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Evaluation of Sustainable Waste Valorization using TreeSoft Set with Neutrosophic Sets

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Abstract
This study proposed a neutrosophic set framework with TreeSoft Set for sustainable waste valorization selection. The neutrosophic set is used to overcome uncertainty and vague information in the evaluation process. The neutrosophic set has three membership degrees: truth, indeterminacy, and falsity. The multi-criteria decision-making (MCDM) methodology deals with various criteria to evaluate waste valorization. The VIKOR method is an MCDM method used to rank the alternatives. The numerical example was created with 12 criteria and 10 alternatives. Three decision-makers and experts are invited to evaluate the requirements and options. We used the bipolar neutrosophic numbers to replace the opinions of experts.

Keywords: TreeSoft Set, Neutrosophic Set; Waste Valorization, Sustainability.

1. Introduction

The expanding number of people and the handling of waste both show positive correlations, with the former rising due to the increased use of goods to meet demands. The increasing prevalence of trash and substantial waste, which includes both organic and inorganic materials, poses significant environmental risks, including releasing greenhouse gases into the atmosphere, contaminating land, and contaminating underground water supplies. The World Bank predicts that 2.01 billion tons of municipal solid waste (MSW) were generated globally in 2018, with 33% of that garbage requiring disposal that is ecologically friendly[1].

According to projections, by 2050, there will be 3.40 billion tonnes of MSW worldwide. Conventional waste disposal techniques, including landfills and incineration, are used worldwide but have an unsustainable quality regarding the financial, ecological, and social aspects because of the significant emissions generated throughout the decomposition process[2]–[4]. Moreover, problems emerging from the waste treatment life cycle, which includes gathering, transferring, treatment, disposal, and the creation of byproducts, exacerbate sustainability concerns. It is crucial to choose a suitable and sustainable waste breakdown method to mitigate the aforementioned difficulties and guarantee the long-term sustainability of the procedure for handling waste[5]–[7].

Multicriteria decision-making (MCDM) approaches are well suited for comparing the outcomes of the options as the prioritization process encompasses many alternatives and several criteria in the evaluation stages. Numerous physical and intangible criteria may be included in MCDM situations. Due to the ambiguity of the language evaluations, using crisp data for intangible criteria in traditional MCDM approaches may result in an adequate evaluation[8]–[11]. As a result, fuzzy sets have been added to these techniques to improve their suitability in unpredictable environments. Zadeh invented fuzzy sets to indicate an element's partial membership in a set[12], [13].

In 1995, Florentin Smarandache extended intuitionistic fuzzy sets to create neurosophic sets to express uncertainty in the data and the decision maker's indecision. Truth, Indeterminacy, and Falsity, which represent the corresponding degrees of truthiness, indeterminacy, and falsity, make up neutrosophic sets. The preciseness of the information is expressed in the neutrosophic sets by truth (degrees of belongingness), falsehood (non-belongingness) values, and indeterminacy (degree of hesitation) values, which indicate the decision maker's hesitancy. Neutronosophic sets enable insufficient data to be characterized in subsets that may be used to discriminate between relativity and completeness. These characteristics indicate uncertainty and indeterminacy[14]–[16].

2. Methodology

This section introduces some definitions of IVNSs.

Definition 1

Let Y be a universe of discourse with a generic element in Y denoted by y. We can define the neutrosophic variable y as $y = (T, I, F)$ where T,I, and F refer to the degrees of truth, indeterminacy, and falsity membership.

$$0 \leq \sup\big(T(y)\big) + \sup\big(I(y)\big) + \sup\big(F(y)\big) \leq 3 \tag{1}$$

We can define the IVNSs as:

$$y = ([T^L, T^U], [I^L, I^U], [F^L, F^U]) \tag{2}$$

Definition 2

We can deneutrosophication of IVNNs as:

$$D(y) = \left(\frac{T_y^L + T_y^U}{2} + \left(1 - \frac{\left(I_y^L + I_y^U\right)}{2}\right) * \ I_y^U - \left(\frac{F_y^L + F_y^U}{2}\right) * \left(1 - F_y^U\right)\right) \tag{3}$$

