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Solving the Minimum Spanning Tree Problem Under Interval-Valued Fermatean Neutrosophic Domain

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Abstract: In classical graph theory, the minimal spanning tree (MST) is a subgraph that lacks cycles and efficiently connects every vertex by utilizing edges with the minimum weights. The computation of a minimum spanning tree for a graph has been a pervasive problem over time. However, in practical scenarios, uncertainty often arises in the form of fuzzy edge weights, leading to the emergence of the Fuzzy Minimum Spanning Tree (FMST). This specialized approach is adept at managing the inherent uncertainty present in edge weights within a fuzzy graph, a situation commonly encountered in real-world applications. This study introduces the initial optimization approach for the Minimum Spanning Tree Problem within the context of interval-valued fermatean neutrosophic domain. The proposed solution involves the adaptation of the Dhouib-Matrix-MSTP (DM-MSTP) method, an innovative technique designed for optimal resolution. The DM-MSTP method operates by employing a column-row navigation strategy through the adjacency matrix. To the best of our knowledge, instances of this specific problem have not been addressed previously. To address this gap, a case study is generated, providing a comprehensive application of the novel DM-MSTP method with detailed insights into its functionality and efficacy.

Keywords: Minimum Spanning Tree Problem; Fermatean Neutrosophic Domain; Dhouib-Matrix-MSTP; Artificial Intelligence; Operations Research; Combinatorial Optimization

1. Introduction

The Minimum Spanning Tree (MST) is a fundamental concept in graph theory, serving as a crucial optimization problem with widespread applications in various fields. At its core, the MST seeks to identify the most efficient way to connect a set of vertices within a graph by utilizing edges with minimal total weights. This problem has garnered significant attention due to its relevance in operational research, communication systems, transportation networks, logistics, supply chain management, image processing, wireless telecommunication, and cluster analysis. Consider a connected graph G=(V,E) where V represents the set of vertices and E is the set of edges. In the context of this graph, a tree denoted as T is considered a spanning tree if it serves as a subgraph of G and encompasses all the nodes present in G. Referred to as a maximal tree subgraph, T stands out among other trees within G as the most extensive, containing the maximum number of arcs.

Conventional spanning trees operate under the assumption of precise and deterministic edge weights, which may not align with the realities of many real-world situations characterized by ambiguous or imprecise information. The introduction of fuzzy logic offers a valuable means to address uncertainty, providing a more accurate model for scenarios where the nature of relationships between nodes is not fully understood or easily quantifiable. The necessity for a Fuzzy Spanning Tree (FST) arises as a response to the limitations of traditional spanning tree models in effectively handling the inherent uncertainties found in practical, real-world scenarios. Recognizing that real-world systems often grapple with imprecise, uncertain, or incomplete information, the adoption of fuzzy logic in spanning tree models emerges as a solution. This approach enhances the models' ability to capture and represent inherent uncertainties, resulting in solutions that are more robust and adaptable across various application domains. Zadeh [34] introduced the concept of fuzzy sets to address ambiguity and uncertainty by employing a degree of membership function. Subsequently, Atanassov [35] proposed intuitionistic fuzzy sets (IFS), capable of managing both membership and non-membership functions, offering enhanced flexibility in dealing with ambiguity.

Building upon these ideas, Smarandache [36] developed the concept of neutrosophic sets (NS), which includes membership, indeterminacy, and non-membership functions. NS gained considerable attention, leading to numerous studies [37]-[41]. In the domain of minimum spanning trees (MST), Agnes et al. [29] formulated a Fuzzy Clustering model, while Oscan et al. [30] addressed uncertain cost and demand parameters with a capacitated fuzzy MST approach. Mohanta et al. [31] proposed an algorithm for intuitionistic fuzzy MST, and Mandal et al. [32] explored robust MST in cancer detection using intuitionistic fuzzy graphs. For neutrosophic fuzzy graphs, Dey et al. [33] implemented an algorithm for MST.

Senapati et al. [42] first introduced Fermatean fuzzy sets as an extension of Pythagorean fuzzy sets to overcome their inherent limitations. Following this, Jeevaraj [43] further developed the concept by proposing interval-valued Fermatean fuzzy sets. Subsequently, Palani et al. [44] elaborated on the topic by presenting a decision-making methodology within the framework of Pythagorean interval-valued Fermatean fuzzy sets. Expanding on this research, Ruan et al. [45] discussed a multi-criteria decision-making approach tailored for interval-valued Fermatean neutrosophic fuzzy sets.

