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Decision Making By Neutrosophic Over Soft Topological Space

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Abstract. The empirical correlation system serves as a crucial tool for unveiling the linear interconnections between two variables. Its significance lies in providing a prominent approach to depict a straightforward relationship without explicitly indicating a causal link between the sets involved. In the current research, an innovative concept of correlations is introduced specifically for Neutrosophic Over Soft Sets ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-sets). This novel framework involves a meticulous examination of basic definitions and operations associated with Neutrosophic Over Soft Sets. Furthermore, the study extends to the introduction of a groundbreaking concept: a topological space integrated with Neutrosophic Over Soft Sets ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-sets). This addition aims to broaden the scope of understanding and application in mathematical contexts.The research does not merely establish theoretical foundations; it also explores various properties and theorems related to the introduced concepts. This is complemented by a series of numerical examples designed to provide clarity and facilitate a comprehensive grasp of the material. To demonstrate the practical application of these concepts, the research utilizes the correlation framework to present a numerical illustration. Specifically, it is applied to determine the top-performing student at GFC School for the academic year 2022-2023, showcasing the real-world relevance and applicability of the proposed methodologies.

Keywords: Neutrosophic Over Soft Set and Neutrosophic Over Soft Topological Space.

1. Introduction

In the course of daily life, uncertainty is a common experience. For instance, when rolling a die or tossing a coin onto an uneven surface, uncertainties emerge. The inception of fuzzy sets was credited to Zadeh [23] (1965), who introduced the notion of membership degrees. Zadeh also laid the foundation for a theory of possibility [24], whereas Bellman et al. [2] delved into decision-making within contexts influenced by fuzziness. Expanding on Zadeh's contributions, Atanassov introduced intuitionist fuzzy sets, concentrating on both degrees of membership and non-membership [1].

Smarandache [19] is attributed with the discovery of neutrosophic sets and the exploration of novel trends and applications within neutrosophic theory. In 1995, Bustince and Burillo [5] investigated the correlation of intuitionist fuzzy sets in scenarios involving interval values. Three potential utilities of neutrosophic sets were postulated by Christianto [6]. In 1999, Molodtsov [14] brought to light the primary result concerning soft sets, with a subsequent finding contributed by Maji et al. [12]. The year 2002 marked the introduction of neutrosophic soft sets by Broumi [3]. Neutrosophic sets have found practicality in medical contexts, as pursued by researchers [17, 18]and also in variouse field [7, 10, 16].Correlation measures are harnessed to discern connections between pairs of variables.

In 2015, Broumi and Deli embarked on an exploration into correlation measures for neutrosophic sets [4]. Radha et al. ushered in the concept of neutrosophic Pythagorean sets and their elevated correlation in the year 2021 [15]. An alternative facet of correlation was ushered into the spotlight by Ye, J., back in 2013 [22]. Wang et al. [21] delved into the realm of single-valued neutrosophic sets. The year 2020 saw Mallick, R., and Pramanik, S. [13] delve into discussions about pentapartitioned neutrosophic sets and their inherent properties. In 2019, Jansi et al. [11] stumbled upon the concept of neutrosophic pythagorean sets featuring both dependent and independent components. Smarandache [20] brought forth the innovative notion of neutrosophic sets replete with over, under, and off limits in 2016. Murugesan et al. [10] (2023) undertake a comparative analysis between Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps to unravel the intricacies of Covid variants. RN Devi and G Muthumari [8] conducted an in-depth study on the realm of topologized domination concerning NOver top graphs in 2021. Moreover, in the subsequent year, 2022, RN Devi etal., introduced a pioneering study on Digital Neutrosophic Topological Spaces, pushing the boundaries of mathematical modeling [9] .

The manuscript introduces innovative concepts: $\mathcal{N}\mathfrak{s}^{\mathfrak{o}}$-set and $\mathcal{N}\mathfrak{s}^{\mathfrak{o}}$-topological space. It also presents measures of correlations for neutrosophic over soft sets, elucidating foundational definitions, operations, and theorems supported by concrete numerical examples. The inclusion of numerical illustrations, drawn from a survey involving five teachers at GFC School, adds practical relevance. This survey aims to identify the top-performing student for the academic year 2022-2023. The manuscript, thus, seamlessly integrates theoretical developments, illustrative examples, and real-world applications, contributing comprehensively to the field of study.

2. **Preliminary**

This section contains basic definition for Neutrosophic Set(NS),Neutrosophic Over Set(NOS),Neutrosophic Soft Set(NSS),Neutrosophic Topological Space(NTS) and Soft Topological Space(STS).

Definition 2.1. Let $\mathcal{H}$ be an non empty set and $\mathcal{J}$ is said to be an NS.Then

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0,1]$ and $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$. Here $\aleph(\mathsf{h}), \eth(\mathsf{h})$ and $\Upsilon(\mathsf{h})$ are degree of true membership,degree of indeterminacy and degree of falsity.

Definition 2.2. Let $\mathcal{J}$ be an NS in $\mathcal{H}$.If $\mathcal{J}$ is said to be an NOS in an non-empty set $\mathcal{H}$ then it has at-least one neutrosophic component is > 1 and no other component are < 0 is defined as,

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

Where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, \Omega]$, $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$ and Ω is said to be over-limit of NOS

Note: $\rho(\mathcal{H})$ is a set of all the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ subset of an non-empty set $\mathcal{H}$

Definition 2.3. Let an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\odot = \{e, \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$ is said to be a Null $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ and $\circledast = \{e, \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$ is said to be an universal $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$.