Definition 3

Let $y_1 = \left([T_{y_1}^L, T_{y_1}^U], [I_{y_1}^L, I_{y_1}^U], [F_{y_1}^L, F_{y_1}^U]\right)$ and $y_2 = \left([T_{y_2}^L, T_{y_2}^U], [I_{y_2}^L, I_{y_2}^U], [F_{y_2}^L, F_{y_2}^U]\right)$ two interval-valued neutrosophic numbers, then some mathematical equations can be defined as:

$$y_1^c = \left([F_{y_1}^L, F_{y_1}^U], [1 - I_{y_1}^U, 1 - I_{y_1}^L], [T_{y_1}^L, T_{y_1}^U]\right) \tag{4}$$

$$y_1 \oplus y_2 = \begin{pmatrix} [T_{y_1}^L + T_{y_2}^L - T_{y_1}^L T_{y_2}^L, T_{y_1}^U + T_{y_2}^U - T_{y_1}^U T_{y_2}^U], \\ [I_{y_1}^L I_{y_2}^L, I_{y_1}^U I_{y_2}^U], \\ [F_{y_1}^L F_{y_2}^L, F_{y_1}^U F_{y_2}^U] \end{pmatrix} \tag{5}$$

$$y_1 \otimes y_2 = \begin{pmatrix} [T^L_{y_1}T^L_{y_2}, T^U_{y_1}T^U_{y_2}], \\ [I^L_{y_1} + I^L_{y_2} - I^L_{y_1}I^L_{y_2}, I^U_{y_1} + I^U_{y_2} - I^U_{y_1}I^U_{y_2}], \\ [F^L_{y_1} + F^L_{y_2} - F^L_{y_1}F^L_{y_2}, F^U_{y_1} + F^U_{y_2} - F^U_{y_1}F^U_{y_2}] \end{pmatrix} \quad (6)$$

Definition 4

We can define the bipolar neutrosophic sets (BNSs) [17]–[19]as:

$$A = \{< x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) >\} \quad (7)$$

Let $y_1 = (T_1^+(x), I_1^+(x), F_1^+(x), T_1^-(x), I_1^-(x), F_1^-(x))$ and $y_2 = (T_2^+(x), I_2^+(x), F_2^+(x), T_2^-(x), I_2^-(x), F_2^-(x))$

$$y_1 + y_2 = \begin{pmatrix} T_1^+ + T_2^+ - T_1^+T_2^+, I_1^+I_2^+, F_1^+F_2^+, \\ -T_1^-T_2^-, -(-I_1^- - I_2^- - I_1^-I_2^-), -(-F_1^- - F_2^- - F_1^-F_2^-) \end{pmatrix} \quad (8)$$

$$y_1 \cdot y_2 = \begin{pmatrix} T_1^+T_2^+, I_1^+ + I_2^+ - I_1^+I_2^+ + F_1^+ + F_2^+ - F_1^+F_2^+, \\ -(-T_1^- - T_2^- - T_1^-T_2^-), -I_1^-I_2^-, -F_1^-F_2^- \end{pmatrix} \quad (9)$$

Let U be a universe disclosure and H a non-empty subset of U, with $P(H)$ $be\ a\ powerset\ of\ H$.

Let TSR be a set of attributes of the problem (criteria),

$$TSR = \{TSR_1, TSR_2, \ldots, TSR_{\mathrm{n}}\}, n \geq 1 \quad (10)$$

Where $TSR_1, TSR_2, \ldots, TSR_{\mathrm{n}}$ are criteria of the first level of the tree.

Each attribute $TSR_1, 1 \leq i \leq n, is\ formed\ by\ sub-attributes$:

$TSR_1 = \{TSR_{1,1}, TSR_{1,2}. \ldots,\}$

$TSR_2 = \{TSR_{2,1}, TSR_{2,2}. \ldots,\}$

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$TSR_{\mathrm{n}} = \{TSR_{\mathrm{n},1}, TSR_{\mathrm{n},2}. \ldots,\}$

Where $TSR_{\mathrm{i,j}}$ are sub-attributes.