Despite these advancements, there has been a notable absence of matrix-based methods for MST in the literature. This gap in existing methods has motivated our work, leading us to extend the concept to the Dhouib-Matrix-MSTP (DM-MSTP) method for interval-valued fermatean fuzzy graphs.

This paper is structured as follows: Section 2 introduces the fundamental concepts of IFS, NFS, FFS, and IVFNS. Section 3 outlines the Dhouib Matrix MST method for IVFNN. Section 4 provides numerical examples, and Section 5 concludes the paper.

2. Preliminaries

Definition 1. The Fermatean fuzzy Set (FFS) $\tilde{F}$ in the universal set X is defined by $\tilde{F} = \{\langle x, \mu_{\tilde{F}}(x), \nu_{\tilde{F}}(x)\rangle : x \in X\}$ where the membership function $\mu_{\tilde{F}}(x)$: X → [0, 1] and the non-membership function $\nu_{\tilde{F}}(x)$: X ⟶ [0, 1] satisfy the condition $[\mu_{\tilde{F}}(x)]^3 + [\nu_{\tilde{F}}(x)]^3 \leq 1$ is said to be the degree of hesitation of x to $\tilde{F}$.

Definition 2. Let X be the universe of discourse. Then $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle : x \in X\}$ is defined as Neutrosophic Fuzzy Set (NFS), where the truth-membership function is represented as $T_N(x): X \to [0,1]$ an indeterminacy-membership function $I_N(x): X \to [0,1]$ and the falsity membership function $F_N(x): X \to [0,1]$ which satisifes the conditions $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3, \forall x \in X$.

Definition 3. A neutrosophic fuzzy set ℓ in the universe X is the form of $\ell = \{\langle u, T_\ell(u), I_\ell(u), F_\ell(u)\rangle : u \in \ell\}$ represents the degree of truth, indeterminacy and falisty-membership of ℓ respectively. The mapping $T_\ell(u): \ell \to [0,1]$, $I_\ell(u): \ell \to [0,1]$, $F_\ell(u): \ell \to [0,1]$ and $0 \leq T_\ell(u)^3 + I_\ell(u)^3 + F_\ell(u)^3 \leq 2$. Here $\ell = (T_\ell, I_\ell, F_\ell)$ is denoted as fermatean neutrosophic number (FNN).

Definition 4. [28] An interval-valued fermatean neutrosophic set (IVFNS) A on the universe of discourse X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, where $T_A(x) = [T^-_A(x), T^+_A(x)]$, $I_A(x) = [I^-_A(x), I^+_A(x)]$, $F_A(x) = [F^-_A(x), F^+_A(x)]$ represents the truth-membership degree, indeterminacy-membership degree and falsity-membership degree, respectively. Hence the mapping $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, $F_A(x): X \to [0,1]$ and $0 \leq (T_A(x))^3 + (F_A(x))^3 \leq 1$ and $0 \leq (I_A(x))^3 \leq 1$ $0 \leq (T_A(x))^3 + (F_A(x))^3 + (I_A(x))^3 \leq 2$ means $0 \leq (T_A(x))^3 + (F_A(x))^3 + (I_A(x))^3 \leq 2 \ \forall \ x \in X$

Definition 5. [28] Let $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U]\rangle$ then the score function $S(x)$ is defined as:

$$S(x) = \frac{\left(T_A^L(x)\right)^3 + \left(T_A^U(x)\right)^3 + \left(I_A^L(x)\right)^3 + \left(I_A^U(x)\right)^3 + \left(F_A^L(x)\right)^3 + \left(F_A^U(x)\right)^3}{2}.$$

3. The Dhouib-Matrix-MSTP method

In a very recent work, the first author invented a new optimal method entitled Dhouib-Matrix-MSTP (DM-MSTP) to optimally solve the Minimum Spanning Tree Problem. The general structure of DM-MSTP is depicted in Figure 1 where two lists (the MST-Path and the Min-Columns) are added to the Adjacency-Matrix.

