NSS
Neutrosophic Sets and Systems, Vol. 67, 2024
University of New Mexico

The indefinite symbolic plithogenic trigonometric integrals

Yaser Ahmad Alhasan[1,*], Suliman Sheen[2], Raja Abdullah Abdulfatah[3] and Mohammed Mustafa Ahmed[4]

[1,*]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, KSA; y.alhasan@psau.edu.sa
[4]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.; S.almleh@psau.edu.sa
[3]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.; r.abdulfatah@psau.edu.sa
[4]Department of mathematics, Faculty of science, Islamic University of Madinah, Madinah 42351, Saudi Arabia; mohamemedm086@gmail.com

***Corresponding author**: y.alhasan@psau.edu.sa

Abstract: This paper discussed the indefinite plithogenic trigonometric integrals, where we presented the integrating products of plithogenic trigonometric function, also studying the plithogenic trigonometric identities, which facilitated finding the integral of the associated formulas. In addition to a set of exercises that clarify each idea.

Keywords: trigonometric integrals; plithogenic trigonometric function; plithogenic trigonometric identities.

---

1. Introduction and Preliminaries

To The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. Plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic set, logic, probability, and statistics [2], in addition to presenting introduction to the symbolic plithogenic algebraic structures (revisited), through which he discussed several ideas, including mathematical operations on plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7].

Paper is divided into four parts. Provides an introduction in the first portion, which includes a review of Plithogenic science. A few definitions of a Plithogenic and operations with plithogenic numbers are covered in the second section. The third section defined plithogenic functions. The paper's conclusion is provided in the fourth section.

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [8-9]. Alhasan, Smarandache and Abdulfatah presented the indefinite symbolic plithogenic integrals [10].

Division of Symbolic Plithogenic Numbers [1]

Let consider two symbolic plithogenic numbers as below:

$$PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$$

$$PN_s = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s$$

$$\frac{PN_r}{PN_s} = \begin{cases} none, one \ \ many & r \geq s \\ \emptyset & r < s \end{cases}$$

This paper covered a number of topics; the indefinite plithogenic trigonometric integrals were covered in the main discussion section after the introduction and preliminary information were presented in the first section. The paper's conclusion is provided in the final section.

Main Discussion

We will consider C the constant of the symbolic plithogenic integral defined as:
$C = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$, where: $a_0, a_1, a_2, \ldots, a_n$ are real numbers.

Integrating products of symbolic plithogenic trigonometric function:

I. $\int PN_r\, sin^m(PN_sx)\, cos^n(PN_sx) \quad dx$, where m and n are positive integers.

To find this integral, we can distinguish the following two cases:

- Case n is odd:
  - split of $cos(PN_sx)$
  - apply $cos^2(PN_sx) = 1 - sin^2(PN_sx)$
  - we substitution $u = sin(PN_sx)$
- Case m is odd:
  - split of $sin(PN_sx)$
  - apply $sin^2(PN_sx) = 1 - cos^2(PN_sx)$
  - we substitution $u = cos(PN_sx)$

Example 1

Find: $\int (P_3 + 1)\, sin^2(P_2 + 2)x\, cos^3(P_2 + 2)x\, dx$
Solution:

$$\int (P_3 + 2)\, sin^2(P_3 + 1)x\, cos^3(P_3 + 1)x\, dx = \int (P_3 + 2)\, sin^2(P_3 + 1)x\, cos^2(P_3 + 1)x \quad cos(P_3 + 1)x\, dx$$

$$= \int (P_3 + 2)\, sin^2(P_3 + 1)x\, (1 - sin^2(P_3 + 1)x) \quad cos(P_3 + 1)x\, dx$$

$$= \int [(P_3 + 2)\, sin^2(P_3 + 1)x - (P_3 + 2)\, sin^4(P_3 + 1)x]\, cos(P_3 + 1)x\, dx$$

By substitution:

$$u = sin(P_3 + 1)x \qquad \Longrightarrow \qquad du = (P_3 + 1)\, cos(P_3 + 1)x \quad dx$$

$$\Longrightarrow \qquad = \int \left[\left(\frac{P_3 + 2}{P_3 + 1}\right) u^2 - \left(\frac{P_3 + 2}{P_3 + 1}\right) u^4 \right] du$$

$$=\left(\frac{P_3+2}{P_3+1}\right)\left(\frac{u^3}{3}-\frac{u^5}{5}\right)+C=\left(2-\frac{1}{2}P_3\right)\left(\frac{sin^3(P_3+1)x}{3}-\frac{sin^5(P_3+1)x}{5}\right)+C$$

where:
$C=a_0+a_1P_1+a_2P_2+a_3P_3$; $a_0,\ a_1,\ a_2,\ a_3$ are real numbers.

