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Exploring Neutrosophic Linear Programming in Advanced Fuzzy Contexts

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Abstract. A neutrosophic set is a mathematical framework that extends fuzzy and intuitionistic sets to handle indeterminate or contradictory information using three components: truth, falsity, and indeterminacy - membership degrees which deal with handling indeterminate, imprecise, and uncertain data, while Extended Fuzzy Theory extends the standard fuzzy set theory to manage more intricate membership degrees. The primary objective of this review is to thoroughly investigate and summarize the existing literature on trapezoidal neutrosophic environments, particularly focusing on aspects such as the NFMOLP Problems in SVTpN environments. Ultimately, this comprehensive review article aims to enhance the understanding of the potential of these integrated methodologies for effectively presenting decision-making amidst complex and uncertain conditions.

Keywords: Fuzzy Linear Programming problems; Linear Programming problems; Uncertainty principle; Membership function; Single valued trapezoidal neutrosophic numbers; NFMOLP- problems

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1. **Introduction**

Linear programming (LP) problem is a powerful mathematical technique in optimization employed to address problems where the objective is to maximize or minimize a linear function while adhering to a set of linear constraints. In linear programming, the decision maker first identifies the decision variables, objective function, and constraints. Various methods find the feasible solution of LPP using the various methods simplex method [1], dual simplex [2], graphical method [3] and so on which satisfy the constraints and optimize the objective value.

The area of linear programming is very broad. The applications of LPP contain various areas such as Production planning [4], transport services [5], water management [6] and so on. Over the years researchers have worked in these areas such as using a modified dual simplex method Patil et al. [7] developed an android application for cattle feed and Zhou et al. [8] introduced the SMSS optimization algorithm for a data-clustering problem, demonstrating its potential and effectiveness with eleven benchmark datasets. Chen et al. [9] presented a parameter identification approach for photovoltaic models using a hybrid adaptive Nelder-Mead simplex algorithm based on eagle strategy. Furthermore, Liero et al. [10] developed a comprehensive theory for the novel class of Optimal Entropy-Transport problems, focusing on nonnegative and finite Radon measures in general topological spaces. In another study, Tang et al. [11] proposed a novel deep learning-based algorithm-based partial channel assignment algorithm to intelligently allocate channels to each link in the SDN-IoT network.

Our intention is that this comprehensive literature review will be very useful for students or researchers to understand both Fuzzy and trapezoidal neutrosophic environments through a single review paper. After reviewing the introduction, we found that there is a significant gap in the study of FLPP. Therefore, we have presented some important methodologies and applications in a unified manner, considering different aspects of uncertainty. As a result, we are compelled to explore the application of the Trapezoidal neutrosophic environments to LPP and provide an updated analysis of applications and methodologies to discourse these gaps. The summary of the trapezoidal neutrosophic environments will support researchers in promoting and discovering improvements and progress in the field of Linear Programming Problems (LPP). Through this comprehensive review paper, we attempt to support students and researchers in developing a profound understanding of Neutrosophic, LR Fuzzy, Intuitionistic, and Hesitant fuzzy models. Ultimately, our work aims to contribute to fostering progress in this area. .

The paper is organized as follows. Section 2 discuss difficulties in the classical linear programming model and Section 3 define the key concepts and definitions of fuzzy theory. In Section 4 introduce the FLPP then after in section 5 discuss some important definitions related to Fuzzy extended theory and Section 6 short literature review on LPP under the neutrosophic principle. Finally in Sction 7 we conclude the review.

1.1. *List of Abbreviations used throughout this paper.*

- **FLP:** stands for ”Fuzzy Linear Programming”
- **ITpNNs:** stands for ”Interval trapezoidal neutrosophic numbers”
- **IFS:** stands for ”Intuitionistic fuzzy Set”
- **IFNs:** stands for ”Intuitionistic fuzzy numbers”

