A Generalization of Neutrosophic Metric Space and Related Fixed Point Results

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Abstract: Neutrosophic set is a generalization of classical sets, fuzzy set, intuitionistic sets etc. A mathematical notion neutrosophic set dealing issues containing inconsistent, indeterminate and imprecise data. In this manuscript, we toss the notion of Neutrosophic b-metric-like spaces and obtain some fixed point results in the sense of Neutrosophic b-metric-like spaces. Our results are improvements of recent results in the existing literature. For the validity of these results some non-trivial examples are imparted.

Keywords: Metric-like spaces; Neutrosophic b-metric-like spaces; fixed point; uniqueness

1. Introduction and Preliminaries

The notion of fuzzy sets (FSs) given by Zadeh [9], this auspicious concept gave a new direction of research and this idea has deeply influenced many scientific fields. In this connectedness, Kramosil and Michalek [10] initiated the notion of fuzzy metric spaces (FMSs) by reformulate the notion of probabilistic metric spaces to FMSs. George and Veeramani [11] derived a Hausdorff topology originated by FMS. Afterward, the utility of FMS appeared in applied sciences such as fixed point theory, image and signal processing, medical imaging, decision-making. Later on, the existence theory of fixed point in FMS has been enriched with a number of altered generalizations. Fuzzy version of Banach contraction principle was given by Garbiec [12] in the sense of FMS. For some necessary concepts please see [13, 14].

In recent times, Harandi [7] originated the notion of metric-like spaces (MLS), which generalized the concept of metric spaces in beautiful manners. In this connectedness, Shukla and Abbas [8] reformulated the notion of (MLS) and originated fuzzy metric-like spaces (FMLS). The approach of intuitionistic FMS was tossed by Park in [2]. Kirişci and Simsek [1] generalized the approach of intuitionistic FMS and tossed the approach of neutrosophic metric space (NMS). Simsek, Kirişci [5] and Sowndrarajan et al. [6] proved some fixed point (FP) results in the setting of NMS.

In this article, we tossed the notion of Neutrosophic b-metric-like space (NBMLS) in which self distance may not be equal to 1, 0 and 0. So, our approach is more generalized in the existing literature. Also this article is enriched with fixed point results and non-trivial examples. For some necessary results see [3, 4, 15]. Authors in [16-20] worked on different generalizations of NMSs and proved several fixed point results.

The main objectives of this manuscript are:

1. To introduce the concept of neutrosophic b-metric-like space.
2. To prove some fixed point results in the sense of neutrosophic b-metric-like space.
3. To enhance existing literature of fuzzy metric spaces and fuzzy fixed point theory.

This manuscript is organized with some rudimentary concepts of FMLS and NMS. The concept of NBMLS is discussed in detail and some fixed point results with non-trivial examples are imparted. A conclusion is provided for the obtained results.

In this section, some basic definitions are given that are helpful to understand the main results.

Shukla and Abbas introduced the concept of FMLS and utilized this idea to investigate fixed point results. Shukla and Abbas defined the notion of FMLS as follows:

Definition 1.1 [8] A 3-tuple $(Q,\Psi,*)$ is said an FMLS if $Q \neq \Phi$ is a random set, $*$ is a continuous t-norm (CTN) and Ψ is a FS on $Q \times Q \times (0,\infty)$ meet the points below for all $\vartheta, \mathfrak{I}, \mu \in Q, \partial, s > 0$:

FL1) $\Psi(\vartheta,\mathfrak{I},\partial) > 0$;

FL2) If $\Psi(\vartheta,\mathfrak{I},\partial) = 1$, then $\vartheta = \mathfrak{I}$;

FL3) $\Psi(\vartheta,\mathfrak{I},\partial) = \Psi(\mathfrak{I},\vartheta,\partial)$;

FL4) $\Psi(\vartheta,\mu,\partial + s) \geq \Psi(\vartheta,\mathfrak{I},\partial) * \Psi(\mathfrak{I},\mu,s)$;

FL5) $\Psi(\vartheta,\mathfrak{I}, \cdot): (0,\infty) \to [0,1]$ is continuous.

Example 1.2 [8] Assume $Q = \mathbb{R}^+$, $a \in \mathbb{R}^+$ and $n > 0$. Define CTN by $\sigma * \kappa = \sigma\kappa$ and the FS Ψ on $Q \times Q \times (0,\infty)$ by

$$\Psi(\vartheta,\mathfrak{I},\partial) = \frac{a\partial}{a\partial + n(\max\{\vartheta,\mathfrak{I}\})}, \forall\, \vartheta,\mathfrak{I} \in Q, \partial > 0.$$

Then $(Q,\Psi,*)$ is a FMLS.

The concept of neutrosophic metric spaces was discussed by Kirişci and Simsek in his work and he defined the said concept as follows:

Definition 1.3 [1] Suppose $Q \neq \emptyset$, assume a six tuple $(Q, \Psi, \Phi, \Omega, *, \circ)$ where $*$ is a CTN, $\circ$ is a continuous t-conorm (CTCN), Ψ, Φ and Ω are (Neutrosophic sets) NSs on $Q \times Q \times (0, \infty)$. If $(Q, \Psi, \Phi, \Omega, *, \circ)$ meet the below circumstances for all $\vartheta, \mathcal{I}, \mu \in Q$ and $\Omega, s > 0$:

(NS1) $\Psi(\vartheta, \mathcal{I}, \partial) + \Phi(\vartheta, \mathcal{I}, \partial) + \Omega(\vartheta, \mathcal{I}, \partial) \leq 3$,

(NS2) $0 \leq \Psi(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS3) $\Psi(\vartheta, \mathcal{I}, \partial) = 1 \iff \vartheta = \mathcal{I}$,

(NS4) $\Psi(\vartheta, \mathcal{I}, \partial) = \Psi(\mathcal{I}, \vartheta, \partial)$,

(NS5) $\Psi\big(\vartheta, \mu, (\partial + s)\big) \geq \Psi(\vartheta, \mathcal{I}, \partial) * \Psi(\mathcal{I}, \mu, s)$,

(NS6) $\Psi(\vartheta, \mathcal{I}, \cdot): [0, \infty) \to [0,1]$ is a continuous,

(NS7) $\lim_{\Omega \to \infty} \Psi(\vartheta, \mathcal{I}, \partial) = 1$,

(NS8) $0 \leq \Phi(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS9) $\Phi(\vartheta, \mathcal{I}, \partial) = 0 \iff \vartheta = \mathcal{I}$,

