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Neutrosophic Inventory System for Decaying Items with Price Dependent Demand

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Abstract. This study presents an empirical investigation of a specific problem in inventory control, notably the management of decaying commodities with demand that is influenced by price. The approach taken in this research incorporates the effective use of trapezoidal neutrosophic numbers. The present model is employed for the objective of finding the neutrosophic optimal total cost and neutrosophic optimal interval of time for the inventory system. Defuzzification process is done with the help of signed distance method. Moreover, a mathematical evaluation is conducted in order to assess the stated model, and the findings are elucidated through the use of a numerical illustration.

Keywords: Neutrosophic sets; Neutrosophic trapezoidal numbers; Neutrosophic total cost; Neutrosophic demand; Neutrosophic purchase cost; Signed distance method.

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1. Introduction

Decay refers to the process by which products undergo damage, deterioration, or disintegration over time, resulting in a gradual depletion of value. This phenomenon is particularly relevant in the scenario of materials that are intended for storage and potential future use. So decaying cannot be avoided in any kind of business scenarios. In the vegetable business, it is difficult to predict the quantity that will be purchased. The surplus supply should always be maintained to make sure the customers get everything they ordered. However, there is always a portion of the quantity that will get wasted. This wastage happens by the time the vegetables are bought back to the other retail stores. To avoid this, we can compromise with those to-be wasted vegetables and sell them for 50-60% of their original price. Thus, we are avoiding the vegetables that are made by the exhaustive farming process.

The rate of decay has a positive correlation with the passage of time. Every cycle has shortages that have been partly accumulated. Kundu and Chakrabarti [18] introduced an Economic Order Quantity (EOQ) Model incorporating demand and partial backlogging of fuzzy type in the study they performed. The present model has been specifically developed to cater to the requirements of a continuous review inventory system that manages deteriorating products, while also considering the effects of time-dependent demand. The inventory model put forth by U. Sushil Kumar and S.Rajput [19] examines the management of deteriorating commodities, implementing factors such as time-dependent demand rates and partial backlogging. In their study, Dutta and Kumar [3] examined a fuzzy inventory model that involves the management of degrading products with shortages, specifically under the circumstance of a full backlog.

In a research study performed by M. Maragatham and P.K. Lakshmidevi [5], a fuzzy inventory model was presented to address the issue of deteriorating products with demand which is centered on price. In their study, Palani and Maragatham [10] proposed a fuzzy inventory model to address the management of time-dependent degrading products. The model takes into account factors such as lead time, stock-dependent demand rate, and shortages. In their study, D. Sharmila and R. Uthayakumar [14] developed an inventory model that utilizes fuzzy logic to handle deteriorating products, shortages, and exponential demand. The notions of neutrosophic set and neutrosophic logic were first put forward by Smarandache [15], who addressed it from the perspective of non-standard analysis. In their article, Mullai and Broumi [6] established a novel inventory model involving the concept of neutrosophy and neglects to account for shortages.

This paper is organised as follows:
In section 2 and 3, the basic definitions and a few assumptions and notations that are very beneficial to expand this proposed model are given. Section 4 deals with the development of inventory model for decaying items with price dependent demand using neutrosophic demand and neutrosophic price are represented by trapezoidal neutrosophic numbers. To defuzzify the model, signed distance method is used. Also the neutrosophic optimal total cost and the neutrosophic optimal interval of time for the plan are determined. Our proposed neutrosophic system is illustrated with a numerical example and also sensitivity analysis is taken. The results are observed and compared graphically in section 5.

2. **Preliminaries:**

In this section, the basic definitions involving intuitionistic fuzzy set, intuitionistic fuzzy number, neutrosophic set, single valued neutrosophic sets and trapezoidal neutrosophic number are outlined.

Definition:1 [11] **Intuitionistic fuzzy set**

Let X denotes the universal set. An intuitionistic fuzzy set(IFS) $\bar{A}$ in X is an object having the form $\bar{A} = \{< x, \mu_A(x), \nu_A(x)) > /x \in X\}$, where $\mu_{\bar{A}} : X \to [0,1]$ and $\nu_{\bar{A}} : X \to [0,1]$ represents the membership and non membership degree of the element x$\in$ X to the set $\bar{A}$(a subset of the set X), respectively. Thus, $\forall$x$\in$ X, we have $0 \leq \mu_{\bar{A}}(x) + \nu_{\bar{A}}(x) \leq 1$.