Definition 2.4. Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameters on $\mathcal{H}$.Consider $\mathcal{A} \subset \mathcal{E}$.The collection $(\lambda, \mathcal{A})$ is an NSS then it is defined as,$\mathfrak{h}$

$$\lambda : \mathcal{A} \to \rho(\mathcal{H})$$

where λ is a mapping and $\rho(\mathcal{H})$ is a collection of all the subsets of NS in $\mathcal{H}$.

Definition 2.5. A neutrosophic topology(NT) τ_{NT} is a collection of subset of an NS $\mathcal{W}$ such that

(i) $\odot, \circledast \in \tau_{NT}$.

(ii) The union of an arbitrary collection τ_{NT} is in τ_{NT}.

(iii) The finite intersection of subsets τ_{NT} is in τ_{NT}.

Then $(\mathcal{W}, \tau_{NT})$ is called neutrosophic topological space(NTS).An element of τ_{NOS} is called an neutrosophic open set and τ_{NCS} is called an neutrosophic closed set.

Definition 2.6. A soft topology(ST) τ_{ST} is a collection of subset of an soft set(SS) $\mathcal{W}$ such that

(i) $\odot, \circledast \in \tau_{ST}$.

(ii) The union of an arbitrary collection τ_{ST} is in τ_{ST}.

(iii) The finite intersection of subsets τ_{ST} is in τ_{ST}.

Then $(\mathcal{W}, \tau_{ST})$ is called soft topological space(STS).An element of τ_{SOS} is called an soft open set and τ_{SCS} is called an soft closed set.

3. Neutrosophic Over Soft Topological Space

Definition 3.1. Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameter on $\mathcal{H}$.Then $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is defined by a set valued function

$$\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} : \mathcal{E} \to \rho(\mathcal{H})$$

where $\rho(\mathcal{H})$ is an set of all $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$.$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is an valued function from the set of parameter $\mathcal{E}$ on $\mathcal{H}$ is defined as

$$\mathcal{J} = (\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E}) = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Definition 3.2. Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ be a two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set.If $\mathcal{J}$ is said to be a subset of $\mathcal{W}$ i.e.,$\mathcal{J} \subseteq \mathcal{W}$ then

$$\aleph_{\mathcal{J}}(\mathsf{h}) \leq \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}) \leq \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \geq \Upsilon_{\mathcal{W}}(\mathsf{h})$$

In other words $\mathcal{W}$ is an super set of $\mathcal{J}$

Definition 3.3. Let $\mathcal{J} \subset \mathcal{W}$ and $\mathcal{W} \subset \mathcal{J}$ then $\mathcal{J} = \mathcal{W}$

Definition 3.4. Let $\mathcal{J}$ and $\mathcal{W}$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set, Then the union,intersection and compliment are defined by

(i)$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(ii)$\mathcal{J} \Omega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(iii)$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

Proposition 3.5. *Let $\mathcal{J}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$. Then*

(i). $\odot^{☾} = \circledast$

(ii). $\circledast^{☾} = \odot$

(iii). $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

Proof. 1. $\odot^{☾} = \circledast$

$$\odot = \{e, \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$$

$$\odot^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \circledast$$

$$\implies \odot^{☾} = \circledast$$

2. $\circledast^{☾} = \odot$

$$\circledast = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\circledast^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \odot$$

$$\Longrightarrow \circledast^{☾} = \odot$$

3. $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J}^{☾})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \Omega - (\Omega - \eth_{\mathcal{J}}(\mathsf{h})), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} = \mathcal{J}$$

$$\Longrightarrow (\mathcal{J}^{☾})^{☾} = \mathcal{J}$$

□

Proposition 3.6. *Let $\mathcal{J}$ and $\mathcal{W}$ be an $\mathcal{N}^{\mathfrak{o}}_{\mathfrak{s}}$-set on $\mathcal{H}$. Then*

(i). $\mathcal{J} \mho \mathcal{J} = \mathcal{J} \Omega \mathcal{J} = \mathcal{J}$

(ii). $\mathcal{J} \mho \mathcal{W} = \mathcal{W} \mho \mathcal{J}$

(iii). $\mathcal{J} \Omega \mathcal{W} = \mathcal{W} \Omega \mathcal{J}$

(iv). $\mathcal{J} \mho \odot = \mathcal{J}$ *and* $\mathcal{J} \mho \circledast = \circledast$

(v). $\mathcal{J} \Omega \odot = \odot$ *and* $\mathcal{J} \Omega \circledast = \mathcal{J}$

Proof. The proof is obvious from the definition. □

Theorem 3.7. *Let $\mathcal{J}$ and $\mathcal{W} \in \mathcal{N}^{\mathfrak{o}}_{\mathfrak{s}}$-set. Then*

(i). $(\mathcal{J} \mho \mathcal{W})^{☾} = \mathcal{J}^{☾} \mho \mathcal{W}^{☾}$

(ii). $(\mathcal{J} \Omega \mathcal{W})^{☾} = \mathcal{J}^{☾} \Omega \mathcal{W}^{☾}$

Proof. (i).By the union definition,

$$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J} \mho \mathcal{W})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \tag{1}$$

By the definition of compliment

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{☾} \mho \mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \tag{2}$$

