac{1-\eta}{\eta}}\right]+\pi_d d y_0 e^{-m\xi}t_1^{\frac{1-\eta}{\eta}}+\pi_p d\left[y_0 e^{-m\xi}t_1^{\frac{1-\eta}{\eta}}\right]\right\},$$

$$\frac{\partial^2 TC}{\partial T^2}=\frac{(1+\eta)}{\eta^2 T^{\frac{1}{\eta}+2}}\left\{\pi_o T^{\frac{1}{\eta}}+hd\left[\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{y(\xi)t_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}\right]+\frac{bd}{2}\left[\frac{t_1^{\frac{1+2\eta}{\eta}}}{1+2\eta}+\frac{y(\xi)t_1^{\frac{1+3\eta}{\eta}}}{3(1+3\eta)}\right]+\frac{\pi_d d y(\xi)t_1^{\frac{1+\eta}{\eta}}}{(1+\eta)}\right.$$
$$\left.+\pi_b d\left[-T t_1^{\frac{1}{\eta}}+\frac{\eta T^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{t_1^{\frac{1+\eta}{\eta}}}{1+\eta}\right]+\pi_p d\left[T^{\frac{1}{\eta}}+\frac{1}{1+\eta}\left(y(\xi)t_1^{\frac{1+\eta}{\eta}}\right)\right]+\xi T^{\frac{1}{\eta}}\right\}$$
$$-\frac{2}{\eta T^{\frac{1}{\eta}+1}}\left\{\frac{(1-\eta)\pi_o T^{\frac{1}{\eta}-2}}{\eta}+\pi_b d\left[T^{\frac{1}{\eta}}-t_1^{\frac{1}{\eta}}\right]+\frac{\pi_p d T^{\frac{1}{\eta}-1}}{\eta}+\frac{\xi T^{\frac{1}{\eta}-1}}{\eta}\right\}$$
$$+\frac{1}{\eta^2 T^{\frac{1}{\eta}}}\left\{(1-\eta)(1-2\eta)\pi_o T^{\frac{1}{\eta}-3}+\pi_b d\eta T^{\frac{1}{\eta}-1}+(1-\eta)\pi_p d T^{\frac{1}{\eta}-2}+(1-\eta)\xi T^{\frac{1}{\eta}-2}\right\}$$

$$\frac{\partial^2 TC}{\partial \xi^2}=\frac{m^2 y_0 e^{-m\xi}}{T^{\frac{1}{\eta}}}\left\{\frac{hdt_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}+\frac{bdt_1^{\frac{1+3\eta}{\eta}}}{6(1+3\eta)}+\frac{\pi_d dt_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{\pi_p d}{1+\eta}t_1^{\frac{1+\eta}{\eta}}\right\}$$

$$\frac{\partial TC}{\partial t_1 \partial T}=\frac{\partial TC}{\partial T \partial t_1}=-\frac{1}{\eta^2 T^{\frac{1}{\eta}+1}}\left\{hd\left[t_1^{\frac{1}{\eta}}+\frac{1}{2}y_0 e^{-m\xi}t_1^{\frac{1+\eta}{\eta}}\right]+\frac{bd}{2}\left[t_1^{\frac{1+\eta}{\eta}}+\frac{1}{3}y_0 e^{-m\xi}t_1^{\frac{1+2\eta}{\eta}}\right]+\pi_d d y_0 e^{-m\xi}t_1^{\frac{1}{\eta}}\right.$$
$$\left.+\pi_b d\left[-Tt_1^{\frac{1-\eta}{\eta}}+t_1^{\frac{1}{\eta}}\right]+\pi_p d\left[y_0 e^{-m\xi}t_1^{\frac{1}{\eta}}\right]\right\}-\frac{\pi_b d t_1^{\frac{1-\eta}{\eta}}}{\eta T^{\frac{1}{\eta}}}$$

$$\frac{\partial TC}{\partial T \partial \xi}=\frac{\partial TC}{\partial \xi \partial T}=\frac{m y_0 e^{-m\xi}}{\eta T^{\frac{1}{\eta}+1}}\left\{\frac{hdt_1^{\frac{1+2\eta}{\eta}}}{2(1+2\eta)}+\frac{bdt_1^{\frac{1+3\eta}{\eta}}}{6(1+3\eta)}+\frac{\pi_d dt_1^{\frac{1+\eta}{\eta}}}{1+\eta}+\frac{\pi_p d}{1+\eta}t_1^{\frac{1+\eta}{\eta}}\right\}$$

$$\frac{\partial TC}{\partial t_1 \partial \xi}=\frac{\partial TC}{\partial \xi \partial t_1}=-\frac{m y_0 e^{-m\xi}}{\eta T^{\frac{1}{\eta}}}\left\{\frac{hdt_1^{\frac{1+\eta}{\eta}}}{2}+\frac{bdt_1^{\frac{1+2\eta}{\eta}}}{6}+\pi_d dt_1^{\frac{1}{\eta}}+\pi_p dt_1^{\frac{1}{\eta}}\right\}$$

Algorithm:

Step 1: Initialize the values for d, η, π_o, h, b, π_p, π_d, π_b, δ, y_0 and m.