Definition:2 [11] **Intuitionistic fuzzy number**

An intuitionistic fuzzy number $\bar{A}$ preserves the following properties:
i) an intuitionistic fuzzy subset of the real line(R)
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\bar{A}}(x)$ =1
iii) a convex set for the membership function $\mu_{\bar{A}}(x)$
i.e., $\mu_{\bar{A}}[\lambda x_1 + (1-\lambda)x_2] \geq min\{\mu_{\bar{A}}(x_1), \mu_{\bar{A}}(x_2), x_1, x_2 \in R\}, \lambda \in [0,1]$
iv) a concave set for the non membership function $\nu_{\bar{A}}(x)$
i.e., $\nu_{\bar{A}}[\lambda x_1 + (1-\lambda)x_2] \leq max\{\nu_{\bar{A}}(x_1), \nu_{\bar{A}}(x_2), x_1, x_2 \in R\}, \lambda \in [0,1]$

Definition:3 [13] **Trapezoidal intuitionistic fuzzy number**

A trapezoidal intuitionistic fuzzy number $\bar{A} = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ is a subset of IFS on the set of R whose membership and non membership are structured as follows:

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a_1}{b_1-a_1}, & \text{when } a_1 \leq x \leq b_1 \\ 1, & \text{when } b_1 \leq x \leq c_1 \\ \frac{d_1-x}{d_1-c_1}, & \text{when } c_1 \leq x \leq d_1 \\ 0, & \text{otherwise} \end{cases} \qquad \nu_{\tilde{A}} = \begin{cases} \frac{b_1-x}{b_1-a_1}, & \text{when } a_1' \leq x \leq b_1 \\ 0, & \text{when } c_1 \leq x \leq b_1 \\ \frac{x-c_1}{d_1-c_1}, & \text{when } c_1 \leq x \leq d_1' \\ 1, & \text{otherwise} \end{cases}$$

Definition:4 [4] **(Neutrosophic set)**

Consider X as an universal set. A neutrosophic set A in X is distinguished by a truth T_A, an indeterminacy I_A and falsity-membership function F_A, where $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It is defined as A=$\{\langle$ x, T_A(x), $I_A(x)$, $F_A(x)$ $\rangle$: x$\in$ X, T_A(x), $I_A(x)$, $F_A(x) \in\ ]0^-, 1^+[\ \}$ and also restriction not allowed on the sum of T_A(x), $I_A(x)$ and $F_A(x)$. So, $0^- \leq T_A$(x)+I_A(x) + F_A(x) $\leq 3^+$.

Definition:5 [4] **(Single valued neutrosophic set)**

Consider X to be the universal set. A single valued neutrosophic set A in X is distinguished by a truth T_A, indeterminacy I_A and falsity-membership function F_A, where T_A(x), I_A(x) and F_A(x) are real standard elements of [0,1]. It is defined in the following way, A={$\langle$ x, T_A(x), $I_A(x)$, F$_A(x)$ $\rangle$:x$\in$ X,T_A(x), $I_A(x)$, $F_A(x)$ $\in$ [0, 1] } and restriction not defined on the sum of T_A(x), I_A(x) and F_A(x). Here, we have 0$\leq$ T_A(x)+I_A(x) + F_A(x) $\leq$ 3.

Definition:6 [4] **(Trapezoidal neutrosophic number)**

A single valued trapezoidal neutrosophic number $\tilde{a}$ = $\langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$ is a unique set of neutrosophic type on the real number set R, whose truth, indeterminacy, and falsity membership are defined as follows:

$$\mu\tilde{a}(x) = \begin{cases} (x-a_1)\mathrm{w}_{\tilde{a}}/(b_1-a_1), & \text{if } a_1 \leq x < b_1 \\ \mathrm{w}_{\tilde{a}}, & \text{if } b_1 \leq x \leq c_1 \\ (d_1-x)\mathrm{w}_{\tilde{a}}/(d_1-c_1), & \text{if } c_1 < x \leq d_1 \\ 0, & \text{if } otherwise \end{cases}$$

$$\nu\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)\mathrm{u}_{\tilde{a}})/(b_1-a_1), & \text{if } a_1 \leq x < b_1 \\ \mathrm{u}_{\tilde{a}}, & \text{if } b_1 \leq x \leq c_1 \\ (x-c_1+(d_1-x)\mathrm{u}_{\tilde{a}})/(d_1-c_1), & \text{if } c_1 < x \leq d_1 \\ 1, & \text{if } otherwise \end{cases}$$

$$\lambda\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)\mathrm{y}_{\tilde{a}})/(b_1-a_1), & \text{if } a_1 \leq x < b_1 \\ \mathrm{y}_{\tilde{a}}, & \text{if } b_1 \leq x \leq c_1 \\ (x-c_1+(d_1-x)\mathrm{y}_{\tilde{a}})/(d_1-c_1), & \text{if } c_1 < x \leq d_1 \\ 1, & \text{if } otherwise \end{cases}$$

respectively.

A positive single-valued trapezoidal neutrosophic number, represented by $\tilde{a} > 0$, is defined as $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$, where $a_1 \geq 0$ and at least $d_1 > 0$. Similarly, in the case when d_1 is less than or equal to 0 and at least a_1 is less than 0, the representation $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$ is referred to as a negative single-valued trapezoidal neutrosophic number, which may be expressed as $\tilde{a}$ <0. A trapezoidal neutrosophic number, denoted as $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$, may represent an uncertain amount within a certain range, roughly corresponding to the interval $[b_1, c_1]$.

Definition:7 [3] **(Signed distance method)**

The signed distance of $\tilde{A}$(a fuzzy set defined on R) is defined as:

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$$

where

$$\begin{aligned} A_\alpha &= [A_L(\alpha), A_R(\alpha)] \\ &= [a+(b-a)\alpha, d-(d-c)\alpha], \alpha\epsilon[0,1], \end{aligned}$$

is α-cut of fuzzy set $\tilde{A}$, which is a closed interval.