The TreeSoft set can be formed by:

$$F: P(Tree(TSR)) \rightarrow P(H) \quad (11)$$

$Tree(TSR)$ is the set of all nodes and leaves from level 1 to level m and $P(Tree(TSR))$ is the power set of the $Tree(TSR)$.

$$Tree(TSR) = \{TSR_i |\ i_1 = 1,2,3, \ldots\} \cup \{TSR_i |\ i_1, i_2 = 1,2,3, \ldots\} \cup \{TSR_i |\ i_1, i_2, i_3 = 1,2,3, \ldots\} \cup \ldots \cup \{TSR_i | i_1, i_2, \ldots\ i_m = 1,2,3, \ldots\} \quad (12)$$

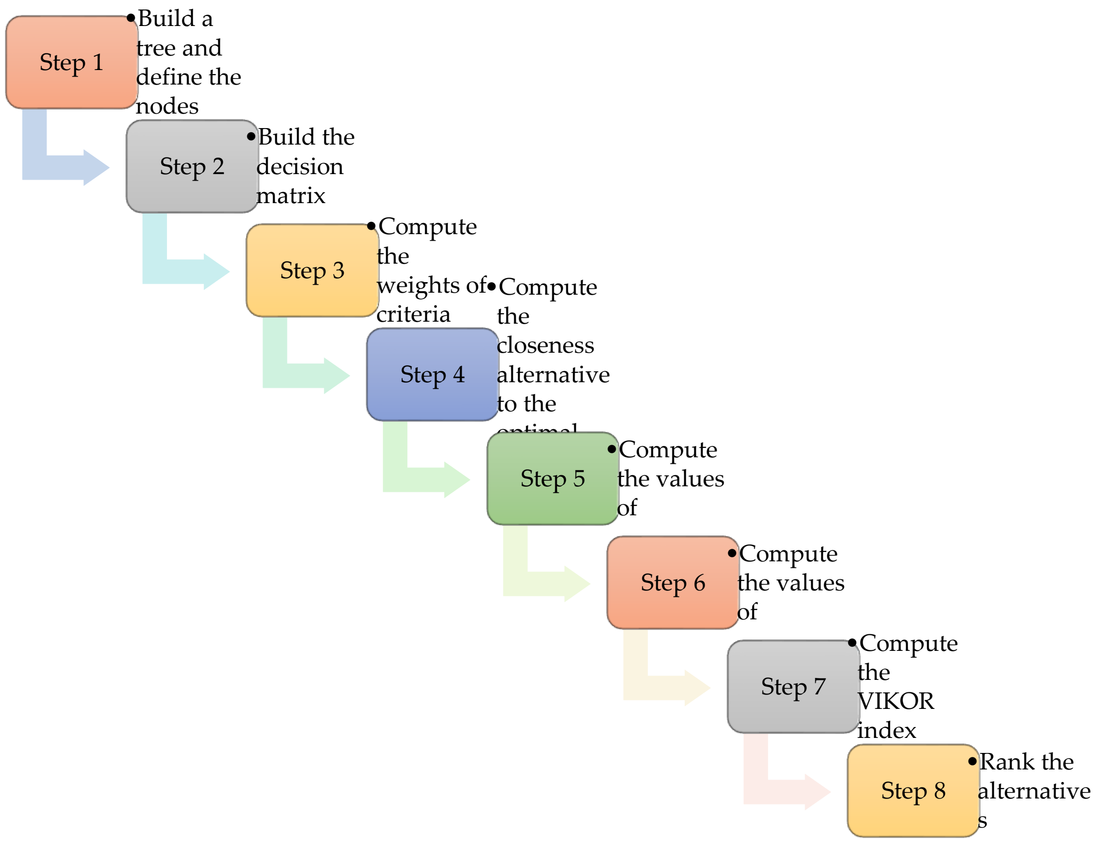

Figure 1. The steps of the proposed methodology.

The next steps of the neutrosophic TreeSoft Set with the VIKOR method as shown in Figure 1.

Step 1. Build a tree and define the nodes [20]–[23].

