Production Planning with The Neutrosophic Fuzzy Multi-Objective Optimization Technique

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Abstract: Businesses want to realize their production at the desired time and quality. In addition, businesses aim to use their existing resources efficiently and increase their earnings. The realization of more than one purpose is achieved by multi-purpose planning of production. This study considered the multi-objective production planning of a company that manufactures spare parts for household appliances. The management of the company wants to minimize its cost, cycle time, defective rates, and material wastage while maximizing its profits in the process of producing all 121 different products. This study presents a solution for this multi-item multi-objective production problem using the intuitionistic fuzzy and neutrosophic fuzzy multi-objective optimization models. The study gives a step-by-step explanation of the methods used to achieve the solution and a comparison detailing the problems of the business and the solutions obtained. The results show that the neutrosophic fuzzy multi-objective optimization model, which is capable of independently handling uncertainty, produces better results than the intuitionistic fuzzy multi-objective optimization model.

Keywords: Multi objective programming, Neutrosophic fuzzy set, Multi-objective optimization, Neutrosophic fuzzy multi-objective optimization, Production planning.

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1. Introduction

Production refers to the entirety of the process that brings together companies, organizations, and individuals that produce, supply, and demand goods and services, the outcome of which are products that create economic benefits. E-commerce and virtual marketplaces that have emerged from the prevailing world economic order rife with increasing competition and technological advances continue to make for a difficult environment for companies. Businesses have to operate under multiple and often conflicting purposes at any given time.

To stay ahead of the competition, businesses have to satisfy the demands of their customers by making their products available in the desired features and dimensions, and at the desired time. This can be achievable through robust production planning. Planning plays a vital role in the organization's race to stay alive and competitive by showing the business its present position and outlining the best way to make use of the available resources to attain organizational goals. Production planning, in its simplest form, is the set of activities involved in determining the product to be manufactured, when and in what form. The production system faces uncertainties resulting from various reasons like the changes in the needs and preferences of people as well as deviations in the production process. In the event of such uncertainty, businesses are faced with the task of overcoming the confusion caused by the unpredictability while also striving to continue their activities under multiple objectives. The occurrence of these uncertainties in the course of operation is an inevitable part of doing business.

Uncertainty refers to a situation in which the information available about a certain situation is incomplete, inaccurate, or doubtful. The basis of uncertainty is the information that people receive from the outside world. Uncertainty causes doubt about the value of a variable, phenomenon, the decision to be made and the conclusion to be drawn. It hinders the functioning of the decision mechanism, and the success of a business is greatly proportional to how well it can handle uncertainties. Under the uncertain circumstances of doing business, it is almost impossible to meet the supply and demand expectations using only the classical approaches. This limitation of classical approaches can be complemented by fuzzy structures that can make uncertainty known in the best possible way.

Fuzzy structures provide an environment in which decision-makers are able to consider the uncertainty in all its aspects leading to more accurate and timely decisions. Fuzzy structures, that have increasingly complemented the limitations of classical approaches in the process of making and activating production plans, have also shown a developing sequence from fuzzy sets to intuitionistic fuzzy sets and neutrosophic fuzzy sets.

This study aims to show that production planning in a production company can be achieved using the neutrosophic fuzzy multi-objective optimization technique. The study thus used neutrosophic fuzzy sets with a multi-objective optimization technique, which has been touted in the literature as a generalization of fuzzy and intuitionistic fuzzy sets. The application followed three different approaches of the neutrosophic fuzzy multi-objective optimization technique; Model I, Model II and Model III. After the three models, the intuitionistic fuzzy multi-objective optimization technique was used to measure the effectiveness of the neutrosophic fuzzy multi-objective optimization technique. The results obtained from both applications were then compared.

2. Literature Review

Businesses are faced with the task of fulfilling the demands of the customers and delivering the desired product attributes in a timely manner. Those that are able to carry out the production process within a certain plan stand to attain a competitive edge in the market. Planning refers to the process of outlining how to achieve the company goals by making the best use of available resources. Production planning enables businesses to coordinate their activities in daily, weekly, monthly and annual timeframes in line with business objectives [1]. The competition that is the result of the various developments going on in the world has made planning an integral part of the business. The optimal of business resources like human, machine, material, and time is tied to the quality of the plan [2].

Businesses are often in operation for more than one purpose. They are faced with multiple objectives such as profit maximization and resource minimization instead of a single objective like cost minimization. The multi-objective nature of the goals combined with product diversity turns production into a complex multi-objective problem. Cheng and Xiao-Bing looked at the production planning problem with reference to the delivery time, production balance, stock, and overtime purposes [3]. Yazdani et al. (2021) examined the minimization of the sustainability function, which includes the total production and process costs and the amount of harmful environmental and social components [4].

Most production planning problems are multi-objective with varying degrees of uncertainty. The fuzzy logic system introduced by Zadeh in the 1960s has proved useful for decision-makers in many areas. Zadeh (1965) stated that the human thought structure, to a great extent, isn't clear, but fuzzy [5]. People express their thoughts in linguistic terms rather than numerical data. This leads to different ways of interpreting different thoughts leading to different decisions and hence uncertainty. Fuzzy systems offer the opportunity to easily model uncertainty. Various researchers have used fuzzy sets to solve multi-objective optimization problems. Wang and Liang developed a fuzzy multi-objective linear programming model to solve multi-item multi-objective aggregate production planning decision problems. In the proposed model, a solution is presented to minimize the total production cost, labor turnover rate, transportation, and ordering costs [6]. Kumawat et al. (2021) came up with a fuzzy multi-objective optimization model that is intended to minimize carbon emissions and energy consumption in sustainable production planning [7]. Komsiyah et al. (2018) employed a fuzzy goal programming technique in furniture production to maximize profit and minimize production and raw material cost [8]. In 1986, Atanassov presented intuitionistic fuzzy sets as an alternative to fuzzy sets. Intuitionistic fuzzy sets, unlike fuzzy sets, define uncertainty using the degree of membership and non-membership, as well as the degree of hesitation. Neutrosophic fuzzy sets by Smarandache in 1995 brought a new approach to solving problems involving uncertainty [9]. Many researchers have contributed to the development of neutrosophic fuzzy sets and the solution to different problems. Some of the studies are given in Table 1.

Table 1 Summary of the Literature

Author	Technique	Goal	Explanation
Bharati and Singh [10]	Intuitionistic Fuzzy Optimization	Multi-objective, production and profit maximization	Agricultural production problem
Ali et al. [11]	Intuitionistic Fuzzy Optimization, Fuzzy Goal Programming	Multi-objective, Profit maximization, minimization carrying cost	Inventory modeling
Hussian et al. [12]	Neutrosophic Linear Programming	Single objective - profit maximization	production enterprise
Abdel-Baset et al. [13]	Neutrosophic Integer Programming Problems	Single objective	Neutrosophic integer programming with numerical examples
Ahmad and Adhami [14]	Fuzzy optimization, Intuitionistic optimization, Neutrosophic optimization	Multi-objective, transportation cost, labor cost, safety cost	Transportation problems
Hu et al. [15]	Intuitionistic Fuzzy programming, Neutrosophic Programming	Multi-objective, normal time production cost, and overtime production cost, inventory cost, order cost, labor cost.	Production Planning Problem
Khan et al. [16]	Intuitionistic Fuzzy Optimization, Neutrosophic Fuzzy Optimization	Multi-profit maximization, production, and carrying cost minimization	Production planning problem in a hardware company
Mondal et al. [17]	Neutrosophic Geometric Programming, Neutrosophic Non-Linear Programming	Single objective - cost minimization	Economic order quantity problem
Roy and Das [18]	Neutrosophic multi-objective linear programming,	Multi-profit, quality, employee satisfaction	Production Planning

	Intuitionistic fuzzy optimization,		

3. Basic Concepts

In this section, are mentioned basic concepts of fuzzy sets, intuitionistic fuzzy sets, and neutrosophic fuzzy.

Definition 1 (Fuzzy Set): The fuzzy set $\tilde{A}$, consisting of x elements defined in the universal set X, is presented as, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. The degree of membership function $\mu_{\tilde{A}}(x)$ is also called the degree of accuracy. The degree of membership function takes values between 0 and 1 and is defined as $\mu_{\tilde{A}}: X \rightarrow [0,1]$ [19].

Definition 2 (Intuitionistic Fuzzy Set): Intuitionistic fuzzy sets, which are the generalization of fuzzy sets, are defined based on two subsets; the membership function $\mu_{A^{\tilde{I}}}(x)$ and the non-membership function $\nu_{A^{\tilde{I}}}(x)$. The intuitionistic fuzzy defined in the universal set X is in the form of $A^{\tilde{I}} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) \mid x \in X\}$. $\mu_{A^{\tilde{I}}}: X \rightarrow [0,1]$ and $\nu_{A^{\tilde{I}}}: X \rightarrow [0,1]$

Intuitionistic fuzzy sets are defined by the degree of hesitation as well as the degree of membership and non-membership. The hesitation index or degree of hesitation is denoted

$\pi_{A^{\tilde{I}}}$. The degree of hesitation is calculated by subtracting the sum of the degrees of membership and non-membership from one. $\pi_{A^{\tilde{I}}} = 1 - \mu_{A^{\tilde{I}}}(x) - \nu_{A^{\tilde{I}}}(x)$ [19].