Remark: For every $a, 0 \in R$, the signed distance $d(a,0) = a$. Definition 4's criteria are as follows: $d(a,0) = a$ is the distance between a and 0 if $0 < a$. The distance $-d(a,0) = -a$ exists between a and 0 if $a < 0$. As a result, we state that the signed distance between a and 0 is $d(a,0) = a$.

3. Notations and Assumption

3.1. *Notations:*

$q^I(t)$ = The intuitionistic fuzzy inventory level at time t^I

d^I = The intuitionistic fuzzy demand rate

α_1^I = The intuitionistic fuzzy decaying rate in $[0, t_1^I]$

M^I = The intuitionistic fuzzy shortage level of inventory

R^I = The intuitionistic fuzzy inventory level at t^I=0

Q^I = Lot size of intuitionistic fuzzy type

t_1^I = The interval of time at which the inventory attains zero

t_2^I = The interval of time at which the shortages are allowed

T^I = The intuitionistic fuzzy time length of the plan

C_h^I = Holding cost per unit item of intuitionistic fuzzy type

C_s^I = The shortage cost per unit item of intuitionistic fuzzy type

P^I = Purchase cost per unit item of intuitionistic fuzzy type

C_o^I = The intuitionistic fuzzy ordering cost per order

$F^I(q)$ = Defuzzified intuitionistic fuzzy total cost

$(TC)^I$ = The intuitionistic fuzzy total cost for the period $[0,T^I]$

(TC) = The total cost for the period [0,T]

$(TC)^F$ = The fuzzy total cost for the period $[0,T^F]$

$q^N(t)$ = The neutrosophic inventory level at time t^N

d^N = The neutrosophic demand rate

α_1^N = The neutrosophic decaying rate in $[0,t_1^N]$

M^N = The neutrosophic shortage level of inventory

R^N = The neutrosophic inventory level at $t^N=0$

Q^N = The neutrosophic lot size

T^N = The neutrosophic time length of the plan

C_h^N = The neutrosophic holding cost per unit item

C_s^N = The neutrosophic shortage cost per unit item

P^N = The neutrosophic purchase cost per unit item

C_o^N = The neutrosophic ordering cost per order

$(TC)^N$ = The neutrosophic total cost for the period $[0,T^N]$

$F^N(q)$ = Defuzzified neutrosophic total cost

3.2. *Assumptions:*

- The neutrosophic demand is related to the unit price as d^N =A(p$^{\beta^N}$) where A > 0, $0 < \beta < 1$.
- The neutrosophic shortages are allowed.
- During the cycle deterioration is not repaired or replaced.
- The neutrosophic purchase cost and neutrosophic demand are considered.
- The neutrosophic lead time is considered to be zero.
- The neutrosophic decay will be instantaneous.

4. **Model Description:**

This section illustrates the intuitionistic fuzzy model and the neutrosophioc model for finding the optimal total cost and optimal time length for the inventory system.

Intuitionistic fuzzy model:

The model presented in this study demonstrates Intuitionistic fuzzy purchasing cost and Intuitionistic fuzzy demand via trapezoidal Intuitionistic fuzzy numbers.

(ie) P^I = $(p_1^I, p_2^I, p_3^I, p_4^I)(p_1'^I, p_2'^I, p_3'^I, p_4'^I)$

D^I = $(D_1^I, D_2^I, D_3^I, D_4^I)(D_1'^I, D_2'^I, D_3'^I, D_4'^I)$

Based on [5], intuitionistic fuzzy total cost is defined as follows:

(TC)^I = $P^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{D^I}{\alpha_1^I})(1 - e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D^I + \alpha_1^I)}}) - \frac{D^I(Q^I - M^I)}{\alpha_1^I(D^I + \alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2D^I} + d^I(Q^I - M^I)(1 - \frac{D^I}{D^I + \alpha_1^I})$

= $(p_1^I, p_2^I, p_3^I, p_4^I, p_1'^I, p_2'^I, p_3'^I, p_4'^I)Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{D_1^I}{\alpha_1^I}, Q^I - M^I + \frac{D_2^I}{\alpha_1^I}, Q^I - M^I + \frac{D_3^I}{\alpha_1^I}, Q^I - M^I + \frac{D_4^I}{\alpha_1^I}, Q^I - M^I + \frac{D_1'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_2'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_3'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_4'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}((Q^I - M^I + \frac{D_1^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_1^I + \alpha_1^I)}}),$

$(Q^I - M^I + \frac{D_2^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_2^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_3^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_3^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_4^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_4^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_1'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_1'^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_2'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_2'^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_3'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_3'^I + \alpha_1^I)}}), (Q^I - M^I + \frac{D_4'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I - M^I)}{(D_4'^I + \alpha_1^I)}}) -$

