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Neutrosophic Doubt Fuzzy Bi-ideal of BS-Algebras

P. Ayesha Parveen[1*] and M. Himaya Jaleela Begum[2]

[1] Assistant Professor(T), Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-12, Tamil Nadu, India;

[2] Assistant Professor, Department of Mathematics, Sadakathullah Appa College, Tirunelveli-11, Tamil Nadu, India;

[1] aabidaaasima@gmail.com and [2] himaya2013@ gmail.com

*Correspondence: aabidaaasima@gmail.com

Abstract. In this research paper, our aim is to introduce the new concept of neutrosophic doubt fuzzy bi-ideal of BS-algebras as an extension of doubt fuzzy bi-ideal of BS-algebras and investigated its algebraic nature. Neutrosophic doubt fuzzy bi-ideal of BS-algebras is also applied in Cartesian product. Finally, we also provide the homomorphic behaviour of Neutrosophic doubt fuzzy bi-ideal of BS-algebras.

Keywords: BS-algebras, Neutrosophic doubt fuzzy bi-ideal, Homomorphism.

1. Introduction

The fuzzy subsets was first introduced by L.A.Zadeh[8]. In 1966, Imai and Iseki gave the idea of BCK-algebras and BCI-algebras[3]. J.Neggers and H.S. Kim initiated the notion of B-algebras[4] which is a generalisation of BCK-algebras. We launched the notion of BS-algebras which is a generalisation of B-algebras and established the notion of Doubt fuzzy bi-ideal of BS-algebras[1]. We also innovated the notion of Neutrosophic fuzzy bi-ideal of BS-algebras[2]. F. Smarandache[5] extented the concept of fuzzy logic to neutrosophic logic which includes indeterminancy. Neutrosophic set theory played a major role in decision making problem, medical diagnosis, robotics, image processing, etc.

The main objective of this paper is to putforth the notion of Neutrosophic Doubt Fuzzy Bi-ideal(NDFB) of BS-algebras and studied their algebraic properties. We obtained the product of neutrosophic doubt fuzzy bi-ideal for BS-algebras. Finally, we studied how to deal with homomorphism of neutrosophic doubt fuzzy bi-ideal for BS-algebras.

2. Preliminaries:

In this Section, some basic definitions are given that are necessary for this paper. Throughout this paper, let $\mathfrak{B}$ denotes BS-algebra.

Definition 2.1 [1] A set BS-algebra $\mathfrak{B}\neq\phi$ with 1 as constant and * as binary operation satisfying the following axioms

(i) $\alpha*\alpha=1$

(ii) $\alpha*1=\alpha$

(iii) $(\alpha*\beta)*\gamma = \alpha*(\gamma*(1*\beta))\ \forall\ \alpha,\beta,\gamma \in \mathfrak{B}$

Definition 2.2 A fuzzy subset Ƒ of $\mathfrak{B}$ is called the fuzzy ideal of $\mathfrak{B}$ if it satisfies

(i) $Ƒ(1) \geq Ƒ(\alpha)$

(ii) $Ƒ(\beta) \geq \{Ƒ(\alpha) \wedge Ƒ(\beta*\alpha)\}\ \forall\ \alpha,\beta \in \mathfrak{B}$

Definition 2.3 [1] A fuzzy subset Ƒ of $\mathfrak{B}$ is called the fuzzy bi-ideal of $\mathfrak{B}$ if it satisfies

(i) $Ƒ(1) \geq Ƒ(\alpha)$

(ii) $Ƒ(\beta*\gamma) \geq \{Ƒ(\alpha) \wedge Ƒ(\alpha*(\beta*\gamma))\}\ \forall\ \alpha,\beta,\gamma \in \mathfrak{B}$

Definition 2.4 A fuzzy set Ƒ of $\mathfrak{B}$ is called the doubt fuzzy ideal of $\mathfrak{B}$ if it satisfies

(i) $Ƒ(1) \leq Ƒ(\alpha)$

(ii) $Ƒ(\beta) \leq \{Ƒ(\alpha) \vee Ƒ(\beta*\alpha))\}\ \forall\ \alpha,\beta \in \mathfrak{B}$

Definition 2.5 [1] A fuzzy set Ƒ of $\mathfrak{B}$ is called the Doubt Fuzzy Bi- ideal(DF) of $\mathfrak{B}$ if it satisfies

(i) $Ƒ(1) \leq Ƒ(\alpha)$

(ii) $Ƒ(\beta*\gamma) \leq \{Ƒ(\alpha) \vee Ƒ(\alpha*(\beta*\gamma))\}\ \forall\ \alpha,\beta,\gamma \in \mathfrak{B}$

Example 2.6 [1] Let $\mathfrak{B}$ = {1, u, v, w} be the set with the following Cayley table

*	1	u	v	w
1	1	u	v	w
u	u	1	w	v
v	v	w	1	u
w	w	v	u	1

Then ($\mathfrak{B}$, *, 1) is a BS-algebra. Then the fuzzy set $Ƒ:\mathfrak{B}\rightarrow[0,1]$ is defined by $Ƒ(1)= Ƒ(\alpha)=0.7$ and $Ƒ(\beta)= Ƒ(\gamma)=0.9$, which is a Doubt Fuzzy(DF) bi-ideal of $\mathfrak{B}$.