and:

$$\frac{P_3+2}{P_3+1}=x_0+x_1P_1+x_2P_2+x_3P_3$$

$$P_3+2=(P_2+1)(x_0+x_1P_1+x_2P_2+x_3P_3)$$

$$P_3+2=x_0P_3+x_1P_3+x_2P_3+x_3P_3+x_0+x_1P_1+x_2P_2+x_3P_3$$

$P_3+2=x_0+x_1P_1+x_2P_2+(x_0+x_1+x_2+2x_3)P_3$, then:

$x_0=2\ ,x_1=0\ ,x_2=0\ ,x_3=-\frac{1}{2}$

hence: $\frac{P_3+2}{P_3+1}=2-\frac{1}{2}P_3$

let's check the answer:

$$\frac{d}{dx}\left[\left(2-\frac{1}{2}P_3\right)\left(\frac{sin^3(P_3+1)x}{3}-\frac{sin^5(P_3+1)x}{5}\right)+C\right]$$

$$=\left(2-\frac{1}{2}P_3\right)\left(\frac{3(P_3+1)sin^2(P_3+1)x.cos(P_3+1)x}{3}-\frac{5(P_3+1)sin^4(P_3+1)x.cos(P_3+1)x}{5}\right)$$

$$=\left(2-\frac{1}{2}P_3\right)(P_3+1)sin^2(P_3+1)x(1-sin^2(P_3+1)x)\,cos(P_3+1)$$

$$=(P_3+2)sin^2(P_3+1)x(cos^2(P_3+1)x)\,cos(P_3+1)$$

$=(P_3+2)sin^2(P_3+1)xcos^3(P_3+1)x$ (The same integral function)

II. $\int tan^m(PN_sx)\,sec^n(PN_sx)\,dx$, where m and n are positive integers.

To find this integral, we can distinguish the following cases:

- Case n is even:
  - split of $sec^2(PN_sx)$
  - apply $sec^2(PN_sx)=1+tan^2(PN_sx)$
  - we substitution $u=tan(PN_sx)$
- Case m is odd:
  - split of $sec(PN_sx)$ $tan(PN_sx)$
  - apply $tan^2(PN_sx)=sec^2(PN_sx)-1$

- we substitution $u = sec(PN_s x)$

➢ Case m even and n odd:

- apply $tan^2(PN_s x) = sec^2(PN_s x) - 1$
- we substitution $u = sec(PN_s x)$ or $u = tan(PN_s x)$, depending on the case.

Example 2

Find: $\int(-3P_7 + 4P_6)\, tan^2(P_6 x) \quad sec^4(P_6 x)x \quad dx$

Solution:

$n = 4$ (even)

$$\int(-3P_7 + 4P_6)\, tan^2(P_6 x) \quad sec^4(P_6 x)\, dx = \int(-3P_7 + 4P_6)\, tan^2(P_6 x) \quad sec^2(P_6 x)\, sec^2(P_6 x)\, dx$$

$$= \int(-3P_7 + 4P_6)(tan^2(P_6 x) + tan^4(P_6 x))\, sec^2(P_6 x) \quad dx$$

by substitution:

$$u = tan(P_6 x) \qquad \Rightarrow \qquad du = P_6\, sec^2(P_6 x) \quad dx$$

$$\Rightarrow \int(-3P_7 + 4P_6)(tan^2(P_6 x) + tan^4(P_6 x))\, sec^2(P_6 x) \quad dx$$

$$= \quad \frac{-3P_7 + 4P_6}{P_6}\int(u^2 + u^4)\, du$$

$$= (x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 - 3P_7)\left(\frac{u^3}{3} + \frac{u^5}{5}\right) + C$$

$$= (x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 - 3P_7)\left(\frac{tan^3(P_6 x)}{3} + \frac{tan^5(P_6 x)}{5}\right) + C$$

where:
$C = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + x_4P_4 + a_5P_5 + a_6P_6 + a_7P_7$; $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are real numbers.

and:

$$\frac{-3P_7 + 4P_6}{P_6} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7$$

$$-3P_7 + 4P_6 = (P_6 x)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7)$$

$$-3P_7 + 4P_6 = x_0P_6 + x_1P_6 + x_2P_6 + x_3P_6 + x_4P_6 + x_5P_6 + x_6P_6 + x_7P_7$$

$$-3P_7 + 4P_6 = (x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6)P_6 + x_7P_7$$

then:

$$x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 4 \;\; and \quad x_7 = -3$$

hence:

$\frac{-3P_7+4P_6}{P_6} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 - 3P_7$

where: $x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 4$

Example 3

Find:

$$\int (P_4 - 3)\, tan^3(P_1 + 1)x\, sec^3(P_1 + 1)x\, dx$$

Solution:

$m = 3$ (odd)

$$\int (P_4 - 3)\, tan^3(P_1 + 1)x\, sec^3(P_1 + 1)x\, dx$$
$$= \int (P_4 - 3)\, tan^2(P_1 + 1)x \quad sec^2(P_1 + 1)x \quad sec(P_1 + 1)x \quad tan(P_1 + 1)x\, dx$$

$$= \int (P_4 - 3)(sec^4(P_1 + 1)x - sec^2(P_1 + 1)x)\, sec(P_1 + 1)x \quad tan(P_1 + 1)x\, dx$$

by substitution:

$$u = sec(P_1 + 1)x \qquad \Longrightarrow \qquad du = (P_1 + 1)\, sec(P_1 + 1)x \quad tan(P_1 + 1)x \quad dx$$

$$\Longrightarrow \int (P_4 - 3)(sec^4(P_1 + 1)x - sec^2(P_1 + 1)x) \quad sec(P_1 + 1)x \quad tan(P_1 + 1)x\, dx$$

$$= \frac{P_4 - 3}{P_1 + 1} \int (u^4 + u^2)\, du$$

$$= \left(-3 + \frac{3}{2}P_1 + \frac{1}{2}P_4\right)\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$$

$$= \left(-3 + \frac{3}{2}P_1 + \frac{1}{2}P_4\right)\left(\frac{sec^5(P_1 + 1)x}{5} - \frac{sec^3(P_1 + 1)x}{3}\right) + C$$

where:
$C = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4$; a_0, a_1, a_2, a_3, a_4 are real numbers.

and:

$\frac{P_4 - 3}{P_1 + 1} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4$

$$P_4 - 3 = (P_1 + 1)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4)$$

$$P_4 - 3 = x_0P_1 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4$$

$$P_4 - 3 = x_0 + (x_0 + 2x_1)P_1 + 2x_2P_2 + 2x_3P_3 + 2x_4P_4$$

then:

$$x_0 = -3 \;\; , x_1 = \frac{3}{2} \;\; , x_2 = 0 \;\; , x_3 = 0 \;\; and \;\;\; x_4 = \frac{1}{2}$$

hence:

$\frac{P_4-3}{P_1+1} = -3 + \frac{3}{2}P_1 + \frac{1}{2}P_4$

III. $\int cot^m(PN_sx)\, csc^n(PN_sx)\, dx$ *, where* m *and* n *are positive integers.*

To find this integral, we can distinguish the following cases:

- Case n is even:
  - split of $csc^2(PN_sx)$
  - apply $csc^2(PN_sx) = 1 + cot^2(PN_sx)$
  - we substitution $u = cot(PN_sx)$
- Case m is odd:
  - split of $csc(PN_sx)$ $cot(PN_sx)$
  - apply $cot^2(PN_sx) = csc^2(PN_sx) - 1$
  - we substitution $u = csc(PN_sx)$
- Case m even and n odd:
  - apply $cot^2(PN_sx) = csc^2(PN_sx) - 1$
  - we substitution $u = csc(PN_sx)$ or $u = cot(PN_sx)$, depending on the case.

Example 6

Find: $\int 2P_9\sqrt{cot(P_8x)}\;\; csc^4(P_8x)\, dx$

Solution:

$n = 4$ (even)

$$\int 2P_9\sqrt{cot(P_8x)}\;\; csc^4(P_8x)\, dx = \int 2P_9\, cot^{1/2}(P_8x)\;\; csc^2(P_8x)\, csc^2(P_8x)\, dx$$

$$= \int 2P_9\left(cot^{1/2}(P_8x) + cot^{3/2}(P_8x)\right) csc^2(P_8x)\;\; dx$$

by substitution:

$$u = cot(P_8x) \qquad \Rightarrow \qquad du = -P_8\, csc^2(P_8x)\;\; dx$$

$$\Rightarrow \int 2P_9\left(cot^{\frac{1}{2}}(P_8x) + cot^{\frac{3}{2}}(P_8x)\right) csc^2(P_8x)\;\; dx$$

$$= \quad -\frac{2P_9}{P_8}\int \left(u^{1/2} + u^{3/2}\right) du$$

$$= -(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7 + x_8P_8 + 2P_9)\left(\frac{2\ \ u^{\frac{3}{2}}}{3} + \frac{2\ \ u^{\frac{5}{2}}}{5}\right) + C$$

$$= (x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7 + x_8P_8 + 2P_9)\left(-\frac{2\ \ cot^{\frac{3}{2}}(P_8x)}{3} - \frac{2\ \ cot^{5/2}(P_8x)}{5}\right) + C$$

where:
$C = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + x_4P_4 + a_5P_5 + a_6P_6 + a_7P_7 + a_8P_8 + a_9P_9$
; $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ are real numbers.