- **TpFn:** stands for ”Trapezoidal Fuzzy Number”
- **SVTpNNs:** stands for ”Single-valued trapezoidal neutrosophic numbers”
- **GTpFn:** stands for ”General Trapezoidal Fuzzy Number”
- **WTpFn:** stands for ”Weighted Trapezoidal Fuzzy Number”
- **TpIFN:** stands for ”Trapezoidal Intuitionistic Fuzzy Number ”
- **MADM:** stands for ”Multiple Attribute Decision Making”
- **NTpLWAA:** stands for ”Neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation”
- **NTpLWGA:** stands for ”Neutrosophic trapezoid linguistic weighted geometric aggregation.”
- **R-NWFS:** stands for ”Robust no-wait flow shop scheduling”
- **NFMOLP:** stands for ”Neutrosophic Fuzzy Multi-Objective Linear Programming”
- **SMSS:** stands for ”Simplex method-based social spider”
- **MOLP:** stands for “Multi-Objective Linear Programming”

2. Discussion Difficulties in The Classical Linear Programming Model

Classical Linear Programming (LP) have several demerits that can be addressed using fuzzy logic. Firstly, classical LP assumes precise and deterministic input data, which can lead to unrealistic and rigid solutions in the face of uncertainties and imprecisions in real-world problems. Fuzzy logic, on the other hand, allows for the representation of vagueness and uncertainty in the input data, enabling a more robust and flexible approach to problem-solving. Secondly, classical LP may struggle to handle subjective and qualitative factors adequately, while fuzzy logic can incorporate linguistic variables and expert opinions through fuzzy set theory, providing a more human-like decision-making framework. Additionally, classical LP may not handle imprecise or ambiguous constraints effectively, leading to suboptimal solutions, whereas fuzzy logic can model and manage fuzzy constraints more naturally, resulting in more meaningful and context-sensitive outcomes. In summary, fuzzy logic addresses the demerits of classical LP by embracing uncertainty, imprecision, and subjectivity, making it a valuable tool for handling complex real-world problems. In specific decision-making situations where multiple factors come into play, some of which may be difficult to quantify precisely, fuzzy logic presents a highly flexible and subtle approach when contrasted with an LP (Linear Programming) model. The versatility of fuzzy logic lies in its capability to encompass a wider array of factors and integrate subjective or qualitative inputs. As a result, fuzzy logic has demonstrated its superiority over Linear Programming models. The pioneering work introducing fuzzy logic was accomplished by Lotfi A. Zadeh [12] in 1965.

3. Fuzzy Theory: Key Concepts and Definitions

Fuzzy set:

Zadeh [12] introduced the thought of fuzzy logic in 1965 to address systems with ill-defined, vague, or incomplete information. If U is a collection of elements denoted by u, then a fuzzy set $\widetilde{F}$ in U is a set of ordered pairs: $\widetilde{F} = \left\{\left(u, \mu_{\widetilde{F}}(u)\right) : u \in U\right\}$ Where $\mu_{\widetilde{F}}(u) : U \to [0,1]$is called grade of membership or the membership function or degree of compatibility or degree of truth of u in $\widetilde{F}$.

Notably, there are different types of fuzzy membership functions, including Triangular and Trapezoidal fuzzy functions, which are defined as below: -

Triangular fuzzy Numbers [13]:

A fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, defined on the universal set as a set of real numbers is said to be a Triangular fuzzy number if its grade of membership function, $\mu_{\widetilde{M}}(n)$is follow the below condition:

(1) $\mu_{\widetilde{M}}(n)$ a strictly increasing and continuous function between the intervals $[\, a_1, a_2]$.

(2) $\mu_{\widetilde{M}}(n)$ a strictly decreasing and continuous function between the intervals $[\, a_2, a_3]$.

(3) $\mu_{\widetilde{M}}(n)$, a continuous function under the interval $[\, 0, 1]$.

And given by

$$\mu_{\widetilde{M}}(n) = \left\{\begin{array}{ll} \frac{n-a_1}{a_2-a_1} & \text{for } a_1 \le n \le a_2 \\ \frac{a_3-n}{a_3-a_2} & \text{for } a_2 \le n \le a_3 \\ 0 & \text{othewise} \end{array}\right\}$$

Trapezoidal Fuzzy Number (TpFN) [14]:

A TpFN can be written as $\widetilde{Tp_f} = \langle m_1, m_2, m_3, m_4 \rangle$ whose membership function $\mu_{\widetilde{Tp_f}}$ as follows:

$$\mu_{\widetilde{Tp_f}}(t) = \begin{cases} \frac{t-m_1}{m_2-m_1}, & m_1 \le t \le m_2; \\ 1, & m_2 \le t \le m_3; \\ \frac{m_4-t}{tf_4-tf_3}, & m_3 \le t \le m_4; \\ 0, & Otherwise \end{cases}$$

$$where \quad m_1, m_2, m_3, m_4 \in \mathbb{R}$$

Weighted Trapezoidal Fuzzy Number [15]:

Let set $\tilde{S}_p = \left(\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega_l^p, \omega_r^p\right)$ (where $\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p$and $\tilde{s}_4^p$ real number with $\tilde{s}_1^p \le \tilde{s}_2^p \le \tilde{s}_3^p \le \tilde{s}_4^p$, $\omega_r^p, \omega_l^p$the right height and the left height of $\tilde{S}_p$) is GTpFn and the grade of membership function is define as :

$$\mu_{\tilde{S}_p}(t) = \left\{\begin{array}{ll} 0 & \text{for } t\tilde{s}_1^p or\ t\tilde{s}_4^p \\ \omega_l^p \cdot \frac{t-\tilde{s}_1^p}{\tilde{s}_2^p-\tilde{s}_1^p} & \text{for } \tilde{s}_1^p \le t \le \tilde{s}_2^p \\ \omega_l^p + \left(\omega_l^p - \omega_l^p\right) \frac{t-\tilde{s}_2^p}{\tilde{s}_3^p-\tilde{s}_2^p} & \text{for } \tilde{s}_2^p \le t \le \tilde{s}_3^p \\ \omega_r^p \cdot \frac{t-\tilde{s}_4^p}{\tilde{s}_3^p-\tilde{s}_4^p} & \text{for } \tilde{s}_3^p \le t \le \tilde{s}_4^p \end{array}\right\}$$

Where $0 \prec \omega_l^p \leq 1$ and $0 \prec \omega_r^p \leq 1$; If $\omega_l^p = \omega_r^p$ then $\tilde{S}_p$become $\tilde{S}_p = (\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega)$ also known as WTpFn.

LR flat Trapezoidal Fuzzy Number [16]:

Let $\tilde{\varphi} = (\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_l, \tilde{\varphi}_r)_{LR}$ is said L-R flat TpFn iff the grade of membership function $\mu_{\tilde{\varphi}}(t)$ is defined by-

$$\mu_{\tilde{\varphi}}(t) = \left\{ \begin{array}{ll} L\left(\frac{\tilde{\varphi}_1 - t}{\tilde{\varphi}_l}\right) & \text{For } t \leq \tilde{\varphi}_1 \text{ and } \tilde{\varphi}_l > 0 \\ R\left(\frac{t - \tilde{\varphi}_2}{\tilde{\varphi}_r}\right) & \text{For } t \geq \tilde{\varphi}_2 \text{ and } \tilde{\varphi}_r > 0 \\ 1 & \text{For } \tilde{\varphi}_1 \leq t \leq \tilde{\varphi}_2 \end{array} \right\}$$

Hence, Fuzzy Logic presents a broader range of considerations and holds vagueness and uncertainty during the optimization solution, rendering it a more advantageous alternative to LP Models in certain scenarios. As demonstrated in Section 2, conventional LP struggles to adequately address uncertainty, leading to decreased accuracy in identifying optimal solutions. To overcome this limitation, Zimmermann proposed fuzzy Linear Programming Problems, which we will thoroughly examine in Section 4.

4. Introduction of FLPP

Fuzzy linear programming (FLP) is a prevailing mathematical framework that extends classical linear programming by including uncertainties and imprecisions through fuzzy logic. The FLP model, first introduced by Zimmermann [17], provides a powerful approach to tackle Linear Programming (LP) problems by integrating fuzzy linear constraints and keeping track of the evolution of related research and advancements. The goal of FLPP is to attain an optimal solution for a given objective function while adhering to a fuzzy set of constraints. In real-life situations, uncertainties are universal, making classical linear programming insufficient for accurately representing complex problems but FLP addresses this limitation by utilizing fuzzy sets and fuzzy numbers to model imprecise data, enabling decision-makers to optimize systems and resources under uncertain conditions. Its significance lies in various practical domains, such as supply chain management [18], financial planning [19], construction projects [20], environmental management [21], and so on. This paper explores various real-world applications to showcase the practical relevance and efficacy of FLPP in handling imprecision and uncertainty. By embracing the inherent fuzziness of real-world problems, FLPP equips decision-makers with a powerful tool to address intricate optimization challenges effectively.