From (1) and (2) we get,

$$(\mathcal{J} \mho \mathcal{W})^{☾} = \mathcal{J}^{☾} \mho \mathcal{W}^{☾}$$

(ii). By the union definition we know that,

$$\mathcal{J} \Omega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J}\Omega\mathcal{W})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (3)$$

By the definition of compliment

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{☾}\Omega\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (4)$$

From (3) and (4) we get,

$$(\mathcal{J}\Omega\mathcal{W})^{☾} = \mathcal{J}^{☾}\Omega\mathcal{W}^{☾} \quad \square$$

Definition 3.8. Let $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ be a neutrosophic over soft topology($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology) in $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\mathcal{J}$ is a collection of subset of an non-empty set $\mathcal{H}$ such that

(i) $\odot, \circledast \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

(ii) The union of an arbitrary collection $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

(iii) The finite intersection of subsets $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

Then $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is called neutrosophic over soft topological space($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space).An element of τ_{NOSOS} is called an neutrosophic over soft open set and τ_{NOSCS} is named an neutrosophic over soft closed set.

Example 3.9. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two students, $\mathcal{A} = \{Puntuality(\mathsf{q})\}$ and $\mathcal{G} \in \tau_{NOSCS}$ such that,

$$\mathcal{Q}(\mathsf{q}) = \{\langle \mathsf{r}_1, 1.2, 0.6, 0.5\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}$$

$(\mathcal{Q}, \mathcal{A}) = \{\mathsf{q} = \{\langle \mathsf{r}_1, 1.2, 0.6, 0.5\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}\}$.

Then,$\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{Q}, \mathcal{A})\}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology on $\mathcal{W}$.

Theorem 3.10. *Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$, then $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$.*

Proof. Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$.

$\implies \odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ and $\odot, \circledast \in \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$

$\implies \odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ $\therefore$ $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$. $\square$

Remark 3.11. In the theorem 2.2 instead of the intersection operation if we use union operation the claim may not be true. It can be seen following example.

Example 3.12. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two mobile phone and $\mathcal{A} = \{batterydurability(\mathsf{q}_1), workingspeed(\mathsf{q}_2)\}$.

Then $(\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A}) \in \tau_{NOSCS}$ such that,

$$\mathcal{R}_1(\mathsf{q}_1) = \{\langle \mathsf{r}_1, 1.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}\}$$
$$\mathcal{R}_1(\mathsf{q}_2) = \{\langle \mathsf{r}_1, 1.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.6\rangle\}\}$$
$$(\mathcal{R}_1, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.6\rangle\}\}\}$$
$$\mathcal{R}_2(\mathsf{q}_1) = \{\langle \mathsf{r}_1, 1.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, 1.2, 0.3, 0.3\rangle\}$$
$$\mathcal{R}_2(\mathsf{q}_2) = \{\langle \mathsf{r}_1 1.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2 1.1, 0.6, 0.6\rangle\}$$
$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, 1.2, 0.3, 0.3\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2, 1.1, 0.6, 0.6\rangle\}\}\}$$

Then, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$ and $\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_2, \mathcal{A})\}$ are two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{W}$.

But $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A})\}$.

Because $(\mathcal{R}_1, \mathcal{A}) \Omega (\mathcal{R}_2, \mathcal{A}) \notin \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.So, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is not $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$.

Definition 3.13. An operators of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ $\mathcal{R} \in \tau_{NOSCS}$, then neutrosophic over soft topological interior and closure are $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ and $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is defined as:

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \mho\{\mathcal{N} : \mathcal{N} \subseteq \mathcal{H} \text{and} \mathcal{N} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$ and

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$.

Proposition 3.14. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space and $\mathcal{R}$ is a subset of $\langle$, then*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the largest NOS open set contained in $\mathcal{R}$.

(ii) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the smallest NOS closed set containing $\mathcal{R}$.

Proof. (i) By the definition of interior, $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$.Let $\mathcal{N}$ be an open set such that $\mathcal{N} \subset \mathcal{R}$. $\because \mathcal{N}$ is open and $\mathcal{N} \subset \mathcal{R}$,then

$\mathcal{N} \subset int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the largest open set contained in $\mathcal{R}$.

(ii) By the closure definition,

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the smallest closed set containing $\mathcal{R}$. □

Theorem 3.15. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in τ_{NOSCS}.Then,*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\odot) = \odot$ and $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\circledast) = \circledast$.

(ii) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \subseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a NOSOS iff $\mathcal{Q} = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{Q})$.

(iv) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{Q})$

(vi) $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cup int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cup \mathcal{Q})$

(vii) $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cap \mathcal{Q}) = int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cap int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

(iii) If $\mathcal{Q}$ is a NOSOS over $\mathcal{H}$, then $\mathcal{Q}$ is itself a NOSOS over $\mathcal{H}$ which contains $\mathcal{Q}$.

So,$\mathcal{Q}$ is the largest $\mathcal{N}_\mathfrak{s}^\circ$ contained in $\mathcal{Q}$

$\implies int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) = \mathcal{Q}$.

Conversely, suppose that $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) = \mathcal{Q}$.then $\mathcal{Q} \in \tau_{\mathcal{N}_\mathfrak{s}^\circ}$.

(iv) Let $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) = \mathcal{Q}$.

Then, $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) = \mathcal{Q}$ from (iii).