$(\frac{D_1^I(Q^I-M^I)}{\alpha_1^I(D_1^I+\alpha_1^I)}, \frac{D_2^I(Q^I-M^I)}{\alpha_1^I(D_2^I+\alpha_1^I)}, \frac{D_3^I(Q^I-M^I)}{\alpha_1^I(D_3^I+\alpha_1^I)}, \frac{D_4^I(Q^I-M^I)}{\alpha_1^I(D_4^I+\alpha_1^I)}, \frac{D_1'^I(Q^I-M^I)}{\alpha_1^I(D_1'^I+\alpha_1^I)}, \frac{D_2'^I(Q^I-M^I)}{\alpha_1^I(D_2'^I+\alpha_1^I)}, \frac{D_3'^I(Q^I-M^I)}{\alpha_1^I(D_3'^I+\alpha_1^I)} \frac{D_4'^I(Q^I-M^I)}{\alpha_1^I(D_4'^I+\alpha_1^I)})]+$
$(\frac{C_s^I M^{2^I}}{2D_1^I},$
$\frac{C_s^I M^{2^I}}{2D_2^I}, \frac{C_s^I M^{2^I}}{2D_3^I}, \frac{C_s^I M^{2^I}}{2D_4^I}, \frac{C_s^I M^{2^I}}{2D_1'^I}, \frac{C_s^I M^{2^I}}{2D_2'^I}, \frac{C_s^I M^{2^I}}{2D_3'^I}, \frac{C_s^I M^{2^I}}{2D_4'^I}) + d^I(Q^I - M^I)(1 -$
$(\frac{D_1^I}{D_1^I+\alpha_1^I}, \frac{D_2^I}{D_2^I+\alpha_1^I}, \frac{D_3^I}{D_3^I+\alpha_1^I}, \frac{D_4^I}{D_4^I+\alpha_1^I}, \frac{D_1'^I}{D_1'^I+\alpha_1^I},$
$\frac{D_2'^I}{D_2'^I+\alpha_1^I}, \frac{D_3'^I}{D_3'^I+\alpha_1^I}, \frac{D_4'^I}{D_4'^I+\alpha_1^I})) = (a^I, b^I, c^I, d^I)(a'^I, b'^I, c'^I, d'^I),$

where

$$a^I = p_1^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_4^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_4^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_1^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_4^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_4^\beta)^I}{(Ap_1^\beta)^I+\alpha_1^I}]$$

$$b^I = p_2^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_3^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_3^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_2^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_3^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_3^\beta)^I}{(Ap_2^\beta)^I+\alpha_1^I}]$$

$$c^I = p_3^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_2^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_2^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_3^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_2^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_2^\beta)^I}{(Ap_3^\beta)^I+\alpha_1^I}]$$

$$d^I = p_4^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_1^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_1^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_4^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_1^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_1^\beta)^I}{(Ap_4^\beta)^I+\alpha_1^I}]$$

$$a'^I = p_1'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_4^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_4^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_1^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_4^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_4^\beta)'^I}{(Ap_1^\beta)'^I+\alpha_1^I}]$$

$$b'^I = p_2'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_3^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_3^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_2^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_3^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_3^\beta)'^I}{(Ap_2^\beta)'^I+\alpha_1^I}]$$

$$c'^{I} = p_3'^{I} Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^{\beta})'^{I}}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^{\beta})'^{I}}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_2^{\beta})'^{I}+\alpha_1^I)}}) + \frac{(Ap_2^{\beta})'^{I}(Q^I-M^I)}{\alpha_1^I((Ap_3^{\beta})'^{I}+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_2^{\beta})'^{I}} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_2^{\beta})'^{I}}{(Ap_3^{\beta})'^{I}+\alpha_1^I}]$$

$$d'^{I} = p_4'^{I} Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^{\beta})'^{I}}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^{\beta})'^{I}}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_1^{\beta})'^{I}+\alpha_1^I)}}) + \frac{(Ap_1^{\beta})'^{I}(Q^I-M^I)}{\alpha_1^I((Ap_4^{\beta})'^{I}+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_1^{\beta})'^{I}} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_1^{\beta})'^{I}}{(Ap_4^{\beta})'^{I}+\alpha_1^I}]$$

The defuzzified intuitionistic fuzzy total cost using signed distance method is given by

$$\text{d}((\text{TC})^I, 0) = \frac{1}{2}[[(p_1^I + p_4^I) + (p_1'^I + p_4'^I)]Q^I + 2C_o^I + C_h^I[\frac{1}{\alpha_1^I}\{(2Q^I - 2M^I + \frac{A[(p_1^{\beta^I} + p_4^{\beta^I}) + (p_1^{\beta'^I} + p_4^{\beta'^I})]}{\alpha_1^I})\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + (Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_4^{\beta^I}} + \frac{1}{p_1^{\beta^I}}) + (\frac{1}{p_4^{\beta'^I}} + \frac{1}{p_1^{\beta'^I}})] - d(Q^I - M^I)A[(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})] + 2d^I(Q^I - M^I)]$$

$$+\frac{1}{4}[[(p_2^I - p_1^I) + (p_2'^I + p_1'^I)]Q^I + C_h^I[\frac{1}{\alpha_1^I}\{(\frac{A}{\alpha_1^I}[(p_2^{\beta^I} - p_1^{\beta^I}) + (p_2^{\beta'^I} - p_1^{\beta'^I})])\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_3^{\beta^I} + p_3^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_3^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_3^{\beta'^I}} - \frac{1}{p_4^{\beta'^I}})] - d^I(Q^I - M^I)A[(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})]$$