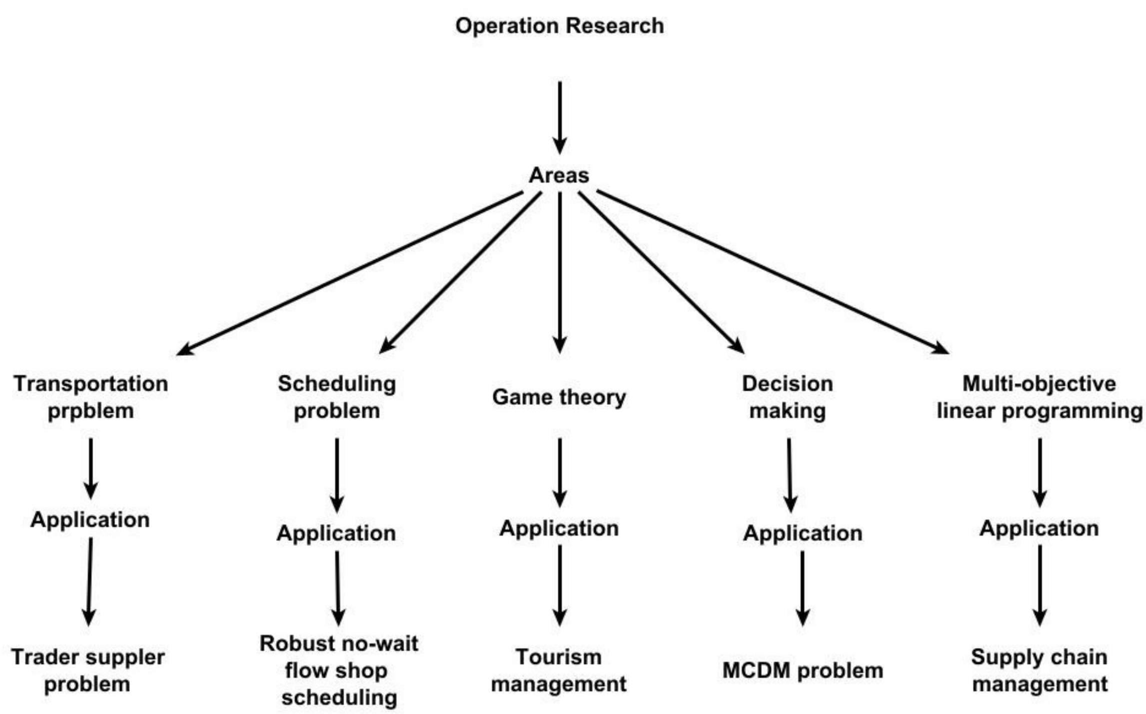
Table 1. Researchers from diverse domains have extensively investigated the real-life applications of FLPP, revealing its profound impact.

Authors	**Year**	**Environment**	**Application**	**Significance**
Ebrahimnejad [22]	2016	Transportation prpblem	Trader suppler problem	The author proposed a novel approach to address the transportation problem by incorporating interval-valued trapezoidal fuzzy numbers to represent transportation costs, supplies, and demands and solve it.
Sun et al. [23]	2021	Scheduling problem	To solve the robust no-wait flow shop scheduling	This article aims to investigate the R-NWFS problem with interval-valued fuzzy processing times, with the objective of minimizing the make span while adhering to an upper bound on the total completion time.
Bhaumik and Roy [24]	2021	Game theory	Tourism management	The primary objective of this article is to devise and examine a matrix game with multiple objectives. The focus lies in resolving the problem within a single-valued neutrosophic environment, utilizing a linguistic approach.
Deli and Karaaslan [25]	2021	Decision making	MCDM problem	Using generalized hesitant trapezoidal fuzzy numbers, which express membership degrees through multiple possible trapezoidal fuzzy numbers, proves to be a more appropriate and effective approach for solving real-life MCDM problems compared to real life problem.