Step 2: Evaluate TC (t_1, T, ξ)

Step 3: Evaluate $\frac{\partial TC}{\partial t_1}$, $\frac{\partial TC}{\partial T}$ and $\frac{\partial TC}{\partial \xi}$.

Step 4: Solve simultaneous equations $\frac{\partial TC}{\partial t_1}=0$, $\frac{\partial TC}{\partial T}=0$ and $\frac{\partial TC}{\partial \xi}=0$.

Step 5: Using the results from step 4, check the sufficiency conditions.

Step 6: If the computed value in step 5 is greater than zero, then move on to step 7 else, move on to step 4.

Step 7: Evaluate TC^* and Q^* using equations (14) and (8) respectively.

Step 8: Stop.

7. Particular cases

Next, we demonstrate how the suggested model may be used to get specific situations for several inventory models developed by other authors.

1. When t_1 = T, $\xi \to 0$, η=1, and b → 0, then the model is reduced to an EOQ model with constant demand, constant holding cost and no shortages.
2. When η = 1, indicating a constant demand function, the proposed model simplifies to the one presented by Dye and Hsieh [9], particularly when their model incorporates complete backlogging.
3. If $\xi \to 0$, b → 0, $\pi_d \to 0$, then the reduced system coincides with the model analyzed by Rajeswari and Vanjikkodi [23] when, in their model, complete backlogging is considered.

8. Numerical illustration and sensitivity analysis

8.1 Crisp Environment

A numerical example is presented here to validate the aforementioned theoretical model. Analyzing the outcomes can offer essential information to a decision-maker. Consider the following inventory system parameters for a certain type of cake, which may deteriorate over time.

d = 100 kilograms (kg), $\eta = 2$, π_o = \$ 250/order, h = \$ 1 /kg/week,
b= \$ 6 /kg/week, π_p = \$12/kg, π_d = \$15/kg/week, π_b = \$18/kg/week,
$y_0 = 0.1$, m=0.05.

The derivatives (16), (17) and (18) are computed. Solving the resulting highly nonlinear equation in MATLAB, yields t_1^* = 1.0292 weeks, T^*= 1.3015 weeks and investment in preservation technology ξ^* = \$ 28.867. Then, the order quantity Q^* = 131.09 kg and the overall expense per unit of time is obtained as TC^*= \$ 1555.6473 by using (8) and (14) respectively. Total cost function's convexity shown in figure 3 – 5.

Sufficiency condition:

$$D_1 = 1119.44 > 0, \quad D_2 = \begin{vmatrix} 1119.44 & -777.62 \\ -777.62 & 824.79 \end{vmatrix} = 318597.68 \; > \; 0,$$

$$D_3 = \begin{vmatrix} 1119.44 & -777.63 & -1.50 \\ -777.63 & 824.79 & 0.3842 \\ -1.50 & 0.3842 & 0.050 \end{vmatrix} = 14805.06 \; > \; 0$$