The tree has more than one level, in the first level, the main criteria and introduced as $SWM_1, SWM_2 \dots, SWM_n$

In the second level, the sub-criteria are introduced as $SWM_{1.1}, SWM_{1.2}, \dots.$ And $SWM_{2.1}, SWM_{2.2}, \dots.$

Step 2. Build the decision matrix

The decision matrix is built by using the information of decision makers and experts between criteria and alternatives.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}; i = 1,2, \dots, m; j = 1,2, \dots, n \quad (13)$$

Step 3. Compute the weights of the criteria.

The weights of the criteria are computed by using the average method.

$$\sum_{j=1}^{n} w_j = 1 \quad (14)$$

Step 4. Compute the closeness alternative to the optimal solution.

$$U_i = \left\{ \sum_{j=1}^{n} \left[\frac{w_j (r_j^* - r_{ij})}{(r_j^* - r_j^-)} \right]^P \right\}^{\frac{1}{p}} i = 1,2, \dots, m; j = 1,2, \dots, n; 1 \leq P \leq \infty \quad (15)$$

Where r_j^* is the best and r_j^- is the worst

$$\begin{cases} r_j^* = \max_i x_{ij} \\ r_j^- = \min_i x_{ij} \end{cases} (positive\ criteria) i = 1,2, \dots, m; j = 1,2, \dots, n \quad (16)$$

$$\begin{cases} r_j^* = \min_i x_{ij} \\ r_j^- = \max_i x_{ij} \end{cases} (negative\ criteria) i = 1,2, \dots, m; j = 1,2, \dots, n \quad (17)$$

Step 5. Compute the values of S_i

$$S_i = \sum_{j=1}^{n} w_j \ \frac{(r_j^* - r_{ij}^-)}{(r_j^* - r_j^-)} \ i = 1,2, \dots, m; j = 1,2, \dots, n \quad (18)$$

Step 6. Compute the values of R_i

$$R_i = \max_j \left[w_j \ \frac{(r_j^* - r_{ij}^-)}{(r_j^* - r_j^-)} \right] i = 1,2, \dots, m; j = 1,2, \dots, n \quad (19)$$

Step 7. Compute the VIKOR index.

$$Q_i = t * \left[\frac{(S_i - S^*)}{S^- - S^*} \right] + (1 - t) * \left[\frac{(R_i - R^*)}{R^- - R^*} \right] i = 1,2, \dots, m; j = 1,2, \dots, n \quad (20)$$

$$S^* = \min_i S_i \, , S^- = \max_i S_i \, , R^* = \min_i R_i \, , R^- = \max_i R_i \quad (21)$$

Where $t = 0.5$

Step 8. Rank the alternatives.

The alternatives are ranked as descending values of Q_i.