(NS10) $\Phi(\vartheta, \mathcal{I}, \partial) = \Phi(\mathcal{I}, \vartheta, \partial)$,

(NS11) $\Phi\big(\vartheta, \mu, b(\partial + s)\big) \leq \Phi(\vartheta, \mathcal{I}, \partial) \circ \Phi(\mathcal{I}, \mu, s)$,

(NS12) $\Phi(\vartheta, \mathcal{I}, \cdot): [0, \infty) \to [0,1]$ is a continuous,

(NS13) $\lim_{\Omega \to \infty} \Phi(\vartheta, \mathcal{I}, \partial) = 0$,

(NS14) $0 \leq \Omega(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS15) $\Omega(\vartheta, \mathcal{I}, \partial) = 0 \iff \vartheta = \mathcal{I}$,

(NS16) $\Omega(\vartheta, \mathcal{I}, \partial) = \Omega(\mathcal{I}, \vartheta, \partial)$,

(NS17) $\Omega\big(\vartheta, \mu, (\partial + s)\big) \leq \Omega(\vartheta, \mathcal{I}, \partial) \circ \Omega(\mathcal{I}, \mu, s)$, (NS18) $\Omega(\vartheta,\mathcal{J},\cdot):[0,\infty)\to[0,1]$ is a continuous,

(NS19) $\lim_{\Omega\to\infty}\Omega(\vartheta,\mathcal{J},\partial)=0$,

(NS20) If $\Omega\le 0$ then $\Psi(\vartheta,\mathcal{J},\partial)=0,\Phi(\vartheta,\mathcal{J},\partial)=1,\Omega(\vartheta,\mathcal{J},\partial)=1.$

Then (Q,Ψ,Φ,Ω) Neutrosophic metric on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ is an NMS.

2. Main Results

In this section, we introduce the concept of NBMLS and prove some FP results.

Definition 2.1 Suppose $Q\neq\emptyset$, assume a six tuple $(Q,\Psi,\Phi,\Omega,*,\circ)$ where $*$ is a CTN, $\circ$ is a CTCN, Ψ,Φ and Ω are NSs on $Q\times Q\times(0,\infty)$. If $(Q,\Psi,\Phi,\Omega,*,\circ)$ meet the below circumstances for all $\vartheta,\mathcal{J},\mu\in Q$ and $\Omega,s>0$:

(NL1) $\Psi(\vartheta,\mathcal{J},\partial)+\Phi(\vartheta,\mathcal{J},\partial)+\Omega(\vartheta,\mathcal{J},\partial)\le 3$,

(NL2) $0\le\Psi(\vartheta,\mathcal{J},\partial)\le 1$,

(NL3) $\Psi(\vartheta,\mathcal{J},\partial)=1$ implies $\vartheta=\mathcal{J}$,

(NL4) $\Psi(\vartheta,\mathcal{J},\partial)=\Psi(\mathcal{J},\vartheta,\partial)$,

(NL5) $\Psi(\vartheta,\mu,b(\partial+s))\ge\Psi(\vartheta,\mathcal{J},\partial)*\Psi(\mathcal{J},\mu,s)$,

(NL6) $\Psi(\vartheta,\mathcal{J},\cdot):[0,\infty)\to[0,1]$ is a continuous,

(NL7) $\lim_{\Omega\to\infty}\Psi(\vartheta,\mathcal{J},\partial)=1$,

(NL8) $0\le\Phi(\vartheta,\mathcal{J},\partial)\le 1$,

(NL9) $\Phi(\vartheta,\mathcal{J},\partial)=0$ implies $\vartheta=\mathcal{J}$,

(NL10) $\Phi(\vartheta,\mathcal{J},\partial)=\Phi(\mathcal{J},\vartheta,\partial)$,

(NL11) $\Phi\big(\vartheta,\mu,b(\partial+s)\big)\le\Phi(\vartheta,\mathcal{J},\partial)\circ\Phi(\mathcal{J},\mu,s)$,

(NL12) $\Phi(\vartheta,\mathcal{J},\cdot):[0,\infty)\to[0,1]$ is a continuous, (NL13) $\lim_{\Omega\to\infty} \Phi(\vartheta,\mathfrak{I},\partial) = 0,$

(NL14) $0 \leq \Omega(\vartheta,\mathfrak{I},\partial) \leq 1,$

(NL15) $\Omega(\vartheta,\mathfrak{I},\partial) = 0$ implies $\vartheta = \mathfrak{I},$

(NL16) $\Omega(\vartheta,\mathfrak{I},\partial) = \Omega(\mathfrak{I},\vartheta,\partial),$

(NL17) $\Omega\big(\vartheta,\mu,b(\partial+s)\big) \leq \Omega(\vartheta,\mathfrak{I},\partial)\circ\Omega(\mathfrak{I},\mu,s),$

(NL18) $\Omega(\vartheta,\mathfrak{I},\cdot):[0,\infty) \to [0,1]$ is a continuous,

(NL19) $\lim_{\Omega\to\infty} \Omega(\vartheta,\mathfrak{I},\partial) = 0,$

(NS20) If $\Omega \leq 0$ then $\Psi(\vartheta,\mathfrak{I},\partial) = 0, \Phi(\vartheta,\mathfrak{I},\partial) = 1, \Omega(\vartheta,\mathfrak{I},\partial) = 1.$

Then (Q,Ψ,Φ,Ω) is known as NBML on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS.

Remark 2.2 In definition (2.1), a set Q is gifted a NBMLS with a CTN (*) and CTCN (∘). (NL3), (NL9) and (NL15) circumstances of NBMLS, that is, the self-distance may not be equal to 1, 0 and 0, i.e., $\Psi(\vartheta,\vartheta,\partial) \neq 1, \Phi(\vartheta,\vartheta,\partial) \neq 0$ and $\Omega(\vartheta,\vartheta,\partial) \neq 0$ for all $\partial > 0$, for some or may be for all $\vartheta \in Q$.

Proposition 2.3 Let (Q,σ) be any BMLS. Then $(Q,\Psi,\Phi,\Omega,*,\circ)$ is a NBMLS, where '*' and '∘' are defined respectively $\sigma * \kappa = \sigma\kappa$ and $\sigma\circ\kappa = \max\{\sigma,\kappa\}$ and NSs Ψ,Φ and Ω are given by

$$\Psi(\vartheta,\mathfrak{I},\partial) = \frac{a\partial^n}{a\partial^n + m\sigma(\vartheta,\mathfrak{I})} \ \forall\, \vartheta,\mathfrak{I} \in Q, \partial > 0,$$
$$\Phi(\vartheta,\mathfrak{I},\partial) = \frac{m\sigma(\vartheta,\mathfrak{I})}{a\partial^n + m\sigma(\vartheta,\mathfrak{I})} \ \forall\, \vartheta,\mathfrak{I} \in Q, \partial > 0,$$
$$\Omega(\vartheta,\mathfrak{I},\partial) = \frac{m\sigma(\vartheta,\mathfrak{I})}{a\partial^n} \ \forall\, \vartheta,\mathfrak{I} \in Q, \partial > 0.$$

Where, $a \in \mathbb{R}^+, m > 0$ and $n \geq 1$.