$\implies int_{\mathcal{N}_\mathfrak{s}^\circ}(int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R})) = int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R})$

(v) Suppose that $\mathcal{R} \subseteq \mathcal{Q}$.As $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \subseteq \mathcal{R} \subseteq \mathcal{Q}.int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R})$ is a Neutrosophic over soft subset of $\mathcal{Q}$

From definition (3.2) we get, $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$.

(vi) It is clear that $\mathcal{R} \subseteq \mathcal{R} \Cup \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{R} \Cup \mathcal{Q}$.

Thus,

$int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cup int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$ and

$int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cup int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$

$\implies int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cup int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cup \mathcal{Q})$ [By (v)].

(vii) Clearly w.k.t.

$int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cap \mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}((R))$ and $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cap \mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}((Q))$[By (v)].

So, that $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cap \mathcal{Q}) \subseteq int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cap int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$

Also,$int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \subseteq \mathcal{R}$ and $int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) \subseteq \mathcal{Q}$ we have

$int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cap int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q}) \subseteq \mathcal{R} \Cap \mathcal{Q}$.

$\implies int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R} \Cap \mathcal{Q}) = int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{R}) \Cap int_{\mathcal{N}_\mathfrak{s}^\circ}(\mathcal{Q})$ □

Example 3.16. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two team in an company and

$\mathcal{A} = \{\text{punctuality}(\mathsf{q}_1), \text{accuracy of target}(\mathsf{q}_2)\}$. Then

$$(\mathcal{R}_1, \mathcal{A}) = \{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.3, 0.5, 0.3\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.4\rangle\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.2, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.5, 0.2\rangle\}\}\}$$

$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.6, 0.6, 0.5\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 1.1, 0.1\rangle, \langle \mathsf{r}_2, 1.5, 0.5, 0.5\rangle\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.1, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.4, 0.6, 0.4\rangle\}\}\}$$

$$(\mathcal{R}_3, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.4, 1.4, 0.2\rangle, \langle \mathsf{r}_2, 1.2, 0.4, 0.2\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 1.1, 0.2\rangle, \langle \mathsf{r}_2, 1.1, 0.4, 0.1\rangle\}\}, \{\mathsf{q}_3 =, \{\langle \mathsf{r}_1, 1.3, 1.4, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.2\rangle\}\}\}$$

Then, $\tau_{\mathcal{N}_s^o} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$.

$int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) = \odot$

$int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A}) = \odot$

Then, $int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) \mho int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A}) = \odot$

$int_{\mathcal{N}_s^o}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = (\mathcal{R}_1, \mathcal{A})$

$\therefore int_{\mathcal{N}_s^o}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) \neq int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) \mho int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A})$

Theorem 3.17. *Let $(\mathcal{H}, \tau_{\mathcal{N}_s^o})$ be a $\mathcal{N}_s^o$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in τ_{NOSCS}.Then,*

(i) $cl_{\mathcal{N}_s^o}(\odot) = \odot$ and $cl_{\mathcal{N}_s^o}(\circledast) = \circledast$.

(ii) $cl_{\mathcal{N}_s^o}(\mathcal{R}) \supseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a NOSCS iff $\mathcal{Q} = cl_{\mathcal{N}_s^o}(\mathcal{Q})$.

(iv) $cl_{\mathcal{N}_s^o}(cl_{\mathcal{N}_s^o}(\mathcal{R})) = cl_{\mathcal{N}_s^o}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies cl_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq cl_{\mathcal{N}_s^o}(\mathcal{Q})$

(vi) $cl_{\mathcal{N}_s^o}(\mathcal{R}) \mho cl_{\mathcal{N}_s^o}(\mathcal{Q}) = cl_{\mathcal{N}_s^o}(\mathcal{R} \mho \mathcal{Q})$

(vii) $cl_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) \subseteq cl_{\mathcal{N}_s^o}(\mathcal{R}) \Omega cl_{\mathcal{N}_s^o}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

Proof of (vi) and (vii) similar to the Theorem 2.3 (vi) and (vii)

(iii) If $\mathcal{R}$ is a NOSCS on $\mathcal{H}$ then $\mathcal{R}$ is itself a NOSCS over $\mathcal{H}$ which contains $\mathcal{R}$.

$\therefore \mathcal{R}$ is a smallest NOSCS containing $\mathcal{R}$. and $\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$.

Conversely, Suppose that $\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$. As. $\mathcal{R}$ is a NOSCS, so $\mathcal{R}$ is a NOSCS over $\mathcal{H}$.

(vi) $\mathcal{R}$ is a NOSCS then by the proof (iii)

$\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$..

(v) Suppose $\mathcal{R} \subseteq \mathcal{Q}$.Then every neutrosophic over soft closed super-set of $\mathcal{Q}$ also contained in $\mathcal{R}$.