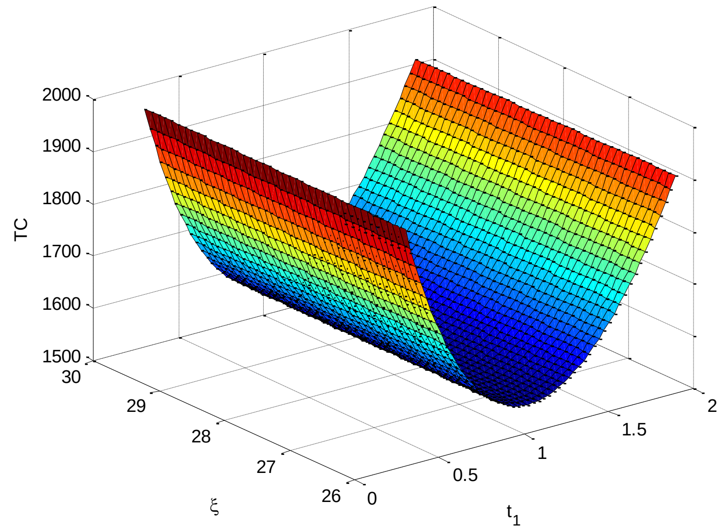

Figure 3. Total cost Vs. ξ and t_1 for fixed T

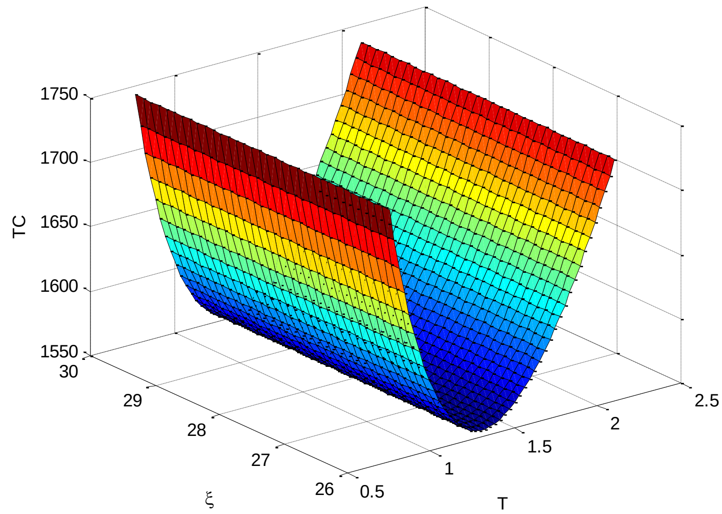

Figure 4. Total cost Vs. ξ and T for fixed t_1

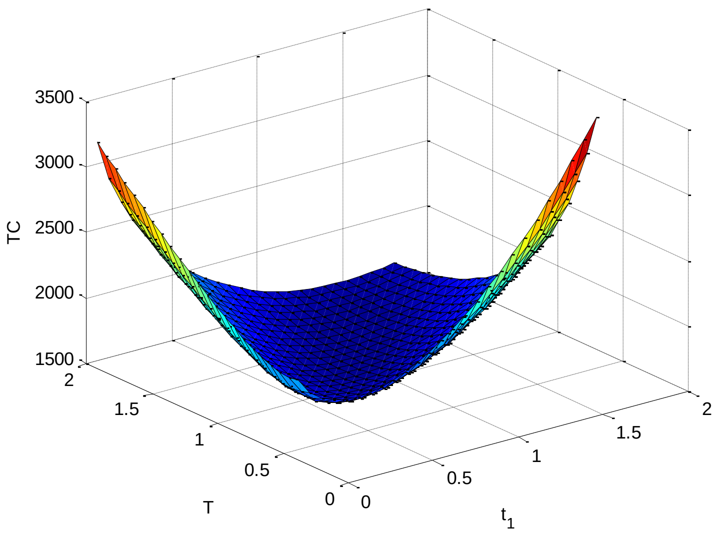

Figure 5. Total cost Vs. t_1 and T for fixed ξ

8.2 Comparitive Study

Cases	**With using preservation technology**					**Without using preservation technology**			
	t_1^*(weeks)	**T^*(weeks)**	**ξ ($)**	**Q^* (kg)**	**TC ($)**	**t_1^*(weeks)**	**T^*(weeks)**	**Q^* (kg)**	**TC ($)**
(i) Present model	1.0292	1.3015	28.867	131.09	1555.6	0.86583	1.1744	120.35	1584.9
(ii) Constant demand (η=1)	0.8996	1.1125	32.342	112.05	1615.5	0.75084	1.0031	103.13	1653.9

By comparing the total cost of the preservation technology model with the non-preservation technology model, it becomes evident that preservation technology lowers costs while extending positive inventory time.

8.3 Triangular Neutrosophic Environment

A numerical illustration has been given utilising TNNs to manifest the impact of imprecise cost parameters on the presented inventory system.

$$\tilde{\pi}_{dN} = \langle (180,250,310),(200,260,320),(150,220,290) \rangle$$

$$\tilde{\pi}_{pN} = \langle (8,12,15),(10,13,16),(6,9,11) \rangle$$

$$\tilde{h}_{N} = \langle (0.8,1,1.5),(0.9,1.3,2),(0.6,0.8,1.4) \rangle$$

$$\tilde{\pi}_{dN} = \langle (10,15,20),(12,17,21),(8,13,16) \rangle$$

$$\tilde{\pi}_{bN} = \langle (15,18,21),(16,20,23),(12,16,18) \rangle$$

Using (1) obtain the de-neutrosophic costs and substitute the obtained values in (16), (17) and (18). And solving the resulting simultaneous equations we get, t_1^* = 1.0166 weeks, T^*= 1.2947 weeks and ξ^* = $ 28.084.

From (8) and (15) the order quantity Q^* = **130.42** kg and the overall expense per unit of time is $\tilde{TC}_N^*$ = $ **1525.43**.

The neutrosophic environment has lower optimal inventory costs than a crisp environment.

8.4 Sensitivity Analysis

Sensitivity analysis of the formulated model is done out for various input parameters in Table 2. Sensitivity analysis is performed to measure the impact of each model limiting factors on the model's outcome. It is done by adjusting the parameters' value from -30% to +30%.

Table 2. Variations in parameter 'y_0', 'm', 'h', 'b', 'π_o', 'π_p', 'π_b', 'π_d'.