Definition 2.7 [6] A Neutrosophic fuzzy set $\mathcal{N}$ on the Universe of discourse X characterised by a truth membership function $Ț_{\mathcal{N}}(\alpha)$, an indeterminacy function $\mathcal{J}_{\mathcal{N}}(\alpha)$ and a falsity membership function $Ƒ_{\mathcal{N}}(\alpha)$ is defined as $\mathcal{N}=\{<\alpha, Ț_{\mathcal{N}}(\alpha), \mathcal{J}_{\mathcal{N}}(\alpha), Ƒ_{\mathcal{N}}(\alpha)>: \alpha \in X\}$ where $Ț_{\mathcal{N}}, \mathcal{J}_{\mathcal{N}}, Ƒ_{\mathcal{N}} : X\longrightarrow[0,1]$ and $0 \leq Ț_{\mathcal{N}}+ \mathcal{J}_{\mathcal{N}}+ Ƒ_{\mathcal{N}} \leq 3$

Definition 2.8 [6] Let $\mathcal{M}$ and $\mathcal{N}$ be the two neutrosophic fuzzy set of X. Then $\alpha \in X$

i) $\mathcal{M} \cup \mathcal{N}$={<$\alpha$, $T_{\mathcal{M}\cup\mathcal{N}}(\alpha)$, $\mathcal{J}_{\mathcal{M}\cup\mathcal{N}}(\alpha)$, $F_{\mathcal{M}\cup\mathcal{N}}(\alpha)$>}, where

$T_{\mathcal{M}\cup\mathcal{N}}(\alpha) = (T_{\mathcal{M}}(\alpha) \vee T_{\mathcal{N}}(\alpha))$; $\mathcal{J}_{\mathcal{M}\cup\mathcal{N}}(\alpha) = (\mathcal{J}_{\mathcal{M}}(\alpha) \wedge \mathcal{J}_{\mathcal{N}}(\alpha))$; $F_{\mathcal{M}\cup\mathcal{N}}(\alpha) = (F_{\mathcal{M}}(\alpha) \wedge F_{\mathcal{N}}(\alpha))$

ii) $\mathcal{M} \cap \mathcal{N}$={<$\alpha$, $T_{\mathcal{M}\cap\mathcal{N}}(\alpha)$, $\mathcal{J}_{\mathcal{M}\cap\mathcal{N}}(\alpha)$, $F_{\mathcal{M}\cap\mathcal{N}}(\alpha)$>}, where

$T_{\mathcal{M}\cap\mathcal{N}}(\alpha) = (T_{\mathcal{M}}(\alpha) \wedge T_{\mathcal{N}}(\alpha))$; $\mathcal{J}_{\mathcal{M}\cap\mathcal{N}}(\alpha) = (\mathcal{J}_{\mathcal{M}}(\alpha) \vee \mathcal{J}_{\mathcal{N}}(\alpha))$; $F_{\mathcal{M}\cap\mathcal{N}}(\alpha) = (F_{\mathcal{M}}(\alpha) \vee F_{\mathcal{N}}(\alpha))$

Definition 2.9 A Neutrosophic Fuzzy Set $\mathcal{N}$ of BS-algebra $\mathfrak{B}$ is called the Neutrosophic Fuzzy Ideal of $\mathfrak{B}$ if $\forall\, \alpha, \beta, \gamma \in \mathfrak{B}$
(i) $T_{\mathcal{N}}(1) \geq T_{\mathcal{N}}(\alpha)$; $\mathcal{J}_{\mathcal{N}}(1) \leq \mathcal{J}_{\mathcal{N}}(\alpha)$; $F_{\mathcal{N}}(1) \leq F_{\mathcal{N}}(\alpha)$;
(ii) $T_{\mathcal{N}}(\beta) \geq \{T_{\mathcal{N}}(\alpha) \wedge T_{\mathcal{N}}(\beta*\alpha)\}$;
$\mathcal{J}_{\mathcal{N}}(\beta) \leq \{\mathcal{J}_{\mathcal{N}}(\alpha) \vee \mathcal{J}_{\mathcal{N}}(\beta*\alpha)\}$;
$F_{\mathcal{N}}(\beta) \leq \{F_{\mathcal{N}}(\alpha) \vee F_{\mathcal{N}}(\beta*\alpha)\}$

Definition 2.10 [2] A Neutrosophic fuzzy set $\mathcal{N}$ of BS-algebra $\mathfrak{B}$ is called the Neutrosophic Fuzzy Bi-ideal of $\mathfrak{B}$ if $\forall\, \alpha, \beta, \gamma \in \mathfrak{B}$
(i) $T_{\mathcal{N}}(1) \geq T_{\mathcal{N}}(\alpha)$; $\mathcal{J}_{\mathcal{N}}(1) \leq \mathcal{J}_{\mathcal{N}}(\alpha)$; $F_{\mathcal{N}}(1) \leq F_{\mathcal{N}}(\alpha)$;
(ii) $T_{\mathcal{N}}(\beta*\gamma) \geq \{T_{\mathcal{N}}(\alpha) \wedge T_{\mathcal{N}}(\alpha*(\beta*\gamma))\}$;
$\mathcal{J}_{\mathcal{N}}(\beta*\gamma) \leq \{\mathcal{J}_{\mathcal{N}}(\alpha) \vee \mathcal{J}_{\mathcal{N}}(\alpha*(\beta*\gamma))\}$;
$F_{\mathcal{N}}(\beta*\gamma) \leq \{F_{\mathcal{N}}(\alpha) \vee F_{\mathcal{N}}(\alpha*(\beta*\gamma))\}$

Definition 2.11 A Neutrosophic fuzzy set $\mathcal{D}$ of BS-algebra $\mathfrak{B}$ is called the Neutrosophic Doubt Fuzzy Ideal of $\mathfrak{B}$ if $\forall\, \alpha, \beta, \gamma \in \mathfrak{B}$
(i) $T_{\mathcal{D}}(1) \leq T_{\mathcal{D}}(\alpha)$; $\mathcal{J}_{\mathcal{D}}(1) \geq \mathcal{J}_{\mathcal{D}}(\alpha)$; $F_{\mathcal{D}}(1) \geq F_{\mathcal{D}}(\alpha)$;
(ii) $T_{\mathcal{D}}(\beta) \leq \{T_{\mathcal{D}}(\alpha) \vee T_{\mathcal{D}}(\beta*\alpha)\}$;
$\mathcal{J}_{\mathcal{D}}(\beta) \geq \{\mathcal{J}_{\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\beta*\alpha)\}$;
$F_{\mathcal{D}}(\beta) \geq \{F_{\mathcal{D}}(\alpha) \wedge F_{\mathcal{D}}(\beta*\alpha)\}$

3. NEUTROSOPHIC DOUBT FUZZY BI-IDEAL (NDFB) OF BS-ALGEBRAS

In this Section, the concept of doubt fuzzy bi-ideal of $\mathfrak{B}$ can be extented to Neutrosophic doubt fuzzy bi-ideal of $\mathfrak{B}$. We proved that the union of two NDFB of $\mathfrak{B}$ is again a NDFB of $\mathfrak{B}$. We also proved that the intersection of two NDFB of $\mathfrak{B}$ is again a NDFB of $\mathfrak{B}$.