$\implies$ super-sets of $\mathcal{Q}$ is also a NOSCS. Thus,

$cl_{\mathcal{N}_s^o}(\mathcal{R}) = cl_{\mathcal{N}_s^o}(\mathcal{Q})$. □

Example 3.18. Let $\mathcal{W} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two team in an company and $\mathcal{A} = \{\text{punctuality}(\mathsf{q}_1), \text{accuracy of target}(\mathsf{q}_2)\}$. Then $\mathcal{R}$,$\mathcal{Q}$ and $\mathcal{V} \in \tau_{NOSOS}$ such that

$$(\mathcal{R}_1, \mathcal{A}) = \{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.3, 0.5, 0.3\rangle, \{\mathsf{q}_2, \{\langle \mathsf{r}_1, 1.1, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.4\rangle\}\}\},$$
$$\{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.2, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.5, 0.2\rangle\}\}\}$$

$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.6, 0.6, 0.5\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 1.1, 0.1\rangle, \langle \mathsf{r}_2, 1.5, 0.5, 0.5\rangle\}\},$$
$$\{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.1, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.4, 0.6, 0.4\rangle\}\}\}$$

$(\mathcal{R}_3, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.4, 1.4, 0.2\rangle, \langle \mathsf{r}_2, 1.2, 0.4, 0.2\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 1.1, 0.2\rangle, \langle \mathsf{r}_2, 1.1, 0.4, 0.1\rangle\}\},$
$\{\mathsf{q}_3 =, \{\langle \mathsf{r}_1, 1.3, 1.4, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.2\rangle\}\}\}$

Then, $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$.

$\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}^{\complement} = \{\circledast, \odot, \mathcal{R}^{\complement}\}$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) = \circledast$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A}) = \circledast$

Then, $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) \mho cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A}) = \circledast$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = \circledast$

$\therefore cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) \mho cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A})$

4. Measure Of Correlation for Neutrosophic Over Soft Set

Definition 4.1. Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set over an non-empty set $\mathcal{H}$ is of the form

$$\mathcal{J} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Then the $\aleph_{\mathfrak{s}}$ correlation coefficient of $\mathcal{J}$ and $\mathcal{W}$ is

$$\varphi(\mathcal{J}, \mathcal{W}) = \frac{\mathcal{K}(\mathcal{J}, \mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J}, \mathcal{J}).\mathcal{K}(\mathcal{W}, \mathcal{W})})} \tag{5}$$

Where

$$\mathcal{K}(\mathcal{J}, \mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{J}, \mathcal{J}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{W}, \mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

Proposition 4.2. *Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$. Then it satisfies the following condition*

1. $0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$
2. $\varphi(\mathcal{J}, \mathcal{W}) = \frac{1}{n}$ *iff* $\mathcal{J} = \mathcal{W}$
3. $\varphi(\mathcal{J}, \mathcal{W}) = 1$ *iff* $\mathcal{J} = \mathcal{W}$ *and* $n = 1$
4. $\varphi(\mathcal{J}, \mathcal{W}) = \varphi(\mathcal{W}, \mathcal{J})$

Proof. 1. $0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$

The definition of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is conclude that $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, \Omega]$ so $0 \leq \varphi(\mathcal{J}, \mathcal{W})$

Now we have to prove $\varphi(\mathcal{J}, \mathcal{W}) \leq 1$

$$\begin{aligned}\mathcal{K}(\mathcal{J}, \mathcal{W}) &= \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right) \\
&= [(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^2.(\eth_{\mathcal{W}}(\mathsf{h}_1))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2] \\
&+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_2))^2.(\eth_{\mathcal{W}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2] \\
&+ \cdots + [(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_n))^2.(\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\end{aligned}$$

By Cauchy-Schwartz inequality, we get

$$\begin{aligned}(\mathcal{K}(\mathcal{J}, \mathcal{W}))^2 \leq &\Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^4] + [(\aleph_{\mathcal{J}}(\mathsf{h}_2))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_2))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^4] + \cdots \\
&+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_n))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^4]\Big).\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_1))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^4] \\
&+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_2))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^4] + \cdots + [(\aleph_{\mathcal{W}}(\mathsf{h}_n))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^4]\Big)\end{aligned}$$

$$\begin{aligned}\leq &\Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^2(\eth_{\mathcal{J}}(\mathsf{h}_1))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2] + [(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2 \\
&+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2(\eth_{\mathcal{J}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2] + \cdots + [(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2 \\
&+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2(\eth_{\mathcal{J}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2]\Big)\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_1))^2(\eth_{\mathcal{W}}(\mathsf{h}_1))^2 \\
&+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2] + [(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_2))^2(\eth_{\mathcal{W}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2] \\
&+ \cdots + [(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\Big) \\
&= (\mathcal{K}(\mathcal{J}, \mathcal{J}))(\mathcal{K}(\mathcal{W}, \mathcal{W})) \\
&\implies (\mathcal{K}(\mathcal{J}, \mathcal{W}))^2 \leq [(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))] \\
&\implies \mathcal{K}(\mathcal{J}, \mathcal{W}) \leq \sqrt{[(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))]}\end{aligned}$$

Then,

$$\mathcal{K}(\mathcal{J}, \mathcal{W}) \leq n\sqrt{[(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))]}$$

$$\therefore 0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$$

2. $\varphi(\mathcal{J}, \mathcal{W}) = \frac{1}{n}$ iff $\mathcal{J} = \mathcal{W}$

given that, $\mathcal{J} = \mathcal{W}$

$$\implies \mathcal{K}(\mathcal{J}, \mathcal{W}) = \mathcal{K}(\mathcal{W}, \mathcal{W}) \tag{6}$$

$$\mathcal{K}(\mathcal{J}, \mathcal{J}) = \mathcal{K}(\mathcal{W}, \mathcal{W}) \tag{7}$$

From (6) and (7), we get

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{1}{n} \tag{8}$$

3. $\varphi(\mathcal{J},\mathcal{W}) = 1$ iff $\mathcal{J} = \mathcal{W}$ and $n = 1$