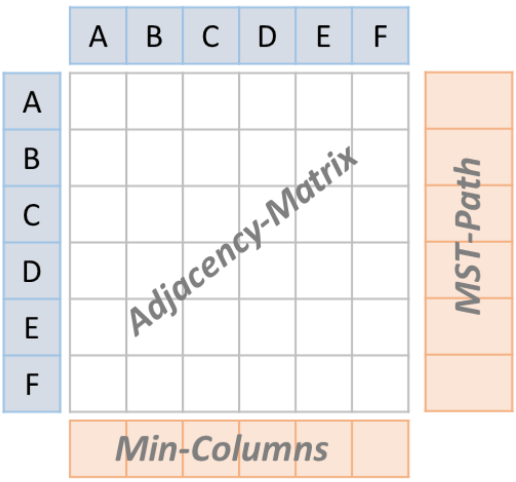

Figure 1. The general structure of DM-MSTP

Indeed, MST-Path gathers the generated components of the spanning tree, Min-Columns is used to drive the research process and Adjacency-Matrix represents the distance between all the vertices.

DM-MSTP is composed of four steps (see Figure 2) and for a more clarification a detailed example will be presented in section 4.

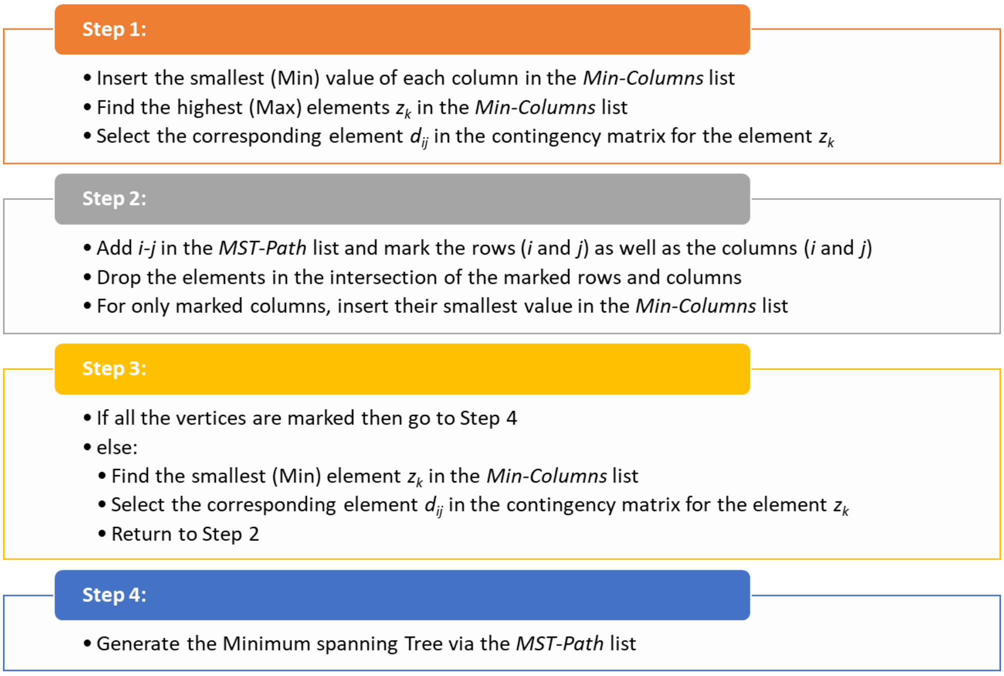

Figure 2. The general structure of DM-MSTP

DM-MSTP is designed under the general concept of Dhouib-Matrix where several optimization methods are developed such as the DM-ALL-SPP to create the shortest path between all-pairs of vertices in (Dhouib, 2024b) and the DM-SPP to solve single-pair, single-source and single-destination Shortest Path Problems (Dhouib, 2023a; Dhouib, 2023b; Dhouib, 2024c; Dhouib, 2024d). in addition, two heuristics (DM-AP1 and DM-AP2) are developed for the Assignment Problems in (Dhouib, 2022a; Dhouib, 2022b; Dhouib, 2023c; Dhouib and Sutikno, 2023) and the DM-TP1 is invented for the Transportation Problems in (Dhouib, 2021a; Dhouib, 2021b). Moreover, two other methods (DM-TSP1 and DM-TSP2) are designed for the Travelling Salesman Problems in (Dhouib, 2021c; Dhouib, 2021d; Dhouib, 2022c; Dhouib et al., 2021; Dhouib et al., 2023). Besides, three innovative metaheuristics are developed: DM4 (Dhouib, 2024e; Dhouib, 2022d; Dhouib and Pezer, 2022; Dhouib and Pezer, 2023; Dhouib, 2023d; Dhouib et al, 2024), DM3 (Dhouib, 2021e; Dhouib and Zouari, 2023a; Dhouib and Zouari, 2023b) and FtN (Dhouib, 2022e).