Definition 3.1 A Neutrosophic fuzzy set $\mathcal{D}$ of BS-algebra $\mathfrak{B}$ is called the Neutrosophic Doubt Fuzzy Bi-ideal (NDFB) of $\mathfrak{B}$ if $\forall\, \alpha, \beta, \gamma \in \mathfrak{B}$
($\mathcal{D}_1$) $T_{\mathcal{D}}(1) \leq T_{\mathcal{D}}(\alpha)$; $\mathcal{J}_{\mathcal{D}}(1) \geq \mathcal{J}_{\mathcal{D}}(\alpha)$; $F_{\mathcal{D}}(1) \geq F_{\mathcal{D}}(\alpha)$;
($\mathcal{D}_2$) $T_{\mathcal{D}}(\beta*\gamma) \leq \{T_{\mathcal{D}}(\alpha) \vee T_{\mathcal{D}}(\alpha*(\beta*\gamma))\}$;
$\mathcal{J}_{\mathcal{D}}(\beta*\gamma) \geq \{\mathcal{J}_{\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}$;
$F_{\mathcal{D}}(\beta*\gamma) \geq \{F_{\mathcal{D}}(\alpha) \wedge F_{\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Theorem 3.2 Let $\mathcal{C}$ and $\mathcal{D}$ be two NDFB of $\mathfrak{B}$. Then $\mathcal{C} \cup \mathcal{D}$ is a NDFB of $\mathfrak{B}$.

Proof

Let $\mathcal{C}$ and $\mathcal{D}$ be two NDFB of $\mathfrak{B}$. For any $\alpha, \beta, \gamma \in \mathfrak{B}$
i) $T_{\mathcal{C}\cup\mathcal{D}}(1) = \{T_{\mathcal{C}}(1) \vee T_{\mathcal{D}}(1)\}$

$\leq \{Ţ_{\mathcal{C}}(\alpha) \vee Ţ_{\mathcal{D}}(\alpha)\}$
$= Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

Therefore, $Ţ_{\mathcal{C}\cup\mathcal{D}}(1) \leq Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

and $\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(1) = \{\mathcal{J}_{\mathcal{C}}(1) \wedge \mathcal{J}_{\mathcal{D}}(1)\}$
$\geq \{\mathcal{J}_{\mathcal{C}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha)\}$
$= \mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

Therefore, $\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(1) \geq \mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

and $Ƒ_{\mathcal{C}\cup\mathcal{D}}(1) = \{Ƒ_{\mathcal{C}}(1) \wedge Ƒ_{\mathcal{D}}(1)\}$
$\geq \{Ƒ_{\mathcal{C}}(\alpha) \wedge Ƒ_{\mathcal{D}}(\alpha)\}$
$= Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

Therefore, $Ƒ_{\mathcal{C}\cup\mathcal{D}}(1) \geq Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha)$

ii) $Ţ_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) = \{Ţ_{\mathcal{C}}(\beta*\gamma) \vee Ţ_{\mathcal{D}}(\beta*\gamma)\}$
$\leq \{\{Ţ_{\mathcal{C}}(\alpha) \vee Ţ_{\mathcal{C}}(\alpha*(\beta*\gamma))\} \vee \{Ţ_{\mathcal{D}}(\alpha) \vee Ţ_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{\{Ţ_{\mathcal{C}}(\alpha) \vee Ţ_{\mathcal{D}}(\alpha)\} \vee \{Ţ_{\mathcal{C}}(\alpha*(\beta*\gamma)) \vee Ţ_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha) \vee Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Therefore, $Ţ_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) \leq \{Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha) \vee Ţ_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

and $\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) = \{\mathcal{J}_{\mathcal{C}}(\beta*\gamma) \wedge \mathcal{J}_{\mathcal{D}}(\beta*\gamma)\}$
$\geq \{\{\mathcal{J}_{\mathcal{C}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}}(\alpha*(\beta*\gamma))\} \wedge \{\mathcal{J}_{\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{\{\mathcal{J}_{\mathcal{C}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha)\} \wedge \{\mathcal{J}_{\mathcal{C}}(\alpha*(\beta*\gamma)) \wedge \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Therefore, $\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) \geq \{\mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

and $Ƒ_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) = \{Ƒ_{\mathcal{C}}(\beta*\gamma) \wedge Ƒ_{\mathcal{D}}(\beta*\gamma)\}$
$\geq \{\{Ƒ_{\mathcal{C}}(\alpha) \wedge Ƒ_{\mathcal{C}}(\alpha*(\beta*\gamma))\} \wedge \{Ƒ_{\mathcal{D}}(\alpha) \wedge Ƒ_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{\{Ƒ_{\mathcal{C}}(\alpha) \wedge Ƒ_{\mathcal{D}}(\alpha)\} \wedge \{Ƒ_{\mathcal{C}}(\alpha*(\beta*\gamma)) \wedge Ƒ_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$
$= \{Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha) \wedge Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Therefore, $Ƒ_{\mathcal{C}\cup\mathcal{D}}(\beta*\gamma) \geq \{Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha) \wedge Ƒ_{\mathcal{C}\cup\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Hence, $\boldsymbol{\mathcal{C}}\cup\boldsymbol{\mathcal{D}}$ is a NDFB of $\mathfrak{B}$

Theorem 3.3 Let $\boldsymbol{\mathcal{C}}$ and $\boldsymbol{\mathcal{D}}$ be two NDFB of $\mathfrak{B}$. Then $\boldsymbol{\mathcal{C}}\cap\boldsymbol{\mathcal{D}}$ is a NDFB of $\mathfrak{B}$.