Definition 3. (Neutrosophic Fuzzy Set): Let X be the universal set and x a general element in this set. The neutrosophic fuzzy set A defined in X is characterized by the membership functions of truth, uncertainty, and inaccuracy.

$$\mu_{A^{\tilde{N}}}(x);\ x \rightarrow]0^-, 1^+[$$

$$\sigma_{A^{\tilde{N}}}(x):\ x \rightarrow]0^-, 1^+[$$

$$\nu_{A^{\tilde{N}}}(x):\ x \rightarrow]0^-, 1^+[$$

The Neutrosophic fuzzy sets, whether real, standard, or non-standard subsets in the $\mu_{A^{\tilde{N}}}(x)$, $\sigma_{A^{\tilde{N}}}(x)$, $\nu_{A^{\tilde{N}}}(x)$ $]0^-, 1^+[$ range are shown as follows.

$$A^{\tilde{N}} = \{< x, \mu_{\tilde{A}}(x), \sigma_{A^{\tilde{N}}}(x), \nu_{\tilde{A}}(x) > \mid x \in X\}$$

There is no limit to $\mu_{A^{\tilde{N}}}(x)$, $\sigma_{A^{\tilde{N}}}(x)$, $\nu_{A^{\tilde{N}}}(x)$ totals and

$0^- \leq sup\ \mu_{A^{\tilde{N}}}(x) + \sup \sigma_{A^{\tilde{N}}}(x) + \sup \nu_{A^{\tilde{N}}}(x) \leq 3^+$

Definition 4. (Single Valued Neutrosophic sets): Let X be the universal set and x a general element in this set. The single-valued neutrosophic set is defined by the truth membership function $\mu_{A^{\tilde{N}}}(X)$, indeterminacy membership function $\sigma_{A^{\tilde{N}}}(X)$ and the falsity membership function $\nu_{A^{\tilde{N}}}(X)$ [20].

$$A^{\tilde{N}} = \{< x, \mu_{\tilde{A}}(x), \sigma_{A^{\tilde{N}}}(x), \nu_{\tilde{A}}(x) > \mid x \in X\}$$

$0 \leq \mu_{A^{\tilde{N}}}(x) + \sigma_{A^{\tilde{N}}}(x) + \nu_{A^{\tilde{N}}}(x) \leq 3$, $\mu_{A^{\tilde{N}}}(x)$, $\sigma_{A^{\tilde{N}}}(x)$ and $\nu_{A^{\tilde{N}}}(x) \in [0,1]$

4. Multi-Objective Optimization Solution Methods

Multi-objective optimization problems arise when there is a need for simultaneous optimization of more than one objective. A general multi-objective optimization model is mathematically represented as [21]:

$Max(Min)f(x) = \{Z_1(x), Z_2(x), ... Z_n(x)\}$

Constraints

$g(x) \leq 0, x \in X$

$g(x) \geq 0, x \in X$

$g(x) = 0, x \in X$

$x \geq 0, x \in X$

There are multiple solution techniques for multi-objective optimization problems including neutrosophic fuzzy and intuitionistic fuzzy multi-objective optimization techniques which will be outlined in the next section.

A. Intuitionistic Fuzzy Multi-Objective Optimization Technique

The intuitionistic fuzzy multi-objective optimization model is denoted as follows [22].

$max = [f_1{}^I(x), f_2{}^I(x), \dots f_{k1}{}^I(x)]$

$min = [f_{k1+1,}{}^I(x), f_{k1+2}{}^I(x), \dots f_k{}^I(x)]$

Constraints

$g_i(x) \leq c_i$, i= 1,2,…, m_1

$g_i(x) \geq c_i$, i= $m_1 + 1, m_1 + 2, \dots, m_2$

$g_i(x) = c_i$, i= $m_2 + 1, m_2 + 2, \dots, m,$

$x \geq 0,$ **(1)**

Where;

x: decision variable,

$f_j{}^I(x)$: Objective function,

$g_l(x)$: Constraint function

The following steps are followed when solving a multi-objective intuitionistic fuzzy optimization problem [10]:

Step 1: The intuitionistic fuzzy multi-objective optimization problem is set up as given in equation (1).

Step 2: One of the objective functions of the problem is chosen randomly and solved using a single-objective classical linear programming technique respecting all the constraints of the problem, and ignoring the other objectives.

Step 3: Step 2 is repeated for all other objective functions. The values of the objective functions and decision variables are obtained. The optimal solutions obtained for each objective function are then used to determine the solution vector $x_1, x_2\ x_3, \dots, x_k$.

Step 4: The solution vectors obtained are then used to get the values of each of the objective functions as in the pay-off matrix shown in Table 2 below [23].

Table 2 *Pay-off Matrix*

Step 5: The lower and upper limits of the membership functions are determined using the values obtained from the pay-off matrix. The lower and upper limits of membership and non-membership functions are calculated differently for different objective functions. Table 3 shows how the lower and upper limits of the membership functions are calculated when the objective functions have a maximum structure [10].

Table 3 *Determination of the Upper and Lower Limits of Intuitionistic Fuzzy Membership Functions with a Maximization Structure*

Objective Function Type	**Objective Function with Maximization Type**	
Membership Function Type	**Lower Limit**	**Upper Limit**
Membership Function (μ)	$L_k^{\mu} = min\{f_k(x_k)\}$	$U_k^{\mu} = max\{f_k(x_k)\}$
Non-Membership Function (ν)	$L_k^{\nu} = L_k^{\mu}$	$U_k^{\nu}=U_k^{\mu} - \lambda(U_k^{\mu} - L_k^{\mu})$

Table 4 shows how to calculate the lower and upper limits of the membership functions where the objective functions have a minimization type [10].

Table 4 *Determination of the Upper and Lower Limits of Intuitionistic Fuzzy Membership Functions with a Minimization Type*

Objective Function Type	Objective Function with Minimization Type	
Membership Function Type	**Lower Limit**	**Upper Limit**
Membership Function (μ)	$L_k^{\mu} = min\{f_k(x_k)\}$	$U_k^{\mu} = max\{f_k(x_k)\}$
Non-Membership Function (ν)	$L_k^{\nu}=L_k^{\mu}+\lambda(U_k^{\mu} - L_k^{\mu})$	$U_k^{\nu} = U_k^{\mu}$

The U_k shown in the equations indicates the upper limit of the relevant objective, in other words, the highest value it can take. L_k indicates the lower limit of the relevant objective, in other words, the lowest value it can take. U_k indicates the best value that the objective function can take in problems with a maximization type as well as the worst value that the objective function will take in problems with a minimization type. The λ value given in the table is determined by the decision maker, provided that it is in the range of $0 < \lambda < 1$ [24]. In our study, the value λ=0,3 was used.

Step 6: Membership and non-membership functions are created with the lower and upper limits obtained in step 5. The determination of membership and non-membership functions differs according to the type of objective functions.

If the objective function has a maximization type, the membership function is defined as shown in equation (2).

$$\mu_{k\tilde{\imath}}(x)= \begin{cases} 0, & if \quad f_k(x) \le L_k^{\mu} \\ \frac{f_k(x)-L_k^{\mu}}{U_k^{\mu}-L_k^{\mu}}, & if \quad L_k^{\mu} \le f_k(x) \le U_k^{\mu} \\ 1, & if \quad f_k(x) \ge U_k^{\mu} \end{cases} \tag{2}$$

If the objective function has a maximization type, the non-membership function is defined as shown in equation (3).

$$\nu_{k\tilde{\imath}}(x)= \begin{cases} 0, & if \quad f_k(x) \ge U_k^{\nu} \\ \frac{U_k^{\nu}(x)-f_k(x)}{U_k^{\nu}-L_k^{\nu}}, & if \quad L_k^{\nu} \le f_k(x) \le U_k^{\nu} \\ 1, & if \quad f_k(x) \le L_k^{\nu} \end{cases} \tag{3}$$

Where;

L_k^{μ}: Lower limit of membership function

U_k^{μ}: Upper limit of membership function

L_k^{ν}: Lower limit of a non-membership function

U_k^{ν}: Upper limit of a non-membership function

$f_k(x)$: k. Function.

Figure 1 shows the membership and non-membership functions for intuitionistic fuzzy sets with a maximization type [10].