4. Numerical examples

In this section, the DM-MSTP is simulated on an undirected graph under interval-valued Fermatean neutrosophic domains where the objective is to create the shortest minimum spanning tree. The interval-valued Fermatean neutrosophic number $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \rangle$ is converted to a crisp number using Equation 1 originally developed by (Broumi et al., 2023):

$$S(x) = \frac{\left(T_A^L(x)\right)^3 + \left(T_A^U(x)\right)^3 + \left(I_A^L(x)\right)^3 + \left(I_A^U(x)\right)^3 + \left(F_A^L(x)\right)^3 + \left(F_A^U(x)\right)^3}{2} \quad (1)$$

Consider the graph in Figure 3, with six vertices and nines edges.

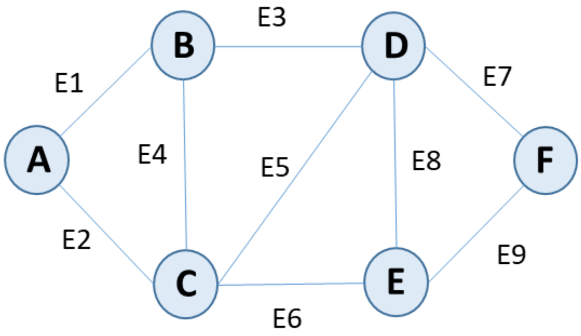

Figure 3. Undirected graph with six vertices and nine edges

The values of the nine edges are represented as interval-valued Fermatean neutrosophic numbers and are given in Table 1.

Table 1. The edge interval-valued Fermatean neutrosophic values

Edge name	Neutrosophic length	Crisp length
E1	⟨[0.4,0.9], [0.5,0.6], [0.1,0.3]⟩	0.5810
E2	⟨[0.5,0.9], [0.4,0.6], [0.1,0.5]⟩	0.6300
E3	⟨[0.2,0.9], [0.3,0.5], [0.1,0.4]⟩	0.4770
E4	⟨[0.2,0.6], [0.4,0.6], [0.8,0.9]⟩	0.8725
E5	⟨[0.1,0.6], [0.5,0.8], [0.6,0.8]⟩	0.7910
E6	⟨[0.2,0.5], [0.4,0.7], [0.6,0.8]⟩	0.6340
E7	⟨[0.1,0.7], [0.5,0.7], [0.6,0.8]⟩	0.7700
E8	⟨[0.3,0.9], [0.3,0.5], [0.2,0.5]⟩	0.5205
E9	⟨[0.5,0.9], [0.2,0.4], [0.1,0.6]⟩	0.5715

The first step is to convert the interval-valued Fermatean neutrosophic set to a crisp one using the score function developed in Equation 1. Figure 4, illustrates the crisp matrix.

$$\begin{bmatrix} \infty & 0.5810 & 0.6300 & \infty & \infty & \infty \\ 0.5810 & \infty & 0.4770 & 0.8725 & \infty & \infty \\ 0.6300 & 0.4770 & \infty & 0.7910 & 0.6340 & \infty \\ \infty & 0.8725 & 0.7910 & \infty & 0.7700 & 0.5205 \\ \infty & \infty & 0.6340 & 0.7700 & \infty & 0.5715 \\ \infty & \infty & \infty & 0.5205 & 0.5715 & \infty \end{bmatrix}$$

Figure 4. The crisp adjacency matrix

DM-MSTP starts by inserting the smallest elements of each column in Min-Columns and selecting the biggest value (0.5810) at column 1. The corresponding element (dBA) in column 1 is selected to indicate that vertices B and A are connected and 'B-A' is archived in MSTP-Path (see Figure 5).