Proof

Let $\boldsymbol{\mathcal{C}}$ and $\boldsymbol{\mathcal{D}}$ be two NDFB of $\mathfrak{B}$. For any $\alpha, \beta, \gamma \in \mathfrak{B}$

(i) $Ţ_{\mathcal{C}\cap\mathcal{D}}(1) = \{Ţ_{\mathcal{C}}(1) \wedge Ţ_{\mathcal{D}}(1)\}$
$\leq \{Ţ_{\mathcal{C}}(\alpha) \wedge Ţ_{\mathcal{D}}(\alpha)\}$
$= Ţ_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

Therefore, $Ţ_{\mathcal{C}\cap\mathcal{D}}(1) \leq Ţ_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

and $\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(1) = \{\mathcal{J}_{\mathcal{C}}(1) \vee \mathcal{J}_{\mathcal{D}}(1)\}$
$\geq \{\mathcal{J}_{\mathcal{C}}(\alpha) \vee \mathcal{J}_{\mathcal{D}}(\alpha)\}$
$= \mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

Therefore, $\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(1) \geq \mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

and $Ƒ_{\mathcal{C}\cap\mathcal{D}}(1) = \{Ƒ_{\mathcal{C}}(1) \vee Ƒ_{\mathcal{D}}(1)\}$
$\geq \{Ƒ_{\mathcal{C}}(\alpha) \vee Ƒ_{\mathcal{D}}(\alpha)\}$
$= Ƒ_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

Therefore, $Ƒ_{\mathcal{C}\cap\mathcal{D}}(1) \geq Ƒ_{\mathcal{C}\cap\mathcal{D}}(\alpha)$

(ii) $\text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\beta*\gamma) = \{\text{Ʈ}_{\mathcal{C}}(\beta*\gamma) \wedge \text{Ʈ}_{\mathcal{D}}(\beta*\gamma)\}$

$\leq \{\{\text{Ʈ}_{\mathcal{C}}(\alpha) \vee \text{Ʈ}_{\mathcal{C}}(\alpha*(\beta*\gamma))\} \wedge \{\text{Ʈ}_{\mathcal{D}}(\alpha) \vee \text{Ʈ}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$

$= \{\{\text{Ʈ}_{\mathcal{C}}(\alpha) \wedge \text{Ʈ}_{\mathcal{D}}(\alpha)\} \vee \{\text{Ʈ}_{\mathcal{C}}(\alpha*(\beta*\gamma)) \wedge \text{Ʈ}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$

$= \{\text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\alpha) \vee \text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\alpha\leq*(\beta*\gamma))\}$

Therefore, $\text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\beta*\gamma) \leq \{\text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\alpha) \vee \text{Ʈ}_{\mathcal{C}\cap\mathcal{D}}(\alpha*(\beta*\gamma))\}$

and $\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\beta*\gamma) = \{\mathcal{J}_{\mathcal{C}}(\beta*\gamma) \vee \mathcal{J}_{\mathcal{D}}(\beta*\gamma)\}$

$\geq \{\{\mathcal{J}_{\mathcal{C}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}}(\alpha*(\beta*\gamma))\} \vee \{\mathcal{J}_{\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$

$= \{\{\mathcal{J}_{\mathcal{C}}(\alpha) \vee \mathcal{J}_{\mathcal{D}}(\alpha)\} \wedge \{\mathcal{J}_{\mathcal{C}}(\alpha*(\beta*\gamma)) \vee \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}\}$

$= \{\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Therefore, $\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\beta*\gamma) \geq \{\mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{C}\cap\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Similarly, $\text{Ƒ}_{\mathcal{C}\cap\mathcal{D}}(\beta*\gamma) \geq \{\text{Ƒ}_{\mathcal{C}\cap\mathcal{D}}(\alpha) \wedge \text{Ƒ}_{\mathcal{C}\cap\mathcal{D}}(\alpha*(\beta*\gamma))\}$

Hence, $\boldsymbol{\mathcal{C}}\cap\boldsymbol{\mathcal{D}}$ is a NDFB of $\mathfrak{B}$

Corollary 3.4 Let $\boldsymbol{\mathcal{D}}_1, \boldsymbol{\mathcal{D}}_2, \ldots\ldots, \boldsymbol{\mathcal{D}}_n$ are NDFB of $\mathfrak{B}$, then $\boldsymbol{\mathcal{D}} = \bigcap_{i=1}^{n} \mathcal{D}_i$ is also a NDFB of $\mathfrak{B}$

Proof

Straight forward using theorem 3.3

Lemma 3.5 For all s, t ∈ I and i be any positive integer, if s = t, then

i) $s^i \leq t^i$

ii) $[(s\wedge t)]^i = (s^i \wedge t^i)$

iii) $[(s\vee t)]^i = (s^i \vee t^i)$

Theorem 3.6 Let $\boldsymbol{\mathcal{D}}$ be a NDFB of $\mathfrak{B}$, then $\boldsymbol{\mathcal{D}}^i=\{<\alpha, \text{Ʈ}_{\mathcal{D}^i}(\alpha), \mathcal{J}_{\mathcal{D}^i}(\alpha), \text{Ƒ}_{\mathcal{D}^i}(\alpha)>: \alpha \in \mathfrak{B}\}$ is a NDFB of $\mathfrak{B}^i$, where i is any positive integer and $\text{Ʈ}_{\mathcal{D}^i}(\alpha) = (\text{Ʈ}_{\mathcal{D}}(\alpha))^i$, $\mathcal{J}_{\mathcal{D}^i}(\alpha) = (\mathcal{J}_{\mathcal{D}}(\alpha))^i$, $\text{Ƒ}_{\mathcal{D}^i}(\alpha) = (\text{Ƒ}_{\mathcal{D}}(\alpha))^i$