Then (Q,Ψ,Φ,Ω) be a Neutrosophic b-metric-like on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS.

Remark 2.4 Note that the above proposition also holds for CTN $\sigma * \kappa = \min\{\sigma, \kappa\}$ and CTCN $\sigma \circ \kappa = \max\{\sigma, \kappa\}$.

Remark 2.5 The proposition (2.3) shows that every BMLS induces a NBMLS. For $a = n = m = 1$ the induced NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is called the standard NBMLS, where $a \in \mathbb{R}^+$

$$\Psi_{bl}(\vartheta, \mathcal{J}, \partial) = \frac{\partial}{\partial + \sigma(\vartheta, \mathcal{J})} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \mathcal{J}, \partial) = \frac{\sigma(\vartheta, \mathcal{J})}{\partial + \sigma(\vartheta, \mathcal{J})} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \mathcal{J}, \partial) = \frac{\sigma(\vartheta, \mathcal{J})}{\partial} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Example 2.6 Let $Q = \mathbb{R}^+, a \in \mathbb{R}^+$ and $m > 0$. Define * by $\sigma * \kappa = \sigma\kappa$ and $\circ$ by $\sigma \circ \kappa = \max\{\sigma, \kappa\}$ and NSs Ψ, Φ and Ω in $Q \times Q \times (0, \infty)$ by

$$\Psi(\vartheta, \mathcal{J}, \partial) = \frac{a\partial}{a\partial + m(\max\{\vartheta, \mathcal{J}\}^2\ )} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \mathcal{J}, \partial) = \frac{m(\max\{\vartheta, \mathcal{J}\}^2\ )}{a\partial + m(\max\{\vartheta, \mathcal{J}\}^2\ )} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \mathcal{J}, \partial) = \frac{m(\max\{\vartheta, \mathcal{J}\}^2\ )}{a\partial} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Then, since $\sigma(\vartheta, \mathcal{J}) = \max\{\vartheta, \mathcal{J}\}^2 \ \ \forall\ \vartheta, \mathcal{J} \in Q$ is a BMLS on Q. Therefore, by proposition (2.3) $(Q, \Psi, \Phi, \Omega, *, \circ)$ is a NBMLS, but self-distance not equal to 1, 0 and 0.

As,

$$\Psi(\vartheta, \vartheta, \partial) = \frac{a\partial}{a\partial + m\vartheta^2} \neq 1 \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \vartheta, \partial) = \frac{m\vartheta^2}{a\partial + m\vartheta^2} \neq 0 \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \vartheta, \partial) = \frac{m\vartheta^2}{a\partial} \neq 0 \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Definition 2.7 A sequence $\{\vartheta_n\}$ is NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be convergent to $\vartheta \in Q$ if

$$\lim_{n \to \infty} \Psi(\vartheta_n, \vartheta, \partial) = \Psi(\vartheta, \vartheta, \partial) \ \forall\ \partial > 0,$$

$$\lim_{n \to \infty} \Phi(\vartheta_n, \vartheta, \partial) = \Phi(\vartheta, \vartheta, \partial) \ \forall\ \partial > 0,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \Omega(\vartheta, \vartheta, \partial)\ \forall\ \partial > 0.$$

Definition 2.8 A sequence $\{\vartheta_n\}$ in a NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be Cauchy sequence (CS) if

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

for all $\partial \geq 0, \varepsilon \geq 1$ exist and is finite.

Definition 2.9 A NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be complete if every CS $\{\vartheta_n\}$ in Q converge to some $\vartheta \in Q$ such that

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta, \partial) = \Psi(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Psi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{ for all } \partial \geq 0, \varepsilon \geq 1,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta, \partial) = \Phi(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{for all } \partial \geq 0, \varepsilon \geq 1,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \Omega(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{ for all } \partial \geq 0, \varepsilon \geq 1.$$

Remark 2.10 In NBMLS, the limit of a convergent sequence may not be unique for instance, for a NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ given in proposition (2.3) with $\sigma(\vartheta, \mathcal{I}) = \max\{\vartheta, \mathcal{I}\}^2$ and $n = a = m = 1$. Define a sequence $\{\vartheta_n\}$ in Q by $\vartheta_n = 1 - \frac{1}{n}, \forall\ n \in \mathbb{N}$. If $\vartheta \geq 1$ then

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\partial}{\partial + \max\{\vartheta_n, \vartheta\}^2} = \frac{\partial}{\partial + \max\{\vartheta, \vartheta\}^2} = \Psi(\vartheta, \vartheta, \partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n, \vartheta\}^2}{\partial + \max\{\vartheta_n, \vartheta\}^2} = \frac{\max\{\vartheta, \vartheta\}^2}{\partial + \max\{\vartheta, \vartheta\}^2} = \Phi(\vartheta, \vartheta, \partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n, \vartheta\}^2}{\partial} = \frac{\max\{\vartheta, \vartheta\}^2}{\partial} = \Omega(\vartheta, \vartheta, \partial)\ \forall \partial > 0.$$

Therefore, the sequence $\{\vartheta_n\}$ converge to all $\vartheta \in Q$ with $\vartheta \geq 1$.

Remark 2.11 In an NBMLS, a convergent sequence may not be Cauchy. Assume an NBMLS $(Q,\Psi,\Phi,\Omega,*,\circ)$ given in above remark (2.10) Define a sequence $\{\vartheta_n\}$ in Q by $\vartheta_n = 1+(-1)^n, \forall\, n \in \mathbb{N}$. If $\vartheta \geq 2$, then

$$\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\partial}{\partial + \max\{\vartheta_n,\vartheta\}^2} = \frac{\partial}{\partial + \max\{\vartheta,\vartheta\}^2} = \Psi(\vartheta,\vartheta,\partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n,\vartheta\}^2}{\partial + \max\{\vartheta_n,\vartheta\}^2} = \frac{\max\{\vartheta,\vartheta\}^2}{\partial + \max\{\vartheta,\vartheta\}^2} = \Phi(\vartheta,\vartheta,\partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n,\vartheta\}^2}{\partial} = \frac{\max\{\vartheta,\vartheta\}^2}{\partial} = \Omega(\vartheta,\vartheta,\partial)\ \forall \partial > 0.$$

Therefore, a sequence $\{\vartheta_n\}$ converges to all $\vartheta \in Q$ with $\vartheta \geq 2$, but it is not a Cauchy sequence as $\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$, $\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$ and $\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$ does not exist.