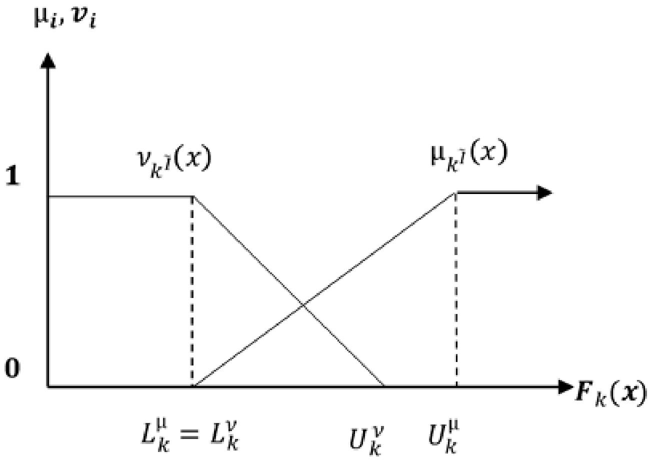

Figure 1. Membership Functions in Intuitionistic Fuzzy Sets with a Maximization Type

If the objective function has a minimization type, the membership function is defined as shown in equation (4).

$$\mu_{k^I}(x) = \begin{cases} 1, & if \quad f_k(x) \leq L_k^{\mu} \\ \frac{U_k^{\mu} - f_k(x)}{U_k^{\mu} - L_k^{\mu}}, & if \quad L_k^{\mu} \leq f_k(x) \leq U_k^{\mu} \\ 0, & if \quad f_k(x) \geq U_k^{\mu} \end{cases} \quad \textbf{(4)}$$

If the objective function has a minimization type, the non-membership function is defined as shown in equation (5).

$$\nu_{k^I}(x) = \begin{cases} 1, & if \quad f_k(x) \geq U_k^{\nu} \\ \frac{f_k(x) - L_k^{\nu}(x)}{U_k^{\nu} - L_k^{\nu}} & if \quad L_k^{\nu} \leq f_k(x) \leq U_k^{\nu} \\ 0, & if \quad f_k(x) \leq L_k^{\mu} \end{cases} \quad \textbf{(5)}$$

Figure 2 shows the membership and non-membership functions for intuitionistic fuzzy sets with a minimization type [25].

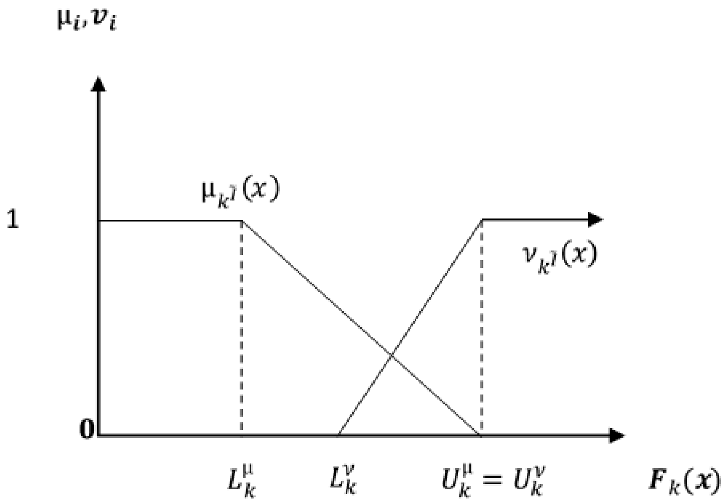

Figure 2. Membership Functions in Intuitionistic Fuzzy Sets with a Minimization Type

In an optimization problem, where there are objectives with a maximization type, the acceptance level, in other words, the degree of membership increases as each function approaches its highest value (U_k^{μ}). Therefore, the decision maker is completely satisfied when the relevant objective reaches the highest value. However, in cases where the objectives have a minimizing type, the satisfaction level increases as each function approaches its lowest value (L_k^{μ}). The decision maker is completely satisfied if all objectives reach their lowest values.

Step 7: After the membership and non-membership functions have been determined, the intuitionistic fuzzy optimization model is shown as in equation (6). The membership function is maximized and the non-membership function is minimized.

$$\begin{aligned}
&Maximum\ \ \mu_{k^{\tilde{I}}}(f_k(x)) \\
&Minimum\ v_{k^{\tilde{I}}}(f_k(x)) \\
&Constraints \\
&\mu_k^{I}(f_k(x)) + v_k^{I}(f_k(x)) \le 1 \\
&\mu_k^{I}(f_k(x)) \ge v_k^{I}(f_k(x)) \\
&v_{k^{\tilde{I}}}(f_k(x)) \ge 0 \\
&g_j\ \le b_j \\
&x \ge 0 \\
&k\ = 1,2, \ldots, k. \\
&j = 1,2, \ldots, m.
\end{aligned} \tag{6}$$

This model is then transformed into the classical linear programming model as shown in equation (7). This transformation is accomplished by pairing the membership function with an additional variable, like α, and the non-membership function with an additional variable, like β.

$$Maximum\,(\alpha-\beta)$$

$$Constraints$$

$$\mu_k^I\big(f_k(x)\big)\geq\alpha$$

$$v_k^I\big(f_k(x)\big)\leq\beta$$

$$\alpha+\beta\leq 1$$

$$\alpha\geq\beta$$

$$\beta\geq 0$$

$$g_j\ \leq b_j$$

$$x\geq 0$$

$$k\,=1,2,\dots,k.$$

$$j=1,2,\dots,m. \quad \textbf{(7)}$$

Variables α and β are defined in the range [0,1]. α represents the smallest membership degree, while β represents the largest non-membership degree. α maximizes the smallest membership degree and β minimizes the largest non-membership degree. Thus, thanks to the linear model obtained, the problems can be easily solved.

B. **Neutrosophic Fuzzy Multi-Objective Optimization Technique**

The Neutrosophic fuzzy multi-objective linear programming problems are shown in equation (8) [26].

$$max=[f_1^N(x),f_2^N(x),\dots,f_{k1}^N(x)\,]$$

$$min=[f_{k_1+1}^N(x),f_{k_1+2}^N(x),\dots,f_k^N(x)\,]$$

$$Constraints$$

$$g_i(x)\leq c_i,\quad i=1,2,\dots,m_1$$

$$g_i(x)\geq c_i,\quad i=m_1+1,m_1+2,\dots,m_2 \quad \textbf{(8)}$$

$$g_i(x) = c_i, \quad i = m_2 + 1, m_2 + 2, \dots, m_2$$

$$x \geq 0$$

The neutrosophic fuzzy multi-objective optimization model, the neutrosophic fuzzy decision set D^N, is defined as the combination of neutrosophic fuzzy objectives (${G_k}^N$) and neutrosophic fuzzy constraints (${C_j}^N$) as in equation (9) [27].

$$D^N = \left(\bigcap_{k=1}^{p} {G_k}^N\right) \cap \left(\bigcap_{k=1}^{q} {C_j}^N\right) = \{x, \mu_{D^N}(x), \sigma_{D^N}(x), \nu_{D^N}(x)\} \quad \textbf{(9)}$$

For $\forall x \in X$, the membership functions of the neutrosophic fuzzy decision set are defined as shown in equation (10).

$$\mu_{D^{\tilde{N}}}(x) = min(\mu_{G1}{}^N(x), \mu_{G2}{}^N(x), \dots, \mu_{\mu_{Gp}}{}^N(x);\ \mu_{C1}{}^N(x), \mu_{C2}{}^N(x), \dots, \mu_{Cq}{}^N(x))$$

$$\sigma_{D^{\tilde{N}}}(x) = min(\sigma_{G1}{}^N(x), \sigma_{G2}{}^N(x), \dots, \sigma_{Gq}{}^N(x);\ \sigma_{C1}{}^N(x), \sigma_{C2}{}^N(x), \dots, \sigma_{Cq}{}^N(x))$$

$$\nu_{D^{\tilde{N}}}(x) = max(\nu_{G1}{}^N(x), \nu_{G2}{}^N(x), \dots, \nu_{Gp}{}^N(x);\ \nu_{C1}{}^N(x), \nu_{C2}{}^N(x), \dots, \nu_{Cq}{}^N(x)) \quad \textbf{(10)}$$

Where;

$\mu_{D^{\tilde{N}}}(x)$: Decision set for the truth membership function,

$\sigma_{D^{\tilde{N}}}(x)$: Decision set for the indeterminacy membership function,

$\nu_{D^{\tilde{N}}}(x)$: Decision set for the falsity membership function,

$\mu_{Gp}{}^{\tilde{N}}(x)$: Goal set for the truth membership function,

$\sigma_{Gp}{}^{\tilde{N}}(x)$: Goal set for the indeterminacy membership function,

$\nu_{Gp}{}^{\tilde{N}}(x)$: Goal set for the falsity membership function,

$\mu_{Cj}{}^{\tilde{N}}(x)$: Constraint set for the truth membership function,

$\sigma_{Cj}{}^{\tilde{N}}(x)$: Constraint set for the indeterminacy membership function,

$\nu_{Cj}{}^{\tilde{N}}(x)$: Constraint set for the falsity membership function.

The neutrosophic fuzzy multi-objective optimization problems given in Equation (10) are transformed into the model specified in Equation (11) by adding variables like α, γ, β to the model after the membership functions of the objectives and constraints are determined.

