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Translation of Neutrosophic INK-Algebra

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Abstract. This study investigates the philosophical foundations of the neutrosophic set($\mathbb{N}_S$), as first proposed by Smarandache. It clarifies the connection between single-valued $\mathbb{N}_s$s and their function as a specialized subset in the larger context of $\mathbb{N}_s$s, particularly in the fields of science and engineering. This paper investigates neutrosophic INK-ideals(NINK-Is) within INK-algebras(INK-A) by using the notion of translation, which is positioned as an extension of intuitionistic fuzzy sets. The notion of translation neutrosophic INK-algebras(NTINK-A) is introduced and their fundamental characteristics are explored. In addition, properties associated with the translation of INK-subalgebra(INK-Ss) and INK-ideals(INK-I) are investigated, along with the dynamics of their multiplications, unions, and intersections in the context of neutrosophic INK-ideals(NINK-I). Further definitions and theorems are added in the article, providing comprehensive insights complexities of NINK-A.

Keywords: INK-algebra; Translation INK-ideal; Neutrosophic translation INK-ideal; Neutrosophic translation INK-subalgebra.

1. Introduction

Zade [9] in 1965 introduced, the idea of fuzzy sets has shown to be useful in addressing uncertainties in a variety of real-world applications, then Imai [31] and Iseki [32], [30] applied the fuzzy set in BCI/BCK-algebras. Atanassov [2] developed the intuitionistic fuzzy set in 1983 as an extension of this concept, and it has since been used in a variety of industries, including financial services, sales analysis, marketing of new products, negotiation techniques, and psychology research. In Mathematica Japonica, an essay by Jun and Meng from 1994 [28] delves into the subject of fuzzy p-ideals in BCI-algebras. The main goal is to comprehend these fuzzy p-ideals' features and attributes about BCI-algebras, algebraic structures having uses in computer science and mathematical logic. Furthermore, neutrosophic probability, set, and logic are included in the notion of neutrosophic, which was first introduced by Smarandache [5]. A philosophical framework called neutrosophic addresses contradiction, indeterminacy, and imperfect knowledge. Jun and Eun Hwan Roh's [7] study, published in Open Mathematics, extends the investigation of neutrosophic ideals in a particular kind of algebraic structure by delving into MBJ-neutrosophic ideals of BCK/BCI-algebras. Kaviyarasu et a. explored fuzzy p-ideals in INK-Algebra and $\mathbb{N}_s$s in the same algebraic structure [10]. The comprehension of fuzzy and neutrosophic notions in the context of INK-Algebra is probably improved by these investigations [11].

The concept of $\mathbb{N}_s$s is presented as a more comprehensive framework that generalizes intuitionistic fuzzy sets in Smarandache's [4]. In [8], Lee, Jun, and Doh published a study on fuzzy multiplications and translations in BCK/BCI-algebras. It probably looks at these operations' characteristics and ramifications to these algebraic structures. Agboola and Davvaz gave an introduction to neutrosophic BCI/BCK algebras [1]. Senapati [20] explores fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras, presumably investigating the connections and implications of fuzzy translations and fuzzy H-ideals in these algebraic structures.

Senapati et a. [21] also published a paper in the Eurasian Mathematical Journal that probably focuses on Atanassov's intuitionistic fuzzy translations of ideals in BCK/BCI-algebras and intuitionistic fuzzy subalgebras, thus expanding the literature on fuzzy translations to intuitionistic fuzzy sets. The 2016 work by Wadei F. Al-Omeri [27] and colleagues investigate the "Ra" operator in ideal topological spaces, presumably looking at its characteristics and uses in the setting of these spaces. Kaviyarasu and Indhira [13]presented on intuitionistic fuzzy INK-ideals of INK-algebras.The study by Mohseni Takallo et al. [16] presents MBJ-neutrosophic structures and their uses in BCK/BCI-algebras, probably investigating the characteristics and importance of these structures to these algebraic systems. The 2018 study by Al-Omeri et al. [26] examines the degree of (L, M)-fuzzy semi-precontinuous and (L, M)-fuzzy semi-preirresolute functions, most likely with an emphasis on their continuity qualities and fuzzy mathematics consequences. The 2019 work by Manokaran et al. [14] addresses MBJNeutrosophic B-Subalgebras, probably examining their characteristics and uses in the context of neutrosophic algebraic structures.

Khalid et al. [15] investigate MBJ-neutrosophic T-ideals on B-algebras, presumably looking into the characteristics and uses of these ideals inside the context of B-algebras.In their 2020 work, Kaviyarasu et al. [12] address the direct product of neutrosophic INKAlgebras, presumably investigating the structure and characteristics of these products with neutrosophic algebraic systems.

Al-Omeri et al. [23] is devoted to cone metric spaces and neutrosophic fixed point theorems; they most likely contain conclusions about the uniqueness and existence of fixed points in

$\mathbb{N}_s$tings. Mixed b-fuzzy topological spaces are covered in Al-Omeri's [24] work, which probably explores the characteristics and uses of these spaces in fuzzy topology and later examines virtually e-I-continuous functions in another article, most likely to examine their continuity qualities and potential uses in mathematical analysis [25]. In their 2021 work, Song et al. [19] use MBJ-Neutrosophic structures to study commutative ideals of BCI-Algebras. They probably investigate these ideals' characteristics and uses to BCI-Algebras. In the Hacettepe Journal of Mathematics and Statistics, Jun's [29] work addresses fuzzy ideal translations in BCK/BCI algebras. This study probably investigates the transformation of fuzzy ideals inside the algebraic structures. Bordbar et al. [3] investigate positive implicative ideals of BCK-algebras based on falling shadows and $\mathbb{N}_s$s. They most likely look into these ideas' characteristics and uses, paying close attention to their implicative elements. The SuperHyperSoft Set and the Fuzzy Extension Super HyperSoft Set are concepts that are introduced in Smarandache's [22], offering a fresh take on set theory with applications in neutrosophic systems. New forms of soft sets, such as HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set, are presented in another work by Smarandache [6]. These new types of soft sets give expansions and enhancements to the theories of current soft sets. The direct product of neutrosophic h-ideal in INK-Algebra is examined in Kaviyarasu and Rajeshwari [17]. It probably looks at the creation and characteristics of such products inside this algebraic structure. Pura Vida Neutrosophic Algebra is the topic of Ranulfo et al. 2023 article, which is expected to present a novel method of algebraic structures with applications in neutrosophic systems [18]. They extended their study to intuitionistic fuzzy translations by examine the interplay between fuzzy translations, extensions, and multiplications. The present paper uses $\mathbb{N}_s$s to study TINK-Is in INK-algebras. The writers investigate the characteristics of translation INK-subalgebra and present the idea of NINK-I. The following aspects are addressed in this discussion:

- Using translations to set up NINK-Ss.
- To discuss NTINK-Is in relation to INK-subalgebras.
- Investigating the relationship between translations of NINK-I and NINK-Ss.
- Presenting the conditions under which a translations of NINK-I can be made from an neutrosophic INK-algebra.
- constructing the translation property for NINK-I.