$$-\frac{1}{4}[[(p_4^I - p_3^I) + (p_4'^I + p_3'^I)]Q^I + C_h^I[\frac{1}{\alpha_1^I}\{(\frac{A}{\alpha_1^I}[(p_4^{\beta^I} - p_3^{\beta^I}) + (p_4^{\beta'^I} - p_3^{\beta'^I})])\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_2^{\beta^I} + p_2^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_1^{\beta^I}} - \frac{1}{p_2^{\beta^I}}) + (\frac{1}{p_1^{\beta'^I}} - \frac{1}{p_2^{\beta'^I}})] - d^I(Q^I - M^I)A[(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})] = \text{F}^I(\text{q})$$

To find the minimum of $D(F^I(q))$ by taking the derivative $D(F^I(q))$, we get

$$\frac{d(F^I(q))}{dM} = \frac{1}{2}[-C_h^I[\frac{2}{\alpha_1^I} + \frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))(\frac{\alpha_1^I}{A(p_4^{\beta^I}+p_4^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + (Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))(\frac{\alpha_1^I}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} -$$

$$\frac{A}{\alpha_1^I}\{(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_4^{\beta^I}} + \frac{1}{p_1^{\beta^I}}) + (\frac{1}{p_4^{\beta'^I}} + \frac{1}{p_1^{\beta'^I}})] + dA[(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})] - 2d^I]$$

$$+\frac{1}{4}[-C_h^I(\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_3^{\beta^I} + p_3^{\beta'^I}))(\frac{\alpha_1^I}{A(p_3^{\beta^I}+p_3^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))(\frac{\alpha_1^I}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} - \frac{A}{\alpha_1^I}\{(\frac{p_3^{\beta^I}}{Ap_2^{\beta}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_3^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_3^{\beta'^I}} - \frac{1}{p_4^{\beta'^I}})] + d^I A[(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})]$$

$$-\frac{1}{4}[-C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))(\frac{\alpha_1^I}{A(p_1^{\beta^I}+p_1^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_2^{\beta^I} + p_2^{\beta'^I}))(\frac{\alpha_1^I}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} + e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} - \frac{A}{\alpha_1^I}\{(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_2^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_1^{\beta'^I}} - \frac{1}{p_2^{\beta'^I}})] + d^I A[(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})]$$

Neutrosophic model:

In this specific type of model, the trapezoidal neutrosophic numbers have been used to describe the neutrosophic purchasing cost and neutrosophic demand.

(ie) P^N = $(p_1^N, p_2^N, p_3^N, p_4^N)(p_1'^N, p_2'^N, p_3'^N, p_4'^N)(p_1''^N, p_2''^N, p_3''^N, p_4''^N)$

D^N = $(D_1^N, D_2^N, D_3^N, D_4^N)(D_1'^N, D_2'^N, D_3'^N, D_4'^N)(D_1''^N, D_2''^N, D_3''^N, D_4''^N)$

Based on [5], neutrosophic total cost is defined as follows:

(TC)^N = $P^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{D^N}{\alpha_1^N})(1 - e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D^N+\alpha_1^N)}}) - \frac{D^N(Q^N-M^N)}{\alpha_1^N(D^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2D^N} + d^N(Q^N - M^N)(1 - \frac{D^N}{D^N+\alpha_1^N})$

= $(p_1^N, p_2^N, p_3^N, p_4^N, p_1'^N, p_2'^N, p_3'^N, p_4', p_N''^N, p_2''^N, p_3''^N, p_4''^N)Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{D_1^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4^N}{\alpha_1^N}, Q^N - M^N + \frac{D_1'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4'^N}{\alpha_1^N} + \frac{D_1''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4''^N}{\alpha_1^N})] -$

$$C_h^N[\frac{1}{\alpha_1^N}((Q^N - M^N + \frac{D_1^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_2^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_3^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_1'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_2'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2'^N+\alpha_1^N)}}),$$

$$(Q^N - M^N + \frac{D_3'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_1''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1''^N+\alpha_1^N)}}), (Q^N-M^N+\frac{D_2''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2''^N+\alpha_1^N)}}), (Q^N-M^N+\frac{D_3''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3''^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4''^N+\alpha_1^N)}}) - (\frac{D_1^N(Q^N-M^N)}{\alpha_1^N(D_1^N+\alpha_1^N)}, \frac{D_2^N(Q^N-M^N)}{\alpha_1^N(D_2^N+\alpha_1^N)},$$