	% change	t_1^*(weeks)	T^*(weeks)	ξ^* ($)	Q^* (kg)	TC^* ($)	% change in TC^*
y_0	-30	1.0292	1.3015	21.733	131.09	1548.5138	-0.4586
	-20	1.0292	1.3015	24.404	131.09	1551.1844	-0.2869
	-10	1.0292	1.3015	26.76	131.09	1553.5400	-0.1355
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0292	1.3015	30.773	131.09	1557.5535	+0.1225
	+20	1.0292	1.3015	32.513	131.09	1559.2937	+0.2344
	+30	1.0292	1.3015	34.114	131.09	1560.8945	+0.3373
m	-30	1.0018	1.28	30.095	129.32	1566.2577	+0.6821
	-20	1.0133	1.289	30.024	130.09	1562.2375	+0.4236
	-10	1.0221	1.296	29.544	130.64	1558.7540	+0.1997
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0349	1.306	28.101	131.45	1552.9498	-0.1734
	+20	1.0397	1.3098	27.304	131.75	1550.5823	-0.3256
	+30	1.0438	1.3129	26.508	132.00	1548.4834	-0.4605
h	-30	1.0579	1.3239	29.477	133.30	1546.7272	-0.5734
	-20	1.0482	1.3163	29.273	132.55	1549.6328	-0.3866
	-10	1.0386	1.3089	29.07	131.82	1552.6073	-0.1954
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0199	1.2942	28.665	130.36	1558.7528	+0.1996
	+20	1.0107	1.2871	28.463	129.66	1561.9210	+0.4033
	+30	1.0017	1.280	28.261	128.96	1565.1528	+0.6110
b	-30	1.1486	1.4042	31.385	141.28	1543.4880	-0.7816
	-20	1.1033	1.365	30.465	137.39	1546.2535	-0.6039
	-10	1.0639	1.3311	29.631	134.02	1550.4057	-0.3369
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	0.9983	1.2753	28.161	128.49	1561.6983	+0.3890
	+20	0.9705	1.2518	27.504	126.16	1568.3684	+0.8177
	+30	0.9453	1.2306	26.89	124.07	1575.5177	+1.2773
π_o	-30	0.8996	1.1226	26.203	113.25	1503.9951	-3.3203
	-20	0.9466	1.1868	27.211	119.65	1519.4851	-2.3246
	-10	0.9896	1.2462	28.09	125.57	1536.9220	-1.2037
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0661	1.3535	29.564	136.27	1575.2259	+1.2586

	+20	1.1006	1.4025	30.195	141.16	1595.3640	+2.5531
	+30	1.1332	1.4491	30.771	145.81	1615.8696	+3.8712
π_p	-30	1.0288	1.3012	26.081	131.20	1192.8696	-23.3200
	-20	1.029	1.3013	27.054	131.16	1313.8390	-15.5439
	-10	1.0291	1.3014	27.981	131.12	1434.7635	-7.7706
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0293	1.3016	29.715	131.06	1676.4936	+7.7682
	+20	1.0294	1.3017	30.529	131.03	1797.3055	+15.5343
	+30	1.0295	1.3017	31.312	130.99	1918.0858	+23.2982
π_b	-30	0.9892	1.3579	27.218	136.83	1545.2022	-0.6714
	-20	1.0052	1.3349	27.886	134.49	1549.2555	-0.4109
	-10	1.0183	1.3165	28.424	132.61	1552.6855	-0.1904
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0384	1.2889	29.238	129.81	1558.2484	+0.1672
	+20	1.0462	1.2783	29.553	128.73	1560.5661	+0.3162
	+30	1.053	1.2692	29.825	127.80	1562.6564	+0.4506
π_d	-30	1.0287	1.3011	25.32	131.23	1552.1105	-0.2274
	-20	1.0289	1.3012	26.573	131.17	1553.3602	-0.1470
	-10	1.0291	1.3014	27.753	131.13	1554.5364	-0.0714
	0	1.0292	1.3015	28.867	131.09	1555.6473	0
	+10	1.0293	1.3016	29.922	131.05	1556.6997	+0.0677
	+20	1.0294	1.3017	30.924	131.02	1557.6995	+0.1319
	+30	1.0295	1.3018	31.879	130.99	1558.6517	+0.1931

Graphical representations of the sensitivity of positive inventory time, total cycle time, order quantity, preservation technology investment and total cost to various factors are provided in Figures 6 – 13.