Proof

Let $\boldsymbol{\mathcal{D}}$ be a NDFB of $\mathfrak{B}$. For any $\alpha, \beta, \gamma \in \mathfrak{B}$

i) $\text{Ʈ}_{\mathcal{D}^i}(1) = (\text{Ʈ}_{\mathcal{D}}(1))^i$

$\leq (\text{Ʈ}_{\mathcal{D}}(\alpha))^i$

$= \text{Ʈ}_{\mathcal{D}^i}(\alpha)$

Therefore, $\text{Ʈ}_{\mathcal{D}^i}(1) \leq \text{Ʈ}_{\mathcal{D}^i}(\alpha)$

and $\mathcal{J}_{\mathcal{D}^i}(1) = (\mathcal{J}_{\mathcal{D}}(1))^i$

$\geq (\mathcal{J}_{\mathcal{D}}(\alpha))^i$

$= \mathcal{J}_{\mathcal{D}^i}(\alpha)$

Therefore, $\mathcal{J}_{\mathcal{D}^i}(1) \geq \mathcal{J}_{\mathcal{D}^i}(\alpha)$

and $\text{Ƒ}_{\mathcal{D}^i}(1) = (\text{Ƒ}_{\mathcal{D}}(1))^i$

$\geq (\text{Ƒ}_{\mathcal{D}}(\alpha))^i$

$= \text{Ƒ}_{\mathcal{D}^i}(\alpha)$

Therefore, $\text{Ƒ}_{\mathcal{D}^i}(1) \geq \text{Ƒ}_{\mathcal{D}^i}(\alpha)$

ii) $\text{Ʈ}_{\mathcal{D}^i}(\beta*\gamma) = (\text{Ʈ}_{\mathcal{D}}(\beta*\gamma))^i$

$\leq [\{\text{Ʈ}_{\mathcal{D}}(\alpha) \vee \text{Ʈ}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}]^i$

$= \{[\text{Ʈ}_{\mathcal{D}}(\alpha)]^i \vee [\text{Ʈ}_{\mathcal{D}}(\alpha*(\beta*\gamma))]^i\}$

$= \{\text{Ʈ}_{\mathcal{D}^i}(\alpha) \vee \text{Ʈ}_{\mathcal{D}^i}(\alpha*(\beta*\gamma))\}$

Therefore, $\text{Ʈ}_{\mathcal{D}^i}(\beta*\gamma) \leq \{\text{Ʈ}_{\mathcal{D}^i}(\alpha) \vee \text{Ʈ}_{\mathcal{D}^i}(\alpha*(\beta*\gamma))\}$

and $\mathcal{J}_{\mathcal{D}^i}(\beta*\gamma) = (\mathcal{J}_{\mathcal{D}}(\beta*\gamma))^i$
$\geq [\{\mathcal{J}_{\mathcal{D}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))\}]^i$
$= \{[\mathcal{J}_{\mathcal{D}}(\alpha)]^i \wedge [\mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma))]^i\}$
$= \{\mathcal{J}_{\mathcal{D}^i}(\alpha) \wedge \mathcal{J}_{\mathcal{D}^i}(\alpha*(\beta*\gamma))\}$
Therefore, $\mathcal{J}_{\mathcal{D}^i}(\beta*\gamma) \geq \{\mathcal{J}_{\mathcal{D}^i}(\alpha) \wedge \mathcal{J}_{\mathcal{D}^i}(\alpha*(\beta*\gamma))\}$
Similarly, we can prove that $\mathrm{F}_{\mathcal{D}^i}(\beta*\gamma) \geq \{\mathrm{F}_{\mathcal{D}^i}(\alpha) \wedge \mathrm{F}_{\mathcal{D}^i}(\alpha*(\beta*\gamma))\}$
Hence $\mathcal{D}^i$ is a NDFB of $\mathfrak{B}^i$

4. PRODUCT OF NEUTROSOPHIC DOUBT FUZZY BI IDEAL OF BS-ALGEBRAS

In this section, the product of NDFB of $\mathfrak{B}$ are defined and corresponding theorems are investigated.

Definition 4.1 Let $\mathcal{C}$ and $\mathcal{D}$ be two neutrosophic doubt fuzzy subsets of $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then the direct product of neutrosophic doubt fuzzy subsets of BS-algebra $\mathfrak{B}_1$ and $\mathfrak{B}_2$ is defined by $\mathcal{C}$x$\mathcal{D}$: $\mathfrak{B}_1 \times \mathfrak{B}_2 \longrightarrow [0,1]$ such that
$\mathcal{C}x\mathcal{D} = \{<(\alpha,\beta), \mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta), \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta), \mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta)>: \alpha\in\mathfrak{B}_1, \beta\in\mathfrak{B}_2\}$, where
$\mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta) = (\mathrm{T}_{\mathcal{C}}(\alpha) \vee \mathrm{T}_{\mathcal{D}}(\beta))$; $\mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta) = (\mathcal{J}_{\mathcal{C}}(\alpha) \wedge \mathcal{J}_{\mathcal{D}}(\beta))$; $\mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha,\beta) = (\mathrm{F}_{\mathcal{C}}(\alpha) \wedge \mathrm{F}_{\mathcal{D}}(\beta))$

Definition 4.2 Let $\mathcal{C}$ and $\mathcal{D}$ be two neutrosophic doubt fuzzy subsets of $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then $\mathcal{C}$x$\mathcal{D}$ is a NDFB of $\mathfrak{B}_1 \times \mathfrak{B}_2$ if it satisfies the following conditions
i) $\mathrm{T}_{\mathcal{C}x\mathcal{D}}(1,1) \leq \mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$; $\mathcal{J}_{\mathcal{C}x\mathcal{D}}(1,1) \geq \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$; $\mathrm{F}_{\mathcal{C}x\mathcal{D}}(1,1) \geq \mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$;
ii) $\mathrm{T}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)*(\gamma_1, \gamma_2)) \leq \{\mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \vee \mathrm{T}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)*(\gamma_1, \gamma_2)))\}$;
$\mathcal{J}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)*(\gamma_1, \gamma_2)) \geq \{\mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \wedge \mathcal{J}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)*(\gamma_1, \gamma_2)))\}$;
$\mathrm{F}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)*(\gamma_1, \gamma_2)) \geq \{\mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \wedge \mathrm{F}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)*(\gamma_1, \gamma_2)))\}$.