Put $n = 1$ in (8) we get,

$$\varphi(\mathcal{J},\mathcal{W}) = 1$$

4. $\varphi(\mathcal{J},\mathcal{W}) = \varphi(\mathcal{W},\mathcal{J})$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{W},\mathcal{J}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{J})}{n(\sqrt{\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{J},\mathcal{J})})}$$

Only we have to prove $\mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$= \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

□

Definition 4.3. Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$ is of the form

$$\mathcal{J} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Then the $\ddot{\Upsilon}_{\mathfrak{s}}$ correlation coefficient of $\mathcal{J}$ and $\mathcal{W}$ is

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\left[min(\mathcal{K}(\mathcal{J},\mathcal{J}),\mathcal{K}(\mathcal{W},\mathcal{W}))\right]} \tag{9}$$

Where

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{J},\mathcal{J})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{W},\mathcal{W})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

Proposition 4.4. *Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$. Then it satisfies the following condition*

1. $0 \leq \varphi^*(\mathcal{J},\mathcal{W}) \leq 1$
2. $\varphi^*(\mathcal{J},\mathcal{W}) = \frac{1}{n}$ *iff* $\mathcal{J}=\mathcal{W}$
3. $\varphi^*(\mathcal{J},\mathcal{W}) = 1$ *iff* $\mathcal{J}=\mathcal{W}$ *and* $n=1$
4. $\varphi^*(\mathcal{J},\mathcal{W}) = \varphi^*(\mathcal{W},\mathcal{J})$

Proof. 1. $0 \leq \varphi^*(\mathcal{J},\mathcal{W}) \leq 1$

By the definition of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set we know that $\aleph,\eth,\Upsilon:\mathcal{H}\to[0,\Omega]$ so $0\leq\varphi^*(\mathcal{J},\mathcal{W})$

Now we have to prove $\varphi(\mathcal{J},\mathcal{W})\leq 1$

$$\mathcal{K}(\mathcal{J},\mathcal{W})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$\begin{aligned}&=[(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2+(\eth_{\mathcal{J}}(\mathsf{h}_1))^2.(\eth_{\mathcal{W}}(\mathsf{h}_1))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2]\\&+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2.(\eth_{\mathcal{W}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]\\&+\cdots+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2.(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\end{aligned}$$

By Cauchy-Schwartz inequality, we get

$$\begin{aligned}(\mathcal{K}(\mathcal{J},\mathcal{W}))^2\leq&\Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^4+(\eth_{\mathcal{J}}(\mathsf{h}_1))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^4]+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^4+(\eth_{\mathcal{J}}(\mathsf{h}_2))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^4]+\cdots\\&+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^4+(\eth_{\mathcal{J}}(\mathsf{h}_n))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^4]\Big).\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^4+(\eth_{\mathcal{W}}(\mathsf{h}_1))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^4]\\&+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^4+(\eth_{\mathcal{W}}(\mathsf{h}_2))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^4]+\cdots+[(\aleph_{\mathcal{W}}(\mathsf{h}_n))^4+(\eth_{\mathcal{W}}(\mathsf{h}_n))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^4]\Big)\end{aligned}$$

$$\begin{aligned}\leq&\Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2+(\eth_{\mathcal{J}}(\mathsf{h}_1))^2(\eth_{\mathcal{J}}(\mathsf{h}_1))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2]+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2\\&+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2(\eth_{\mathcal{J}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2]+\cdots+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2\\&+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2(\eth_{\mathcal{J}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2]\Big)\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2+(\eth_{\mathcal{W}}(\mathsf{h}_1))^2(\eth_{\mathcal{W}}(\mathsf{h}_1))^2\\&+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2]+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2+(\eth_{\mathcal{W}}(\mathsf{h}_2))^2(\eth_{\mathcal{W}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]\\&+\cdots+[(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\Big)\end{aligned}$$

$$=(\mathcal{K}(\mathcal{J},\mathcal{J}))(\mathcal{K}(\mathcal{W},\mathcal{W}))$$

$$\implies(\mathcal{K}(\mathcal{J},\mathcal{W}))^2\leq[(\mathcal{K}(\mathcal{J},\mathcal{W}))(\mathcal{K}(\mathcal{J},\mathcal{W}))]$$

Then,

$$\mathcal{K}(\mathcal{J},\mathcal{W}) \leq n\big[min[(\mathcal{K}(\mathcal{J},\mathcal{W})),(\mathcal{K}(\mathcal{J},\mathcal{W}))]\big]$$

2. $\varphi(\mathcal{J},\mathcal{W}) = \frac{1}{n}$ iff $\mathcal{J} = \mathcal{W}$

given that, $\mathcal{J} = \mathcal{W}$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{W}) \tag{10}$$

$$\mathcal{K}(\mathcal{J},\mathcal{J}) = \mathcal{K}(\mathcal{W},\mathcal{W}) \tag{11}$$

From (10) and (11), we get

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{1}{n} \tag{12}$$

3. $\varphi^*(\mathcal{J},\mathcal{W}) = 1$ iff $\mathcal{J} = \mathcal{W}$ and $n = 1$