A summary of the fundamental ideas of INK-algebra and $\mathbb{N}_s$s that are necessary for understanding the discussions in this paper is given in the second section. $\mathbb{N}_s$s are used to study TINK-Ss and INK-I in the third section. This investigation is continued in Section four.

2. Basic Definitions

The basic components required to understand this paper are included in this section.

Definition 2.1. [11] In algebra $(\gimel, \star, 0)$ is known as a INK-algebra if it fulfils the following criteria for any $\check{l}, \check{m}, \check{n} \in \gimel$.

(1) $((\check{l} \star \check{m}) \star (\check{l} \star \check{n})) \star (\check{n} \star \check{m}) = 0$
(2) $(((\check{l} \star \check{n}) \star (\check{m} \star \check{n})) \star (\check{l} \star \check{m}) = 0$
(3) $\check{l} \star 0 = \check{l}$
(4) $\check{l} \star \check{m} = 0$ and $\check{m} \star \check{l} = 0 \Longleftrightarrow \check{l} = \check{m}$

where $\star$ is a binary operation and the 0 is a constant of $\gimel$.

Example 2.2. Consider INK-algebra $\gimel = \{0, 1, 2, 3, 4\}$ with the following cayclay table.

$\cdot$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	b	0
c	c	a	c	0	a
d	d	d	d	d	0

Table 1. INK-algebra

Definition 2.3. [11] If $\check{l} \star \check{m}$ in Ş, a non-empty subset Ş of an INK-algebra $(\gimel, \star, 0)$ is said to be an INK-s of $\gimel$.

Definition 2.4. [12] Consider the INK-algebra $(\gimel, \star, 0)$. In INK-I of $\gimel$ is a non-empty subset I of $\gimel$ that satisfies,

(1) $0 \in$ Į
(2) $((\check{n} \star \check{l}) \star (\check{n} \star \check{m})) \in$ Į and $\check{m} \in$ Į imply $\check{l} \in$ Į, $\forall\, \check{l}, \check{m}, \check{n} \in \gimel$.

Definition 2.5. [4] In a non-void set $\gimel$, a $\mathbb{N}_s$ A is the form's structure $A = \left\{ \left(\gimel, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l}) \right) | \check{l} \in \gimel \right\}$, where $\Bbbk_A^{T_r} : \gimel \to [0,1]$ is a truth membership function $\Bbbk_A^{I_d} : \gimel \to [0,1]$ is a indeterminate membership function and $\Bbbk_A^{F_s} : \gimel \to [0,1]$ is a false membership function.

Definition 2.6. [12] A $\mathbb{N}_s$ $A = \left\langle \Bbbk^T, \Bbbk^I, \Bbbk^F \right\rangle$ is known as a NINK-S if it meets all the requirements of INK-algebra,

1. $\left(\Bbbk_A^{T_r}\right)(\check{l} \star \check{m}) \geq \wedge \left\{ \left(\Bbbk_A^{T_r}\right)(\check{l}), \left(\Bbbk_A^{T_r}\right)(\check{m}) \right\}$
2. $\left(\Bbbk_A^{I_d}\right)(\check{l} \star \check{m}) \geq \wedge \left\{ \left(\Bbbk_A^{I_d}\right)(\check{l}), \left(\Bbbk_A^{I_d}\right)(\check{m}) \right\}$
3. $\left(\Bbbk_A^{F_s}\right)(\check{l} \star \check{m}) \leq \vee \left\{ \left(\Bbbk_A^{F_s}\right)(\check{l}), \left(\Bbbk_A^{F_s}\right)(\check{m}) \right\}, \ \forall\, \check{l}, \check{m}, \in \gimel.$

A	0	1	2	3	4
$\Bbbk^{Tr}$	0.5	0.4	0.2	0.5	0.1
$\Bbbk^{I_d}$	0.6	0.3	0.1	0.4	0.1
$\Bbbk^{F_s}$	0.26	0.35	0.45	0.55	0.59

TABLE 2. NINK-S

Example 2.7. Define $A = \langle \Bbbk^{Tr}, \Bbbk^{I_d}, \Bbbk^{F_s} \rangle$ be an neutrosophic subset of $\daleth$ in Table 1.

Then A is a neutrosophic INK-S of $\daleth$.

Definition 2.8. [12] Let $\daleth$ be a INK-algebra. $\mathbb{N}_s$ $A = \left\{ \left(\daleth, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l}) \right) | \check{l} \in \daleth \right\}$ in $\daleth$ is referred to as N-I of $\daleth$, if it meets the following criteria

(1) $\Bbbk_A^{T_r}(0) \geq \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(0) \geq \Bbbk_A^{I_d}(\check{l})$ and $\Bbbk_A^{F_s}(0) \leq \Bbbk_A^{F_s}(\check{l})$
(2) $\Bbbk_A^{T_r}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{T_r}(\check{l} \star \check{m}), \Bbbk_A^{T_r}(\check{m}) \right\}$
(3) $\Bbbk_A^{I_d}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{I_d}(\check{l} \star \check{m}), \Bbbk_A^{I_d}(\check{m}) \right\}$
(4) $\Bbbk_A^{F_s}(\check{l}) \leq \vee \left\{ \Bbbk_A^{F_s}(\check{l} \star \check{m}), \Bbbk_A^{F_s}(\check{m}) \right\}$. $\forall\ \check{l}, \check{m} \in \daleth$.