	A	B	C	D	E	F	
A		0.5810	0.6300				B-A
B	**0.5810**		0.4770	0.8725			
C	0.6300	0.4770		0.7910	0.6340		
D		0.8725	0.7910		0.7700	0.5205	
E			0.6340	0.7700		0.5715	
F				0.5205	0.5715		
	0.5810	0.4770	0.4770	0.5205	0.5715	0.5205	

B-A

Figure 5. The vertices B and A are connected

Besides, the rows and columns (B and A) are selected (see elements with the yellow color in Figure 6) and the values of elements in the intersection are dropped. Next, Min-Columns is initiated with the smallest element of only the selected column (with the yellow color), the smallest value is selected (0.4770) and its corresponding element (dCB) is designated. Thus, vertex C is connected to vertex B and 'C-B' is archived in MSTP-Path.

	A	B	C	D	E	F	
A			0.6300				B-A
B			0.4770	0.8725			C-B
C	0.6300	**0.4770**		0.7910	0.6340		
D		0.8725	0.7910		0.7700	0.5205	
E			0.6340	0.7700		0.5715	
F				0.5205	0.5715		
	0.6300	**0.4770**					

C-B

Figure 6. The vertices C and B are connected

Next, row and column C are selected and the value of the elements in the intersection of A, B and C are dropped (see Figure 7). Also, the smallest elements for each selected column are inserted in Min-Columns, their smallest value (0.6340) is selected and its corresponding element (dEC) is identified. Then, vertex E is connected to vertex C and 'E-C' is added to MSTP-Path.

	A	B	C	D	E	F	
A							B-A
B				0.8725			C-B
C				0.7910	0.6340		E-C
D		0.8725	0.7910		0.7700	0.5205	
E			**0.6340**	0.7700		0.5715	
F				0.5205	0.5715		
		0.8725	**0.6340**				

E-C

Figure 7. The vertices E and C are connected

Following, row and column E are selected and the value of the elements in the intersection are discarded (see Figure 8). Similarly, Min-Columns is initiated, the smallest value (0.5715) is selected and its corresponding element (dFE) is identified. Then, vertex F is connected to vertex E and 'F-E' is added to MSTP-Path.

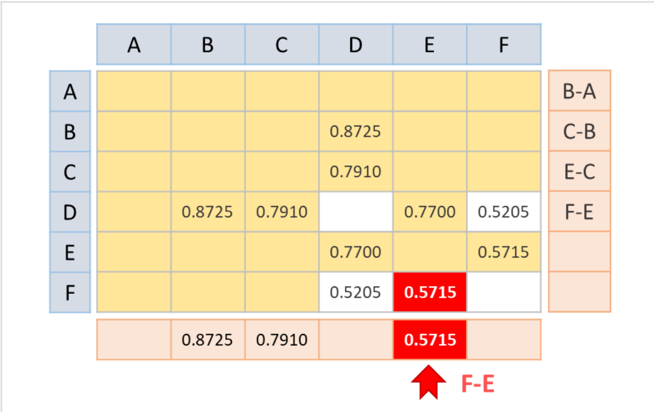

Figure 8. The vertices F and E are connected

Subsequent, row and column E are selected and the value of the elements in the intersection are discarded (see Figure 9). Similarly, the Min-Columns is initiated, the smallest value (0.5205) is selected and its correspondent element is (dDF) identified. Then, vertex DE is connected to vertex F and 'D-F' is added to MSTP-Path.

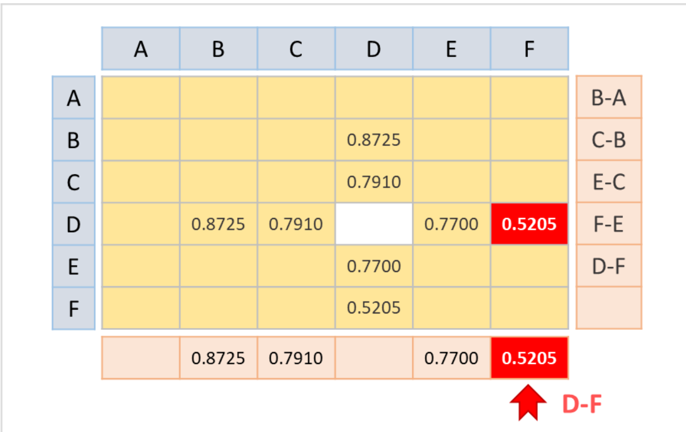

Figure 9. The vertices D and F are connected

Finally, row and column D are selected and the value of the elements in the intersection are discarded. After the above steps, the adjacency matrix is empty and the minimum spanning tree can be generated from MSTP-Path with a total cost of 0.5810+0.4770+0.6340+0.5715+0.5205 = 2.784. Clearly, from Figure 10 you can see the proposed solution generated by DM-SPP.