Theorem 2.12 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS such that

$$\lim_{\partial\to\infty} \Psi(\vartheta,\jmath,\partial) = 1\,,\ \lim_{\partial\to\infty} \Phi(\vartheta,\jmath,\partial) = 0 \ \text{and}\ \lim_{\partial\to\infty} \Omega(\vartheta,\jmath,\partial) = 0$$

for all $\vartheta,\jmath \in Q, \partial > 0$ and $\xi: Q \to Q$ be a mapping fulfill the circumstances

$$\Psi(\xi\vartheta,\xi\jmath,\alpha\partial) \geq \Psi(\vartheta,\jmath,\partial), \qquad \Phi(\xi\vartheta,\xi\jmath,\alpha\partial) \leq \Phi(\vartheta,\jmath,\partial)$$

$$\text{and } \Omega(\xi\vartheta,\xi\jmath,\alpha\partial) \leq \Omega(\vartheta,\jmath,\partial), \tag{1}$$

for all $\vartheta,\jmath \in Q, \partial > 0$, where $\alpha \in (0,1)$. Then ξ has a unique FP $\pi \in Q$ and

$$\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0 \text{ and } \Omega(\pi,\pi,\partial) = 0 \ \ \forall\ \partial > 0.$$

Proof: Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. For a random $\vartheta_0 \in Q$, define a sequence $\{\vartheta_n\}$ in Q by

$$\vartheta_1 = \xi\vartheta_0\ ,\ \vartheta_2 = \xi^2\vartheta_0 = \xi\vartheta_1, \dots, \vartheta_n = \xi^n\vartheta_0 = \xi\vartheta_{n-1} \text{ for all } n \in \mathbb{N}.$$

If $\vartheta_n = \vartheta_{n-1}$ for some $n \in \mathbb{N}$ then ϑ_n is a FP of ξ. We suppose that $\vartheta_n \neq \vartheta_{n-1}$ for all $n \in \mathbb{N}$. For $\partial > 0$ and $n \in \mathbb{N}$, we obtain from (1) that

$$\Psi(\vartheta_n,\vartheta_{n+1},\partial) \geq \Psi(\vartheta_{n+1},\vartheta_n,\alpha\partial) = \Psi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial) \geq \Psi(\vartheta_n,\vartheta_{n-1},\partial),$$

$$\Phi(\vartheta_n,\vartheta_{n+1},\partial)\le\Phi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Phi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\le\Phi(\vartheta_n,\vartheta_{n-1},\partial)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+1},\partial)\le\Omega(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Omega(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\le\Omega(\vartheta_n,\vartheta_{n-1},\partial),$$

for all $n\in\mathbb{N}$ and $\partial>0.$ Therefore, by using the above inequalities, we obtain that

$$\Psi(\vartheta_{n+1},\vartheta_n,\partial)\ge\Psi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Psi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\ge\Psi(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Psi(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\ge\Psi\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\ge\cdots\ge\Psi\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right),\tag{2}$$

$$\Phi(\vartheta_{n+1},\vartheta_n,\partial)\le\Phi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Phi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\le\Phi(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Phi(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\le\Phi\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\le\cdots\le\Phi\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right)\tag{3}$$

and

$$\Omega(\vartheta_{n+1},\vartheta_n,\partial)\le\Omega(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Omega(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\le\Omega(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Omega(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\le\Omega\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\le\cdots\le\Omega\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right)\tag{4}$$

for all $n\in\mathbb{N}$, $\varepsilon\ge1$ and ∂>0. We obtain that

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\ge\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right),$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\le\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\le\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right).$$

Ongoing in this track, we deduce

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\ge\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\cdots*\Psi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right)$$

and

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\le\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\cdots\circ\Phi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right),$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\le\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\cdots\circ\Omega\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right).$$

Using (2), (3) and (4) in the above inequality, we deduce

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\ge\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right)*\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right)*\cdots*\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right),\tag{5}$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right) \circ \Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right) \circ \cdots \circ \Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right), \tag{6}$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right) \circ \Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right) \circ \cdots \circ \Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right) \tag{7}$$

We know that $\lim_{n\to\infty} \Psi(\vartheta,\mathfrak{I},\partial) = 1$, $s \lim_{n\to\infty} \Phi(\vartheta,\mathfrak{I},\partial) = 0, \forall\, \vartheta,\mathfrak{I} \in Q$ and $\partial > 0, \alpha \in (0,1)$. So, from (5), (6) and (7) we deduce that

$$\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 1 * 1 * \cdots * 1 = 1, \forall\, \partial > 0, \varepsilon \geq 1,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 0 \circ 0 \circ \cdots \circ 0 = 0, \forall\, \partial > 0, \varepsilon \geq 1,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 0 \circ 0 \circ \cdots \circ 0 = 0, \forall\, \partial > 0, \varepsilon \geq 1,$$

Hence, $\{\vartheta_n\}$ is a CS. The completeness of the NBMLS

$(Q,\Psi,\Phi,\Omega,*,\circ)$ agrees that there exists $\pi \in Q$ such that

$$\lim_{n\to\infty} \Psi(\vartheta_n,\pi,\partial) = \lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = \Psi(\pi,\pi,\partial) = 1, \forall\, \partial > 0, \varepsilon \geq 1, \tag{8}$$

$$\lim_{n\to\infty} \Phi(\vartheta_n,\pi,\partial) = \lim_{n\to\infty} \Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = \Phi(\pi,\pi,\partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1, \tag{9}$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n,\pi,\partial) = \lim_{n\to\infty} \Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = \Omega(\pi,\pi,\partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1. \tag{10}$$