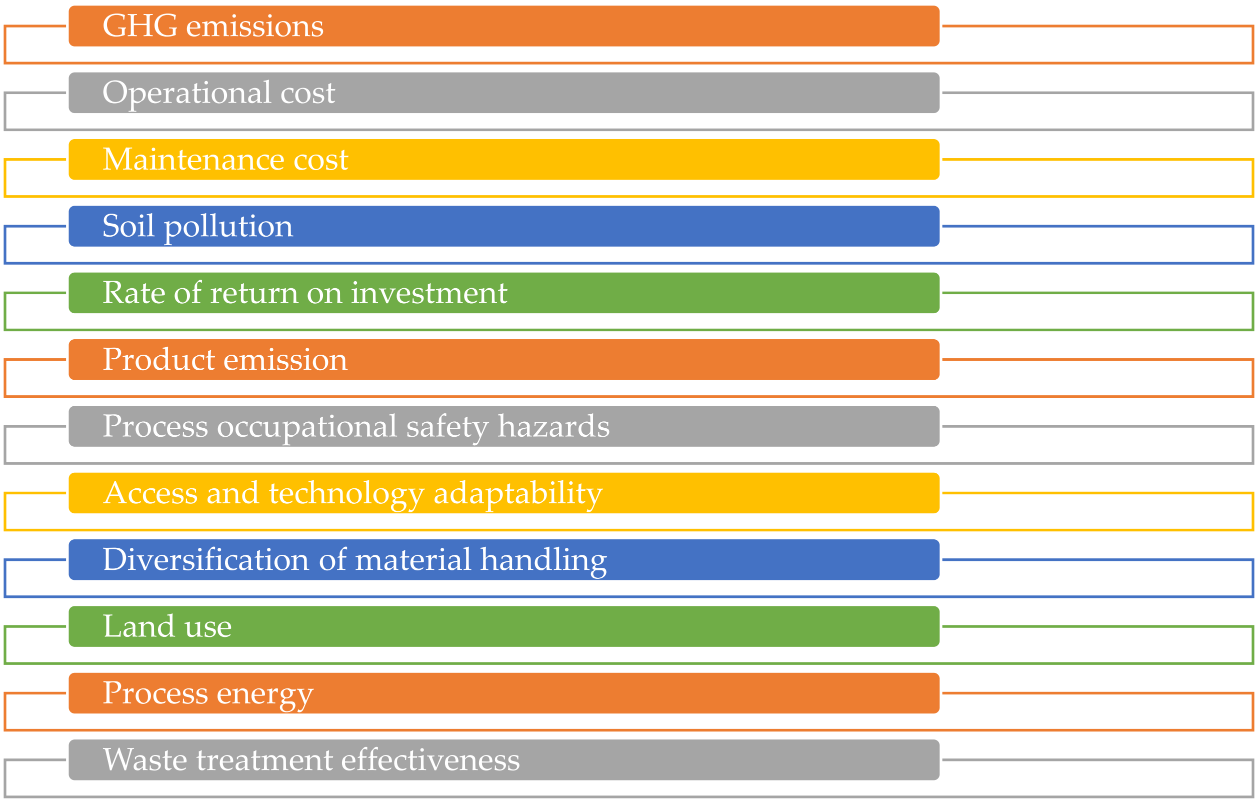

Figure 2. The list of used criteria.

3. Numerical Example

This section shows the results of TreeSoft with the BNS and VIKOR methods. This study used 12 criteria as shown in Figure 2 and ten alternatives. Three experts used the bipolar neutrosophic numbers (BNNs) to evaluate the criteria and alternatives.

Step 1. Build a tree and define the nodes.

The tree has more than one level, in the first level, the main criteria and introduced as $SWM_1, SWM_2 \ldots, SWM_n$

In the second level, the sub-criteria are introduced as $SWM_{1.1}, SWM_{1.2}$, ... And $SWM_{2.1}, SWM_{2.2}$, ...

Step 2. Build the decision matrix using Eq. (13). Table 1 shows the decision matrix.

Table 1. The decision matrix.

	SWM1	SWM2	SWM3	SWM4	SWM5	SWM6	SWM7	SWM8	SWM9	SWM10	SWM11	SWM12
SWA1	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA2	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA3	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA4	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA5	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]
SWA6	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA7	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA8	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA9	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA10	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]

	SWM1	SWM2	SWM3	SWM4	SWM5	SWM6	SWM7	SWM8	SWM9	SWM10	SWM11	SWM12
SWA1	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA2	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA3	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA4	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA5	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]
SWA6	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA7	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA8	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA9	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]
SWA10	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]

	SWM1	SWM2	SWM3	SWM4	SWM5	SWM6	SWM7	SWM8	SWM9	SWM10	SWM11	SWM12
SWA1	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA2	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]
SWA3	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA4	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]
SWA5	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]
SWA6	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]
SWA7	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]
SWA8	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]
SWA9	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]
SWA10	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.90, 0.10, 0.10, −0.40, −0.80, −0.90]	[0.80, 0.50, 0.50, −0.30, −0.80, −0.80]	[0.50, 0.50, 0.50, −0.50, −0.50, −0.50]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]	[0.10, 0.80, 0.70, −0.90, −0.20, −0.10]	[0.40, 0.20, 0.70, −0.50, −0.20, −0.10]