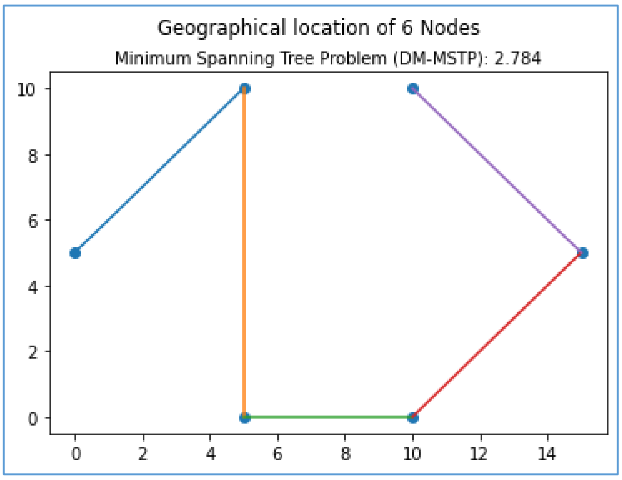

Figure 10. The minimum spanning tree generated by DM-MSTP

5. Conclusions

This paper introduces enhancements to the Dhouib-Matrix-MSTP (DM-MSTP) method to optimize the Minimum Spanning Tree Problem within an interval-valued Fermatean neutrosophic framework. The DM-MSTP method, known for its efficiency, incorporates two key lists: the Path-Memory list, which stores spanning tree edges, and the Min-Column list, used to determine the next column to activate. Additionally, the paper includes a detailed case study illustrating the application of the method. Future research will explore the application of DM-MSTP to other variants of the Minimum Spanning Tree Problem, such as the capacitated version, and its extension to other neutrosophic domains.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Dhouib S (2024a) Innovative Method to Solve the Minimum Spanning Tree Problem: The Dhouib-Matrix-MSTP (DM-MSTP). Results in Control and Optimization, 14:100359. https://doi.org/10.1016/j.rico.2023.100359.
2. Dhouib S (2024b) An Original Optimal All-Pairs Shortest Path Method: The Dhouib-Matrix-ALL-SPP (DM-ALL-SPP). Data Science and Management In press. https://doi.org/10.1016/j.dsm.2024.01.005.
3. Dhouib S (2024c) Intelligent Path Planning for Cognitive Mobile Robot Based on Dhouib-Matrix-SPP Method. Cognitive Robotics, Vol. 4:62-73, https://doi.org/10.1016/j.cogr.2024.02.001.
4. Dhouib S (2024d) Faster than Dijkstra and A* Methods for the Mobile Robot Path Planning Problem Using Four Movement Directions: The Dhouib-Matrix-SPP-4. Advances in Transdisciplinary Engineering, Mechatronics and Automation Technology 46:284-290. https://doi.org/10.3233/ATDE231116.
5. Dhouib S (2023a) An optimal method for the Shortest Path Problem: The Dhouib-Matrix-SPP (DM-SPP). Results in Control and Optimization 12:100269. https://doi.org/10.1016/j.rico.2023.100269.
6. Dhouib S (2023b) Shortest Path Planning via the Rapid Dhouib-Matrix-SPP (DM-SPP) Method for the Autonomous Mobile Robot. Results in Control and Optimization 13:100299. https://doi.org/10.1016/j.rico.2023.100299.
7. Dhouib S (2022a) An Intelligent Assignment Problem Using Novel Heuristic: The Dhouib-Matrix-AP1 (DM-AP1): Novel Method for Assignment Problem. International Journal of Intelligent Systems and Applications in Engineering 10(1):135-141.
8. Dhouib S. (2022b) Novel Optimization Method for Unbalanced Assignment Problems with Multiple Jobs: The Dhouib-Matrix-AP2. Intelligent Systems with Applications 17:200179. https://doi.org/10.1016/j.iswa.2023.200179.

