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# Operations of Single Valued Neutrosophic Coloring

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**Abstract:** Smarandache introduced the concept of Neutrosophic which deals with membership, non-membership and indeterminacy values. Wang discussed the Single Valued Neutrosophic sets in 2010. Single Valued Neutrosophic graph was introduced by Broumi and in 2019 Single Valued Neutrosophic coloring was introduced. In this paper, some properties of the Single Valued Neutrosophic Coloring of Strong Single Valued Neutrosophic graph, Complete Single Valued Neutrosophic graph and Complement of Single Valued Neutrosophic graphs are discussed.

**Keywords:** single-valued neutrosophic graphs; single-valued neutrosophic vertex coloring; strong single-valued neutrosophic graph; complete single-valued neutrosophic graph.

## 1. Introduction

Francis Guthrie's four-color conjecture was reasoned for the development of the new branch of graph coloring in graph theory. Graph coloring is assigning labels to the vertices or edges or both vertices and edges. Distinct vertices received different colors are called proper coloring. Graph coloring technique used in many areas like telecommunication, scheduling, computer networks etc.

Most of the problems are not only deals the accurate values, sometimes handle vague values. Fuzzy sets were introduced by Zadeh [29] in 1965, dealt imprecise values in his work. Fuzzy graph theory concept was developed by Rosenfeld [25] in 1975. Munoz et al. [27] in 2004 and Eslahchi, Onagh [19] in 2006 discussed the fuzzy chromatic number and its properties.

Kassimir T. Atanassov [11] introduced the concept of intuitionistic fuzzy sets in 1986 and intuitionistic fuzzy graph in 1999. The intuitionistic graphs are handled membership and non-membership values. Vague set concept introduced by Gau and Buehrer [21] in 1993. In 2014, Akram et al. [9] discussed vague graphs and further work extended by Borzooei et al. [12, 13]. Vertex and Edge coloring of Vague graphs were introduced by Arindam Dey et al. [10] in 2018.

Neutrosophic set was introduced by F. Smarandache [25] in 1998, it's a generalization of the intuitionistic fuzzy set. It consists of membership value, indeterminacy value and non-membership value. Neutrosophic logic play a vital role in several of the real valued problems like law, medicine, industry, finance, engineering, IT, etc. Wang et al. [28] worked on Single valued neutrosophic sets in 2010. Strong Neutrosophic graph and its properties were introduced and discussed by Dhavaseelan et al. [20] in 2015 and Single valued neutrosophic concept introduced in 2016 by Akram and Shahzadi [6, 7, 8]. Broumi et al. [14, 15, 16, 17, 18] extended their works in single valued neutrosophic graphs, interval valued neutrosophic graphs (IVNG) and bipolar neutrosophic graphs. Abdel-Basset et al. used Neutrosophic concept in their papers [1, 2, 3, 4, 5] to find the decisions for some real-life operation research and IoT-based enterprises in 2019. In 2019, Jan et al. [23] have reviewed the following definitions: Interval-Valued Fuzzy Graphs (IVFG), Interval-Valued Intuitionistic Fuzzy Graphs (IVIFG), Complement of IVFG, SVNG, IVNG and the complement of SVNG and IVNG. They have modified those definitions, supported with some examples. Neutrosophic graphs happen to play a vital role in the building of neutrosophic models. Also, these graphs can be used in networking, Computer technology, Communication, Genetics, Economics, Sociology, Linguistics, etc., when the concept of indeterminacy is present.

In this research paper, the bounds of single valued neutrosophic vertex coloring for SVNG, Complement of SVNG are determined and discussed some more operations on SVNG.

**Definition 1.1.** [26] Let X be a space of points(objects). A neutrosophic set A in X is characterized by truth-membership function $t_A(x)$, an indeterminacy-membership function $i_A(x)$ and a falsity-membership function $f_A(x)$. The functions $t_A(x)$, $i_A(x)$, and $f_A(x)$, are real standard or non-standard subsets of $]0^-, 1^+[$. That is, $t_A(x): X \to ]0^-, 1^+[$ , $i_A(x): X \to ]0^-, 1^+[$ and $f_A(x): X \to ]0^-, 1^+[$ and $0^- \leq t_A(x) + i_A(x) + f_A(x) \leq 3^+$.