Theorem 4.3 Let $\mathcal{C}$ and $\mathcal{D}$ be two NDFB of $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then $\mathcal{C}$x$\mathcal{D}$ is a NDFB of $\mathfrak{B}_1 \times \mathfrak{B}_2$

Proof

Let $\mathcal{C}$ and $\mathcal{D}$ be two NDFB of $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively.
Let $(\alpha_1, \alpha_2), (\beta_1, \beta_2), (\gamma_1, \gamma_2) \in \mathfrak{B}_1 \times \mathfrak{B}_2$
i) We have $\mathrm{T}_{\mathcal{C}x\mathcal{D}}(1,1) = \{\mathrm{T}_{\mathcal{C}}(1) \vee \mathrm{T}_{\mathcal{D}}(1)\}$
$\leq \{\mathrm{T}_{\mathcal{C}}(\alpha_1) \vee \mathrm{T}_{\mathcal{D}}(\alpha_2)\}$
$= \mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
Therefore, $\mathrm{T}_{\mathcal{C}x\mathcal{D}}(1,1) \leq \mathrm{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
and $\mathcal{J}_{\mathcal{C}x\mathcal{D}}(1,1) = \{\mathcal{J}_{\mathcal{C}}(1) \wedge \mathcal{J}_{\mathcal{D}}(1)\}$
$\geq \{\mathcal{J}_{\mathcal{C}}(\alpha_1) \wedge \mathcal{J}_{\mathcal{D}}(\alpha_2)\}$
$= \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
Therefore, $\mathcal{J}_{\mathcal{C}x\mathcal{D}}(1,1) \geq \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
and $\mathrm{F}_{\mathcal{C}x\mathcal{D}}(1,1) = \{\mathrm{F}_{\mathcal{C}}(1) \wedge \mathrm{F}_{\mathcal{D}}(1)\}$
$\geq \{\mathrm{F}_{\mathcal{C}}(\alpha_1) \wedge \mathrm{F}_{\mathcal{D}}(\alpha_2)\}$
$= \mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
Therefore, $\mathrm{F}_{\mathcal{C}x\mathcal{D}}(1,1) \geq \mathrm{F}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)$
ii) Then $\mathrm{T}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)*(\gamma_1, \gamma_2)) = \mathrm{T}_{\mathcal{C}x\mathcal{D}}(\beta_1*\gamma_1, \beta_2*\gamma_2)$
$= \{\mathrm{T}_{\mathcal{C}}(\beta_1*\gamma_1) \vee \mathrm{T}_{\mathcal{D}}(\beta_2*\gamma_2)\}$
$\leq [\{\mathrm{T}_{\mathcal{C}}(\alpha_1) \vee \mathrm{T}_{\mathcal{C}}(\alpha_1*(\beta_1*\gamma_1))\} \vee \{\mathrm{T}_{\mathcal{D}}(\alpha_2) \vee \mathrm{T}_{\mathcal{D}}(\alpha_2*(\beta_2*\gamma_2))\}]$
$= [\{\mathrm{T}_{\mathcal{C}}(\alpha_1) \vee \mathrm{T}_{\mathcal{D}}(\alpha_2)\} \vee \{\mathrm{T}_{\mathcal{C}}(\alpha_1*(\beta_1*\gamma_1)) \vee \mathrm{T}_{\mathcal{D}}(\alpha_2*(\beta_2*\gamma_2))\}]$

$=\{\text{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \vee \text{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1*(\beta_1*\gamma_1)) , (\alpha_2*(\beta_2*\gamma_2))\}$

$= \{\text{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \vee \text{T}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)* (\gamma_1, \gamma_2)))\}$

Therefore, $\text{T}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)* (\gamma_1, \gamma_2)) \leq \{\text{T}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \vee \text{T}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)* (\gamma_1, \gamma_2)))\}$

and $\mathcal{J}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)* (\gamma_1, \gamma_2)) = \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\beta_1*\gamma_1, \beta_2*\gamma_2)$

$= \{\mathcal{J}_{\mathcal{C}}(\beta_1*\gamma_1) \wedge \mathcal{J}_{\mathcal{D}}(\beta_2*\gamma_2)\}$

$\geq [\{\mathcal{J}_{\mathcal{C}}(\alpha_1) \wedge \mathcal{J}_{\mathcal{C}}(\alpha_1*(\beta_1*\gamma_1))\} \wedge\{\mathcal{J}_{\mathcal{D}}(\alpha_2) \wedge \mathcal{J}_{\mathcal{D}}(\alpha_2*(\beta_2*\gamma_2))\}]$

$= [\{\mathcal{J}_{\mathcal{C}}(\alpha_1) \wedge \mathcal{J}_{\mathcal{D}}(\alpha_2)\} \wedge \{\mathcal{J}_{\mathcal{C}}(\alpha_1*(\beta_1*\gamma_1)) \wedge \mathcal{J}_{\mathcal{D}}(\alpha_2*(\beta_2*\gamma_2))\}]$

$=\{\mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2)\} \wedge\{ \mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1*(\beta_1*\gamma_1)) \wedge (\alpha_2*(\beta_2*\gamma_2))\}$

$= \{\mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1 ,\alpha_2)\}\wedge\{\mathcal{J}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)* (\gamma_1, \gamma_2)))\}$

Therefore, $\mathcal{J}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)* (\gamma_1, \gamma_2)) \geq \{\mathcal{J}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \wedge \mathcal{J}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)* (\gamma_1, \gamma_2)))\}$

Similarly we can easily prove that,

$\text{F}_{\mathcal{C}x\mathcal{D}}((\beta_1, \beta_2)* (\gamma_1, \gamma_2)) \geq \{\text{F}_{\mathcal{C}x\mathcal{D}}(\alpha_1, \alpha_2) \wedge \text{F}_{\mathcal{C}x\mathcal{D}}((\alpha_1, \alpha_2)*((\beta_1, \beta_2)* (\gamma_1, \gamma_2)))\}$

Hence $\mathcal{C}x\mathcal{D}$ is a NDFB of $\mathfrak{B}_1 \text{ x } \mathfrak{B}_2$

5. HOMOMORPHISM OF NDFB OF $\mathfrak{B}$

In this section, the homomorphic behaviour of NDFB of $\mathfrak{B}$ are defined and related theorems are discussed.