Continued on next page

Table 1 – Continued the Literature survey

Authors	**Year**	**Environment**	**Application**	**Significance**
Hassanpour et al. [26]	2023	Multi-objective linear programming	Supply chain management	This paper aims to address a challenging problem of intuitionistic fuzzy multi-objective linear programming, where objective functions and constraints involve intuitionistic fuzzy parameters and using linear ranking function to transform the intuitionistic fuzzy parameters into a crisp problem.



Different subfield of operation research and applications

Throughout our continuous search, we have widely scrutinized the distinct attributes of fuzzy concept. However, it is essential to acknowledge the existence of particular challenges associated with this approach, which necessitate a comprehensive discussion in the subsequent section.

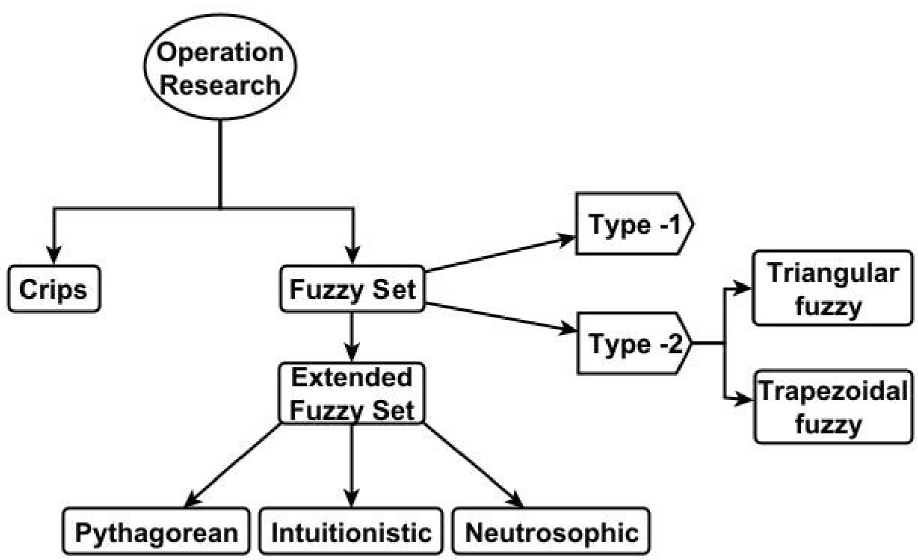
4.1. *Some Challenges of The Fuzzy Linear Programming:*

FLP play important role in decision making problem but Fuzzy Linear Programming (FLP) poses several challenges due to the inherent uncertainty and imprecision associated with fuzzy logic. Additionally, solving problems in fuzzy logic can be computationally intensive, especially when dealing with non-linear functions. As the number of variables and constraints increases, finding optimal solutions becomes more difficult, which can cause scalability issues. It is evident from the above literature that fuzzy theory alone is not sufficient to address uncertainty effectively. Therefore, researchers have developed several theories to overcome these challenges. These theories include Intuitionistic theory (1983) by Atanassov, neutrosophic theory (1990) by Samarandche, Pythagorean theory (2013) by Yager, so on.

5. **Some Important Definitions, Introduction Related to Fuzzy Extended Theory**

The Extended Fuzzy Principle starts by providing an extensive overview of the fundamental concepts and principles underlying fuzzy logic. The key growths in the extended fuzzy set include the following types:

- Intuitionistic
- Neutrosophic
- Pythagorean and so on.