**Definition 1.2.** [7] A single-valued neutrosophic graphs (SVNG) G = (X, Y) is a pair where X: N → [0,1] is a single-valued neutrosophic set on N and Y: N × N → [0,1] is a single-valued neutrosophic relation on N such that

$$t_Y(xy) \leq \min\{t_X(x), t_X(y)\},$$

$$i_Y(xy) \leq \min\{i_X(x), i_X(y)\},$$

$$f_Y(xy) \leq \max\{f_X(x), f_X(y)\},$$

for all x, y ∈ N. X and Y are called the single-valued neutrosophic vertex set of G and the single-valued neutrosophic edge set of G, respectively. A single-valued neutrosophic relation Y is said to be symmetric if $t_Y(xy) = t_Y(yx)$, $i_Y(xy) = i_Y(yx)$ and $f_Y(xy) = f_Y(yx)$, for all x,y ∈ N. Single-valued neutrosophic be abbreviated here as SVN.

## 2. Single-Valued Neutrosophic Vertex Coloring (SVNVC)

In this section, discussed the bounds of SVNVC for the resultant SVNG by some operations on SVNG, CSVNG and complement of SVNG. Also discussed some theorems.

**Definition 2.1. [**24] A family $\Gamma = \{\gamma_1, \gamma_2, \dots, \gamma_k\}$ of SVN fuzzy set is called a k-SVNVC of a SVNG G = (X, Y) if

1. $\vee \gamma_i(x) = X, \ \forall x \in X$
2. $\gamma_i \wedge \gamma_j = 0$

3. For every incident vertices of edge xy of G, $\min\{\gamma_i(m_1(x)), \gamma_i(m_1(y))\} = 0$, $\min\{\gamma_i(i_1(x)), \gamma_i(i_1(y))\} = 0$ *and* $\max\{\gamma_i(n_1(x)), \gamma_i(n_1(y))\} = 1, (1 \le i \le k)$.

This k-SVNVC of G is denoted by $\chi_v(G)$, is called the SVN chromatic number of the SVNG G.

**Definition 2.2** A SVNG G = (X, Y) is called complete single-valued neutrosophic graph (CSVNG) if the following conditions are satisfied:

$$t_Y(xy) = \min\{t_X(x), t_X(y)\},$$
$$i_Y(xy) = \min\{i_X(x), i_X(y)\},$$
$$f_Y(xy) = \max\{f_X(x), f_X(y)\},$$

for all x, y ∈ X.

For any single value neutrosophic subgraph H of SVNG G, $\chi_v(H) \le \chi_v(G)$

Theorem 2.3.

For any SVNG with n vertices $\chi_v(G) \le n$.

Proof:

By the observation that the CSVNG with n vertices has the SVNVC is n. All the other graphs with n vertices are subgraphs of the CSVNG, it is clear by the above observation. Hence $\chi_v(G) \le n$.

**Definition 2.4** Let $G_1 = (X_1, Y_1)$ and $G_2 = (X_2, Y_2)$ be single-valued neutrosophic graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The union G1 ∪ G2 is defined as a pair (X, Y) such that

$$t_X(x) = \begin{cases} t_{X_1}(x), & if\ x \in V_1 and\ x \notin V_2, \\ t_{X_2}(x), & if\ x \in V_2 and\ x \notin V_1, \\ \max\left(t_{X_1}(x), t_{X_2}(x)\right), & if\ x \in V_1 \cap V_2. \end{cases}$$

$$i_X(x) = \begin{cases} i_{X_1}(x), & if\ x \in V_1 and\ x \notin V_2, \\ i_{X_2}(x), & if\ x \in V_2 and\ x \notin V_1, \\ \max\left(i_{X_1}(x), i_{X_2}(x)\right), & if\ x \in V_1 \cap V_2. \end{cases}$$