Now, we examine that $\pi \in Q$ is a FP of ξ. We have

$$\begin{aligned}
\Psi(\pi,\xi\pi,\partial) &\geq \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) * \Psi\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial > 0,\\
&= \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) * \Psi\left(\xi\vartheta_n,\xi\pi,\frac{\partial}{2b}\right)\\
&\geq \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) * \Psi\left(\vartheta_n,\pi,\frac{\partial}{2b\alpha}\right),\\
\Phi(\pi,\xi\pi,\partial) &\leq \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Phi\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial > 0,\\
&= \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Phi\left(\xi\vartheta_n,\xi\pi,\frac{\partial}{2b}\right)\\
&\leq \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Phi\left(\vartheta_n,\pi,\frac{\partial}{2b\alpha}\right)
\end{aligned}$$

and

$$\Omega(\pi,\xi\pi,\partial) \leq \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial > 0,$$
$$= \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\xi\vartheta_{n},\xi\pi,\frac{\partial}{2b}\right)$$
$$\leq \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n},\pi,\frac{\partial}{2b\alpha}\right).$$

Taking limit as $n \to +\infty$, and by (8), (9) and (10), we get

$$\Psi(\pi,\xi\pi,\partial) = 1 * 1 = 1,$$

$$\Phi(\pi,\xi\pi,\partial) = 0 \circ 0 = 0$$

and

$$\Omega(\pi,\xi\pi,\partial) = 0 \circ 0 = 0.$$

That is, π is a FP of ξ,

$$\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0 \text{ and } \Omega(\pi,\pi,\partial) = 0, \forall\, \partial > 0.$$

Now, for proving the uniqueness of FP, assume that γ and π are two FPs of ξ, then by (1), we get

$$\Psi(\pi,\gamma,\partial) = \Psi(\xi\pi,\xi\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Psi(\pi,\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0,$$
$$\Phi(\pi,\gamma,\partial) = \Phi(\xi\pi,\xi\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Phi(\pi,\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0$$

and

$$\Omega(\pi,\gamma,\partial) = \Omega(\xi\pi,\xi\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Omega(\pi,\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0.$$

We get

$$\Psi(\pi,\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha^n}\right), \forall\, n \in \mathbb{N},$$
$$\Phi(\pi,\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha^n}\right), \forall\, n \in \mathbb{N}$$

and

$$\Omega(\pi,\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha^n}\right), \forall\, n \in \mathbb{N}.$$

Taking limit as $n \to +\infty$ and applying the circumstance $\lim_{\partial\to\infty} \Psi(\vartheta,\mathcal{I},\partial) = 1$ and $\lim_{\partial\to\infty} \Phi(\vartheta,\mathcal{I},\partial) = 0$ and $\lim_{\partial\to\infty} \Omega(\vartheta,\mathcal{I},\partial) = 0$, so $\pi = \gamma$, hence the FP is unique.

Example 2.13 Assume $Q = [0,1]$, CTN and CTCN respectively defined as $\sigma * \kappa = \sigma\kappa$ and $\sigma \circ \kappa = \max\{\sigma,\kappa\}$. Also, Ψ, Φ and Ω are defined as

$$\Psi(\vartheta,\mathcal{I},\partial) = \frac{\partial}{\partial + \max\{\vartheta,\mathcal{I}\}^2}\ \forall\, \vartheta,\mathcal{I} \in Q, \partial > 0,$$
$$\Phi(\vartheta,\mathcal{I},\partial) = \frac{\max\{\vartheta,\mathcal{I}\}^2}{\partial + \max\{\vartheta,\mathcal{I}\}^2}\ \forall\, \vartheta,\mathcal{I} \in Q, \partial > 0,$$
$$\Omega(\vartheta,\mathcal{I},\partial) = \frac{\max\{\vartheta,\mathcal{I}\}^2}{\partial}\ \forall\, \vartheta,\mathcal{I} \in Q, \partial > 0.$$

Then $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. Define $\xi: Q \to Q$ by

$$\xi\vartheta = \begin{cases} 0, & \vartheta \in \left[0,\frac{1}{2}\right] \\ \frac{\vartheta}{4}, & \vartheta \in \left(\frac{1}{2},1\right]. \end{cases}$$

Now,

$$\lim_{\partial\to\infty} \Psi(\vartheta,\mathcal{I},\partial) = \lim_{\partial\to\infty} \frac{\partial}{\partial + \max\{\vartheta,\mathcal{I}\}^2} = 1,$$
$$\lim_{\partial\to\infty} \Phi(\vartheta,\mathcal{I},\partial) = \lim_{\partial\to\infty} \frac{\max\{\vartheta,\mathcal{I}\}^2}{\partial + \max\{\vartheta,\mathcal{I}\}^2} = 0,$$
$$\lim_{\partial\to\infty} \Omega(\vartheta,\mathcal{I},\partial) = \lim_{\partial\to\infty} \frac{\max\{\vartheta,\mathcal{I}\}^2}{\partial} = 0.$$

For $\alpha \in \left[\frac{1}{2},1\right)$, we have four cases:

Case.1) If $\vartheta,\mathcal{I} \in \left[0,\frac{1}{2}\right]$, then $\xi\vartheta = \xi\mathcal{I} = 0$.

Case.2) If $\vartheta \in \left[0,\frac{1}{2}\right]$ and $\mathcal{I} \in \left(\frac{1}{2},1\right]$, then $\xi\vartheta = 0$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{4}$.

Case.3) If $\vartheta,\mathcal{I} \in \left(\frac{1}{2},1\right]$, then $\xi\vartheta = \frac{\vartheta}{4}$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{4}$.

Case.4) If $\vartheta \in \left(\frac{1}{2},1\right]$ and $\mathcal{I} \in \left[0,\frac{1}{2}\right]$, then $\xi\vartheta = \frac{\vartheta}{4}$ and $\xi\mathcal{I} = 0$.

From all 4 cases, we obtain that

$$\Psi(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\geq\Psi(\vartheta,\mathcal{J},\partial),$$

$$\Phi(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\leq\Phi(\vartheta,\mathcal{J},\partial),$$

$$\Omega(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\leq\Omega(\vartheta,\mathcal{J},\partial).$$

Hence all circumstances of Theorem 2.12 are fulfilled and 0 is the unique FP of ξ. Also,

$$\Psi(\pi,\pi,\partial)=\Psi(0,0,\partial)=1,\forall\ \partial>0,$$

$$\Phi(\pi,\pi,\partial)=\Phi(0,0,\partial)=0,\forall\ \partial>0,$$

$$\Omega(\pi,\pi,\partial)=\Omega(0,0,\partial)=0,\forall\ \partial>0.$$

Definition 2.14 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS. A mapping $\xi:Q\to Q$ is named to be NBML contractive (NBMLC) if $a\in(0,1)$ such that

$$\frac{1}{\Psi(\xi\vartheta,\xi\mathcal{J},\partial)}-1\leq a\left[\frac{1}{\Psi(\vartheta,\mathcal{J},\partial)}-1\right],\qquad \Phi(\xi\vartheta,\xi\mathcal{J},\partial)\leq a\Phi(\vartheta,\mathcal{J},\partial)$$

$$\text{and } \Omega(\xi\vartheta,\xi\mathcal{J},\partial)\leq a\Omega(\vartheta,\mathcal{J},\partial) \tag{11}$$

for all $\vartheta,\mathcal{J}\in Q$ and $\partial>0$. Here, a is called the NBMLC constant of ξ.