Different types of fuzzy extended principle

Intuitionistic Fuzzy:

Intuitionistic fuzzy are part of extension Fuzzy sets that offer more information about the degree of ambiguity through non- grade of membership. Intuitionistic fuzzy first introduced by Atanassov [27] in 1986 to deal with uncertainty using non- grade of membership and grade of membership functions. The key definition and concept of intuitionistic is discuss below:

Intuitionistic fuzzy Set [27]

A set$\tilde{\theta}_{IF}$ on H is defined as $\tilde{\theta}_{IF} = \{(h, [(\rho(h), \upsilon(h))]) : h \in H\}$ Where $\rho_{IF}(h) : H \to [0,1]$ is membership functions and $\upsilon_{IF}(h) : H \to [0,1]$ is non-membership function. And $\rho_{IF}(h), \upsilon_{IF}(h)$ satisfies the following relation:

$$0 \le \rho_{IF}(h) + \upsilon_{IF}(h) \le 1$$

Intuitionistic Fuzzy Number [28]:

An intuitionistic fuzzy number is represented as,$\tilde{M}_k$with the grade of membership function $\sigma_{\tilde{M}_k}(t)$and non- grade of membership function $\eta_{\tilde{M}_k}(t)$. The following properties hold:

(1) For all, the non- grade of membership function is concave, meaning that$t_1, t_2 \in t, \eta_{\tilde{M}_k}(\lambda_k t_1 + (1-\lambda_k)t_2) \le \max\{\eta_{\tilde{M}_k}(t_1), \eta_{\tilde{M}_k}(t_2)\}$, where $\lambda_k \in [0,1]$.
(2) For all, the membership function is convex, meaning that $t_1, t_2 \in t, \sigma_{\tilde{M}_k}(\lambda_k t_1 + (1-\lambda_k)t_2) \ge \min\{\sigma_{\tilde{M}_k}(t_1), \sigma_{\tilde{M}_k}(t_2)\}$, where $\lambda_k \in [0,1]$.
(3) The number is considered normal as there exists a $t_0 \in t$value such that $\sigma_{\tilde{M}_k}(t_0) = 1$ and $\eta_{\tilde{M}_k}(t_0) = 0$.
(4) It is classified as an intuitionistic fuzzy subset of the real line.

In summary, an intuitionistic fuzzy shows concavity for its non- grade of membership function, convexity for its membership function, satisfies the normal property, and can be categorized as an intuitionistic fuzzy subset within the real set.

Trapezoidal Intuitionistic Fuzzy Number (TpIFN) [29]:

Trapezoidal Intuitionistic fuzzy number denoted as $\widetilde{ITF}$, which possesses a grade of membership $\tau_{\widetilde{ITS}}(\psi)$, and a non- grade of membership $\gamma_{\widetilde{ITF}}(\psi)$ functions follow as:

$$\tau_{\widetilde{ITS}}(\psi) = \begin{cases} \frac{(\psi-q)}{(n-q)}\tau_{\widetilde{ITF}}, & \text{q}\le\psi\le r; \\ \tau_{\widetilde{ITF}}, & \text{r}\le\psi\le s; \\ \frac{(t-\psi)}{(t-s)}\tau_{\widetilde{ITF}}, & \text{s}<\psi\le t; \\ 0 & \text{Otherwise.} \end{cases}$$

$$\gamma_{\widetilde{ITF}}(\psi) = \begin{cases} \frac{(r-\psi)+\nu_{\widetilde{ITF}}(\psi-q_1)}{(r-m_1)}\gamma_{\widetilde{ITF}}, & \text{q}\le\psi\le r; \\ \gamma_{\widetilde{ITF}}, & \text{r}\le\psi\le s; \\ \frac{(\psi-s)+\nu_{\widetilde{ITF}}(t_1-\psi)}{(t_1-o)}\gamma_{\widetilde{ITF}}, & \text{s}<\psi\le t; \\ 0, & \text{Otherwise.} \end{cases}$$

Where $0 \leq \tau_{\widetilde{ITF}}(\alpha) \leq 1; 0 \leq \gamma_{\widetilde{ITF}}(\alpha) \leq 1$; and $\tau_{\widetilde{ITF}} + \gamma_{\widetilde{ITF}} \leq 1; q, r, s, t \in \mathbb{R}$. Then $\widetilde{ITF} = \left\langle \left([q, r, s, t]\, ; \tau_{\widetilde{ITF}} \right), \left([q_1, r, s, t_1]; \gamma_{\widetilde{ITF}} \right) \right\rangle$ is called as an intuitionistic trapezoidal fuzzy number.