Put $n = 1$ in (12) we get,

$$\varphi^*(\mathcal{J},\mathcal{W}) = 1$$

4. $\varphi^*(\mathcal{J},\mathcal{W}) = \varphi(\mathcal{W},\mathcal{J})$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{W},\mathcal{J}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{J})}{n\big[min[\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{J},\mathcal{J})]\big]}$$

Only we have to prove $\mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n}\big((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\big)$$

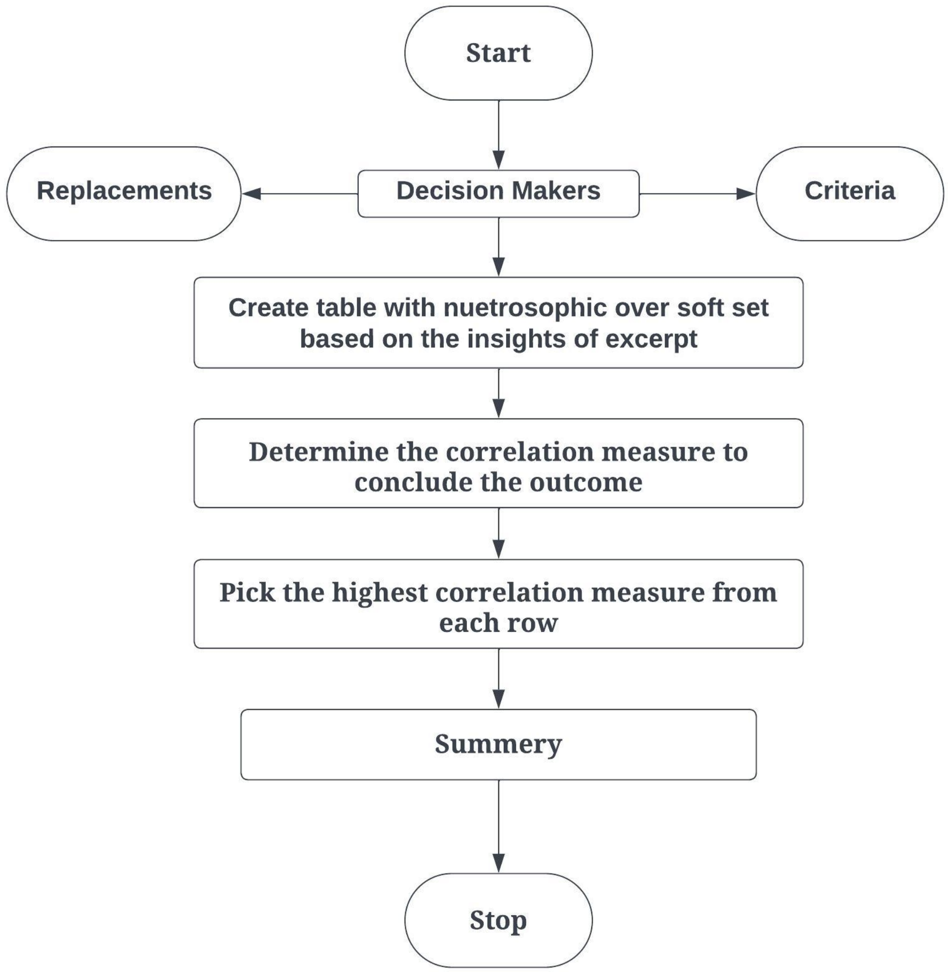
$$= \sum_{\varrho=1}^{n}\big((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\big) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\therefore \varphi^*(\mathcal{J},\mathcal{W}) = \varphi^*(\mathcal{W},\mathcal{J})$$

□

5. Flow Chart To Solving $\mathcal{N}_{\mathfrak{s}}^{o}$-set Using Correlation Measure



6. Numerical Illustration

Assume an effective instance that helps the awarding committee to make a decision to find top-performing student of the year 2022-2023.top-performing students are selected not only by their education also with many criteria.

Similar situation arises for GFC school they are conducting an competition to select a top-performing student. Now they have to superior one student out of three. So for this situation we are applying $\aleph_{\mathfrak{s}}$ and $\ddot{Y}_{\mathfrak{s}}$ correlation for the set $\mathcal{N}_{\mathfrak{s}}^{o}$-set.

Replacement and Criteria:

Let us take three students as S, R, Y .Required Qualities as Education, Self Discipline, Honesty and Awards as First Place, Second Place and Third Place.[Replacement={Students,Awards} and Criteria={Education, Self Discipline, Honesty}]

Where, Education={clearance of all subject,knowledge over subject,general knowledge,team work,creativity in project work,involvement in school educational and sports programs}

Self Discipline= {acceptance,willpower,hard work,persistence,punctuality,regular practice,behaviour}

Honesty={truthfulness,trust towards the student,sincere in following rules and regulation}

Analized Data:

X	Education	Self Discipline	Honesty
S	(1.8,0.7,0.6)	(1.5,0.3,0.8)	(1.6,0.9,0.4)
Y	(1.2,0.7,0.9)	(1.6,0.7,0.6)	(1.4,0.6,0.3)
R	(1.3,1.1,0.5)	(0.8,1.5,0.6)	(1.4,0.8,0.8)