$Max\ \alpha$

$Max\ \gamma$

$Min\ \beta$

$Constraints$

$\mu_{G_k^N}(x) \geq \alpha$

$\mu_{C_k^N}(x) \geq \alpha$

$\sigma_{G_k^N}(x) \geq \gamma$

$\sigma_{C_k^N}(x) \geq \gamma$

$\nu_{G_k^N}(x) \leq \beta$

$\nu_{C_k^N}(x) \leq \beta$

$k = 1,2, \dots, p.$

$\alpha + \gamma + \beta \leq 3$

$\alpha \geq \beta$

$\alpha \geq \gamma$

$\alpha, \gamma, \beta \in [0,1]$

(11)

The α, γ, β variables added to the model can be defined as the degree of satisfaction of the membership functions of the objective functions. These variables are useful in transforming the neutrosophic fuzzy multi-objective optimization model into a single-objective optimization model making it easy to solve neutrosophic fuzzy multi-objective programming problems with linear programming techniques [18].
The steps followed in solving the multi-objective neutrosophic fuzzy optimization problem are outlined hereunder [18, 16].

Step 1: The neutrosophic fuzzy multi-objective optimization problem is formulated as shown in equation (8).

Step 2: Any one of the objective functions of the problem is selected and solved using the classical linear programming technique following all the constraints of the problem until ideal solutions are obtained while ignoring the other objectives.

Step 3: Step 2 is repeated for all other objective functions, and the values of the objective functions and decision variables are determined.

Step 4: To create membership functions, goals, and constraints are first determined. For this, the pay-off matrix (pay-off table) is obtained using the ideal solutions obtained in Step 2 [18].

$$\begin{vmatrix} f_1(x^1)^* \; f_2(x^1) \ldots \ldots \ldots f_p(x^1) \\ f_1(x^2) \; f_2(x^2)^* \ldots \ldots \ldots f_p(x^2) \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ f_1(x^p) \; f_2(x^p) \ldots \ldots \ldots f_p(x^p)^* \end{vmatrix}$$

The diagonal values of the pay-off matrix given above show the best value that each objective can get. In multi-objective optimization problems, it is easy to find the best value an objective can get. On the contrary, it is not easy to find the worst solution value of a goal. The pay-off matrix, which is obtained by optimizing each objective independently, allows one to easily find the worst value that an objective can get. In this way, the intervals in which the objective functions are found can be easily calculated.

Step 5: The best and worst values for each objective function are obtained from the pay-off matrix. These values are then used to determine the lower and upper limits of the membership functions.

The lower and upper limits of the truth, indeterminacy, and falsity functions differ according to the type of the objective function. Table 5 gives the formulas for calculating the lower and upper limit values of the membership functions for objective functions with a maximum type.

Table 5 *Determination of the Upper and Lower Limits of Neutrosophic Fuzzy Membership Functions with a Maximization Type*

Objective Function Type	**Objective Function with a Maximization Type**	
Membership Function	**Lower Limit**	**Upper Limit**
Truth Membership Function (μ)	$L_k^{\mu}= \min\{f_k(x_r^*)\}$	$U_k^{\mu}=\max\{f_k(x_r^*)\}$
Indeterminacy Membership Function (σ)	$L_k^{\sigma}=L_k^{\mu}+\lambda_1(U_k^{\mu} - L_k^{\mu})$	$U_k^{\sigma}=U_k^{\mu}$
Falsity Membership Function (ν)	$L_k^{\nu} = L_k^{\mu}$	$U_k^{\nu}=L_k^{\mu} + \lambda_2(U_k^{\mu} - L_k^{\mu})$

Table 6 gives the formulas for calculating the lower and upper limit values of the membership functions for objective functions with a minimum type.

Table 6 *Determination of the Upper and Lower Limits of Neutrosophic Fuzzy Membership Functions with a Minimization Type*

Objective Function Type	**Objective Function with a Minimization Type**	
Membership Function	**Lower Limit**	**Upper Limit**
Truth Membership Function (μ)	$L_k^{\mu}= \min\{f_k(x_r^*)\}$	$U_k^{\mu} = \max\{f_k(x_r^*)\}$
Indeterminacy Membership Function (σ)	$L_k^{\sigma}=L_k^{\mu}$	$U_k^{\sigma}=L_k^{\mu}+\lambda_1(U_k^{\mu} - L_k^{\mu})''$
Falsity Membership Function (ν)	$L_k^{\nu}=L_k^{\mu}+\lambda_2(U_k^{\mu} - L_k^{\mu})$	$U_k^{\nu} = U_k^{\mu}$

The L_k and U_k in the equations show the lower and upper limits of each objective. U_k (upper), shows the best value for maximization problems, and L_k (lower) shows the best value for minimization problems. Regardless of whether the objective function is in the maximization or minimization type, each objective value falls between the lower limit and the upper limit, as shown in equation (12) [28].

$$L_k \leq f_k(x) \leq U_k \tag{12}$$

The λ_1 and λ_2 values in the equations given in Tables 5 and 6 are the tolerance variables chosen by the decision maker to determine the indeterminacy and falsity membership functions, respectively [16]. These values are in the range [0,1] and are taken to be λ_1= 0,5 and λ_2=0,3 for this study.

Step 6: The truth, indeterminacy, and falsity functions can be formed using the lower and upper limits obtained in Table 5 and Table 6. The determination of the membership functions is done according to the type of objective functions as shown in the equations below [18].

For objective functions with a maximization type, the truth membership function is defined as shown in equation (13).

$$\mu_k(f_k(x)) = \begin{cases} 0 & , \quad f_k(x) \leq L_k^{\mu} \quad ise \\ \dfrac{f_k(x) - L_k^{\mu}}{U_k^{\mu} - L_k^{\mu}} & , \quad L_k^{\mu} \leq f_k(x) \leq U_k^{\mu} \quad ise \\ 1 & , \quad f_k(x) \geq U_k^{\mu} \quad ise \end{cases} \tag{13}$$

If the objective function has a maximization type, the indeterminacy membership function is defined as shown in equation (14).

$$\sigma_k(f_k(x)) = \begin{cases} 0 & , \quad f_k(x) \leq L_k^{\sigma} \quad ise \\ \dfrac{f_k(x) - L_k^{\sigma}}{U_k^{\sigma} - L_k^{\sigma}} & , \quad L_k^{\sigma} \leq f_k(x) \leq U_k^{\sigma} \quad ise \\ 1 & , \quad f_k(x) \geq U_k^{\sigma} \quad ise \end{cases} \tag{14}$$

For objective functions with a maximization type, the falsity membership function is defined as shown in equation (15).

$$\nu_k(f_k(x)) = \begin{cases} 1 & , \quad f_k(x) \leq L_k^{\nu} \quad ise \\ \dfrac{U_k^{\nu} - f_k(x)}{U_k^{\nu} - L_k^{\nu}} & , \quad L_k^{\nu} \leq f_k(x) \leq U_k^{\nu} \quad ise \\ 0 & , \quad f_k(x) \geq U_k^{\nu} \quad ise \end{cases} \tag{15}$$

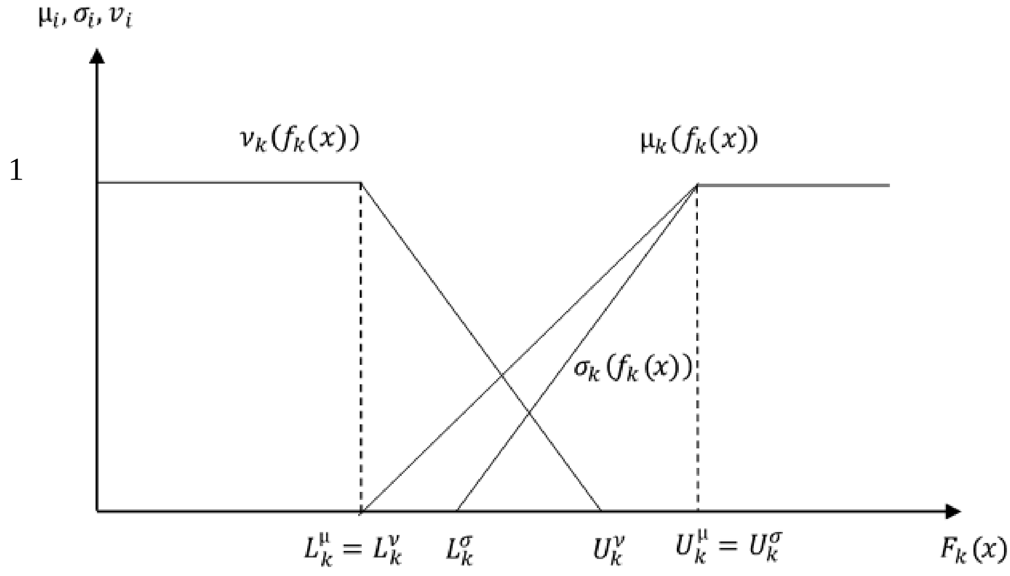

Figure 3 *Membership Functions in Neutrosophic Fuzzy Sets with Maximization Type*

Figure 3 shows the truth, 'indeterminacy, and falsity membership functions for neutrosophic fuzzy sets with a maximization type [29].