$$f_X(x) = \begin{cases} f_{X_1}(x), & if\ x \in V_1 and\ x \notin V_2, \\ f_{X_2}(x), & if\ x \in V_2 and\ x \notin V_1, \\ \min\left(f_{X_1}(x), f_{X_2}(x)\right), & if\ x \in V_1 \cap V_2. \end{cases}$$

$$t_Y(xy) = \begin{cases} t_{Y_1}(xy), & if\ xy \in E_1 and\ x \notin E_2, \\ t_{Y_2}(xy), & if\ xy \in E_2 and\ x \notin E_1, \\ \max\left(t_{Y_1}(x), t_{Y_2}(x)\right), & if\ x \in E_1 \cap E_2. \end{cases}$$

$$i_Y(xy) = \begin{cases} i_{Y_1}(xy), & if\ xy \in E_1 and\ x \notin E_2, \\ i_{Y_2}(xy), & if\ xy \in E_2 and\ x \notin E_1, \\ \max\left(i_{Y_1}(x), i_{Y_2}(x)\right), & if\ x \in E_1 \cap E_2. \end{cases}$$

$$f_Y(xy) = \begin{cases} f_{Y_1}(xy), & if\ xy \in E_1 and\ x \notin E_2, \\ f_{Y_2}(xy), & if\ xy \in E_2 and\ x \notin E_1, \\ \min\left(f_{Y_1}(x), f_{Y_2}(x)\right), & if\ x \in E_1 \cap E_2. \end{cases}$$

For any SVNGs $G_1 = (X_1, Y_1)$ and $G_2 = (X_2, Y_2)$, $\chi_v(G_1 \cup G_2) = max\{\chi_v(G_1), \chi_v(G_2)\}$.

**Definition 2.5 [8]** The complement of a SVNG G = (X, Y) is a SVNG $\bar{G} = (\bar{X}, \bar{Y})$, where

1. $\bar{X} = X$
2. $\bar{t_X}(x) = t_X(x), \bar{i_X}(x) = i_X(x), \bar{f_X}(x) = f_X(x)\ for\ all\ x \in X$

3. $\bar{t_X}(xy) = \begin{cases} \min\{t_X(x), t_X(y)\} & if \ \ t_Y(xy) = 0 \\ \min\{t_X(x), t_X(y)\} - t_Y(xy) & if \ \ t_Y(xy) > 0 \end{cases}$

$\bar{i_X}(xy) = \begin{cases} \min\{i_X(x), i_X(y)\} & if \ \ i_Y(xy) = 0 \\ \min\{i_X(x), i_X(y)\} - i_Y(xy) & if \ \ i_Y(xy) > 0 \end{cases}$

$\bar{f_X}(xy) = \begin{cases} \max\{f_X(x), f_X(y)\} & if \ \ f_Y(xy) = 0 \\ \max\{f_X(x), f_X(y)\} - f_Y(xy) & if \ \ f_Y(xy) > 0 \end{cases}$

for all $x, y \in X$.

**Theorem 2.6.** For any SVNG $G$ with $n$ vertices, $2\sqrt{n} \leq \chi_v(G) + \chi_v(\bar{G}) \leq 2n$ and $n \leq \chi_v(G)\chi_v(\bar{G}) \leq n^2$.

Let every vertex of G has n − 1 adjacent vertices, then by the definition of complement of SVNG each vertex of $\bar{G}$ has the lesser than or equal to n − 1 adjacent vertices. Hence, the inequalities true for all SVNG. Thus, $2\sqrt{n} \leq \chi_v(G) + \chi_v(\bar{G}) \leq 2n$ and $n \leq \chi_v(G)\chi_v(\bar{G}) \leq n^2$.

Definition 2.7.

A SVNG G = (X, Y) is called strong single-valued neutrosophic graph (SSVNG) if the following conditions are satisfied:

$$t_Y(xy) = \min\{t_X(x), t_X(y)\},$$
$$i_Y(xy) = \min\{i_X(x), i_X(y)\},$$
$$f_Y(xy) = \max\{f_X(x), f_X(y)\},$$

for all (x,y) ∈ Y .