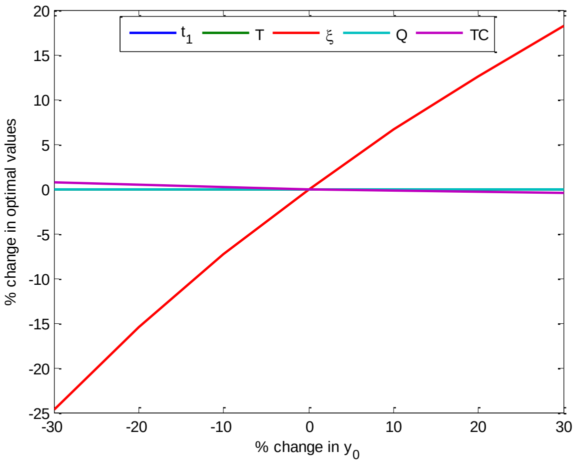

Figure 6. Effect of ‘y_0’ on optimal values

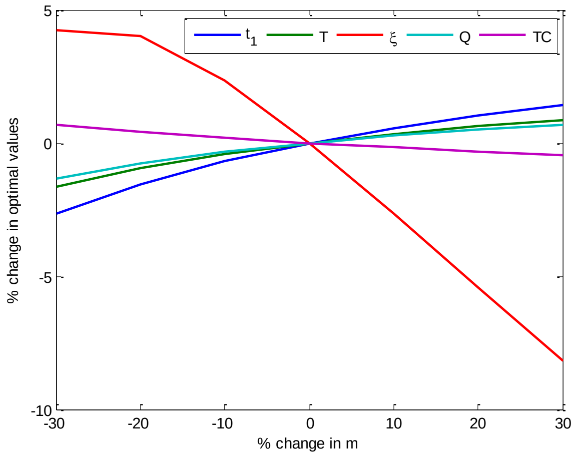

Figure 7. Effect of ‘m’ on optimal values

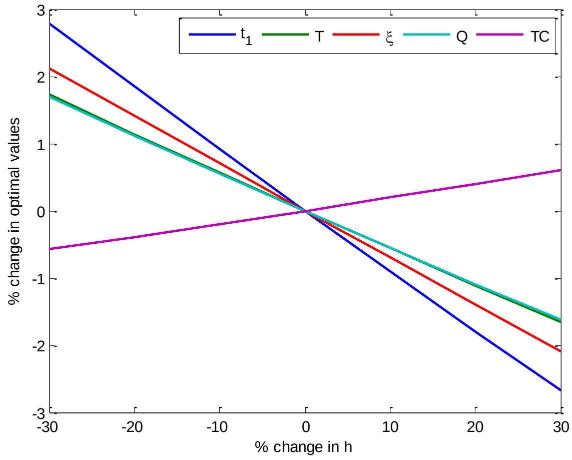

Figure 8. Effect of 'h' on optimal values

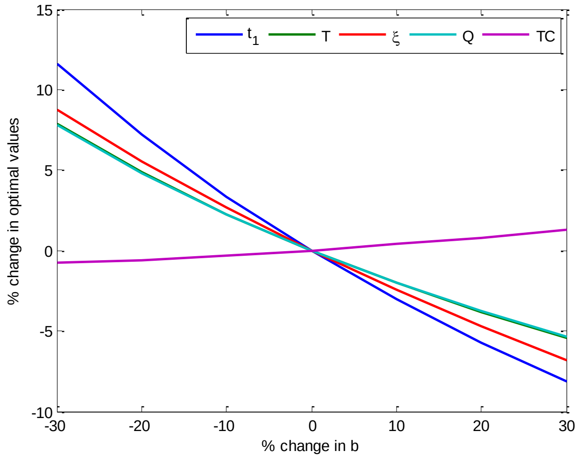

Figure 9. Effect of 'b' on optimal values

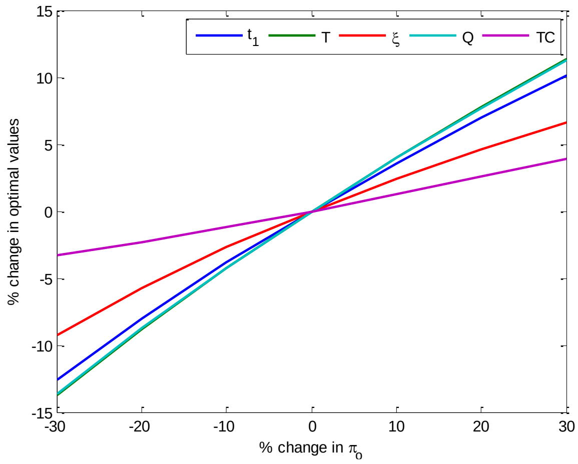

Figure 10. Effect of 'π_o' on optimal values

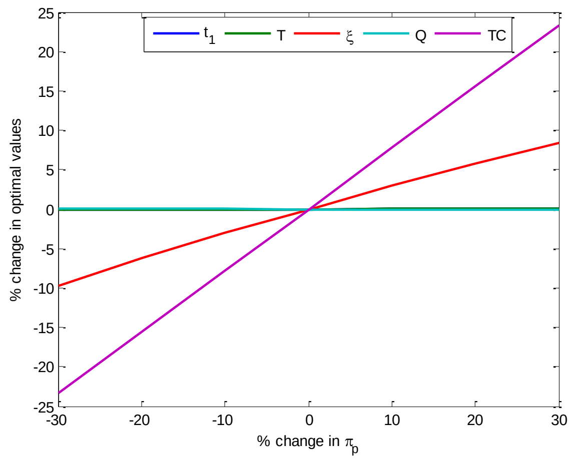

Figure 11. Effect of 'π_p' on optimal values

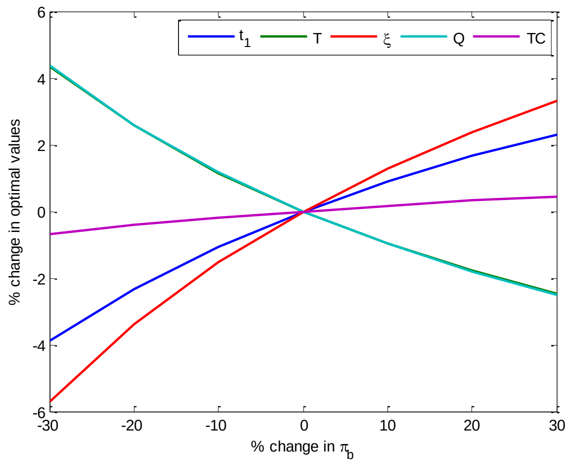

Figure 12. Effect of 'π_b' on optimal values

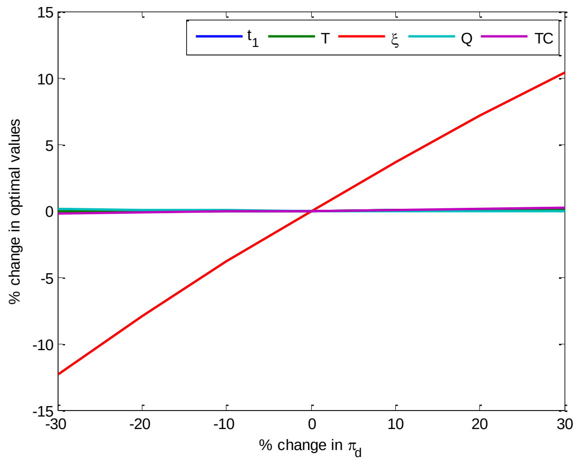

Figure 13. Effect of 'π_d' on optimal values

8.5 Results and discussion:

The following can be seen by carefully examining the table above:

1. As the parameters' values y_0, h, b, π_o, π_p, π_b, π_d increase, the optimal $TC^*(t_1, T, \xi)$ also increases. Conversely, when the corresponding parameter values y_0, h, b, π_o, π_p, π_b, π_d decrease, $TC^*(t_1, T, \xi)$ also decreases. A reduction in the parameter m leads to an increase in $TC^*(t_1, T, \xi)$, whereas the optimal $TC^*(t_1, T, \xi)$ decreases when the parameter m increases.

2. The parameters π_o and π_p have a high degree of sensitivity with regard to changes in $TC^*(t_1, T, \xi)$. While the parameters y_0, m, h, b, π_b and π_d are not as sensitive to changes in $TC^*(t_1, T, \xi)$.
3. As the parameters' values y_0, π_o, π_p, π_b, and π_d increase, the optimal ξ^* also increases. Conversely, when the corresponding parameter values y_0, π_o, π_p, π_b, and π_d decreases, ξ^* also decreases. As the parameters' values m, h, and b increase, the optimal ξ^* decreases, whereas the optimal ξ^* increases when the parameters m, h, and b increase.
4. As the parameters' values m, π_o, π_p, π_d increase, positive inventory time interval (t_1^*) and cycle time (T^*) also increase. Conversely, when the corresponding parameter values m, π_o, π_p, π_d decrease, t_1^*, T^* also decreases. As the parameters h and b increase, the optimal t_1^*, T^* decreases. Conversely, the optimal t_1^*, T^* increases when the parameters h and b decrease. As the parameter π_b increase, the optimal t_1^* increases and T^* decreases. t_1^*, T^* remain constant regardless of any variation in the parameter y_0.