Theorem 2.15 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS and $\xi:Q\to Q$ be a NBMLC mapping with a NBMLC constant a, then ξ has a unique FP $\pi\in Q$ so that $\Psi(\pi,\pi,\partial)=1$, $\Phi(\pi,\pi,\partial)=0$ and $\Omega(\pi,\pi,\partial)=0$, for all $\partial>0$.

Proof: Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. For a random $\vartheta_0\in Q$, express a sequence $\{\vartheta_n\}$ in Q by

$$\vartheta_1=\xi\vartheta_0\ ,\ \vartheta_2=\xi^2\vartheta_0=\xi\vartheta_1,\ldots,\vartheta_n=\xi^n\vartheta_0=\xi\vartheta_{n-1}\ \text{for all } n\in\mathbb{N}.$$

If $\vartheta_n=\vartheta_{n-1}$ for some $n\in\mathbb{N}$, then ϑ_n is a FP of ξ. We suppose that $\vartheta_n\neq\vartheta_{n-1}$ for all $n\in\mathbb{N}$. For $\partial>0$ and $n\in\mathbb{N}$, we obtain from (11)

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}-1=\frac{1}{\Psi(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)}-1\le a\left[\frac{1}{\Psi(\vartheta_{n-1},\vartheta_n,\partial)}-1\right].$$

We have

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}\le\frac{a}{\Psi(\vartheta_{n-1},\vartheta_n,\partial)}+(1-a),\forall\,\partial>0,$$

$$=\frac{a}{\Psi(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)}+(1-a)\le\frac{a^2}{\Psi(\vartheta_{n-2},\vartheta_{n-1},\partial)}+a(1-a)+(1-a).$$

Ongoing in this track, we obtain

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}\le\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+a^{n-1}(1-a)+a^{n-2}(1-a)+\cdots+a(1-a)+(1-a)$$

$$\le\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(a^{n-1}+a^{n-2}+\cdots+1)(1-a)$$

$$\le\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(1-a^n).$$

We have

$$\frac{1}{\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(1-a^n)}\le\ \Psi(\vartheta_n,\vartheta_{n+1},\partial),\forall\,\partial>0,n\in\mathbb{N}. \tag{12}$$

Now,

$$\Phi(\vartheta_n,\vartheta_{n+1},\partial)=\Phi(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)\le a\Phi(\vartheta_{n-1},\vartheta_n,\partial)=a\Phi(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)$$

$$\le a^2\Phi(\vartheta_{n-2},\vartheta_{n-1},\partial)\le\cdots\le a^n\Phi(\vartheta_0,\vartheta_1,\partial) \tag{13}$$

and

$$\Omega(\vartheta_n,\vartheta_{n+1},\partial)=\Omega(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)\le a\Omega(\vartheta_{n-1},\vartheta_n,\partial)=a\Omega(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)$$

$$\le a^2\Omega(\vartheta_{n-2},\vartheta_{n-1},\partial)\le\cdots\le a^n\Omega(\vartheta_0,\vartheta_1,\partial) \tag{14}$$

Now, for $\varepsilon\ge 1$ and $n\in\mathbb{N}$, we get

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\ge\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\ge\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\Psi\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right).$$

Ongoing in this track, we derive

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\ge\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\cdots*\Psi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right),$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\le\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\le\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\Phi\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right) \circ \Omega\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\leq \Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right) \circ \Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right) \circ \Omega\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right).$$

Ongoing in this track, we deduce that

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right) \circ \Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right) \circ \cdots \circ \Omega\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right)$$

By using (12), (13) and (14) in the above inequality, we have

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \geq \frac{1}{\frac{a^n}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)}+(1-a^n)} * \frac{1}{\frac{a^{n+1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2}\right)}+(1-a^{n+1})} * \cdots$$

$$* \frac{1}{\frac{a^{n+\varepsilon-1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)}+(1-a^{n+\varepsilon-1})},$$

$$\geq \frac{1}{\frac{a^n}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)}+1} * \frac{1}{\frac{a^{n+1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2}\right)}+1} * \cdots * \frac{1}{\frac{a^{n+\varepsilon-1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)}+1},$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq a^n\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right) \circ a^{n+1}\Phi\left(\vartheta_1,\vartheta_2,\frac{\partial}{b^2}\right) \circ \cdots \circ a^{n+\varepsilon-1}\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right),$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq a^n\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right) \circ a^{n+1}\Omega\left(\vartheta_1,\vartheta_2,\frac{\partial}{b^2}\right) \circ \cdots \circ a^{n+\varepsilon-1}\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)$$

Here, $a \in (0,1)$, we deduce from the above expression that

$$\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 1 \text{ for all } \partial > 0, \varepsilon \geq 1,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 0 \text{ for all } \partial > 0, \varepsilon \geq 1,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = 0 \text{ for all } \partial > 0, \varepsilon \geq 1.$$

That is, $\{\vartheta_n\}$ is a CS in $(Q,\Psi,\Phi,\Omega,*,\circ)$. By the completeness of $(Q,\Psi,\Phi,\Omega,*,\circ)$. There is $\pi \in Q$, such that

$$\lim_{n\to\infty} \Psi(\vartheta_n,\pi,\partial) = \lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) = \lim_{n\to\infty} \Psi(\pi,\pi,\partial) = 1, \forall\, \partial > 0, \varepsilon \geq 1. \tag{15}$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \pi, \partial) = \lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) = \lim_{n\to\infty} \Phi(\pi, \pi, \partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1. \tag{16}$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \pi, \partial) = \lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) = \lim_{n\to\infty} \Omega(\pi, \pi, \partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1. \tag{17}$$