Observation 2.8

For any SSVNG $G$ with $n$ vertices, $2\sqrt{n} \leq \chi_v(G) + \chi_v(\bar{G}) \leq \mathrm{n} + 1$ and $n \leq \chi_v(G)\chi_v(\bar{G}) \leq (\frac{n+1}{2})^2$.

Given that $G$ is SSVNG and the complement of $G$ is defined by $\bar{G} = (\bar{X}, \bar{Y})$, where

1. $\bar{X} = X$
2. $\bar{t_X}(x) = t_X(x), \bar{i_X}(x) = i_X(x), \bar{f_X}(x) = f_X(x) \ for \ all \ x \in X$
3. $\bar{t_X}(xy) = \begin{cases} \min\{t_X(x), t_X(y)\} & if \ \ t_Y(xy) = 0 \\ 0 & if \ \ t_Y(xy) > 0 \end{cases}$

$\bar{i_X}(xy) = \begin{cases} \min\{i_X(x), i_X(y)\} & if \ \ i_Y(xy) = 0 \\ 0 & if \ \ i_Y(xy) > 0 \end{cases}$

$\bar{f_X}(xy) = \begin{cases} \max\{f_X(x), f_X(y)\} & if \ \ f_Y(xy) = 0 \\ 0 & if \ \ f_Y(xy) > 0 \end{cases}$

for all $x, y \in X$. Hence, the above inequalities hold.

**Theorem 2.9.** For a path graph $P_n$, $\chi_v(P_n) = 2$ where $n \geq 2$.

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of SVN fuzzy sets defined on V as follows:

$$\gamma_1(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \ i = odd \\ (0,0,1) & for \ i = even \end{cases}$$

$$\gamma_2(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \ i = even \\ (0,0,1) & for \ i = odd \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2\}$ fulfilled the conditions of SVNVC of the graph $G$. Hence the SVN chromatic number of $P_n$ is $\chi_v(P_n) = 2$.

**Theorem 2.10.** For a cycle graph $C_n$, $\chi_v(C_n) = \begin{cases} 2 \; if \; n = even \\ 3 \; if \; n = odd \end{cases}$ where $n \geq 3$.

For n is odd:

Let $\Gamma = \{\gamma_1, \gamma_2, , \gamma_3\}$ be a family of SVN fuzzy sets defined on V as follows:

$$\gamma_1(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \; i = 1,3,5, \dots, n-2 \\ (0,0,1) & for \; others \end{cases}$$

$$\gamma_2(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \; i = 2,4,6, \dots, n-1 \\ (0,0,1) & for \; others \end{cases}$$

$$\gamma_3(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \; i = n \\ (0,0,1) & for \; others \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, , \gamma_3\}$ fulfilled the conditions of SVNVC of the graph G. Hence the SVN chromatic number $\chi_v(C_n) = 3$.

For n is even:

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of SVN fuzzy sets defined on V as follows:

$$\gamma_1(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \; i = odd \\ (0,0,1) & for \; i = even \end{cases}$$

$$\gamma_2(x_i) = \begin{cases} (t(x_i), i(x_i), f(x_i)) & for \; i = even \\ (0,0,1) & for \; i = odd \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2\}$ fulfilled the conditions of SVNVC of the graph G. Hence the SVN chromatic number $\chi_v(C_n) = 2$.

**Theorem 2.11.** For any graph SVNG, $\chi_v(G) \leq \Delta(G) + 1$.

Here $\Delta(G)$ denotes the number of edges incident with a vertex of SVNG G, hence the result is true for all SVNG.

## 3. Conclusions

Graph Coloring is an useful technique to solve many real life problems which are easily converted as graph models. SVNG is dealt with vague and imprecise values. Single Valued Neutrosophic Coloring concept was introduced by the authors in [24]. In this paper, we discussed few more results of SVNVC using CSVNG and Complement of SVNG. We have an idea to extend the concept of SVNVC with irregular coloring and dominating coloring technique in future.

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**Conflicts of Interest**

The authors declare no conflict of interest.

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