If the objective function has a minimization type, the membership functions are defined as shown in the equations below [16].

The truth membership function where the objective function has a minimization type is defined as shown in equation (16).

$$\mu_k(f_k(x)) = \begin{cases} 1 & , \quad f_k(x) \leq L_k^{\mu} \quad ise \\ \dfrac{U_k^{\mu} - f_k(x)}{U_k^{\mu} - L_k^{\mu}} & , \quad L_k^{\mu} \leq f_k(x) \leq U_k^{\mu} \quad ise \\ 0 & , \quad f_k(x) \geq U_k^{\mu} \quad ise \end{cases} \tag{16}$$

The indeterminacy membership function where the objective function has a minimization type is defined as shown in equation (17).

$$\sigma_k(f_k(x)) = \begin{cases} 1 & , \quad f_k(x) \leq L_k^{\sigma} \quad ise \\ \dfrac{U_k^{\sigma} - f_k(x)}{U_k^{\sigma} - L_k^{\sigma}} & , \quad L_k^{\sigma} \leq f_k(x) \leq U_k^{\sigma} \quad ise \\ 0 & , \quad f_k(x) \geq U_k^{\sigma} \quad ise \end{cases} \tag{17}$$

The falsity membership function where the objective function has a minimization type is defined as shown in equation (18).

$$\nu_k(f_k(x)) = \begin{cases} 0 & , \quad f_k(x) \leq L_k^{\nu} \quad ise \\ \dfrac{f_k(x) - L_k^{\nu}}{U_k^{\nu} - L_k^{\nu}} & , \quad L_k^{\nu} \leq f_k(x) \leq U_k^{\nu} \quad ise \\ 1 & , \quad f_k(x) \geq U_k^{\nu} \quad ise \end{cases} \tag{18}$$

Where;

L_k^{μ}: The lower limit for the truth membership function

U_k^{μ}: The upper limit for the truth membership function

L_k^{σ}: The lower limit for the indeterminacy membership function

U_k^{σ}: The upper limit for the indeterminacy membership function

L_k^{ν}: The lower limit for the falsity membership function

U_k^{ν}: The upper limit for the falsity membership function

$f_k(x): k.$ Goal function

Figure 4 shows the truth, indeterminacy, and falsity membership functions for neutrosophic fuzzy sets with a minimization type [16].

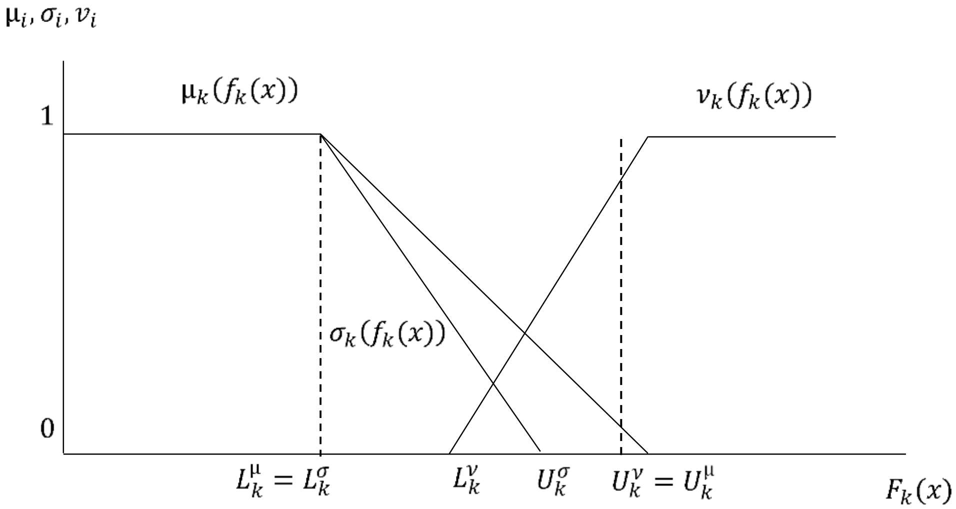

Figure 4 *Membership Functions for Neutrosophic Fuzzy Sets with a Minimization Type*

Step 7: After determining the neutrosophic fuzzy membership functions, the neutrosophic fuzzy multi-objective optimization model is transformed into a single-objective linear optimization model. This transformation is achieved by adding α, γ, β variables to the model. There are different ways of optimizing the membership functions in the neutrosophic fuzzy multi-objective optimization technique

In the neutrophic fuzzy multi-objective optimization technique, membership functions can be optimized in different ways. In this study, we used three models, Model I, Model II, and Model III which are described below.

Model I

According to the Model I approach in Equation (19), the truth membership and indeterminacy membership functions are maximized and the falsity membership function is minimized when solving neutrosophic fuzzy multi-objective optimization problems [30].

$$Max\ (\alpha + \gamma - \beta)$$
$$Constraints$$
$$\mu_k(f_k(x)) \geq \alpha$$
$$\sigma_k(f_k(x)) \geq \gamma$$
$$\nu_k(f_k(x)) \leq \beta$$
$$0 \leq \sigma + \gamma + \beta \leq 3$$
$$\alpha \geq \gamma$$
$$\alpha \geq \beta$$
$$\alpha, \beta, \gamma \in [0,1]$$
$$x \geq 0 \qquad \textbf{(19)}$$

Model II

According to the Model II approach in Equation (20), the truth membership function is maximized while indeterminacy membership and the falsity membership functions are minimized when solving neutrosophic fuzzy multi-objective optimization problems [30].

$$Max\ (\alpha - \gamma - \beta)$$

$$Constraints$$

$$\mu_k(f_k(x)) \geq \alpha$$

$$\sigma_k(f_k(x)) \leq \gamma$$

$$\nu_k(f_k(x)) \leq \beta$$

$$0 \leq \alpha + \gamma + \beta \leq 3$$

$$\alpha \geq \gamma$$

$$\alpha \geq \beta$$

$$\alpha, \gamma, \beta \in [0,1]$$

$$Ax \leq b$$

$$x \geq 0 \qquad \textbf{(20)}$$

Model III

According to the Model III approach in Equation (21), the truth, the indeterminacy, and the falsity membership functions are maximized when solving neutrosophic fuzzy multi-objective optimization problems [18].

$$Max\ (\alpha + \gamma + \beta)$$

$$Constraints$$

$$\mu_k(f_k(x)) \geq \alpha$$

$$\sigma_k(f_k(x)) \geq \gamma$$

$$\nu_k(f_k(x)) \leq \beta$$

$$0 \leq \alpha + \gamma + \beta \leq 3$$

$$\alpha \geq \gamma$$

$$\alpha \geq \beta$$

$$\alpha, \gamma, \beta \ \in [0,1]$$

$$Ax \leq b$$

$$x \geq 0 \qquad \textbf{(21)}$$

5. Multi-Objective Production Planning Problem in a Manufacturing Enterprise

Businesses can stay ahead of the competition by consistently fulfilling the demands of the customers promptly and with the desired features. Production planning plays an integral part in the success of this goal. Planning helps a business take stock of its current situation and decide on the best way to achieve its goals using the available resources. Production planning refers to the set of activities entailed in the determination of the product, the quantity, and the time of production.

This study sought to draw up a production schedule for a company located in the Turkish province of Eskişehir, and manufacturing parts for household appliances. The company was founded in 1994 and uses plastic as its main raw material. The production process requires the use of 62 different pieces of plastic materials which the company secures from different suppliers. The plastic pieces are loaded into different injection machines depending on the product to be made. The company has 53 injection machines to turn plastic particles into the parts that are needed. The flow chart in Figure 5 below shows the production flow for the company.