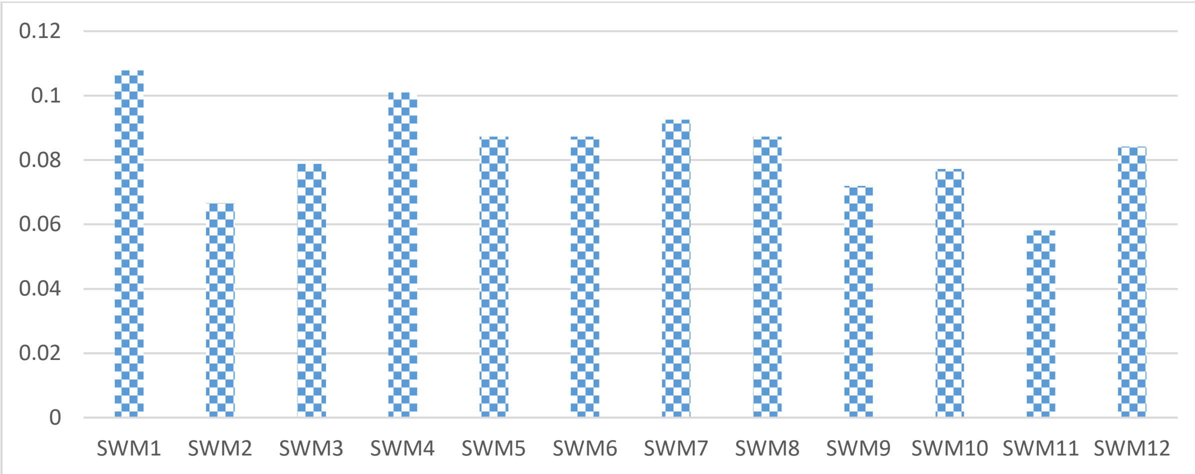

Figure 3. The weights of criteria.

Step 3. Compute the weights of criteria as shown in Figure 3.

Step 4. Compute the closeness alternative to the optimal solution using Eq. (15).

Step 5. Compute the values of S_i using Eq. (18).

Step 6. Compute the values of R_i using Eq. (19).

Step 7. Compute the VIKOR index using Eqs. (20 and 21)

Step 8. Rank the alternatives as shown in Figure 4. Alternative 6 is the best and alternative 8 is the worst.

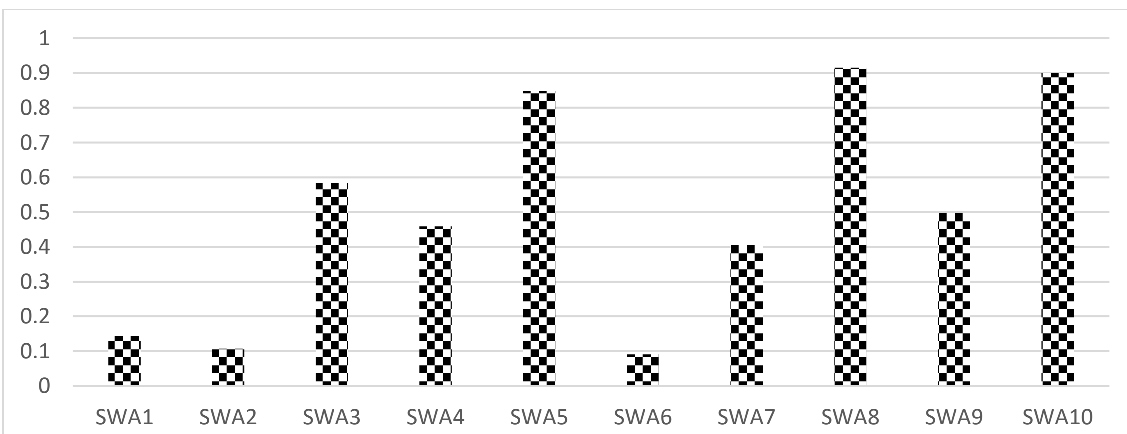

Figure 4. The values of the VIKOR index.

4. Conclusions

This study used the MCDM methodology to evaluate waste valorization. The MCDM methodology is used to deal with various criteria. The VIKOR method is used to rank the alternatives. The MCDM methodology is integrated with a neutrosophic set to deal with uncertainty in the evaluation process. The neutrosophic set and MCDM methodology integrated with TreeSoft Set in the evaluation process. Three decision-makers and experts are invited to rank the criteria and alternatives. We used the BNNs to replace the opinions of experts. Three decision matrices are created using the VIKOR method. We obtain the crisp value in each decision matrix and combine it to get one matrix. We used 12 criteria and 10 alternatives in this study.

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