9. Dhouib S (2023c) Unravelling the Assignment Problem under Intuitionistic Triangular Fuzzy Environment by the Novel Heuristic Dhouib-Matrix-AP1. Yugoslav Journal of Operations Research 33(3):467-480. https://doi.org/10.2298/YJOR220915005D.
10. Dhouib S, Sutikno T (2023) Solving the trapezoidal fuzzy assignment problem using the novel Dhouib-Matrix-AP1 heuristic. Bulletin of Electrical Engineering and Informatics 12(2):950-957. https://doi.org/10.11591/eei.v12i2.4855.
11. Dhouib S (2021a) Solving the Single-Valued Trapezoidal Neutrosophic Transportation Problems through the Novel Dhouib-Matrix-TP1 Heuristic. Mathematical Problems in Engineering 2021:3945808. https://doi.org/10.1155/2021/3945808.
12. Dhouib S (2021b) Solving the Trapezoidal Fuzzy Transportation Problems Via New Heuristic the Dhouib-Matrix-TP1. International Journal of Operations Research and Information Systems 12(4):1-16. https://doi.org/10.4018/IJORIS.294119.
13. Dhouib S (2021c) Neutrosophic Triangular Fuzzy Travelling Salesman Problem Based on Dhouib-Matrix-TSP1 Heuristic. International Journal of Computer and Information Technology 10(5):180-183. https://doi.org/10.24203/ijcit.v10i5.154.
14. Dhouib S (2021d) Optimization of Travelling Salesman Problem on Single Valued Triangular Neutrosophic Number using Dhouib-Matrix-TSP1 Heuristic. International Journal of Engineering 34(12):2642-2647. https://doi.org/ 10.5829/IJE.2021.34.12C.09.
15. Dhouib S (2022c) Novel Heuristic for New Pentagonal Neutrosophic Travelling Salesman Problem. Neutrosophic Sets and Systems 51:344-359. https://doi.org/10.5281/zenodo.7135315.
16. Dhouib S, Broumi S, Lathamaheswari M (2021) Single Valued Trapezoidal Neutrosophic Travelling Salesman Problem with Novel Greedy Method: The Dhouib-Matrix-TSP1 (DM-TSP1). International Journal of Neutrosophic Science, 17, 2, 144-157. https://doi.org/10.54216/IJNS.170205.
17. Dhouib S., Zouari A., Dhouib S. and Chabchoub H. (2023) "Integrating the Artificial Bee Colony Metaheuristic with Dhouib-Matrix-TSP1 heuristic for Holes Drilling Problems", Journal of Industrial and Production Engineering, Vol. 40, No. 3, pp. 177-187, DOI: 10.1080/21681015.2022.2158499.
18. Dhouib S (2024e) Multi-Start Constructive Heuristic through Descriptive Statistical Metrics: The Dhouib-Matrix-4 (DM4) Metaheuristic. International Journal of Operational Research, In press. https://doi.org/10.1504/IJOR.2021.10045069.
19. Dhouib S., Kharrat A., Loukil T. and Chabchoub H. (2024) "Enhancing the Dhouib-Matrix-4 Metaheuristic to Generate the Pareto Non-Dominated Set Solutions for Multi-objective Travelling Salesman Problem: The DM4-PMO Method". Results in Control and Optimization, In press, DOI. https://doi.org/10.1016/j.rico.2024.100402.
20. Dhouib S (2022d) Finding the Shortest Holes Drilling Path in Printed Circuit Board via the Dhouib-Matrix-4 Technique. Advances in Transdisciplinary Engineering, Mechatronics and Automation Technology 33:396-401. https://doi.org/10.3233/ATDE221192.
21. Dhouib S, Pezer D (2022) A Novel Metaheuristic Approach for Drilling Process Planning Optimization: Dhouib-Matrix-4 (DM4). International Journal of Artificial Intelligence 20(2):80-92.
22. Dhouib S, Pezer D (2023) Increasing the Performance of Computer Numerical Control Machine via the Dhouib-Matrix-4 Metaheuristic. Inteligencia Artificial 26(71):142-152. https://doi.org/10.4114/intartif.vol26iss71pp142-152.
23. Dhouib S (2023d) Hierarchical Coverage Repair Policies Optimization by Dhouib-Matrix-4 (DM4) Metaheuristic for Wireless Sensor Networks using Mobile Robot. International Journal of Engineering 36(12):2153-2160. https://doi.org/10.5829/ije.2023.36.12c.03
24. Dhouib S (2021e) Novel Metaheuristic Based on Iterated Constructive Stochastic Heuristic: Dhouib-Matrix-3 (DM3). Applied Computational Intelligence and Soft Computing 2021:7761993. https://doi.org/10.1155/2021/7761993.
25. Dhouib S, Zouari A (2023a) Adaptive Iterated Stochastic Metaheuristic to Optimize Holes Drilling Path in Food Industry: The Adaptive-Dhouib-Matrix-3 (A-DM3). Engineering Applications of Artificial Intelligence 120:105898. https://doi.org/10.1016/j.engappai.2023.105898.
26. Dhouib S, Zouari A (2023b) Optimizing the Non-Productive Time of Robotic Arm for Drilling Circular Holes Network Patterns via the Dhouib-Matrix-3 Metaheuristic. International Journal of Mechatronics and Manufacturing Systems 16(1). https://doi.org/10.1504/IJMMS.2023.10054319.
27. Dhouib S (2022e) Holes Drilling Route Optimization in Printed Circuit Board Using Far-to-Near Metaheuristic. International Journal of Strategic Engineering 5(1):1-12. https://doi.org/10.4018/IJoSE.301568.
28. Broumi, Said, et al. "Interval-valued fermatean neutrosophic graphs." Collected Papers. Volume XIII: On various scientific topics (2022): 496.