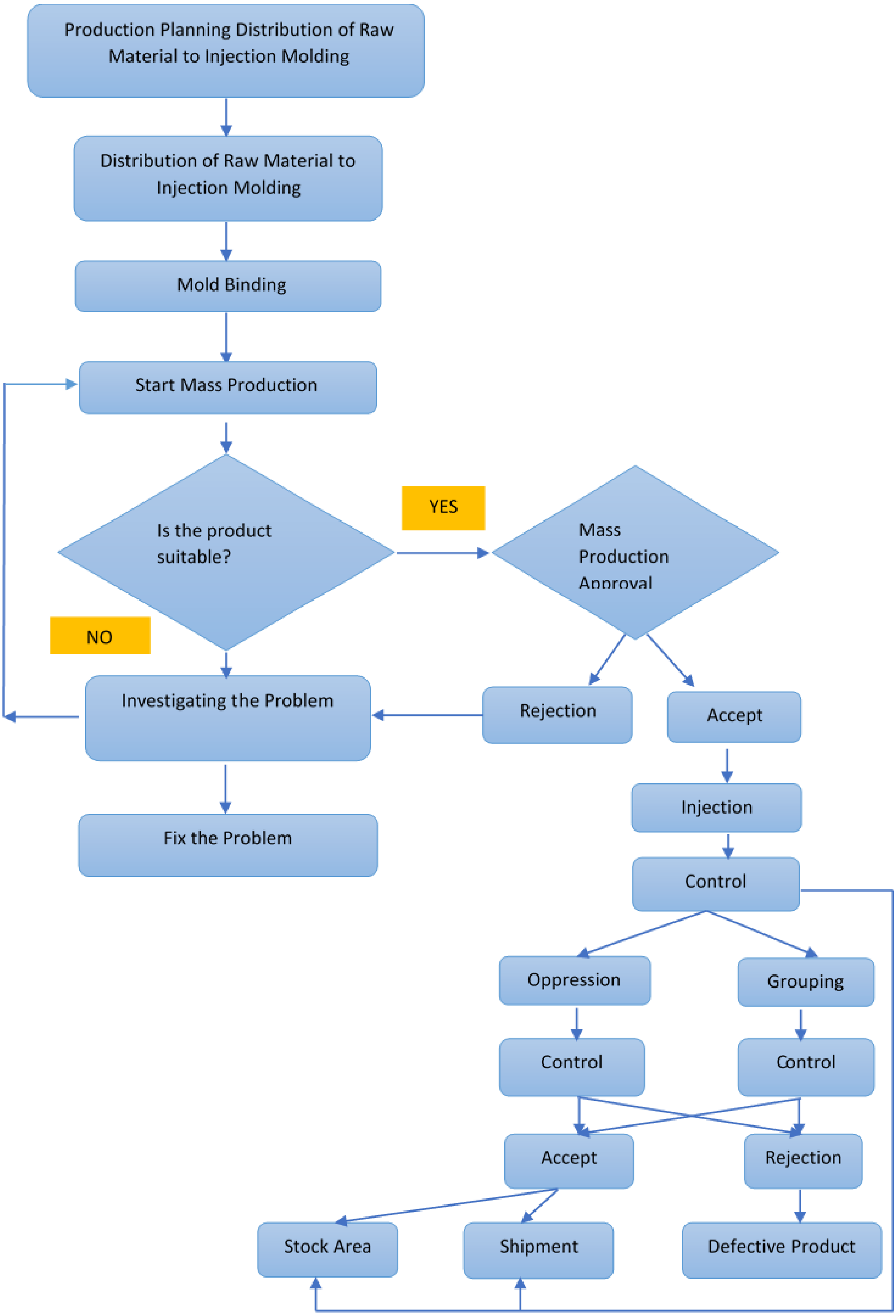
Production Planning Distribution of Raw Material to Injection Molding
Distribution of Raw Material to Injection Molding
Mold Binding
Start Mass Production
Is the product suitable?
YES
Mass Production Approval
NO
Investigating the Problem
Rejection
Accept
Fix the Problem
Injection
Control
Oppression
Grouping
Control
Control
Accept
Rejection
Stock Area
Shipment
Defective Product

Figure 5 *Work Flow Diagram of a Household Appliances Manufacturing Plant*

Terminology

Indices

j : Product type ($j = 1,2, \ldots ,121$)

i: Raw material index ($i = 1,2, \ldots ,62$)

r: Machine index ($r = 1,2, \ldots ,53$)

Decision Variables

x_j :Manufactured items

Parameters

k_j = The profit per unit for one unit x_j (currency)

c_j = production cost of one unit x_j of the product (currency)

s_j = necessary time for the production of one unit x_j of the product (second)

d_j = Defective product rate

f_j = amount of material waste in the production one unit x_j (gram)

T_j = j. The demand for the product

N_i =i. The amount of raw material (gram)

h_{ij} = One unit j. necessary for the product N amount of raw materials (gram)

g_j = To group one unit x_j product necessary time (second)

b_j = To print one unit x_j product necessary time (second)

M_r= r. Machine production capacity (unit)

G = Grouping workshop total labor time

B= Print workshop total labor time

Problem Objectives

The production planning for the production facility takes multiple objectives as its basis. The objectives are outlined below.

1. Production Cost (Z_1): The management of the company where this study was conducted wants the production process to run in a way that minimizes the total cost of production. The general representation of the objective function Z_1 is as shown in equation (22).

$$Min Z_1 = \sum_{j=1}^{n} c_j x_j \quad (22)$$

2. Profit (Z_2): The management wants to achieve a production process that maximizes the profit it gets from the sale of its products. The general representation of the objective function Z_2 is as shown in equation (23).

$$Max Z_2 = \sum_{j=1}^{n} k_j x_j \quad (23)$$

3. Cycle Time (Z_3): Cycle time, in its simplest form, is the time for a product to emerge from the production line. In this study, cycle time refers to the time a product takes to go through the production process, i.e. raw materials go through the injection machine and come out as a product.

The management wants to minimize the total cycle time of the products. The objective function Z_3 is generally represented as shown in equation (24).

$$MinZ_3 = \sum_{j=1}^{n} s_j x_j \tag{24}$$

4. Defective Rates. (Z_4): Breakdown of machinery and equipment, changes in the amount and quality of raw materials, and shortcomings in the competence and skills of the personnel among other reasons may hinder the production process from achieving the desired quality of products, leading to defective products. It is the objective of management to minimize the rate of defective products leaving the process. The general representation of the objective function Z_4 is as shown in equation (25).

$$MinZ_4 = \sum_{j=1}^{n} d_j x_j \tag{25}$$

5. Material Waste Amount (Z_5): The management wants the material loss in the production process to be at the lowest possible level. The general representation of the objective function Z_5 is as shown in equation (26).

$$MinZ_5 = \sum_{j=1}^{n} f_j x_j \tag{26}$$

Constraints:

The production model of the company has 237 constraints under 5 groups. The constraints are outlined below.

1. Demand Constraints

$$x_j \geq T_j$$

2. Raw Material Constraints

$$h_{ij} x_j \leq N_i$$

3. Machine Constraints

$$x_j \leq M_r$$

4. Labor Time Constraint at the Grouping Workshop

$$g_j x_j \leq G$$

5. Print Workshop Total Labor Time Constraint

$$b_j x_j \leq B$$

After these explanations, the multi-objective linear production model for the company is theoretically established as shown in equation (27).

$$\begin{aligned} MinZ_1 &= \textstyle\sum_{j=1}^{n} c_j x_j \\ MinZ_2 &= \textstyle\sum_{j=1}^{n} k_j x_j \\ MinZ_3 &= \textstyle\sum_{j=1}^{n} s_j x_j \\ MinZ_4 &= \textstyle\sum_{j=1}^{n} d_j x_j \end{aligned} \tag{27}$$

$MinZ_5 = \sum_{j=1}^{n} f_j x_j$

$Constraints$

$x_j \geq T_j$

$h_{ij} x_j \leq N_i$

$m_j x_j \leq M_r$

$g_j x_j \leq G$

$b_j x_j \leq B$

and $x_j \geq 0$ and x_j integer j=1,2,…,121.

To solve the multi-objective optimization model, where indeterminacy is in question, using the neutrosophic fuzzy multi-objective optimization technique, the model needs to be converted to a neutrosophic fuzzy type. Each of the objectives in the production model was independently solved, without including the others, using the LINGO 19 package program, and concerning all the constraints of the production model. The constraints of the objective functions were found as follows.

$2.329.133 \leq MinZ_1 \leq 2.742.388$

$1.599.267 \leq MaxZ_2 \leq 1.760.236$

$25.501.660 \leq MinZ_3 \leq 27.526.357$

$99.861{,}89 \leq MinZ_4 \leq 104.081{,}90$

$5.574{,}678 \leq MinZ_5 \leq 5.790{,}102$

The pay-off matrix obtained from solving each objective in the production model independently of the other objectives is given in Table 7.