Definition 2.9. [12] Let $\daleth$ be a INK-A. $\mathbb{N}_S A = \left\{ \left(\daleth, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l}) \right) | \check{l} \in \daleth \right\}$ in $\daleth$ is referred to as NINK-I of $\daleth$, if it meets the following criteria

(1) $\Bbbk_A^{T_r}(0) \geq \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(0) \geq \Bbbk_A^{I_d}(\check{l})$ and $\Bbbk_A^{F_s}(0) \leq \Bbbk_A^{F_s}(\check{l})$
(2) $\Bbbk_A^{T_r}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{T_r}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{T_r}(\check{m}) \right\}$
(3) $\Bbbk_A^{I_d}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{I_d}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{I_d}(\check{m}) \right\}$
(4) $\Bbbk_A^{F_s}(\check{l}) \leq \vee \left\{ \Bbbk_A^{F_s}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{F_s}(\check{m}) \right\}$, $\forall\ \check{l}, \check{m}, \check{n} \in \daleth$.

3. NTINK-S

For the sake of simplicity, we shall use the symbol $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s} \right)$ for the neutrosophic subset $A = \left(\check{l}, \Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s}; \check{l} \in \daleth \right)$. Throughout this paper, we take $T_r, Id = 1 - sup \left\{ \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}) | \check{l} \in \daleth \right\}$ and $Fs = inf \left\{ \Bbbk_A^{F_s}(\check{l}) | \check{l} \in \daleth \right\}$ for any $\mathbb{N}_s$ $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s} \right)$ of $\daleth$.

Definition 3.1. A $\mathbb{N}_s$ $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s} \right)$ be a $\mathbb{N}_s$ of $\daleth$ and $\varrho, \zeta, \nu \in [0, Ts]$. An object having the form $\left(A_{\varrho,\zeta,\nu}^{T_r,Id,Fs} \right)^{Ts} = \left\langle \left(\Bbbk_A^{T_r} \right)_{\varrho}^{Ts}, \left(\Bbbk_A^{I_d} \right)_{\zeta}^{Ts}, \left(\Bbbk_A^{F_s} \right)_{\nu}^{Ts} \right\rangle$ is called a neutrosophic ϱ, ζ, ν translations of $\mathbb{N}_s$, if it satisfies $\left(\Bbbk_A^{T_r} \right)_{\varrho}^{Ts}(\check{l}) = \Bbbk_A^{T_r}(\check{l}) + \varrho$, $\left(\Bbbk_A^{I_d} \right)_{\zeta}^{Ts}(\check{l}) = \Bbbk_A^{I_d}(\check{l}) + \zeta$ and $\left(\Bbbk_A^{F_s} \right)_{\nu}^{Ts}(\check{l}) = \Bbbk_A^{F_s}(\check{l}) - \nu$.

Definition 3.2. A $\mathbb{N}_s$ $A = \left\langle \Bbbk^{Tr}, \Bbbk^{I_d}, \Bbbk^{F_s} \right\rangle$ is known as a NT INK-S if it meets all the requirements of INK-algebra,

1. $\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l} \star \check{m}) = \wedge \left\{ \left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l}), \left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{m}) \right\}$
2. $\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l} \star \check{m}) = \wedge \left\{ \left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}), \left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{m}) \right\}$
3. $\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l} \star \check{m}) = \vee \left\{ \left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}), \left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{m}) \right\}$.

Example 3.3. Consider INK-algebra in Table 2. Let $T_s = 0.5$ and we take $\varrho = 0.4, \zeta = 0.3$ *and* $\nu = 0.2$, then the neutrosophic ϱ, ζ ν translation is define by $(\Bbbk_A^{T_r})_{\varrho}, (\Bbbk_A^{I_d})_{\zeta}$ *and* $(\Bbbk_A^{F_s})_{\nu}$ of A in $\beth$.

A	0	1	2	3	4
$(\Bbbk_A^{T_r})_{\varrho}$	0.9	0.8	0.6	0.9	0.5
$(\Bbbk_A^{I_d})_{\zeta}$	0.9	0.6	0.4	0.7	0.4
$(\Bbbk_A^{F_s})_{\nu}$	0.16	0.15	0.25	0.35	0.39

TABLE 3. NT INK-Algebra

Then A is a NT INK-S of $\beth$.

Definition 3.4. Let A be a $\mathbb{N}_s$ of $\beth$ and $\varrho, \zeta, \nu \in [0, 1]$. An object having the form $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{M} = \left\langle \left(\Bbbk_A^{T_r}\right)_{\varrho}^{M}, \left(\Bbbk_A^{I_d}\right)_{\zeta}^{M}, \left(\Bbbk_A^{F_s}\right)_{\nu}^{M} \right\rangle$ is called a N-M of A if, $\left(\Bbbk_A^{T_r}\right)_{\varrho}^{M}(\check{l}) = (\Bbbk_A^{T_r})^M \cdot \varrho$, $\left(\Bbbk_A^{I_d}\right)_{\zeta}^{M}(\check{l}) = (\Bbbk_A^{I_d})^M \cdot \zeta$ and $\left(\Bbbk_A^{F_s}\right)_{\nu}^{M}(\check{l}) = (\Bbbk_A^{F_s})^M \cdot \nu$, for all $\check{l} \in A$.

Example 3.5. Consider INK-algebra in Table 2. Let $T_s = 0.4$ and we take $\varrho = 0.3, \zeta = 0.2$ *and* $\nu = 0.1$, then the neutrosophic $\varrho, \zeta,$ ν multiplication is define by $(\Bbbk_A^{T_r})_{\varrho}^M, (\Bbbk_A^{I_d})_{\zeta}^M, (\Bbbk_A^{F_s})_{\nu}^M)$ of A in $\beth$.

A	0	1	2	3	4
$(\Bbbk_A^{T_r})_{\varrho}^M$	0.15	0.12	0.06	0.15	0..03
$(\Bbbk_A^{I_d})_{\zeta}^M$	0.12	0.06	0.02	0.08	0.02
$(\Bbbk_A^{F_s})_{\nu}^M$	0.036	0.035	0.045	0.055	0.059

TABLE 4. N-M INK-A

Then A is a NTINK-S of $\beth$.