29. Vathy-Fogarassy, A., Balázs Feil, and János Abonyi. "Minimal spanning tree based fuzzy clustering." Proc World Acad Sci Eng Tech 8 (2005): 7-12.
30. Öncan, Temel. "Design of capacitated minimum spanning tree with uncertain cost and demand parameters." Information Sciences 177.20 (2007): 4354-4367.
31. Mohanta, Kartick, et al. "An algorithmic approach for finding minimum spanning tree in a intuitionistic fuzzy graph." Proceedings of 32nd International Conference on. Vol. 63. 2019.
32. Nithya, S., S. Bhuvaneswari, and S. Senthil. "Robust minimal spanning tree using intuitionistic fuzzy c-means clustering algorithm for breast cancer detection." Am. J. Neural Netw. Appl 5.1 (2019).
33. Dey, Arindam, et al. "A new algorithm for finding minimum spanning trees with undirected neutrosophic graphs." Granular Computing 4 (2019): 63-69.
34. L. A. Zadeh, "Information and control," Fuzzy sets, vol. 8, no. 3, pp. 338–353, 1965.
35. K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy sets and Systems, vol. 20, no. 1, pp. 87–96, 1986
36. Smarandache, Florentin. (1999). A unifying field in Logics: Neutrosophic Logic. Philosophy. American Research Press. 1-141.
37. S. Dey and G. C. Ray, "Properties of redefined neutrosophic composite relation," Neutrosophic Systems with Applications, vol. 7, pp. 1–12, 2023.
38. S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Computation of shortest path prblem in a network with sv-trapezoidal neutrosophic numbers," in Advanced Mechatronic Systems (ICAMechS), 2016 International Conference on. IEEE, 2016, pp. 417–422
39. Broumi, Said, et al. (2022). Interval-valued fermatean neutrosophic graphs. Collected Papers. Volume XIII: On various scientific topics: 496.
40. Broumi, Said, et al. (2019). Shortest path problem in fuzzy, intuitionistic fuzzy and neutrosophic environment: an overview. Complex and Intelligent Systems. 5, 371-378.
41. Broumi, S., S. krishna Prabha, & Vakkas Ulucay. (2023). Interval-Valued Fermatean Neutrosophic Shortest Path Problem via Score Function. Neutrosophic Systems With Applications, 11, 1-10.
42. Senapati, T., Yager, R.R. Fermatean fuzzy sets. J Ambient Intell Human Comput 11, 663–674 (2020). https://doi.org/10.1007/s12652-019-01377-0
43. Jeevaraj, S. "Ordering of interval-valued Fermatean fuzzy sets and its applications." Expert Systems with Applications 185 (2021): 115613.
44. Palanikumar, M., et al. "Multi-criteria group decision making method in Pythagorean interval-valued neutrosophic fuzzy soft soft using VIKOR approach." International Journal of Neutrosophic Science 22.1 (2023): 104-113.
45. Ruan, Chuanyang, and Xiangjing Chen. "Probabilistic Interval-Valued Fermatean Hesitant Fuzzy Set and Its Application to Multi-Attribute Decision Making." Axioms 12.10 (2023): 979.

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