Neutrosophic:

A neutrosophic set is a mathematical framework that extends fuzzy and intuitionistic sets to handle indeterminate or contradictory information using three components: falsity - membership, indeterminacy-membership, and truth-membership degrees which handling indeterminate, imprecise, and uncertain data. Neutrosophic theory introduced by Florentin Smarandache in the 1990s [30], is a remarkable fusion of classical and fuzzy logic. In neutrosophic logic, are considered as false (F), true (T), or indeterminate (I). The key definition and concept of neutrosophic is discuss below:

Neutrosophic Set [13]:

A set $\widetilde{neuK}$ in the universal set U, is said to be Neutrosophic Set if $\widetilde{neuK} = \left\{ \left(u, \left[t_{\widetilde{neuK}}(u), i_{\widetilde{neuK}}(u), f_{\widetilde{neuK}}(u) \right] \right) : u \in U \right\}$ Where $f_{\widetilde{neuK}}(u) : U \to [0,1]$,$i_{\widetilde{neuK}}(u) : U \to [0,1]$, and $t_{\widetilde{neuK}}(u) : U \to [0,1]$ are the Falsity, indeterminacy, and truth membership function respectively and $f_{\widetilde{neuK}}(u), i_{\widetilde{neuK}}(u), t_{\widetilde{neuK}}(u)$ satisfy the following relation.

$$0 \leq sup\left\{ t_{\widetilde{neuK}}(u) \right\} + sup\left\{ i_{\widetilde{neuK}}(u) \right\} + sup\left\{ f_{\widetilde{neuK}}(u) \right\} \leq 3$$

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Single valued trapezoidal neutrosophic number (SVTpNN) [31]:

A set $\widetilde{neuT_p}$ is said to be SVTpNN if the set $\widetilde{neuT_p}$ is defined as: $\widetilde{neuT_p} = \left\{ \left((p_1, p_2, p_3, p_4)\, ; \left[\phi_{T_p}, \varphi_{T_p}, \gamma_{T_p} \right] \right) : p_1, p_2, p_3, p_4 \in \mathbb{R} \right\}$ where truth-membership, indeterminacy-membership, and a falsity-membership are defined as respectively:

$$t_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{(t-p_1)\phi_{T_p}}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \phi_{T_p} & p_2 \leq t \leq p_3 \\ \frac{(p_4-t)\phi_{T_p}}{p_4-p_3} & p_3 \leq t \leq p_4 \\ 0, & \text{Otherwice} \end{array} \right\}$$

$$i_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{\left(p_2-t+(t-p_1)\varphi_{T_p}\right)}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \phi_{T_p} & p_2 \leq t \leq p_3 \\ \frac{\left(t-p_3+(p_4-t)\varphi_{T_p}\right)}{p_4-p_3} & p_3 \leq t \leq p_4 \\ 1, & \text{Otherwice} \end{array} \right\}$$

$$f_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{\left(p_2-t+(t-p_1)\gamma_{T_p}\right)}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \gamma_{T_p} & \mathrm{p}_2 \leq t \leq p_3 \\ \frac{\left(t-p_3+(p_4-t)\gamma_{T_p}\right)}{p_4-p_3} & \mathrm{p}_3 \leq t \leq p_4 \\ 1, & \text{Otherwice} \end{array} \right\}$$

Where, $0 \leq \phi_{T_p}, \varphi_{T_p}, \gamma_{T_p} \leq 1$ and $0 \leq \phi_{T_p} + \varphi_{T_p} + \gamma_{T_p} \leq 3$

6. Different Environment of the Neutrosophic Linear Programming Problem in different real-life problems

Table 2. Researchers from various domains have widely investigated the real-life applications of NFLPP, revealing its deep impact.

Authors	Year	Environment	Application	Significance
Alrefaei et al. [32]	2014	Supply Chain Management	Trapezoidal Neutrosophic Numbers	This paper proposes a flexible approach to fuzzy linear programming, which is employed to address the supply chain management challenges faced by a steel manufacturing company.
Broumi et al. [33]	2016	Decision making	Trapezoidal neutrosophic numbers.	This paper introduces a novel MADM method utilizing the Single valued NTpLWAA operator and the Single valued NTpLWGA operator.
Biswas et al. [34]	2018	Decision making	ITpNNs	The author proposes the development of a MADM strategy utilizing interval trapezoidal neutrosophic numbers (ITpNNs) as the distance measure.
Hamiden Khalifa [35]	2020	Assignment problem	Single-valued TpNN	The objective of this article is to optimize the multi-objective assignment problem using neutrosophic numbers.
Badr et al. [36]	2021	LPP	Trapezoidal neutrosophic numbers	Badr et al. proposed a new Exterior Point Simplex Algorithm to Solve the Neutrosophic Linear Programming Problems.