$$\frac{D_3^N(Q^N-M^N)}{\alpha_1^N(D_3^N+\alpha_1^N)} \frac{D_4^N(Q^N-M^N)}{\alpha_1^N(D_4^N+\alpha_1^N)}, \frac{D_1'^N(Q^N-M^N)}{\alpha_1^N(D_1'^N+\alpha_1^N)}, \frac{D_2'^N(Q^N-M^N)}{\alpha_1^N(D_2'^N+\alpha_1^N)}, \frac{D_3'^N(Q^N-M^N)}{\alpha_1^N(D_3'^N+\alpha_1^N)}, \frac{D_4'^N(Q^N-M^N)}{\alpha_1^N(D_4'^N+\alpha_1^N)}, \frac{D_1''^N(Q^N-M^N)}{\alpha_1^N(D_1''^N+\alpha_1^N)}, \frac{D_2''^N(Q^N-M^N)}{\alpha_1^N(D_2''^N+\alpha_1^N)}, \frac{D_3''^N(}{\alpha_1^N(D}$$

$$\frac{D_4''^N(Q^N-M^N)}{\alpha_1^N(D_4''^N+\alpha_1^N)})] + (\frac{C_s^N M^{2^N}}{2D_1^N}, \frac{C_s^N M^{2^N}}{2D_2^N}, \frac{C_s^N M^{2^N}}{2D_3^N}, \frac{C_s^N M^{2^N}}{2D_4^N}, \frac{C_s^N M^{2^N}}{2D_1'^N}, \frac{C_s^N M^{2^N}}{2D_2'^N}, \frac{C_s^N M^{2^N}}{2D_3'^N}, \frac{C_s^N M^{2^N}}{2D_4'^N}, \frac{C_s^N M^{2^N}}{2D_1''^N}, \frac{C_s^N M^{2^N}}{2D_2''^N}, \frac{C_s^N M^{2^N}}{2D_3''^N}, \frac{C_s^N M^{2^N}}{2D_4''^N}) + d^N(Q^N - M^N)(1 - (\frac{D_1^N}{D_1^N+\alpha_1^N}, \frac{D_2^N}{D_2^N+\alpha_1^N}, \frac{D_3^N}{D_3^N+\alpha_1^N}, \frac{D_4^N}{D_4^N+\alpha_1^N}, \frac{D_1'^N}{D_1'^N+\alpha_1^N}, \frac{D_2'^N}{D_2'^N+\alpha_1^N}, \frac{D_3'^N}{D_3'^N+\alpha_1^N}, \frac{D_4'^N}{D_4'^N+\alpha_1^N}, \frac{D_1''^N}{D_1''^N+\alpha_1^N}, \frac{D_2''^N}{D_2''^N+\alpha_1^N}, \frac{D_3''^N}{D_3''^N+\alpha_1^N}, \frac{D_4''^N}{D_4''^N+\alpha_1^N}))$$

$$= (a^N, b^N, c^N, d^N)(a'^N, b'^N, c'^N, d'^N)(a''^N, b''^N, c''^N, d''^N),$$

where

$$a^N = p_1^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)^N}{(Ap_1^\beta)^N+\alpha_1^N}]$$

$$b^N = p_2^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)^N}{(Ap_2^\beta)^N+\alpha_1^N}]$$

$$c^N = p_3^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)^N}{(Ap_3^\beta)^N+\alpha_1^N}]$$

$$d^N = p_4^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)^N}{(Ap_4^\beta)^N+\alpha_1^N}]$$

$$a'^N = p_1'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)'^N}{(Ap_1^\beta)'^N+\alpha_1^N}]$$

$$b'^N = p_2'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)'^N}{(Ap_2^\beta)'^N+\alpha_1^N}]$$

$$c'^N = p_3'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)'^N}{(Ap_3^\beta)'^N+\alpha_1^N}]$$

$$d'^N = p_4'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)'^N}{(Ap_4^\beta)'^N+\alpha_1^N}]$$

$$a''^N = p_1''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)''^N}{(Ap_1^\beta)''^N+\alpha_1^N}]$$

$$b''^N = p_2''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)''^N}{(Ap_2^\beta)''^N+\alpha_1^N}]$$

$$c''^N = p_3''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)''^N}{(Ap_3^\beta)''^N+\alpha_1^N}]$$

$d''^N = p_4''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)''^N}{(Ap_4^\beta)''^N+\alpha_1^N}]$

The defuzzified neutrosophic total cost using signed distance method is given by

d((TC)^N, 0) $=\frac{1}{2}[[(p_1^N + p_4^N) + (p_1''^N + p_4''^N)]Q^N + 2C_o^N + C_h^N[\frac{1}{\alpha_1^N}\{(2Q^N - 2M^N + \frac{A[(p_1^{\beta^N}+p_4^{\beta^N})+(p_1^{\beta''^N}+p_4^{\beta''^N})]}{\alpha_1^N})\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + (Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_4^{\beta^N}} + \frac{1}{p_1^{\beta^N}}) + (\frac{1}{p_4^{\beta''^N}} + \frac{1}{p_1^{\beta''^N}})] - d(Q^N - M^N)A[(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})] + 2d^N(Q^N - M^N)]$

$+\frac{1}{4}[[(p_2^N - p_1^N) + (p_2''^N + p_1''^N)]Q^N + C_h^N[\frac{1}{\alpha_1^N}\{(\frac{A}{\alpha_1^N}[(p_2^{\beta^N} - p_1^{\beta^N}) + (p_2^{\beta''^N} - p_1^{\beta''^N})])\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_3^{\beta^N}+p_3^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_3^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_3^{\beta''^N}} - \frac{1}{p_4^{\beta''^N}})] - d^N(Q^N - M^N)A[(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})]$