5. As the parameters' values m and π_o increase, optimal Q^* also increases. Conversely, when the corresponding parameter values m, and π_o decrease, so does Q^*. As the parameters' values h, b, π_p, π_b and π_d increase, the optimal Q^* decreases. Conversely, the optimal Q^* increases when h, b, π_p, π_b and π_d values decrease. Q^* remain constant regardless of any variation in the parameter y_0.

The sensitivity analysis provides the following managerial insights:

1. When the deterioration rate (y_0) increases, the optimal ξ^*, $TC^*(t_1, T, \xi)$ increases. Therefore, the retailer aims to minimize deterioration-related losses by enhancing their investment in preservation technology. Unnecessary high investments in preservation technology should be avoided by the retailer when facing a lower deterioration rate, as observed in the study by Khanna et al. [14].
2. When the holding cost (h, b) increases, t_1^*, T^*, ξ^* and Q^* decrease, whereas $TC^*(t_1, T, \xi)$ increases. Therefore, when the holding cost is high, the retailer should maintain only a limited and essential amount of inventory. Additionally, it is advisable to decrease the expenditure allocated to item preservation, as highlighted in the study by Khanna et al. [14].
3. When the ordering cost (π_o) increases, optimal ξ^*, Q^*, $TC^*(t_1, T, \xi)$ increase. Hence retailer should increase the quantity to be ordered when π_o is high, as indicated in the study by Mahapatra et al. [27].
4. When the backordered cost (π_b) increases, ξ^* and $TC^*(t_1, T, \xi)$ increases, whereas Q^* decreases. To minimize total costs, the retailer should increase the order quantity when π_b is lower, as evidenced in the study by Singh and Sharma [9].
5. When the purchase cost (π_p), deterioration cost (π_d) increases, Q^* decreases, whereas the optimal t_1^*, T^*, ξ^* and $TC^*(t_1, T, \xi)$ increases. In practice, as the retailer trims these expenses, the overall cost decreases, as demonstrated in the study by Das et al. [15] along with a reduction in total profit.
6. When the effectiveness parameter (m), increases, t_1^*, T^*, and Q^* increases, whereas ξ^* and $TC^*(t_1, T, \xi)$ decreases. Retailers should implement enhanced and high-quality preservation techniques, thereby minimizing total costs through reduced preservation technology investments. It is also recommended to consider placing larger orders for extended durations, which coincides with the findings of the research by Khanna et al. [14].