Theorem 3.6. *Let A be a $\mathbb{N}_s$ of $\beth$ such that the NT $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a nINK-S of $\beth$, for some $\varrho, \zeta, \nu \in [0, T]$. Then A is a NINK-S of $\beth$.*

Proof. Let $\left(A^{T_r,Id,Fs}\right)^{Ts}_{\varrho,\zeta,\nu}$ is a NINK-S of ℷ. For some $\varrho,\zeta,\nu \in [0, Ţ]$.

$$\begin{aligned}
\Bbbk_A^{T_r}(\check{l}\star\check{m})+\varrho &= (\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{l}\star\check{m})\\
&\geq \wedge\left\{(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{l}),(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{\Bbbk_A^{T_r}(\check{l})+\varrho,\Bbbk_A^{T_r}(\check{m})+\varrho\right\}\\
&= \wedge\left\{\Bbbk_A^{T_r}(\check{l}),\Bbbk_A^{T_r}(\check{m})\right\}+\varrho\\
&= \wedge\left\{\Bbbk_A^{T_r}(\check{l}),\Bbbk_A^{T_r}(\check{m})\right\}\\
\Bbbk_A^{I_d}(\check{l}\star\check{m})+\zeta &= (\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{l}\star\check{m})\\
&\geq \wedge\left\{(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{l}),(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{\Bbbk_A^{I_d}(\check{l})+\zeta,\Bbbk_A^{I_d}(\check{m})+\zeta\right\}\\
&= \wedge\left\{\Bbbk_A^{I_d}(\check{l}),\Bbbk_A^{I_d}(\check{m})\right\}+\zeta\\
&= \wedge\left\{\Bbbk_A^{I_d}(\check{l}),\Bbbk_A^{I_d}(\check{m})\right\}\\
\Bbbk_A^{F_s}(\check{l}\star\check{m})-\nu &= (\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{l}\star\check{m})\\
&\leq \vee\left\{(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{l}),(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{m})\right\}\\
&= \vee\left\{\Bbbk_A^{F_s}(\check{l})-\nu,\Bbbk_A^{F_s}(\check{m})-\nu\right\}\\
&= \vee\left\{\Bbbk_A^{F_s}(\check{l}),\Bbbk_A^{F_s}(\check{m})\right\}-\nu\\
&= \vee\left\{\Bbbk_A^{F_s}(\check{l}),\Bbbk_A^{F_s}(\check{m})\right\}
\end{aligned}$$

□

Theorem 3.7. *If A be a NINK-S of ℷ,then the N-M of A is a NINK-S a of ℷ for all* $\varrho,\zeta,\nu \in [0,1]$.

Proof. Assume that $A=\left\langle \Bbbk_A^{T_r},\Bbbk_A^{I_d},\Bbbk_A^{F_s}\right\rangle$ be a NINK-S of ℷ, $\forall\varrho,\zeta,\nu \in [0,1]$.

$$\begin{aligned}
(\Bbbk_A^{T_r})_{\varrho}^{M}(\check{l}\star\check{m}) &= \varrho\cdot\Bbbk_A^{T_r}(\check{l}\star\check{m})\\
&= \varrho\cdot\wedge\left\{\Bbbk_A^{T_r}(\check{l}),\Bbbk_A^{T_r}(\check{m})\right\}
\end{aligned}$$

$$\begin{aligned}
&= \wedge\left\{\varrho \cdot \mathbb{k}_A^{T_r}(\check{l}), \varrho \cdot \mathbb{k}_A^{T_r}(\check{m})\right\} \\
&\geq \wedge\left\{(\mathbb{k}_A^{T_r})_\varrho^M(\check{l}), (\mathbb{k}_A^{T_r})_\varrho^M(\check{m})\right\} \\
(\mathbb{k}_A^{I_d})_\zeta^M(\check{l} \star \check{m}) &= \zeta \cdot \mathbb{k}_A^{I_d}(\check{l} \star \check{m}) \\
&= \zeta \cdot \wedge\left\{\mathbb{k}_A^{I_d}(\check{l}), \mathbb{k}_A^{I_d}(\check{m})\right\} \\
&= \wedge\left\{\zeta \cdot \mathbb{k}_A^{I_d}(\check{l}), \zeta \cdot \mathbb{k}_A^{I_d}(\check{m})\right\} \\
&\geq \wedge\left\{(\mathbb{k}_A^{I_d})_\zeta^M(\check{l}), (\mathbb{k}_A^{I_d})_\zeta^M(\check{m})\right\} \\
(\mathbb{k}_A^{F_s})_\nu^M(\check{l} \star \check{m}) &= \nu \cdot \mathbb{k}_A^{F_s}(\check{l} \star \check{m}) \\
&= \nu \cdot \vee\left\{\mathbb{k}_A^{F_s}(\check{l}), \mathbb{k}_A^{F_s}(\check{m})\right\} \\
&= \vee\left\{\nu \cdot \mathbb{k}_A^{F_s}(\check{l}), \nu \cdot \mathbb{k}_A^{F_s}(\check{m})\right\} \\
&\leq \vee\left\{(\mathbb{k}_A^{F_s})_\nu^M(\check{l}), (\mathbb{k}_A^{F_s})_\nu^M(\check{m})\right\}
\end{aligned}$$

Hence $(\mathbb{k}_A^{T_r})_\varrho^M, (\mathbb{k}_A^{I_d})_\zeta^M$ and $(\mathbb{k}_A^{F_s})_\nu^M$ is a multilication of neutrosophic INK-S of $\daleth$. □

4. **Translation of NINK-ideal**

In this section, we define NTINK-Is in INK-algebra and investigate its properties.

Definition 4.1. A neutrosophic set $A = \left\langle \mathbb{k}^{T_r}, \mathbb{k}^{I_d}, \mathbb{k}^{F_s} \right\rangle$ is called a NINK-I of INK-A if,

(1) $\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(0) \geq \left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{l})\right\}, \left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(0) \geq \left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l})\right\}$ *and* $\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(0) \leq \left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l})\right\}$

(2) $\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{l}) \geq \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\}$

(3) $\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l}) \geq \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m}), \left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{m})\right\}$

(4) $\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l}) \leq \vee\left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m}), \left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{m})\right\}$

Theorem 4.2. *If the nutrosophic translation $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a NINK-I of $\daleth$ for some $\varrho, \zeta, \nu \in [0, T]$, it must be a neutrosophic ideal of $\daleth$.*

Proof. Let $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ be NT of $\daleth$.Then we have

$$\begin{aligned}
(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\} \\
&\textit{put } \check{n} = 0
\end{aligned}$$

$$
\begin{aligned}
\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l}) &\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{m})\right\}\\
\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l}) &\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l}\star\check{m}),\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{m})\right\}\\
\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}) &\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{m})\right\}\\
& put\ \check{n}=0\\
\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}) &\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{m})\right\}\\
\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}) &\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}\star\check{m}),\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{m})\right\}\\
\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}) &\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{m})\right\}\\
& put\ \check{n}=0\\
\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}) &\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{m})\right\}\\
\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}) &\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}\star\check{m}),\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{m})\right\}
\end{aligned}
$$