and:

$$\frac{2P_9}{P_8} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7 + x_8P_8 + x_9P_9$$

$$2P_9 = (P_8)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7 + x_8P_8 + x_9P_9)$$

$$2P_9 = x_0P_8 + x_1P_8 + x_2P_8 + x_3P_8 + x_4P_8 + x_5P_8 + x_6P_8 + x_7P_8 + x_8P_8 + x_9P_9$$

$$2P_9 = (x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8)P_8 + x_9P_9$$

then:

$$x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 0 \quad and \quad x_9 = 2$$

hence:

$$\frac{2P_9}{P_8} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 + x_6P_6 + x_7P_7 + x_8P_8 + 2P_9$$

where: $x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 0$

Plithogenic trigonometric identities:

1) $sin(PN_rx)\,cos(PN_sx) = \frac{1}{2}\ \ [sin(PN_r + PN_s)x + sin(PN_r - PN_s)x]$

2) $cos(PN_rx)\,sin(PN_sx) = \frac{1}{2}\ \ [sin(PN_r + PN_s)x - sin(PN_r - PN_s)x]$

3) $cos(PN_rx)\,cos(PN_sx) = \frac{1}{2}\ \ [cos(PN_r + PN_s)x + cos(PN_r - PN_s)x]$

4) $sin(PN_rx)\,sin(PN_sx) = \frac{-1}{2}\ \ [cos(PN_r + PN_s)x - cos(PN_r - PN_s)x]$

Example 7

Find:

$$1)\int P_3 sin(P_2+3)x\, cos(2P_1-1)\,x \quad dx=\int \frac{1}{2}P_3 \quad [sin(P_2+2P_1+2)x+sin(P_2-2P_1+4)x]\,dx$$

$$=\frac{1}{2}\left[-\left(\frac{P_3}{P_2+2P_1+2}\right)cos(P_2+2P_1+2)x-\left(\frac{P_3}{P_2-2P_1+4}\right)cos(P_2-2P_1+4)x\right]+C$$

$$=\frac{1}{2}\left[-\frac{1}{5}P_3 cos(P_2+2P_1+2)x-\frac{1}{3}P_3\, cos(P_2-2P_1+4)x\right]+C$$

$$=-\frac{1}{10}P_3 cos(P_2+2P_1+2)x-\frac{1}{6}P_3\, cos(P_2-2P_1+4)x+C$$

where:
$C=a_0+a_1P_1+a_2P_2+a_3P_3$; a_0,a_1,a_2,a_3 are real numbers.

and:

$$\frac{P_3}{P_2+2P_1+2}=x_0+x_1P_1+x_2P_2+x_3P_3$$

$$P_3=(P_2+2P_1+2)(x_0+x_1P_1+x_2P_2+x_3P_3+x_0+x_1P_1+x_2P_2+x_3P_3+x_0+x_1P_1+x_2P_2+x_3P_3)$$

$$P_3=2x_0+2x_1P_1+2x_2P_2+2x_3P_3+2x_0P_1+2x_1P_1+2x_2P_2+2x_3P_3+x_0P_2+x_1P_2+x_2P_2+x_3P_3$$

$$P_3=2x_0+(2x_0+4x_1)P_1+(x_0+x_1+5x_2)P_2+5x_3P_3$$

then:

$$x_0=0\ ,\ x_1=0\ ,\ x_2=0\ ,\ x_3=\tfrac{1}{5}$$

hence:

$$\frac{P_3}{P_2+2P_1+3}=\frac{1}{5}P_3$$

and we have:

$$\frac{P_3}{P_2-2P_1+4}=x_0+x_1P_1+x_2P_2+x_3P_3$$

$$P_3=(P_2-2P_1+4)(x_0+x_1P_1+x_2P_2+x_3P_3+x_0+x_1P_1+x_2P_2+x_3P_3+x_0+x_1P_1+x_2P_2+x_3P_3)$$