$-\frac{1}{4}[[(p_4^N - p_3^N) + (p_4''^N + p_3''^N)]Q^N + C_h^N[\frac{1}{\alpha_1^N}\{(\frac{A}{\alpha_1^N}[(p_4^{\beta^N} - p_3^{\beta^N}) + (p_4^{\beta''^N} - p_3^{\beta''^N})])\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_2^{\beta^N}+p_2^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_1^{\beta^N}} - \frac{1}{p_2^{\beta^N}}) + (\frac{1}{p_1^{\beta''^N}} - \frac{1}{p_2^{\beta''^N}})] - d^N(Q^N - M^N)A[(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})] = F^N(M)$

To find the minimum of $D(F^N(q))$ by taking the derivative $D(F^N(q))$, we get

$\frac{d(F^N(q))}{dM} = \frac{1}{2}[-C_h^N[\frac{2}{\alpha_1^N} + \frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))(\frac{\alpha_1^N}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + (Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))(\frac{\alpha_1^N}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_4^{\beta^N}} + \frac{1}{p_1^{\beta^N}}) + (\frac{1}{p_4^{\beta''^N}} + \frac{1}{p_1^{\beta''^N}})] + dA[(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})] - 2d^N]$

$$+\frac{1}{4}[-C_h^N(\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_3^{\beta^N}+p_3^{\beta''^N}))(\frac{\alpha_1^N}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N} + p_4^{\beta''^N}))(\frac{\alpha_1^N}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_3^{\beta^N}}{Ap_2^{\beta}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_3^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_3^{\beta''^N}} - \frac{1}{p_4^{\beta''^N}})] + d^N A[(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})]$$

$$-\frac{1}{4}[-C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))(\frac{\alpha_1^N}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_2^{\beta^N} + p_2^{\beta''^N}))(\frac{\alpha_1^N}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} + e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_2^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_1^{\beta''^N}} - \frac{1}{p_2^{\beta''^N}})] + d^N A[(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})]$$

5. **Numerical example:**

The relationship between the supply of a commodity at a constant rate of 10 units per day and its unit price may be expressed using the variables A and β. In this case, A represents 100 units and β is equal to 0.02. Supplies of varying quantities can be obtained as needed, with each order incurring a cost of 1000 units. The inventory is managed in lots of 250 units, with a purchase cost of Rs.45 per unit. Additionally, the rate of decay is 0.2. Determine the optimum total cost of the inventory.

Solution:

Let α_1^N = 0.2, C_o^N = 1000 per order, Q^N = 250, d^N = 10, A = 100,

β = 0.02, P^N = (40, 43, 47, 49)(36, 39, 50, 54)(33, 35, 55, 57),

D^N = (85, 95, 109, 112,)(65, 72, 115, 119)(45, 55, 120, 122).

Table 4.1: Analysis of a neutrosophic model as holding and shortage costs increases

S.No	C_h^N	C_s^N	Q$^{N^*}$	(TC)	(TC)^F	(TC)^I	(TC)^N
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10068943.8	20135637.6	7934942.9	5033909.4
2	(13,14,15) (12,14,16) (11,14,17)	(12,16,19) (11,16,20) (10,16,21)	133.33	11762410.9	23522389.08	9269053.4	5880597.27
3	(15,16,17) (14,16,18) (13,16,19)	(13,18,20) (11,18,21) (10,18,22)	132.35	13454174.6	26905733.96	10601822	6726433.47
4	(17,18,19) (16,18,20) (15,18,21)	(14,20,23) (13,20,23) (12,20,24)	131.57	15144874.7	30286951.97	11933752.72	7571737.99
5	(19,20,21) (18,20,22) (17,20,23)	(15,22,25) (14,22,26) (13,22,27)	130.95	16834859.8	33666740.03	13265120.08	8469470.75

Table 4.2: Evaluation of the neutrosophic model as the shortage cost rises

S.No	C_h^N	C_s^N	Q^{N*}	(TC)	$(TC)^F$	$(TC)^I$	$(TC)^N$
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10091627.4	20188962.3	7934942.9	5047240.57
2	(11,12,13) (10,12,14) (9,12,15)	(12,16,19) (11,16,20) (10,16,21)	142.85	9944768.5	19894995.01	7819537.9	4973748.75
3	(11,12,13) (10,12,14) (9,12,15)	(13,18,20) (11,18,21) (10,18,22)	150	9746243.2	19497670.67	7663514.21	4874417.65
4	(11,12,13) (10,12,14) (9,12,15)	(14,20,23) (13,20,23) (12,20,24)	156.25	9518236.6	1904134.63	7484311.84	4760343.65
5	(11,12,13) (10,12,14) (9,12,15)	(15,22,25) (14,22,26) (13,22,27)	161.76	9274965.4	18554548.8	7293106.85	4638637.2