□

Theorem 4.3. *Let the NT $\left(A^{T_r,I_d,F_s}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a NINK-I of $\gimel$, for $\varrho,\zeta,\nu\in[0,1]$. If $\check{l}\leq\check{m}$, then $(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{l})\geq(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m}),(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{l})\geq(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})$ and $(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{l})\leq(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{m})$.*

Proof. Let $\check{l},\check{m},\check{n}\in\gimel$ we have,

$$
\begin{aligned}
(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{l}) &\geq \wedge\left\{(\Bbbk_A^{T_r})_{\varrho}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{l}\star\check{m}),(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\Bbbk_A^{T_r})_{\varrho}^{Ts}(0),(\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\right\}\\
&= (\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\\
(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{l}) &\geq \wedge\left\{(\Bbbk_{L}^{I})_{\zeta}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{l}\star\check{m}),(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\Bbbk_A^{I_d})_{\zeta}^{Ts}(0),(\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})\right\}\\
&= (\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})
\end{aligned}
$$

$$
(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{l}) \leq \vee\left\{(\Bbbk_A^{F_s})_{\nu}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{m})\right\}
$$

$$\begin{aligned}
&= \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}\star\check{m}), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\}\\
&= \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(0), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\}\\
&= (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})
\end{aligned}$$

□

Theorem 4.4. *Let A is a NINK-I of $\beth$, then the NT $\left(A^{T_r,I_d,F_s}\right)^{Ts}_{\varrho,\zeta,\nu}$ of A is a NINK-S of $\beth$.*

Proof.

$$\begin{aligned}
(\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{T_r}(\check{l}\star\check{m})+\varrho\\
&= \wedge\left\{\Bbbk_A^{T_r}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{T_r}(\check{m})\right\}+\varrho \quad (Def.4.1\ in\ (2))\\
&= \wedge\left\{\Bbbk_A^{T_r}(((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{T_r}(\check{m})\right\}+\varrho\\
&= \wedge\left\{\Bbbk_A^{T_r}(0), \Bbbk_A^{T_r}(\check{m})\right\}+\varrho\\
&\geq \wedge\left\{\Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{T_r}(\check{m})\right\}+\varrho\\
(\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}\star\check{m}) &\geq \wedge\left\{(\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}), (\Bbbk_A^{T_r})_\varrho^{Ts}(\check{m})\right\}+\varrho\\
(\Bbbk_A^{I_d})_\zeta^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{I_d}(\check{l}\star\check{m})+\zeta\\
&= \wedge\left\{\Bbbk_A^{I_d}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{I_d}(\check{m})\right\}+\zeta \quad (Def.4.1\ in\ (3))\\
&= \wedge\left\{\Bbbk_A^{I_d}(((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{I_d}(\check{m})\right\}+\varrho\\
&= \wedge\left\{\Bbbk_A^{I_d}(0), \Bbbk_A^{I_d}(\check{m})\right\}+\zeta\\
&\geq \wedge\left\{\Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{I_d}(\check{m})\right\}+\zeta\\
&\geq \wedge\left\{(\Bbbk_A^{I_d})_\zeta^{Ts}(\check{l}), (\Bbbk_A^{I_d})_\zeta^{Ts}(\check{m})\right\}+\zeta\\
(\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{F_s}(\check{l}\star\check{m})+\nu\\
&= ma\check{l}\left\{\Bbbk_A^{F_s}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{F_s}(\check{m})\right\}-\nu\\
&= \vee\left\{\Bbbk_A^{F_s}(((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{F_s}(\check{m})\right\}-\nu \qquad (Def.4.1\ in\ (4))\\
&= \vee\left\{\Bbbk_A^{F_s}(0), \Bbbk_A^{F_s}(\check{m})\right\}-\nu\\
&\leq \vee\left\{\Bbbk_A^{F_s}(\check{l}), \Bbbk_A^{F_s}(\check{m})\right\}-\nu\\
&\leq \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\}+\nu.
\end{aligned}$$

□

Theorem 4.5. *Every neutrosphic Translation $(\Bbbk_A^{T_r})_{\varrho,\zeta,\nu}^{Ts}$ of A is a NINK-ideal of $\gimel$, if A is a NTINK-I of $\gimel$,for all $\varrho,\zeta,\nu \in [0,T]$.*

Proof. Assume that A is a NINK-I of $\gimel$ and let $\varrho,\zeta,\nu \in [0,T]$.

$$\begin{aligned}
(\Bbbk_A^{T_r})_{\varrho}^{Ts}(0) &= (\Bbbk_A^{T_r})(\check{l}) + \varrho \\
&\geq \wedge\left\{(\Bbbk_A^{T_r})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{T_r})(\check{n})\right\} + \varrho \\
&= \wedge\left\{(\Bbbk_A^{T_r})((\check{n}\star\check{l})\star(\check{n}\star\check{m})) + \varrho, (\Bbbk_A^{T_r})(\check{m}) + \varrho\right\} \\
(\Bbbk_A^{T_r})_{\varrho}^{Ts}(0) &= \wedge\left\{(\Bbbk_A^{T_r})_{\varrho}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{T_r})_{\varrho}^{Ts}(\check{m})\right\}. \\
(\Bbbk_A^{I_d})_{\varrho}^{Ts}(0) &= (\Bbbk_A^{I_d})(\check{l}) + \zeta \\
&\geq \wedge\left\{(\Bbbk_A^{I_d})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{I_d})(\check{m})\right\} + \zeta \\
&= \wedge\left\{(\Bbbk_A^{I_d})((\check{n}\star\check{l})\star(\check{n}\star\check{m})) + \zeta, (\Bbbk_A^{I_d})(\check{m}) + \zeta\right\} \\
(\Bbbk_A^{I_d})_{\zeta}^{Ts}(0) &= \wedge\left\{(\Bbbk_A^{I_d})_{\zeta}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{I_d})_{\zeta}^{Ts}(\check{m})\right\}. \\
(\Bbbk_A^{F_s})_{\nu}^{Ts}(0) &= (\Bbbk_A^{F_s})(\check{l}) - \nu \\
&\leq \vee\left\{(\Bbbk_A^{F_s})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{F_s})(\check{m})\right\} - \nu \\
&= \vee\left\{(\Bbbk_A^{F_s})((\check{n}\star\check{l})\star(\check{l}\star\check{m})) + \nu, (\Bbbk_A^{F_s})(\check{m}) - \nu\right\} \\
(\Bbbk_A^{F_s})_{\Bbbk}^{Ts}(0) &= \vee\left\{(\Bbbk_A^{F_s})_{\nu}^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\Bbbk_A^{F_s})_{\nu}^{Ts}(\check{m})\right\}.
\end{aligned}$$