Table 7 *The Pay-off Matrix for the Production Plan*

	Z_1	Z_2	Z_3	Z_4	Z_5
Min Z_1	2.329.133	1.599.267	25.501.660	99.861,89	5.574,678
Max Z_2	2.742.388	1.760.236	27.526.357	104.081,90	5.790,102
Min Z_3	2.329.133	1.599.267	25.501.660	99.861,89	5.574,678
Min Z_4	2.329.133	1.599.267	25.501.660	99.861,89	5.574,678
Min Z_5	2.329.133	1.599.267	25.501.660	99.861,89	5.574,678

To use the neutrosophic programming with the obtained constraints, the truth, indeterminacy, and falsity membership functions of each objective were constructed as follows.

$$\mu_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) \leq 2.329.133 \\ \dfrac{2.742.388 - Z_1(x)}{413.255} & if \quad 2.329.133 \leq Z_1(x) \leq 2.742.388 \\ 0 & if \quad Z_1(x) \geq 2.742.388 \end{cases}$$

$$\sigma_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) \leq 2.329.133 \\ \dfrac{2.535.760{,}5 - Z_1(x)}{206.627.5} & if \quad 2.329.133 \leq Z_1(x) \leq 2.535.760{,}5 \\ 0 & if \quad Z_1(x) \geq 2.535.760{,}5 \end{cases}$$

$$\nu_1(Z_1(x)) = \begin{cases} 0 & if \quad Z_1(x) \leq 2.453.109.5 \\ \dfrac{Z_1(x) - 2.453109,5}{289.278,5} & if \quad 2.453.109,5 \leq Z_1(x) \leq 2.742.388 \\ 1 & if \quad Z_1(x) \geq 2.742.388 \end{cases}$$

$$\mu_2(Z_2(x)) = \begin{cases} 0 & , \quad if \quad Z_2(x) \leq 1.599.267 \\ \dfrac{Z_2(x) - 1.599.267}{160.969} & , \quad if \quad 1.599.267 \leq Z_2(x) \leq 1.760.236 \\ 1 & , \quad if \quad Z_2(x) \geq 1.760.236 \end{cases}$$

$$\sigma_2(Z_2(x)) = \begin{cases} 0, & if \quad Z_2(x) \leq 1,679,751,5 \\ \dfrac{Z_2(x) - 1.679.751,5}{80.484,5}, & if \quad 1.679.751,5 \leq Z_2(x) \leq 1.760.236 \\ 1, & if \quad Z_2(x) \geq 1.760.236 \end{cases}$$

$$\nu_2(Z_2(x)) = \begin{cases} 1 & , \quad if \quad Z_2(x) \leq 1.599.267 \\ \dfrac{1.647.557,7 - Z_2(x)}{48.290,7} & , \quad if \quad 1.599.267 \leq Z_2(x) \leq 1.647.557,7 \\ 0 & , \quad if \quad Z_2(x) \geq 1.647.557,7 \end{cases}$$

$$\mu_3(Z_3(x)) = \begin{cases} 1 & , \quad if \quad Z_3(x) \leq 25.501.660 \\ \dfrac{27.526.357 - Z_3(x)}{2.024.697} & , \quad if \quad 25.501.660 \leq Z_3(x) \leq 27.526.357 \\ 0 & , \quad if \quad Z_3(x) \geq 27.526.357 \end{cases}$$

$$\sigma_3(Z_3(x)) = \begin{cases} 1, & if \quad Z_3(x) \leq 25.501.660 \\ \dfrac{26.514.008,5 - Z_3(x)}{1.012.348,5}, & if \quad 25.501.660 \leq Z_3(x) \leq 26.514.008,5 \\ 0, & if \quad Z_3(x) \geq 25.501.660 \end{cases}$$

$$\nu_3(Z_3(x)) = \begin{cases} 0 & , \quad if \quad Z_3(x) \leq 26.109.0691 \\ \dfrac{Z_3(x) - 26.109.0691}{1.417.287,9} & , \quad if \quad 26.109.0691 \leq Z_3(x) \leq 27.526.357 \\ 1 & , \quad if \quad Z_3(x) \geq 27.526.357 \end{cases}$$

$$\mu_4(Z_4(x)) = \begin{cases} 1 & , \ if \quad Z_4(x) \le 99.861,89 \\ \dfrac{104.081,90 - Z_4(x)}{4.220,01} & , \ if \quad 99.861,89 \le Z_4(x) \le 104.081,90 \\ 0 & , \ if \quad Z_4(x) \ge 104.081,90 \end{cases}$$

$$\sigma_4(Z_4(x)) = \begin{cases} 1, & if \quad Z_4(x) \le 99.861,89 \\ \dfrac{101.971,895 - Z_4(x)}{2.110,005}, & if \quad 99.861,89 \le Z_4(x) \le 101.127,895 \\ 0, & if \quad Z_4(x) \ge 101.127,895 \end{cases}$$

$$v_4(Z_4(x)) = \begin{cases} 0 & , \ if \quad Z_4(x) \le 101.127,893 \\ \dfrac{Z_4(x) - 101.127,893}{2.954,007} & , \ if \quad 101.127,893 \le Z_4(x) \le 104.081,90 \\ 1 & , \ if \quad Z_4(x) \ge 104.081,90 \end{cases}$$

$$\mu_5(Z_5(x)) = \begin{cases} 1 & , \ if \quad Z_5(x) \le 5.574,678 \\ \dfrac{5.790,102 - Z_5(x)}{215,424} & , \ if \quad 5574,678 \le Z_5(x) \le 5.790,102 \\ 0 & , \ if \quad Z_5(x) \ge 5.790,102 \end{cases}$$

$$\sigma_5(Z_5(x)) = \begin{cases} 1, & if \quad Z_5(x) \le 5.574,678 \\ \dfrac{5.682,39 - Z_5(x)}{107,712}, & if \quad 5.574,678 \le Z_5(x) \le 5.682,39 \\ 0, & if \quad Z_5(x) \ge 5.682,39 \end{cases}$$

$$v_5(f_5(x)) = \begin{cases} 0 & , \ if \quad Z_5(x) \le 5.639,3052 \\ \dfrac{Z_5(x) - 5.639,3052}{150,7968} & , \ if \quad 5.639,3052 \le Z_5(x) \le 5.790,102 \\ 1 & , \ if \quad Z_5(x) \ge 5.790,102 \end{cases}$$

The neutrosophic fuzzy multi-objective optimization model was transformed into the classical linear programming model using the created membership functions. After determining the neutrosophic fuzzy membership functions, the neutrosophic fuzzy multi-objective optimization problems were solved independently using the Model I, Model II, and Model III approaches outlined in the previous section.

6. Problem Solution and Findings

In this section, the solution of the production problem with different approaches will be discussed.

a- Neutrosophic Fuzzy Multi-Objective Optimization Model I Approach

According to the Neutrophic fuzzy multi-objective optimization technique Model I approach, the truth membership and indeterminacy membership functions are maximized and the falsity membership function is minimized as follows.

$\boldsymbol{Max}\ (\boldsymbol{\alpha}+\boldsymbol{\gamma}-\boldsymbol{\beta})$

$Constraints$

$\mu_1(Z_1(x)) \geq \alpha$

$\sigma_1(Z_1(x)) \geq \gamma$

$\nu_1(Z_1(x)) \leq \beta$

$\mu_2(Z_2(x)) \geq \alpha$

$\sigma_2(Z_2(x)) \geq \gamma$

$\nu_2(Z_2(x)) \leq \beta$

$\mu_3(Z_3(x)) \geq \alpha$

$\sigma_3(Z_3(x)) \geq \gamma$

$\nu_3(Z_3(x)) \leq \beta$

$\mu_4(Z_4(x)) \geq \alpha$

$\sigma_4(Z_4(x)) \geq \gamma$

$\nu_4(Z_4(x)) \leq \beta$

$\mu_5(Z_5(x)) \geq \alpha$

$\sigma_5(Z_5(x)) \geq \gamma$

$\nu_5(Z_5(x)) \leq \beta$

$x_j \geq T_j$

$h_{ij}x_j \leq N_i$

$m_j x_j \leq M_r$

$g_j x_j \leq G$

$b_j x_j \leq B$

$0 \leq \sigma + \gamma + \beta \leq 3$

$\alpha \geq \gamma$

$\alpha \geq \beta$

$\alpha, \beta, \gamma \in [0,1]$

and $x_j \geq 0$ and x_j integer j=1,2,…,121

b- Neutrosophic Fuzzy Multi-Objective Optimization Model II Approach

According to the Model II approach, which is a neutrosophic fuzzy multi-objective optimization technique, the constraints of the membership functions of the production model do not change, rather it is the degree of optimization of the membership functions that changes. Accordingly, the maximization of the truth membership function and the minimization of the indeterminacy and the falsity functions are shown below.

$\boldsymbol{Max}\ (\boldsymbol{\alpha}-\boldsymbol{\gamma}-\boldsymbol{\beta})$

$Constraints$

$\mu_1(Z_1(x)) \geq \alpha$

$\sigma_1(Z_1(x)) \leq \gamma$

$\nu_1(Z_1(x)) \leq \beta$

$\mu_2(Z_2(x)) \geq \alpha$

$\sigma_2(Z_2(x)) \leq \gamma$
$\nu_2(Z_2(x)) \leq \beta$
$\mu_3(Z_3(x)) \geq \alpha$
$\sigma_3(Z_3(x)) \leq \gamma$
$\nu_3(Z_3(x)) \leq \beta$
$\mu_4(Z_4(x)) \geq \alpha$
$\sigma_4(Z_4(x)) \leq \gamma$
$\nu_4(Z_4(x)) \leq \beta$
$\mu_5(Z_5(x)) \geq \alpha$
$\sigma_5(Z_5(x)) \leq \gamma$
$\nu_5(Z_5(x)) \leq \beta$
$x_j \geq T_j$
$h_{ij} x_j \leq N_i$
$m_j x_j \leq M_r$
$g_j x_j \leq G$
$b_j x_j \leq B$
$0 \leq \sigma + \gamma + \beta \leq 3$
$\alpha \geq \gamma$
$\alpha \geq \beta$
$\alpha, \beta, \gamma \in [0,1]$
and $x_j \geq 0$ and x_j integer j=1,2,…,121

c- Neutrosophic Fuzzy Multi-Objective Optimization Model III Approach

According to the Neutrosophic fuzzy multi-objective optimization technique Model III approach, the maximization of the truth, indeterminacy, and falsity membership functions are obtained as follows.