Definition 5.1 Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be two BS-algebras and h:$\mathfrak{B}_1 \longrightarrow \mathfrak{B}_2$ be a function.

i) If $\mathcal{D}$ is a NDFB in $\mathfrak{B}_2$, then the preimage of $\mathcal{D}$ under h denoted by $h^{-1}(\mathcal{D})$ is the NDFB in $\mathfrak{B}_1$ is defined by $h^{-1}(\mathcal{D})=\{<(\alpha), h^{-1}(\text{T}_{\mathcal{D}}(\alpha)), h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha)), h^{-1}(\text{F}_{\mathcal{D}}(\alpha)) >: \alpha\in\mathfrak{B}\}$,

where $h^{-1}(\text{T}_{\mathcal{D}}(\alpha)) = \text{T}_{\mathcal{D}}(h(\alpha))$; $h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha)) = \mathcal{J}_{\mathcal{D}}(h(\alpha))$; $h^{-1}(\text{F}_{\mathcal{D}}(\alpha)) = \text{F}_{\mathcal{D}}(h(\alpha))$;

Theorem 5.2 Let h: $\mathfrak{B}_1\rightarrow \mathfrak{B}_2$ be an epimorphism of BS-algebras if $\mathcal{D}$ is a NDFB of $\mathfrak{B}_2$, then the pre image of $\mathcal{D}$ under h is also a NDFB of $\mathfrak{B}_1$.

Proof

Let $\mathcal{D}$ is a NDFB of $\mathfrak{B}_2$. Let $\alpha,\beta,\gamma \in \mathfrak{B}_1$

Now, $h^{-1}(\text{T}_{\mathcal{D}}(1)) = \text{T}_{\mathcal{D}}(h(1))$

$\leq \text{T}_{\mathcal{D}}(h(\alpha))$

$= h^{-1}(\text{T}_{\mathcal{D}}(\alpha))$

Therefore $h^{-1}(\text{T}_{\mathcal{D}}(1)) \leq h^{-1}(\text{T}_{\mathcal{D}}(\alpha))$

and $h^{-1}(\mathcal{J}_{\mathcal{D}}(1)) = \mathcal{J}_{\mathcal{D}}(h(1))$

$\geq \mathcal{J}_{\mathcal{D}}(h(\alpha))$

$= h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha))$

Therefore $h^{-1}(\mathcal{J}_{\mathcal{D}}(1)) \geq h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha))$

and $h^{-1}(\text{F}_{\mathcal{D}}(1)) = \text{F}_{\mathcal{D}}(h(1))$

$\geq \text{F}_{\mathcal{D}}(h(\alpha))$

$= h^{-1}(\text{F}_{\mathcal{D}}(\alpha))$

Therefore $h^{-1}(\text{F}_{\mathcal{D}}(1)) \geq h^{-1}(\text{F}_{\mathcal{D}}(\alpha))$

ii) Again, $h^{-1}(\text{T}_{\mathcal{D}}(\beta*\gamma)) = \text{T}_{\mathcal{D}}(h(\beta*\gamma))$

$= \text{T}_{\mathcal{D}}(h(\beta)*h(\gamma))$

$\leq \{\text{T}_{\mathcal{D}}(h(\alpha)) \vee \text{T}_{\mathcal{D}}(h(\alpha)*[h(\beta)*h(\gamma)])\}$

$= \{\text{T}_{\mathcal{D}}(h(\alpha)) \vee \text{T}_{\mathcal{D}}(h(\alpha*(\beta*\gamma)))\}$

Therefore, $h^{-1}(\text{T}_{\mathcal{D}}(\beta*\gamma)) \leq \{h^{-1}(\text{T}_{\mathcal{D}}(\alpha)) \vee h^{-1}(\text{T}_{\mathcal{D}}(\alpha*(\beta*\gamma)))\}$

and $h^{-1}(\mathcal{J}_{\mathcal{D}}(\beta*\gamma)) = \mathcal{J}_{\mathcal{D}}(h(\beta*\gamma))$
$= \mathcal{J}_{\mathcal{D}}(h(\beta)*h(\gamma))$
$\geq \{\mathcal{J}_{\mathcal{D}}(h(\alpha)) \wedge \mathcal{J}_{\mathcal{D}}(h(\alpha)*[h(\beta)*h(\gamma)])\}$
$= \{\mathcal{J}_{\mathcal{D}}(h(\alpha)) \wedge \mathcal{J}_{\mathcal{D}}(h(\alpha*(\beta*\gamma)))\}$
Therefore, $h^{-1}(\mathcal{J}_{\mathcal{D}}(\beta*\gamma)) \geq \{h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha)) \wedge h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha*(\beta*\gamma)))\}$
and $h^{-1}(\digamma_{\mathcal{D}}(\beta*\gamma)) = \digamma_{\mathcal{D}}(h(\beta*\gamma))$
$= \digamma_{\mathcal{D}}(h(\beta)*h(\gamma))$
$\geq \{\digamma_{\mathcal{D}}(h(\alpha)) \wedge \digamma_{\mathcal{D}}(h(\alpha)*[h(\beta)*h(\gamma)])\}$
$= \{\digamma_{\mathcal{D}}(h(\alpha)) \wedge \digamma_{\mathcal{D}}(h(\alpha*(\beta*\gamma)))\}$
Therefore, $h^{-1}(\digamma_{\mathcal{D}}(\beta*\gamma)) \geq \{h^{-1}(\digamma_{\mathcal{D}}(\alpha)) \wedge h^{-1}(\digamma_{\mathcal{D}}(\alpha*(\beta*\gamma)))\}$
Hence $h^{-1}(\boldsymbol{\mathcal{D}})$is a NDFB of $\mathfrak{B}_1$.