□

Theorem 4.6. *Let A be a neutrosophic subset of $\gimel$ such that neutrosophic ϱ,ζ,ν-multiplication of A is a neutrosophic INK-ideal $\gimel$ for some $\varrho,\zeta,\nu \in [0,1]$, then A is a neutrosophic INK-ideal of $\gimel$.*

Proof. Assume that $(\Bbbk_A^{T_r})_{\varrho,\zeta,\nu}^{M}$ is a NINK-ideal of $\gimel$,for some $\varrho,\zeta,\nu \in [0,1]$.
Let $\varrho,\zeta,\nu \in [0,1]$. Then

$$\begin{aligned}
\varrho\cdot(\Bbbk_A^{T_r})(0) &= (\Bbbk_A^{T_r})_{\varrho}^{M}(0) \\
&\geq (\Bbbk_A^{T_r})_{\varrho}^{M}(0) \\
&= \varrho\cdot(\Bbbk_A^{T_r})(\check{l})
\end{aligned}$$

$$
\begin{aligned}
(\Bbbk_A^{T_r})(0) &\geq (\Bbbk_A^{T_r})(\check{l}) \\
\varrho \cdot (\Bbbk_A^{T_r})(\check{l}) &= (\Bbbk_A^{T_r})_\varrho^M(\check{l}) \\
&\geq \wedge \left\{ \left(\Bbbk_A^{T_r}\right)_\varrho^M ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_{\text{Ł}}^{T}\right)_\varrho^M (\check{m}) \right\} \\
&\geq \wedge \left\{ \varrho \cdot \left(\Bbbk_A^{T_r}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \varrho \cdot \left(\Bbbk_A^{T_r}\right) (\check{m}) \right\} \\
\varrho \cdot (\Bbbk_A^{T_r})(\check{l}) &= \varrho \cdot \wedge \left\{ \left(\Bbbk_A^{T_r}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{T_r}\right) (\check{m}) \right\} \\
(\Bbbk_A^{T_r})(\check{l}) &= \wedge \left\{ \left(\Bbbk_A^{T_r}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{T_r}\right) (\check{m}) \right\} \\
\zeta \cdot (\Bbbk_A^{I_d})(0) &= (\Bbbk_A^{I_d})_\zeta^M(0) \\
&\geq (\Bbbk_A^{I_d})_\zeta^M(0) \\
&= \zeta \cdot (\Bbbk_A^{I_d})(\check{l}) \\
(\Bbbk_A^{I_d})(0) &\geq (\Bbbk_A^{I_d})(\check{l}) \\
\zeta \cdot (\Bbbk_A^{I_d})(\check{l}) &= (\Bbbk_A^{I_d})_\zeta^M(\check{l}) \\
&\geq \wedge \left\{ \left(\Bbbk_A^{I_d}\right)_\zeta^M ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{I_d}\right)_\zeta^M (\check{m}) \right\} \\
&\geq \wedge \left\{ \zeta \cdot \left(\Bbbk_A^{I_d}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \zeta \cdot \left(\Bbbk_A^{I_d}\right) (\check{m}) \right\} \\
\zeta \cdot (\Bbbk_A^{I_d})(\check{l}) &= \zeta \cdot \wedge \left\{ \left(\Bbbk_A^{I_d}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{I_d}\right) (\check{m}) \right\} \\
(\Bbbk_A^{I_d})(\check{l}) &= \wedge \left\{ \left(\Bbbk_A^{I_d}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{I_d}\right) (\check{m}) \right\} \\
\nu \cdot (\Bbbk_A^{F_s})(0) &= (\Bbbk_A^{F_s})_\nu^M(0) \\
&\leq (\Bbbk_A^{F_s})_\varrho^M(0) \\
&= \nu \cdot (\Bbbk_A^{F_s})(\check{l}) \\
(\Bbbk_A^{F_s})(0) &\leq (\Bbbk_A^{F_s})(\check{l}) \\
\nu \cdot (\Bbbk_A^{F_s})(\check{l}) &= (\Bbbk_A^{F_s})_\nu^M(\check{l}) \\
&\leq \vee \left\{ \left(\Bbbk_A^{F_s}\right)_\nu^M ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{F_s}\right)_\nu^M (\check{m}) \right\} \\
&\leq \vee \left\{ \nu \cdot \left(\Bbbk_A^{F_s}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \nu \cdot \left(\Bbbk_A^{F_s}\right) (\check{m}) \right\} \\
\nu \cdot (\Bbbk_A^{F_s})(\check{l}) &= \nu \cdot \vee \left\{ \left(\Bbbk_A^{F_s}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{F_s}\right) (\check{m}) \right\} \\
(\Bbbk_A^{F_s})(\check{l}) &= \vee \left\{ \left(\Bbbk_A^{F_s}\right) ((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{F_s}\right) (\check{m}) \right\}
\end{aligned}
$$

Hence $(\Bbbk_A^{T_r})_\varrho^M, (\Bbbk_A^{I_d})_\zeta^M$ and $(\Bbbk_A^{F_s})_\nu^M$ is a multilication of NINK-ideal of $\beth$. □

Theorem 4.7. *If the NT $\left(A^{T_r, I_d, F_s}\right)_{\varrho,\zeta,\nu}^M$ of A is a NINK-ideal of $\beth$, $(\varrho, \zeta, \nu) \in [0, 1]$, then A is a NINK-S of $\beth$.*