$$P_3=4x_0+4x_1P_1+4x_2P_2+4x_3P_3-2x_0P_1-2x_1P_1-2x_2P_2-2x_3P_3+x_0P_2+x_1P_2+x_2P_2+x_3P_3$$

$$P_3=4x_0+(-2x_0+2x_1)P_1+(x_0+x_1+3x_2)P_2+3x_3P_3$$

then:

$$x_0=0\ ,\ x_1=0\ ,\ x_2=0\ ,\ x_3=\tfrac{1}{3}$$

hence:

$$\frac{P_3}{P_2-2P_1+4}=\frac{1}{3}P_3$$

$$2)\int P_2 cos(P_2x)\,cos(2P_2x) \quad dx=\int \frac{1}{2}P_2[cos(3P_2x)+cos(P_2x)] \quad dx$$

$$=\frac{1}{2}[(x_0+x_1P_1+x_2P_2)sin(3P_2x)+(x_0+x_1P_1+x_2P_2)sin(P_2x)]+C$$

where:
$C=a_0+a_1P_1+a_2P_2$; a_0,a_1,a_2 are real numbers.

and:

$$\frac{P_2}{3P_2}=x_0+x_1P_1+x_2P_2$$

$$P_2=(3P_2)(x_0+x_1P_1+x_2P_2)$$

$$P_2=3x_0P_2+3x_1P_2+3x_2P_2$$

$$P_2=(3x_0+3x_1+3x_2)P_2$$

then:

$$x_0+x_1+x_2=\frac{1}{3}$$

hence:

$\frac{P_2}{3P_2}=x_0+x_1P_1+x_2P_2$, where: $x_0+x_1+x_2=\frac{1}{3}$

and we have:

$$\frac{P_2}{P_2}=\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2$$

$$P_2=(P_2)(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2)$$

$$P_2=\acute{x}_0P_2+\acute{x}_1P_2+\acute{x}_2P_2$$

$$P_3=(\acute{x}_0+\acute{x}_1+\acute{x}_2)P_2$$

then:

$$\acute{x}_0+\acute{x}_1+\acute{x}_2=1$$

hence:

$\frac{P_2}{P_2}=\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2$, where: $\acute{x}_0+\acute{x}_1+\acute{x}_2=1$

3) $\int P_4sin(P_7+3)x\ sin(P_6-3)x\ \ dx=\int\frac{-1}{2}P_4[cos(P_7+P_6+6)x-\cos(P_7-P_6)x]\ \ dx$

$$=dose\ \ not\ \ exist$$

because:

$$\frac{P_4}{P_7+P_6+6}=(dose\ \ not\ \ exist)$$

$$and\ \ \frac{P_4}{P_7-P_6}=(dose\ \ not\ \ exist)$$

5. Conclusions

In this paper, we studied the indefinite plithogenic trigonometric integrals by presenting integration methods specific to the plithogenic field; we concluded that we could obtain an integration that does not exist in the plithogenic field, as the division process is not possible according to the concept presented by Smarandache.

Acknowledgments "This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2024/R/1445)".

References

[1] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)". Neutrosophic Sets and Systems, Volume 53, PP: 653-665, 2023

[2] Smarandache, F., "Plithogeny, Plithogenic Set, Logic, Probability, and Statistics", 2017.

[3] Smarandache, F., "An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures", Journal of Fuzzy Extension and Applications, Volume 4, pp. 48-55, 2023.

[4] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[5] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.

[6] Abobala, M., and Allouf, A., " On A Novel Security Scheme for The Encryption and Decryption Of 2×2 Fuzzy Matrices with Rational Entries Based on The Algebra of Neutrosophic Integers and El-Gamal Crypto-System", Neutrosophic Sets and Systems, vol.54, 2023.

[7] M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, "Introduction to Symbolic 2-Plithogenic Probability Theory," Galoitica: Journal of Mathematical Structures and Applications, vol. 7, no. 1, 2023.

[8] Alhasan,Y., "The neutrosophic integrals and integration methods", Neutrosophic Sets and Systems, Volume 43, pp. 290-301, 2021.

[9] Alhasan,Y., "The definite neutrosophic integrals and its applications", Neutrosophic Sets and Systems, Volume 49, pp. 277-293, 2022.

[10] Alhasan,Y., Smarandache,F., Abdulfatah, R., "The indefinite symbolic plithogenic integrals", Neutrosophic Sets and Systems, Volume 60, pp. 29-38, 2023.

Received: Feb 1, 2024. Accepted: April 25, 2024