$\boldsymbol{Max\ (\alpha + \gamma + \beta)}$
$Constraints$
$\mu_1(Z_1(x)) \geq \alpha$
$\sigma_1(Z_1(x)) \geq \gamma$
$\nu_1(Z_1(x)) \leq \beta$
$\mu_2(Z_2(x)) \geq \alpha$
$\sigma_2(Z_2(x)) \geq \gamma$
$\nu_2(Z_2(x)) \leq \beta$
$\mu_3(Z_3(x)) \geq \alpha$
$\sigma_3(Z_3(x)) \geq \gamma$
$\nu_3(Z_3(x)) \leq \beta$
$\mu_4(Z_4(x)) \geq \alpha$
$\sigma_4(Z_4(x)) \geq \gamma$
$\nu_4(Z_4(x)) \leq \beta$
$\mu_5(Z_5(x)) \geq \alpha$
$\sigma_5(Z_5(x)) \geq \gamma$
$\nu_5(Z_5(x)) \leq \beta$

$x_j \geq T_j$
$h_{ij} x_j \leq N_i$
$m_j x_j \leq M_r$
$g_j x_j \leq G$
$b_j x_j \leq B$
$0 \leq \sigma + \gamma + \beta \leq 3$
$\alpha \geq \gamma$
$\alpha \geq \beta$
$\alpha, \beta, \gamma \in [0,1]$
and $x_j \geq 0$ and x_j integer j=1,2,…,121

d- Intuitionistic Fuzzy Multi-Objective Optimization

The intuitionistic fuzzy membership and non-membership functions of the objectives of the production problem obtained from the pay-off matrix in Table 7 are given below.

$$\mu_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) \leq 2.329.133 \\ \dfrac{2.742.388 - Z_1(x)}{413.255} & if \quad 2.329.133 \leq Z_1(x) \leq 2.742.388 \\ 0 & if \quad Z_1(x) \geq 2.742.388 \end{cases}$$

$$\nu_1(Z_1(x)) = \begin{cases} 0 & if \quad Z_1(x) \leq 2.453.109.5 \\ \dfrac{Z_1(x) - 2.453109,5}{289.278,5} & if \quad 2.453.109,5 \leq Z_1(x) \leq 2.742.388 \\ 1 & if \quad Z_1(x) \geq 2.742.388 \end{cases}$$

$$\mu_2(Z_2(x)) = \begin{cases} 0 & , \; if \quad Z_2(x) \leq 1.599.267 \\ \dfrac{Z_2(x) - 1.599.267}{160.969} & , \; if \quad 1.599.267 \leq Z_2(x) \leq 1.760.236 \\ 1 & , \; if \quad Z_2(x) \geq 1.760.236 \end{cases}$$

$$\nu_2(Z_2(x)) = \begin{cases} 1 & , \; if \quad Z_2(x) \leq 1.599.267 \\ \dfrac{1.711.945,3 - Z_2(x)}{48.290,7} & , \; if \quad 1.599.267 \leq Z_2(x) \leq 1.711.945,3 \\ 0 & , \; if \quad Z_2(x) \geq 1.711.945,3 \end{cases}$$

$$\mu_3(Z_3(x)) = \begin{cases} 1 & , \; if \quad Z_3(x) \leq 25.501.660 \\ \dfrac{27.526.357 - Z_3(x)}{2.024.697} & , \; if \quad 25.501.660 \leq Z_3(x) \leq 27.526.357 \\ 0 & , \; if \quad Z_3(x) \geq 27.526.357 \end{cases}$$

$$\nu_3(Z_3(x)) = \begin{cases} 0 & , \; if \quad Z_3(x) \leq 26.109.0691 \\ \dfrac{Z_3(x) - 26.109.069,1}{1.417.287,9} & , \; if \quad 26.109.0691 \leq Z_3(x) \leq 27.526.357 \\ 1 & , \; if \quad Z_3(x) \geq 27.526.357 \end{cases}$$

$$\mu_4(Z_4(x)) = \begin{cases} 1 & , \ if \quad Z_4(x) \leq 99.861,89 \\ \dfrac{104.081,90 - Z_4(x)}{4.220,01} & , \ if \quad 99.861,89 \leq Z_4(x) \leq 104.081,90 \\ 0 & , \ if \quad Z_4(x) \geq 104.081,90 \end{cases}$$

$$v_4(f_4(x)) = \begin{cases} 0 & , \ if \quad Z_4(x) \leq 101.127,893 \\ \dfrac{Z_4(x) - 101.127,893}{2.954,007} & , \ if \quad 101.127,893 \leq Z_4(x) \leq 104.081,90 \\ 1 & , \ if \quad Z_4(x) \geq 104.081,90 \end{cases}$$

$$\mu_5(Z_5(x)) = \begin{cases} 1 & , \ if \quad Z_5(x) \leq 5.574,678 \\ \dfrac{5.790,102 - Z_5(x)}{215,424} & , \ if \quad 5574,678 \leq Z_5(x) \leq 5.790,102 \\ 0 & , \ if \quad Z_5(x) \geq 5.790,102 \end{cases}$$

$$v_5(f_5(x)) = \begin{cases} 0 & , \ if \quad Z_5(x) \leq 5.639,3052 \\ \dfrac{Z_5(x) - 5.639,3052}{150,7968} & , \ if \quad 5.639,3052 \leq Z_5(x) \leq 5.790,102 \\ 1 & , \ if \quad Z_5(x) \geq 5.790,102 \end{cases}$$

$\boldsymbol{Max\ (\alpha - \beta)}$

$Constraints$

$\mu_1(Z_1(x)) \geq \alpha$

$v_1(Z_1(x)) \leq \beta$

$\mu_2(Z_2(x)) \geq \alpha$

$v_2(Z_2(x)) \leq \beta$

$\mu_3(Z_3(x)) \geq \alpha$

$v_3(Z_3(x)) \leq \beta$

$\mu_4(Z_4(x)) \geq \alpha$

$v_4(Z_4(x)) \leq \beta$

$\mu_5(Z_5(x)) \geq \alpha$

$v_5(Z_5(x)) \leq \beta$

$x_j \geq T_j$

$h_{ij}x_j \leq N_i$

$m_j x_j \leq M_r$

$g_j x_j \leq G$

$b_j x_j \leq B$

$0 \leq \sigma + \beta \leq 1$

$\alpha \geq \beta$

and $x_j \geq 0$ and x_j integer j=1,2,…,121

The production problem is created and solved with intuitionistic fuzzy sets as given in equation 7. Comparison results are given in Table 8.

Table 8 *Comparison of the Results of the Objective Functions from Different Solution Techniques*

Solution Techniques / Objective Functions	Neutrosophic Fuzzy Multi-Objective Optimization Technique Model I Approach	Neutrosophic Fuzzy Multi-Objective Optimization Technique Model II Approach	Neutrosophic Fuzzy Multi-Objective Optimization Technique Model III Approach	Intuitionistic Fuzzy Multi-Objective Optimization Technique
Z_1	2.517.602	2.517.905	2.517.602	2.546.347
Z_2	1.686.825	1.686.707	1.686.825	1.696.381
Z_3	26.425.010	26.426.520	26.425.010	26.565.750
Z_4	101.786,4	101.789,6	101.786,4	102.079,4
Z_5	5.658,624	5.673,082	5.658.620	5.680,644

According to the solution results in Table 8, the neutrosophic fuzzy multi-objective optimization technique under Models I-II-III gave more optimal results for the objective functions than the intuitionistic fuzzy multi-objective optimization technique. However, the objective function Z_2, which seeks profit maximization, was found to be more optimal under the intuitionistic fuzzy multi-objective optimization technique. The analysis further shows that Model I and Model III in which the indeterminacy membership function is maximized produced more effective results than Model II in which the indeterminacy membership function is minimized.

7. CONCLUSION

This study examined the multi-objective planning problem and sought a solution for a manufacturing company that produces parts for household appliances. The study drew up a one-month multi-objective, multi-item production planning problem and formulated a solution using intuitionistic fuzzy and neutrosophic multi-objective programming. The results of the study show that the neutrosophic fuzzy programming approach provides a better solution than the intuitionistic fuzzy approach.

The study demonstrates the applicability of intuitionistic fuzzy and neutrosophic fuzzy set approaches in multi-objective, multi-item complex production planning problems. Future studies could try different types of fuzzy sets to solve similar and complex multi-objective problems.

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