Proof. Let us assume that $\left(A^{T_r,I_d,F_s}\right)^M_{\varrho,\zeta,\nu}$ of A is a NINK-ideal of $\beth$. Then

$$\begin{aligned}
\varrho\cdot(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= (\Bbbk_A^{T_r})^M_\varrho(\check{l}\star\check{m})\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\} \quad (Def\ 4.1.\ (2))\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho(0),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&= \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{l}),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&= \wedge\left\{\left(\varrho\cdot\Bbbk_A^{T_r}\right)(\check{l}),\varrho\cdot\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
\varrho\cdot(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= \varrho\cdot\wedge\left\{\left((\Bbbk_A^{T_r}\right)(\check{l}),\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= \wedge\left\{\left((\Bbbk_A^{T_r}\right)(\check{l}),\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
\zeta\cdot(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= (\Bbbk_A^{I_d})^M_\zeta(\check{l}\star\check{m})\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\} \quad (Def\ 4.1.\ (3))\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta(0),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&= \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{l}),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&= \wedge\left\{\left(\zeta\cdot\Bbbk_A^{I_d}\right)(\check{l}),\zeta\cdot\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
\zeta\cdot(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= \zeta\cdot\wedge\left\{\left((\Bbbk_A^{I_d}\right)(\check{l}),\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= \wedge\left\{\left((\Bbbk_A^{I_d}\right)(\check{l}),\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
\nu\cdot(\Bbbk_A^{F_s})(\check{l}\star\check{m}) &= (\Bbbk_A^{F_s})^M_\nu(\check{l}\star\check{m})\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\} \quad (Def\ 4.1.\ (4))\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu(0),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&= \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{l}),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&= \vee\left\{\left(\nu\cdot\Bbbk_A^{F_s}\right)(\check{l}),\nu\cdot\left(\Bbbk_A^{F_s}\right)(\check{m})\right\}\\
\nu\cdot(\Bbbk_A^{F_s})(\check{l}\star\check{m}) &= \nu\cdot\vee\left\{\left((\Bbbk_A^{F_s}\right)(\check{l}),\left(\Bbbk_A^{F_s}\right)(\check{m})\right\}
\end{aligned}$$

$$(\Bbbk_A^{F_s})(\check{l} \star \check{m}) = \vee\left\{\left((\Bbbk_A^{F_s}\right)(\check{l}), \left(\Bbbk_{\S}^{F}\right)(\check{m})\right\}$$

□

Theorem 4.8. *Intersection and union of any two neutrosophic translations of a NINK-ideal of ℷ is also a NINK-ideal of ℷ*

Proof. Let $\left(A^{T_r,I_d,F_s}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ be two neutrosophic translations of a NINK-ideal of ℷ where $(\varrho_1,\zeta_1,\nu_1,\nu_2,\varrho_2,\zeta_2) \in [0,1]$. Assume that $\varrho_1 \leq \varrho_2, \zeta_1 \leq \zeta_2$ and $\nu_1 \geq \nu_2$. Then $\left(A^{T_r,Id,Fs}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ are NINK-ideal of ℷ.

$$\begin{aligned}
((\Bbbk_A^{T_r})_{\varrho_1} \cap (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \wedge\{(\Bbbk_A^{T_r})_{\varrho_1}(\check{l}), (\Bbbk_A^{T})_{\varrho_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \varrho_1, \Bbbk_A(\check{l}) + \varrho_2\} \\
&= \Bbbk_A(\check{l}) + \varrho_1 \\
((\Bbbk_A^{T_r})_{\varrho_1} \cap (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \Bbbk_A^{T_r}(\check{l}). \\
((\Bbbk_A^{I_d})_{\zeta_1} \cap (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \wedge\{(\Bbbk_A^{I_d})_{\zeta_1}(\check{l}), (\Bbbk_A^{I})_{\zeta_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \zeta_1, \Bbbk_A(\check{l}) + \zeta_2\} \\
&= \Bbbk_A(\check{l}) + \zeta_1 \\
((\Bbbk_A^{I_d})_{\zeta_1} \cap (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \Bbbk_A^{I_d}(\check{l}). \\
((\Bbbk_A^{F_s})_{\nu_1} \cap (\Bbbk_A^{T})_{\nu_2})(\check{l}) &= \vee\{(\Bbbk_A^{F_s})_{\nu_1}(\check{l}), (\Bbbk_A^{F})_{\nu_2}(\check{l})\} \\
&= \vee\{(\Bbbk_A(\check{l}) + \nu_1, \Bbbk_A(\check{l}) + \nu_2\} \\
&= \Bbbk_A(\check{l}) + \nu_1 \\
((\Bbbk_A^{F_s})_{\nu_1} \cap (\Bbbk_A^{F})_{\nu_2})(\check{l}) &= \Bbbk_A^{F_s}(\check{l}).
\end{aligned}$$

and

$$\begin{aligned}
((\Bbbk_A^{T_r})_{\varrho_1} \cup (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \wedge\{(\Bbbk_A^{T_r})_{\varrho_1}(\check{l}), (\Bbbk_A^{T})_{\varrho_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \varrho_1, \Bbbk_A(\check{l}) + \varrho_2\} \\
&= \Bbbk_A(\check{l}) + \varrho_1 \\
((\Bbbk_A^{T_r})_{\varrho_1} \cup (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \Bbbk_A^{T_r}(\check{l}). \\
((\Bbbk_A^{I_d})_{\zeta_1} \cup (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \wedge\{(\Bbbk_A^{I_d})_{\zeta_1}(\check{l}), (\Bbbk_A^{I})_{\zeta_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \zeta_1, \Bbbk_A(\check{l}) + \zeta_2\} \\
&= \Bbbk_A(\check{l}) + \zeta_1 \\
((\Bbbk_A^{I_d})_{\zeta_1} \cup (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \Bbbk_A^{I_d}(\check{l}).
\end{aligned}$$

$$\begin{aligned}((\mathbb{k}_A^{F_s})_{\nu_1} \cup (\mathbb{k}_A^{T})_{\nu_2})(\check{l}) &= \vee\{(\mathbb{k}_A^{F_s})_{\nu_1}(\check{l}), (\mathbb{k}_A^{F})_{\nu_2}(\check{l})\} \\ &= \vee\{(\mathbb{k}_A(\check{l}) + \nu_1, \mathbb{k}_A(\check{l}) + \nu_2\} \\ &= \mathbb{k}_A(\check{l}) + \nu_1 \\ ((\mathbb{k}_A^{F_s})_{\nu_1} \cup (\mathbb{k}_A^{F})_{\nu_2})(\check{l}) &= \mathbb{k}_A^{F_s}(\check{l}).\end{aligned}$$

Hence $\left(A^{T_r,I_d,F_s}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ are NINK-ideal of $\beth$. $\square$

5. Conclusion

This paper introduces and explores the translation of NINK-ideals within the framework of INK-algebras, shedding light on some of their valuable properties. The investigation establishes connections between neutrosophic fuzzy translations and the N-Ms inherent in these INK-ideals. This research lays the groundwork for further exploration into the theory of INK-algebras. Looking ahead, our future studies on the neutrosophic structure of INK-algebras may include the following topics:

(1) Translation of neutrosophic soft a-ideals in INK-algebras
(2) Product of translation of neutrosophic soft d-ideals in INK-algebras
(3) Exploring the translation dynamics of neutrosophic soft INK-ideals in INK-algebras. We believe that these areas of investigation will contribute to the ongoing development of this field.