9. Conclusion

A model has been developed to manage inventory with power demand patterns and deteriorating products, underscoring the importance of the preservation technology investment function in controlling deterioration rates. This paper makes a significant contribution to business knowledge and practice by helping to reduce losses related to deterioration. The representation of cost parameters as TNNs enables retailers to obtain accurate results, allowing them to make more appropriate and efficient decisions in inventory management. The retailer's total neutrosophic cost has been de-neutrosophied using the removal area method. The convexity of the total cost function guarantees the minimization of overall costs. The total cost with preservation technology investment is TC* = $1555.6. In contrast, the total cost without considering preservation technology is TC* = $1584.9, exceeding the overhead cost. This observation underscores the justification for investing in preservation strategies to mitigate deterioration and reduce overall inventory costs. Sensitivity analysis results demonstrate the novel contribution of this study to inventory management, offering valuable insights for decision-makers and practitioners seeking efficient and cost-effective inventory control strategies.

The following are some suggestions for further research: by considering non-instantaneous and time-dependent deteriorating items, partially backlogged shortages, including the demand that depends on selling price and advertisement and develop by considering different types of uncertainties such as intuitionistic fuzzy, Pythagorean fuzzy, etc.

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References

1. Whitin, T. M. The Theory of inventory management, (2nd edition), Princeton University Press, Princeton, 1957.
2. Ghare, P.M.; Schrader, G.F. An inventory model for exponentially deteriorating items. Journal of Industrial Engineering, 1963, 14(2), pp. 238-243.
3. Sachan, R. S. On (T, S_i) policy inventory model for deteriorating items with time proportional demand. Journal of the Operational Research Society, 1984, 35(11), pp. 1013-1019. https://doi.org/10.1057/jors.1984.197
4. Shah, N. H.; Soni, H. N.; Patel, K. A. Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega, 2013, 41(2), pp. 421-430. https://doi.org/10.1016/j.omega.2012.03.002
5. Hatibaruah, A.; Saha, S. An inventory model for two-parameter Weibull distributed ameliorating and deteriorating items with stock and advertisement frequency dependent demand under trade credit and preservation technology. OPSEARCH, 2023, 1-52. https://doi.org/10.1007/s12597-023-00629-0
6. Hsu, P. H.; Wee, H. M.; Teng, H. M. Preservation technology investment for deteriorating inventory. International Journal of Production Economics, 2010, 124(2), pp. 388-394. https://doi.org/10.1016/j.ijpe.2009.11.034
7. Dye, C. Y.; Hsieh, T. P. An optimal replenishment policy for deteriorating items with effective investment in preservation technology. European Journal of Operational Research, 2012, 218(1), pp. 106-112. https://doi.org/10.1016/j.ejor.2011.10.016
8. He, Y.; Huang, H. Optimizing inventory and pricing policy for seasonal deteriorating products with preservation technology investment. Journal of Industrial engineering, 2013, Article ID 793568, pp. 1-7. https://doi.org/10.1155/2013/793568
9. Singh, S. R.; Sharma, S. A global optimizing policy for decaying items with ramp-type demand rate under two-level trade credit financing taking account of preservation technology. Advances in Decision Sciences, 2013, pp. 1-12. https://doi.org/10.1155/2013/126385
10. Zhang, J.; Liu, G.; Zhang, Q.; Bai, Z. Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. Omega, 2015, 56, pp. 37-49.
11. Mishra, U.; Cárdenas-Barrón, L. E.; Tiwari, S.; Shaikh, A. A.; Treviño-Garza, G. An inventory model under price and stock dependent demand for controllable deterioration rate with szhortages and preservation technology investment. Annals of Operations Research, 2017, 254, pp. 165-190. https://doi.org/10.1007/s10479-017-2419-1