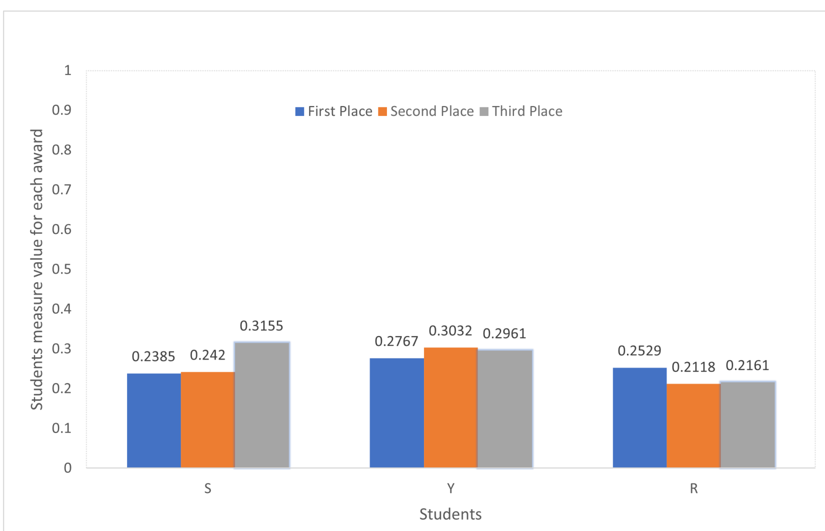
Table 1. Relation between Students and Required Qualities

Z	First Place	Second Place	Third Place
Education	(0.9,1.5,0.4)	(0.5,1.3,0.6)	(1.5,0.5,0.6)
Self Discipline	(1.1,0.5,0.3)	(1.5,0.6,0.7)	(1.4,0.3,0.7)
Honesty	(1.3,0.7,0.8)	(1.1,0.7,0.4)	(0.9,0.8,0.6)

Table 2. Relation between Required Qualities and Places

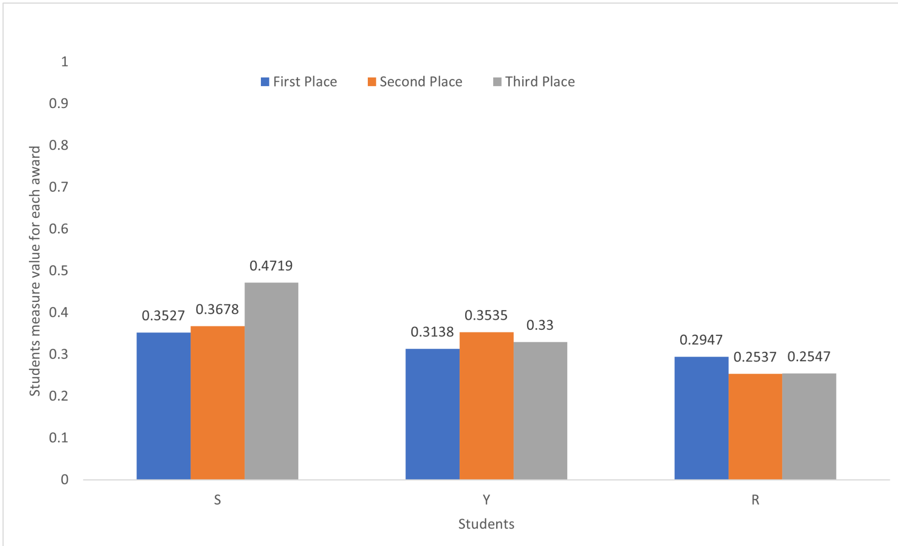
φ	First Place	Second Place	Third Place
S	0.2385	0.2420	**0.3155**
Y	0.2767	**0.3032**	0.2961
R	**0.2529**	0.2118	0.2161

Table 3. $\aleph_5$ Correlation between Students and Places



φ^*	First Place	Second Place	Third Place
S	0.3527	0.3678	**0.4719**
Y	0.3138	**0.3535**	0.3300
R	**0.2947**	0.2537	0.2547

TABLE 4. $\ddot{\mathrm{Y}}_{\mathfrak{s}}$ Correlation between Students and Places



Student	Place
S	Third Place
Y	Second Place
R	First Place

TABLE 5. Summery

Thus the student **R** got first place and awarded for the top-performing student of the year 2022-2023.Also, Student **Y** and **S** got second and third place.

7. Conclusion

This manuscript breaks new ground by offering a novel perspective on correlation within the realm of Neutrosophic Over Soft Sets, contributing to the advancement of theoretical frameworks in this specialized mathematical field. In addition to presenting a comprehensive exploration of various operational characteristics inherent to Neutrosophic Over Soft Sets, the manuscript introduces a fresh analytical approach by proposing a novel formula to quantify correlation. The $\aleph_{\mathfrak{s}}$ correlation and $\ddot{\mathrm{Y}}_{\mathfrak{s}}$ correlation extend beyond traditional measures, providing a nuanced understanding of the relationships within this mathematical framework.

Furthermore, the manuscript demonstrates the practical applicability of the introduced correlation measures through an illustrative scenario. In this scenario, Students **R**, **Y**, and **S** secure the first, second, and third positions, respectively, culminating in the designation of Student **R** as the top-performing student for the academic year 2022-2023. This synthesis of theoretical innovation and practical application not only adds depth to the study of Neutrosophic Over Soft Sets but also underscores the real-world relevance of the proposed measures. As such, this manuscript holds significant implications for both theoretical researchers and practitioners seeking advanced analytical tools within this mathematical domain.

In a similar vein, the utilization of the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set correlation measure extends across a variety of domains, encompassing fields like medicine, industry, and construction.

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