Now, we examine that π is a FP for ξ. By using (11) we get

$$\frac{1}{\Psi(\xi\vartheta_n, \xi\pi, \partial)} - 1 \leq a\left[\frac{1}{\Psi(\vartheta_n, \pi, \partial)} - 1\right] = \frac{a}{\Psi(\vartheta_n, \pi, \partial)} - a,$$

$$\frac{1}{\frac{a}{\Psi(\vartheta_n, \pi, \partial)} + 1 - a} \leq \Psi(\xi\vartheta_n, \xi\pi, \partial).$$

applying the above expression, we deduce

$$\Psi(\pi, \xi\pi, \partial) \geq \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \Psi\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \Psi\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\geq \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \frac{1}{\frac{a}{\Psi\left(\vartheta_n, \pi, \frac{\partial}{2b}\right)} + 1 - a},$$

$$\Phi(\pi, \xi\pi, \partial) \leq \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Phi\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Phi\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\leq \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ a\Phi\left(\vartheta_n, \pi, \frac{\partial}{2b}\right),$$

and

$$\Omega(\pi, \xi\pi, \partial) \leq \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Omega\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\leq \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ a\Omega\left(\vartheta_n, \pi, \frac{\partial}{2b}\right)$$

Taking limit as $n \to \infty$ and using (15), (16) and (17) in the above inequalities, we examine $\Psi(\pi, \xi\pi, \partial) = 1$, $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$, therefore, $\xi\pi = \pi$. That is, π is a FP of ξ and $\Psi(\pi, \pi, \partial) = 1$, $\Phi(\pi, \pi, \partial) = 0$ and $\Omega(\pi, \pi, \partial) = 0$ for all $\partial > 0$.

Now, for proving the uniqueness of the FP π of ξ. Let γ be another FP of ξ, such that $\Psi(\pi,\gamma,$ t)$\neq 1, \Phi(\pi,\gamma,\partial) \neq 0$ and $\Omega(\pi,\gamma,\partial) \neq 0$ for some $\partial > 0$. It monitors from (11) that

$$\frac{1}{\Psi(\pi,\gamma,\partial)} - 1 = \frac{1}{\Psi(\xi\pi,\xi\gamma,\partial)} - 1$$
$$\leq a\left[\frac{1}{\Psi(\pi,\gamma,\partial)} - 1\right] < \frac{1}{\Psi(\pi,\gamma,\partial)} - 1,$$

$$\Phi(\pi,\gamma,\partial) = \Phi(\xi\pi,\xi\gamma,\partial) \leq a\Phi(\pi,\gamma,\partial) < \Phi(\pi,\gamma,\partial),$$

and

$$\Omega(\pi,\gamma,\partial) = \Omega(\xi\pi,\xi\gamma,\partial) \leq a\Omega(\pi,\gamma,\partial) < \Omega(\pi,\gamma,\partial),$$

a contradiction.

That is, we have $\Psi(\pi,\gamma,\partial) = 1, \Phi(\pi,\gamma,\partial) = 0$ and $\Omega(\pi,\gamma,\partial) = 0$, for all $\partial > 0$, and hence $\pi = \gamma$.

Corollary 2.16 Assume $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS and $\xi: Q \to Q$ be a mapping satisfying

$$\frac{1}{\Psi(\xi^n\vartheta,\xi^n\jmath,\partial)} - 1 \leq a\left[\frac{1}{\Psi(\vartheta,\jmath,\partial)} - 1\right],$$
$$\Phi(\xi^n\vartheta,\xi^n\jmath,\partial) \leq a\Phi(\vartheta,\jmath,\partial),$$

and

$$\Omega(\xi^n\vartheta,\xi^n\jmath,\partial) \leq a\Omega(\vartheta,\jmath,\partial)$$

for some $n \in \mathbb{N}, \forall\, \vartheta,\jmath \in Q, \partial > 0$, where $0 < a < 1$. Then ξ has a unique FP $\pi \in Q$ and $\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0\ \forall\, \partial > 0$.

Proof: $\pi \in Q$ is the unique FP of ξ^n by applying theorem 2.15, and $\Psi(\pi,\pi,\partial) = 1$, $\Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0\ \forall\, \partial > 0$. $\xi\pi$ is also a FP of ξ^n as $\xi^n(\xi\pi) = \xi\pi$ and from Theorem 2.15, $\xi\pi = \pi$, π is the unique FP, therefore, the unique FP of ξ is also the unique FP of ξ^n.

Example 2.17 Assume $Q = [0,2]$, CTN and CTCN respectively defined as $\sigma * \kappa = \sigma\kappa$ and $\sigma \circ \kappa = \max\{\sigma, \kappa\}$, given Ψ, Φ and Ω as

$$\Psi(\vartheta, \mathcal{J}, \partial) = \frac{\partial}{\partial + \max\{\vartheta, \mathcal{J}\}^2} \ \forall\, \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \mathcal{J}, \partial) = \frac{\max\{\vartheta, \mathcal{J}\}^2}{\partial + \max\{\vartheta, \mathcal{J}\}^2} \ \forall\, \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \mathcal{J}, \partial) = \frac{\max\{\vartheta, \mathcal{J}\}^2}{\partial} \ \forall\, \vartheta, \mathcal{J} \in Q, \partial > 0.$$

for all $\vartheta, \mathcal{J} \in Q$ and $\partial > 0$.Then $(Q, \Psi, \Phi, \Omega, *, \circ)$ is a complete NMLS. Define $\xi: Q \to Q$ as

$$\xi\vartheta = \begin{cases} 0, & \vartheta = 1 \\ \frac{\vartheta}{5}, & \vartheta \in [0,1) \\ \frac{\vartheta}{7}, & \vartheta \in (1,2]. \end{cases}$$

Then we have 8 cases:

Case.1) If $\vartheta = \mathcal{J} = 1$, then $\xi\vartheta = \xi\mathcal{J} = 0$.

Case.2) If $\vartheta = 1$ and $\mathcal{J} \in [0,1)$, then $\xi\vartheta = 0$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{5}$.

Case.3) If $\vartheta = 1$ and $\mathcal{J} \in (1,2]$, then $\xi\vartheta = 0$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{7}$.

Case.4) If $\vartheta \in [0,1)$ and $\mathcal{J} \in (1,2]$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{7}$.

Case.5) If $\vartheta \in [0,1)$ and $\mathcal{J} \in [0,1)$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{5}$.

Case.6) If $\vartheta \in [0,1)$ and $\mathcal{J} = 1$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{J} = 0$.

Case.7) If $\vartheta \in (1,2]$ and $\mathcal{J} = 1$, then $\xi\vartheta = \frac{\vartheta}{7}$ and $\xi\mathcal{J} = 0$.

Case.8) If $\vartheta \in (1,2]$ and $\mathcal{J} \in (1,2]$, then $\xi\vartheta = \frac{\vartheta}{7}$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{7}$.