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References

1. A. A. A. Agboola and B. Davvaz, Introduction to Neutrosophic BCI/BCKAlgebras,Information.International Journal of Mathematics and Mathematical Sciences, 2015,130. https://doi.org/10.1155/2015/370267.
2. Atanassov, K.T, Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 1986, 20, 87-96.
3. Bordbar, Hashem; Xiao Long Xin; Rajab Ali Borzooei; and Young Bae Jun, Positive implicative ideals of BCK-algebras based on neutrosophic sets and falling shadows. Neutrosophic Sets and Systems, 2022, 48(1).
4. F. Smarandache, Neutrosophic set-A generalization of the intuitionistic fuzzy set. Internat. J. Pure Appl. Math. 2005, 24, 287297.
5. F. Smarandache, Neutrosophy, Neutrosophic Probability, Set, and Logic. Pro Quest Information and Learning, Ann Arbor, Michigan, USA. 1998.
6. F Smarandache, New Types of Soft Sets HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set: An Improved Version. Neutrosophic Systems With Applications,2023, 8, 3541. https://doi.org/10.61356/j.nswa.2023.41

7. Jun, Young Bae and Roh, Eun Hwan. MBJ-neutrosophic ideals of BCK/BCI-algebras, Open Mathematics, 1999, 17(1), 588-601.
8. Lee, K. J., Y. B. Jun, M. I. Doh, Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras. Commun. Korean Math.Soc., 2009, 24, 35336.
9. L. A. Zadeh,Fuzzy sets. Information and Control., 1965,338-353.
10. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Fuzzy p-ideal in INK-Algebra. Journal of Xian University of Architecture and Technology, 2000, 12, 4746-4752.
11. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Neutrosophic set in INK-Algebra, Adv. Math., Sci. J., 2000, 9, 43454352.
12. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Direct product of Neutrosophic INKAlgebras. Infinite Study., 2020, 38.
13. M Kaviyarasu and K Indhira 2017 IOP Conf. Ser.: Mater. Sci. Eng. 263 042142.DOI 10.1088/1757-899X/263/4/042142.
14. Manokaran, Surya, and Muralikrishna Prakasam. On MBJNeutrosophic B-Subalgebra. Neutrosophic Sets and Systems, 2019, 28, 216-227.
15. Mohsin Khalid , Neha Andaleeb Khalid , Rakib Iqbal, MBJ-neutrosophic T-ideal on B-algebra. International Journal of Neutrosophic Science, 2020, 1(1), 29-39.
16. M. Mohseni Takallo., R.A. Borzooei., Young Bae Jun.,MBJ-neutrosophic structures and its applications in BCK/BCI-algebras. Neutrosophic Sets and Systems, 2018, 23.
17. M.Kaviyarasu , Rajeshwari M., Direct Product of Neutrosophic h-ideal in INK-Algebra. International Journal of Neutrosophic Science, 2023, 20(1), 119-127. Doi : https://doi.org/10.54216/IJNS.200110.
18. Ranulfo Paiva Barbosa , & Smarandache, F. Pura Vida Neutrosophic Algebra. Neutrosophic Systems With Applications, 2023, 9, 101106. https://doi.org/10.61356/j.nswa.2023.68
19. Song, S.-Z.; Ozturk, M.A.; Jun, Y.-B. Commutative Ideals of BCI-Algebras Using MBJ-Neutrosophic Structures. Mathematics, 2021, 9, 2047.
20. Senapati, T., Fuzzy Translations of Fuzzy H-ideals in BCK/BCI-Algebras. Journal of the Indonesian Mathematical Society, 2015, 21(1), 4558.
21. Senapati,T.,M. Bhowmik,M.Pal, B. Davvaz. Atanassovs intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras. Eurasian Mathematical Journal, 2015, 6 (1), 96114.
22. Smarandache, F.,Foundation of the SuperHyperSoft Set and the Fuzzy Extension Super HyperS oft Set: A New Vision. Neutrosophic Systems With Applications,2023, 11, 4851. https://doi.org/10.61356/j.nswa.2023.95
23. WadeiF. Al-Omeri, Saeid Jafari and Smarandache, F. (2020). Neutrosophic Fixed Point Theorems and Cone Metric Spaces. Neutrosophic Sets and Systems, 2020, 31, 250-265.
24. Wadei Faris Al-Omeri., On Mixed b-Fuzzy Topological Spaces. International Journal of Fuzzy Logic and Intelligent Systems, 20(3), 242-246.
25. Wadei Faris Al-Omeri., On almost e-I-continuous functions. Demonstratio Mathematica, 2021, 54(1), 168-177.
26. Wadei F. Al-Omeri , O. H. Khalil and A. Ghareeb, Degree of (L, M)-fuzzy semi-precontinuous and (L, M)-fuzzy semi preirresolute functions. Demonstr. Math., 51(1), 182197.
27. Wadei F. Al-Omeri.et.al., On Ra operator in ideal topological spaces. Creat. Math. Inform, 2016, 25, 2016,1-10.
28. Y.B. Jun., J. Meng, Fuzzy p-ideals in BCI-algebras. Mathematica Japonica, 1994, 40, 271282.
29. Y.B.Jun, Translations of fuzzy ideals in BCK/BCI algebras. Hacettepe Journal of Mathematics and Statistics, 2021, 40, 349358.
30. Iseki, K. On BCI-algebras. Math. Semin. Notes 1980.

31. Imai, Y.; Iseki, K. On axiom systems of propositional calculi XIV. Proc. Jpn. Acad. 1966, 42, 1922.
32. Iseki, K.; Tanaka, S. An introduction to the theory of BCK algebras. Math. Jpn. 1978, 23, 126.

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