6.1. *The LPP under the Neutrosophic Principle*

Kamal et al. [37]

The author introduced a Multi-Objective Transportation Problem in which the objective functions are characterized as Type-2 TpFn and demand are expressed as multi-choice and supply are expressed as probabilistic random variables in constraints. Additionally, the study considers the " rate of decrement in profit and rate of increment in Transportation Cost (TC) on transporting the products from destinations to sources due to" factors affecting such as late deliveries, product damage, weather conditions, and other problems, which add to the overall cost. Given the presence of these uncertainties, obtaining a direct optimum solution becomes challenging. Hence, the first step is to convert these uncertainties into a crisp equivalent form. The aim of this article to transform type 2 TpFn into the classical equivalents by using two-phase defuzzification technique. Multi-choice variables are converted into equivalent values using the Stochastic Programming (SP) approach and probabilistic random variables are converted into equivalent values using the binary variables approach. The supply parameters are assumed to follow various types of probabilistic distributions, such as Weibull, Extreme value, Cauchy, and Pareto, while demand parameters are assumed as Normal distribution.

Hosseinzadeh and Tayyebi [38]

This paper presents an effective optimization and modeling framework for Fuzzy Multi-Objective neutrosophic Linear Programming Problem. The proposed framework employs SVTpNNs in Neural Networks to represent the coefficients of the right-hand side parameters, constraints, and objective functions. The main objective is to transform NFMOLP problem into an equivalent crisp MOLP Problem using a ranking function of SVTpNNs. To avoid decision deadlock situations within a hierarchical structure, the proposed method defines appropriate membership functions for each objective function based on the best and worst values of the objectives. This approach aims to select an optimal compromise solution that maximizes the degree of acceptance and minimizes the degree of rejection to some extent, considering all objectives simultaneously. The concept of fuzzy decision sets is employed to achieve this goal effectively. The practical applicability of the proposed method is demonstrated through the resolution of a transportation problem, highlighting its usefulness in real-world scenarios. Industries and business organizations dealing with imprecise and contradictory information can benefit from this approach to handle complex decision-making tasks efficiently.

Fathya and Ammar [39]

In this research, Fathya and Ammar [39] propose an innovative collaborative strategy for addressing multi-objective neutrosophic multi-level linear programming (MNMLP) problems, utilizing the harmonic mean technique. The coefficients of the objective function and the coefficients of constraints are neutrosophic numbers of the level decision makers. To transform the MNMLP problem effectively, we employ the interval programming technique, which splits it into two classical MMLP problems. One of these problems features all coefficients of neutrosophic numbers as upper approximations, while the other uses as lower approximations. Subsequently, they employ the harmonic mean method to amalgamate the various objectives from each classical problem converted into a single objective. By solving the crisp linear single-objective programming problem, they obtain a chosen solution for MNMLP problems. Fathya and Ammar (Fathy & Ammar, 2023) illustrate the practicality of our research through an application that addresses the optimization of the cost for a multi-objective of the transportation problem with a neutrosophic environment.

7. Conclusion:

In conclusion, this review article highlights the integration of Neutrosophic Theory and trapezoidal neutrosophic environments in the situation of LP problems. A neutrosophic theory is a mathematical framework that extends fuzzy and intuitionistic sets to handle uncertain, imprecise, and indeterminate information using three components: truth, falsity, and indeterminacy. On the other hand, Extended Fuzzy Theory extends the traditional fuzzy theory to manage more intricate grade of membership degrees. The primary objective of this review is to summarize and analyze the prevailing literature on trapezoidal neutrosophic environments, particularly focusing on aspects such as the NFMOLP Problems in SVTpN environments. This comprehensive review article aims to enhance the understanding of the potential of these integrated methodologies for effectively presenting decision-making problems amidst complex and uncertain conditions.

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