12. Li, G.; He, X.; Zhou, J.; Wu, H. Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. Omega, 2018, 84, pp. 114-126. https://doi.org/10.1016/j.omega.2018.05.001
13. Das, S. C.; Zidan, A. M.; Manna, A. K.; Shaikh, A. A.; Bhunia, A. K. An application of preservation technology in inventory control system with price dependent demand and partial backlogging. Alexandria Engineering Journal, 2020, 59(3), pp. 1359-1369. https://doi.org/10.1016/j.aej.2020.03.006
14. Khanna, A.; Pritam, P.; Jaggi, C. K. Optimizing preservation strategies for deteriorating items with time-varying holding cost and stock-dependent demand. Yugoslav Journal of Operations Research, 2020, 30(2), pp. 237-250. https://doi.org/10.2298/YJOR1902
15. Bhawaria, S.; Rathore, H. Production inventory model for deteriorating items with hybrid-type demand and partially backlogged shortages. In Mathematical Modeling, Computational Intelligence Techniques and Renewable Energy: Proceedings of the Second International Conference, MMCITRE 2021, Springer Singapore, pp. 229-240. https://doi.org/10.1007/978-981-16-5952-2_20
16. Naddor, E. Inventory Systems, John Wiley and Sons, New York, 1996.
17. Datta, T. A.; Pal, A. K. Order level inventory system with power demand pattern for items with variable rate of deterioration. Indian Journal of Pure and Applied Mathematics, 1988, 19(11), pp. 1043-1053.
18. Lee, W. C.; Wu, J. W. An EOQ model for items with Weibull distributed deterioration, shortages and power demand pattern. International Journal of Information and Management Sciences, 2002, 13(2), pp. 19-34.
19. Dye, C. Y. A Note on" An EOQ Model for Items with Weibull Distributed Deterioration, Shortages and Power Demand Pattern". International Journal of Information and Management Sciences, 2004, pp. 15, 81-84.
20. Rajeswari, N.; Vanjikkodi, T. Deteriorating inventory model with power demand and partial backlogging. International Journal of Mathematical Archive, 2011, 2(9), pp. 1495-1501. http://www.ijma.info/index.php/ijma/article/view/510/258
21. San-José, L. A.; Sicilia, J.; Pando, V.; Alcaide-López-de-Pablo, D. An inventory system with time-dependent demand and partial backordering under return on inventory investment maximization. Computers and Operations Research, 2022, 145, 105861. https://doi.org/10.1016/j.cor.2022.105861
22. San-José, L. A.; Sicilia, J.; Cárdenas-Barrón, L. E.; González-de-la-Rosa, M. A sustainable inventory model for deteriorating items with power demand and full backlogging under a carbon emission tax. International Journal of Production Economics, 2023, 109098. https://doi.org/10.1016/j.ijpe.2023.109098
23. Posner, M. J. M.; Yansouni, B. A class of inventory models with customer impatience. Naval Research Logistics Quarterly, 1972, 19(3), pp. 483-492. https://doi.org/10.1002/nav.3800190308
24. Abad, P. L. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management science, 1996, 42(8), pp. 1093-1104. https://doi.org/10.1287/mnsc.42.8.1093
25. Valliathal, M.; Uthayakumar, R. A comparative study on two-warehouse inventory model for deteriorating items with shortages. International Journal of Procurement Management, 2013, 6(1), pp. 39-57. https://doi.org/10.1504/IJPM.2013.050609
26. Mashud, A. H. M. An EOQ deteriorating inventory model with different types of demand and fully backlogged shortages. International Journal of Logistics Systems and Management, 2020, 36(1), pp. 16-45. https://doi.org/10.1504/IJLSM.2020.107220
27. Mahapatra, A. S.; Dasgupta, A.; Shaw, A. K.; Sarkar, B. An inventory model with uncertain demand under preservation strategy for deteriorating items. RAIRO-Operations Research, 2022, 56(6), 4251-4280. https://doi.org/10.1051/ro/2022145
28. Smarandache, F. Neutrosophy: Neutrosophic probability, set and logic. Rehoboth, USA: American Research Press, 1998.
29. Mullai, M.; Broumi, S. Neutrosophic inventory model without shortages. Asian Journal of Mathematics and Computer Research, 2018, pp. 214-219. https://www.ikprress.org/index.php/AJOMCOR/article/view/948
30. Mullai, M.; Surya, R. Neutrosophic EOQ model with price break. Neutrosophic sets and systems, 2018, 19, pp. 24-29. https://fs.unm.edu/NSS2/index.php/111/article/view/350

31. Pal, S.; Chakraborty, A. Triangular neutrosophic based production reliability model of deteriorating item with ramp type demand under shortages and time discounting. Neutrosophic Sets and Systems, 2020, 35(1), pp. 347-367. https://digitalrepository.unm.edu/nss_journal/vol35/iss1/20
32. Mondal, B.; Garai; A.; Mukhopadhyay, A.; Majumder, S. K. Inventory policies for seasonal items with logistic-growth demand rate under fully permissible delay in payment: a neutrosophic optimization approach. Soft Computing, 2021, 25, pp. 3725-3750. https://doi.org/10.1007/s00500-020-05402-9
33. Sugapriya, C.; Lakshmi, V.; Nagarajan, D.; Broumi, S. Two-warehouse system for trapezoidal bipolar neutrosophic disparate expeditious worsen items with power demand pattern. Neutrosophic Sets and Systems, 2022, 48, pp. 86-99. https://digitalrepository.unm.edu/nss_journal/vol48/iss1/7
34. Bhavani, G. D.; Georgise, F. B.; Mahapatra, G. S.; Maneckshaw, B. Neutrosophic cost pattern of inventory system with novel demand incorporating deterioration and discount on defective items using particle swarm algorithm. Computational Intelligence and Neuroscience, 2022, Article ID 7683417. https://doi.org/10.1155/2022/7683417
35. Mohanta, K.; Jha, A. K.; Dey, A.; Pal, A. An application of neutrosophic logic on an inventory model with two-level partial trade credit policy for time-dependent perishable products. Soft Computing, 2023, 27(8), pp. 4795-4822. https://doi.org/10.1007/s00500-022-07619-2
36. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets. In: Multispace & Multistructure. Neutrosophic Transdisciplinarity, North-European Scientific Publishers, 2010, vol 4, pp. 410–413.
37. Chakraborty, A.; Mondal, S. P.; Ahmadian, A.; Senu, N.; Alam, S.; Salahshour, S. Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications. Symmetry, 2018, 10(8), 327. https://doi.org/10.3390/sym10080327

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