Definition 5.3 [1] Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be two BS-algebras h: $\mathfrak{B}_1 \rightarrow \mathfrak{B}_2$ be a homomorphism. Then h(1) =1

Theorem 5.4 Let h: $\mathfrak{B}_1 \rightarrow \mathfrak{B}_2$ be a homomorphism of BS-algebras if $\boldsymbol{\mathcal{D}}$ is a NDFB of $\mathfrak{B}_1$, then $h(\boldsymbol{\mathcal{D}})$is a NDFB of $\mathfrak{B}_2$.

Proof

Let $\alpha_1, \alpha_2, \alpha_3 \in \mathfrak{B}_1$ and $\beta_1, \beta_2, \beta_3 \in \mathfrak{B}_2$ such that $h(\alpha_1)= \beta_1$, $h(\alpha_2)= \beta_2$, $h(\alpha_3)= \beta_3$
Now, $\mathrm{T}_{\mathcal{D}}(\beta_1)= \mathrm{T}_{\mathcal{D}}(h(\alpha_1))$
$= h^{-1}(\mathrm{T}_{\mathcal{D}}(\alpha_1))$
$\geq h^{-1}(\mathrm{T}_{\mathcal{D}}(1))$
$= \mathrm{T}_{\mathcal{D}}(h(1))$
$= \mathrm{T}_{\mathcal{D}}(1)$
Therefore, $\mathrm{T}_{\mathcal{D}}(\beta_1) \geq \mathrm{T}_{\mathcal{D}}(1)$
And $\mathcal{J}_{\mathcal{D}}(\beta_1)= \mathcal{J}_{\mathcal{D}}(h(\alpha_1))$
$= h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha_1))$
$\leq h^{-1}(\mathcal{J}_{\mathcal{D}}(1))$
$= \mathcal{J}_{\mathcal{D}}(h(1))$
$= \mathcal{J}_{\mathcal{D}}(1)$
Therefore, $\mathcal{J}_{\mathcal{D}}(\beta_1) \leq \mathcal{J}_{\mathcal{D}}(1)$
And $\digamma_{\mathcal{D}}(\beta_1)= \digamma_{\mathcal{D}}(h(\alpha_1))$
$= h^{-1}(\digamma_{\mathcal{D}}(\alpha_1))$
$\leq h^{-1}(\digamma_{\mathcal{D}}(1))$
$= \digamma_{\mathcal{D}}(h(1))$
$= \digamma_{\mathcal{D}}(1)$
Therefore, $\digamma_{\mathcal{D}}(\beta_1) \leq \digamma_{\mathcal{D}}(1)$
ii) Again, $\mathrm{T}_{\mathcal{D}}(\beta_2* \beta_3) = \mathrm{T}_{\mathcal{D}}(h(\alpha_2)* h(\alpha_3))$
$= h^{-1}(\mathrm{T}_{\mathcal{D}}(\alpha_2*\alpha_3))$
$\leq \{h^{-1}(\mathrm{T}_{\mathcal{D}}(\alpha_1)) \vee h^{-1}(\mathrm{T}_{\mathcal{D}}(\alpha_1*(\alpha_2*\alpha_3)))\}$
$=\{\mathrm{T}_{\mathcal{D}}(h(\alpha_1)) \vee \mathrm{T}_{\mathcal{D}}(h(\alpha_1*(\alpha_2*\alpha_3)))\}$
$=\{\mathrm{T}_{\mathcal{D}}(h(\alpha_1)) \vee \mathrm{T}_{\mathcal{D}}(h(\alpha_1)*(h(\alpha_2)*h(\alpha_3)))\}$
$=\{\mathrm{T}_{\mathcal{D}}(\beta_1) \vee \mathrm{T}_{\mathcal{D}}(\beta_1*(\beta_2*\beta_3))\}$
Therefore, $\mathrm{T}_{\mathcal{D}}(\beta_2* \beta_3) \leq \{\mathrm{T}_{\mathcal{D}}(\beta_1) \vee \mathrm{T}_{\mathcal{D}}(\beta_1*(\beta_2*\beta_3))\}$
And $\mathcal{J}_{\mathcal{D}}(\beta_2* \beta_3) = \mathcal{J}_{\mathcal{D}}(h(\alpha_2)* h(\alpha_3))$

$= h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha_2 * \alpha_3))$
$\geq \{h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha_1)) \wedge h^{-1}(\mathcal{J}_{\mathcal{D}}(\alpha_1 * (\alpha_2 * \alpha_3)))\}$
$= \{\mathcal{J}_{\mathcal{D}}(h(\alpha_1)) \wedge \mathcal{J}_{\mathcal{D}}(h(\alpha_1 * (\alpha_2 * \alpha_3)))\}$
$= \{\mathcal{J}_{\mathcal{D}}(h(\alpha_1)) \wedge \mathcal{J}_{\mathcal{D}}(h(\alpha_1) * (h(\alpha_2) * h(\alpha_3)))\}$
$= \{\mathcal{J}_{\mathcal{D}}(\beta_1) \wedge \mathcal{J}_{\mathcal{D}}(\beta_1 * (\beta_2 * \beta_3))\}$

Therefore, $\mathcal{J}_{\mathcal{D}}(\beta_2 * \beta_3) \geq \{\mathcal{J}_{\mathcal{D}}(\beta_1) \wedge \mathcal{J}_{\mathcal{D}}(\beta_1 * (\beta_2 * \beta_3))\}$

Similarly, $\mathcal{F}_{\mathcal{D}}(\beta_2 * \beta_3) \geq \{\mathcal{F}_{\mathcal{D}}(\beta_1) \wedge \mathcal{F}_{\mathcal{D}}(\beta_1 * (\beta_2 * \beta_3))\}$

Hence $h(\mathcal{D})$ is a NDFB of $\mathfrak{B}_2$.

Conclusion

In this research paper, the notion of Neutrosophic doubt fuzzy bi-ideal (NDFB) of BS-algebras $\mathfrak{B}$ are introduced and studied their algebraic properties. We obtained the Cartesian product of neutrosophic doubt fuzzy bi-ideal(NDFB) for BS-algebras $\mathfrak{B}$. Finally, we studied how to deal with homomorphism in neutrosophic doubt fuzzy bi-ideal(NDFB) for BS-algebras $\mathfrak{B}$.

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