Table 4.3: Analysis for neutrosophic model with increase of holding cost

S.No	C_h^N	C_s^N	Q^N	(TC)	(TC)^F	(TC)^I	(TC)^N
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10112447.15	20220784.4	7934942.9	5055196.1
2	(13,14,15) (12,14,16) (11,14,17)	(11,14,17) (9,14,18) (8,14,19)	124.66	11865307.06	23726156.3	9310188.89	5931539.07
3	(15,16,17) (14,16,18) (13,16,19)	(11,14,17) (9,14,18) (8,14,19)	116.67	13497261.3	26989754.3	10590591.44	6747438.57
4	(17,18,19) (16,18,20) (15,18,21)	(11,14,17) (9,14,18) (8,14,19)	109.37	15012882.6	30020718.7	11779679.57	7505179.67
5	(19,20,21) (18,20,22) (17,20,23)	(11,14,17) (9,14,18) (8,14,19)	102.94	16416561.1	32823899.9	12880818	8205974.98

6. Sensitivity Analysis

In this section, the neutrosophic total cost values are compared graphically.

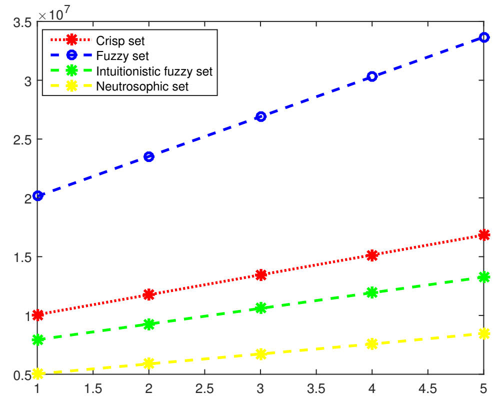

FIGURE 1. A graphical representation of the neutrosophic model as holding and shortages costs increase

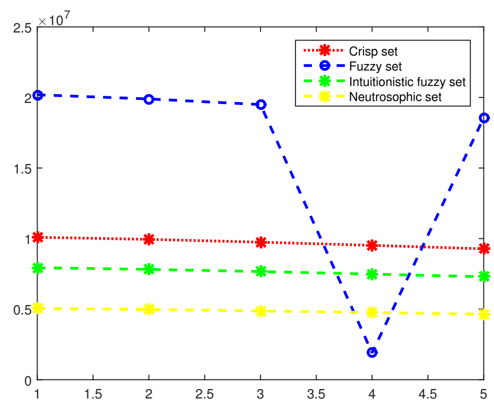

FIGURE 2. Graphical representation for neutrosophic model with increase of shortage cost

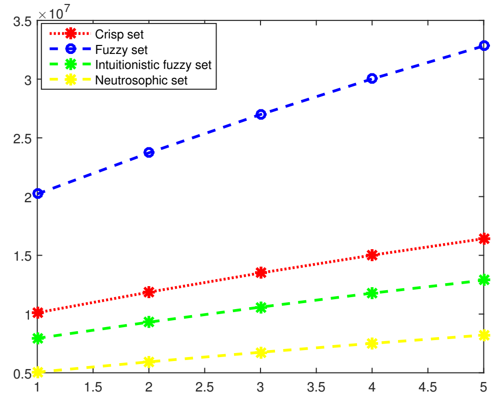

FIGURE 3. Graphical representation for neutrosophic model with increase of holding cost

Observations:

From Tables 1, 2 and 3 in the above descriptive model, it should be noted that compared to the crisp model, the neutrosophic model is very effective. The average total cost obtained in the neutrosophic inventory model is less than the crisp model. Thus, it is possible to claim that making use of neutrosophic sets delivers a more optimal approach to resolving inventory models compared to the conventional crisp, fuzzy, and intuitionistic fuzzy models.

- As neutrosophic holding costs and neutrosophic shortage costs decrease, the total cost of neutrosophics gradually increases.
- Neutrosophic total cost decreases if neutrosophic shortage cost increases.
- There is a slight change in the neutrosophic total cost compared to the neutrosophic holding cost in Table 1.

Conclusion:

A neutrosophic inventory model should be implemented to resolve the administration of decomposing products whose demand is determined by cost. The solution to shortages can be determined by applying the concept of fuzzification to both neutrosophic demand and neutrosophic purchasing cost, using trapezoidal neutrosophic numbers. The signed distance method is used for defuzzification process, to provide a unique optimal solution which will minimize the total cost of the item. Therefore, it can possibly concluded that the neutrosophic inventory method yields improved results compared to other readily accessible models. In future, this study could be extended for shortages and partial backlogging in neutrosophic inventory models.

Acknowledgement

The article has been written with the joint financial support of RUSA-Phase 2.0 grant sanctioned vide letter No.F 24-51/2014-U, Policy (TN Multi-Gen), Dept. of Edn. Govt. of India, Dt.09.10.2018, UGC-SAP (DRS-I) vide letter No.F.510/8/DRS-I/2016(SAP-I) Dt. 23.08.2016 and DST (FST - level I) 657876570 vide letter No.SR/FIST/MS-I/2018/17 Dt. 20.12.2018.

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Received: Jan 9, 2024. Accepted: Mar 30, 2024