All above cases satisfy the NBMLC:

$$\frac{1}{\Psi(\xi\vartheta, \xi\mathcal{J}, \partial)} - 1 \le a\left[\frac{1}{\Psi(\vartheta, \mathcal{J}, \partial)} - 1\right],$$
$$\Phi(\xi\vartheta, \xi\mathcal{J}, \partial) \le a\Phi(\vartheta, \mathcal{J}, \partial),$$
$$\Omega(\xi\vartheta, \xi\mathcal{J}, \partial) \le a\Omega(\vartheta, \mathcal{J}, \partial)$$

with $a \in \left[\frac{1}{2}, 1\right)$ the NBMLC constant. Hence ξ is a NBMLC mapping with $a \in \left[\frac{1}{2}, 1\right)$. All circumstances of theorem 2.15 are fulfilled. Also, 0 is the unique FP of ξ and $\Psi(0,0,\partial) = 1, \Phi(0,0,\partial) = 0$ and $\Omega(0,0,\partial) = 0, \forall\, \partial > 0.$

Theorem 2.18 Let $(Q, \Psi, \Phi, \Omega, *, \circ)$ be a complete NBMLS and $\xi: Q \to Q$ be a NBMLC mapping with an NBMLC constant a. Suppose that their exist $\pi \in Q,$ such that $\Psi(\pi, \xi\pi, \partial) \geq \Psi(\vartheta, \xi\vartheta, \partial)$, $\Phi(\pi, \xi\pi, \partial) \leq \Phi(\vartheta, \xi\vartheta, \partial)$ and $\Omega(\pi, \xi\pi, \partial) \leq \Omega(\vartheta, \xi\vartheta, \partial)$ for all $\vartheta \in Q$ and $\partial > 0,$ we claim that $\Psi(\pi, \xi\pi, \partial) = 1$, $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$ for all $\pi \in Q$ and $\partial > 0,$ then ξ has a unique FP $\pi \in Q$ so that $\Psi(\pi, \pi, \partial) = 1,$ $\Phi(\pi, \pi, \partial) = 0$ and $\Phi(\pi, \pi, \partial) = 0$ for all $\partial > 0.$

Proof: Let $\Psi_\vartheta(\partial) = \Psi(\vartheta, \xi\vartheta, \partial),$ $\Phi_\vartheta(\partial) = \Phi(\vartheta, \xi\vartheta, \partial)$ and $\Omega_\vartheta(\partial) = \Omega(\vartheta, \xi\vartheta, \partial)$ for all $\vartheta \in Q$ and $\partial > 0.$ Then by the assumption $\Psi_\pi(\partial) \geq \Psi_\vartheta(\partial)$, $\Phi_\pi(\partial) \leq \Phi_\vartheta(\partial)$ and $\Omega_\pi(\partial) \leq \Omega_\vartheta(\partial)$ for all $\vartheta \in Q$ and $\partial > 0.$ We claim that $\Psi(\pi, \xi\pi, \partial) = 1,$ $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$ for all $\partial > 0.$ Indeed, if $\Psi_\pi(\partial) = \Psi(\pi, \xi\pi, \partial) < 1$, $\Phi_\pi(\partial) = \Phi(\pi, \xi\pi, \partial) > 0$ and $\Omega_\pi(\partial) = \Omega(\pi, \xi\pi, \partial) > 0$ for some $\partial > 0,$ then it examine from (11) that

$$\frac{1}{\Psi_{\xi\pi}(\partial)} - 1 = \frac{1}{\Psi(\xi\pi, \xi\, \xi\pi, \partial)} - 1$$
$$\leq a\left[\frac{1}{\Psi(\pi, \xi\pi, \partial)} - 1\right] = a\left[\frac{1}{\Psi_\pi(\partial)} - 1\right] < \left[\frac{1}{\Psi_\pi(\partial)} - 1\right],$$
$$\Phi_{\xi\pi}(\partial) = \Phi(\xi\pi, \xi\, \xi\pi, \partial) \leq a[\Phi(\pi, \xi\pi, \partial)] = a[\Phi_\pi(\partial)] < \Phi_\pi(\partial),$$
$$\Omega_{\xi\pi}(\partial) = \Omega(\xi\pi, \xi\, \xi\pi, \partial) \leq a[\Omega(\pi, \xi\pi, \partial)] = a[\Omega_\pi(\partial)] < \Omega_\pi(\partial).$$

That is $\Psi_\pi(\partial) \leq \Psi_{\xi\pi}(\partial), \xi\pi \in Q$ a contradiction. Therefore, we have $\Psi_\vartheta(\partial) = \Psi(\pi, \xi\pi\ \partial) = 1,$ $\Phi_\vartheta(\partial) = \Phi(\pi, \xi\pi\ \partial) = 0$ and $\Omega_\vartheta(\partial) = \Omega(\pi, \xi\pi\ \partial) = 0$ for all

$\partial > 0$, and so $\xi\pi = \pi$. Following the similar argument as in theorem (2.15), uniqueness of FP of ξ follows. If $\Psi(\pi,\pi,\partial) < 1, \Phi(\pi,\pi,\partial) > 0$ and $\Omega(\pi,\pi,\partial) > 0$ for some $\partial > 0$, then from (11), we have

$$\frac{1}{\Psi(\pi,\pi,\partial)} - 1 = \frac{1}{\Psi(\xi\pi,\xi\pi,\partial)} - 1 \leq a\left[\frac{1}{\Psi(\pi,\pi,\partial)} - 1\right] < \left[\frac{1}{\Psi(\pi,\pi,\partial)} - 1\right],$$

$$\Phi(\pi,\pi,\partial) = \Phi(\xi\pi,\xi\pi,\partial) \leq a[\Phi(\pi,\pi,\partial)] < \Phi(\pi,\pi,\partial),$$

$$\Omega(\pi,\pi,\partial) = \Omega(\xi\pi,\xi\pi,\partial) \leq a[\Omega(\pi,\pi,\partial)] < \Omega(\pi,\pi,\partial),$$

a contradiction. Therefore, $\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0$.

Remark 2.19 In the above theorem it is shown that in an NBMLS, the self-Neutrosophic distance of the FP of a NBMLC mapping with a NBMLC constant a, is always 1, 0 and 0. That is, the degree of self-nearness of the fixed point of a NBML contractive mapping is perfect.

3. Conclusion

In this article, the concept of neutrosophic b-metric-like spaces is introduced with some fixed point results and non-trivial examples. This work is more generalized in the existing literature. This work can easily extend in the structure of neutrosophic extended b-metric-like space, controlled neutrosophic b-metric-like spaces and many other structures.

Authors' contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

Conflicts of interest

The authors declare no conflict of interest.

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Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

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