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Single-Valued Pentapartitioned Neutrosophic Soft Set

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Abstract:

Soft set (SS) and neutrosophic set (NS) are important mathematical concepts to deal with uncertainty. NS was further extended to pentapartitioned neutrosophic set (PNS) to deal with uncertainty comprehensively. In order to expand the concept of single-valued pentapartitioned neutrosophic set (SVPNS) and SS, the paper aims to introduce the concept of single-valued pentapartitioned neutrosophic soft set (SVPNS-Set) by adding the ideas of SS and SVPNS together. Furthermore, based on SVPNS-Set, several definitions, examples, properties, propositions and theorems have been established.

Keywords: Single-Valued Neutrosophic Set; SVPN-Set; SVPNS-Set.

1. Introduction:

In 1999, Molodtsov [1] grounded the idea of SS theory. Afterwards, Ali et al. [2] presented some new operations on SS. In 2002, Maji et al. [3] proposed a decision making strategy under the SS environment. SS theory was further studied by Maji et al. [4] in 2003. In 1998, Smarandache [5] introduced the notion of NS by combining the notions of Fuzzy Set (FS) [6] and Intuitionistic FS (IFS) [7]. Afterwards, the concept of bipolar NS was introduced by Deli et al. [8] in 2015. Biswas et al. [9] established the multi-attribute decision making (MADM) strategy using entropy and Grey Relational Analysis (GRA) in the context of NS theory. Later on, Biswas et al. [10] presented the MADM strategy dealing with unknown weight information in the NS environment. Pramanik et al. [11] presented the cross-entropy based MADM strategy in the NS setting. Later on, Maji [12] presented the idea of Single-Valued Neutrosophic Soft Set (SVNS-Set) in the year 2013 by combining the notions of NS and SS. Thereafter, Maji [13] further studied SVNS-Set in 2013. Many researchers around the globe proposed MADM strategies such as TOPSIS [14], GRA [15, 16, 17], etc. Later on, Karaaslan [18] presented two algorithms for group decision making in SVNS-Set environment. Das et al. [19] presented a group decision making strategy based on neutrosophic soft matrix (NSM) and relative weights of experts applying the notion of SVNS-Set. Jha et al. [20] applied SVNS-Set to present a stock tending analysis. Pramanik et al. [21] proposed the TOPSIS based MADM strategy under the single-valued neutrosophic soft expert set environment. Subsequently, the neutrosophic bipolar vague soft set was introduced by Mukherjee and Das [22], who also suggested a MADM strategy for this setting. In 2017, Bera and Mahapatra [23] grounded the idea of topology on SVNS-Sets. Later on, Bera and Mahapatra [24] further studied the neutrosophic soft topological space. Thereafter, the notion of separation axioms via neutrosophic soft topological space was introduced by Aras et al. [25] in 2019. Neurosophic soft compactness via neutrosophic simply soft open set was introduced by Das and Pramanik [26] in 2020. Mehmood et al. [27] grounded the concept of neutrosophic soft α-open set via neutrosophic soft topological space. Smarandache [28] expanded the concept of the SS in 2018 to include the hyper SS and plithogenic hyper SS.

NS and multi-valued neutrosophic refined logic [30] were expanded in 2020 by Mallick and Pramanik [29] by including contradiction, ignorance, and unknown components in order to handle indeterminacy and uncertainty thoroughly. This gave rise to the PNS. Das et al. [31] proposed the GRA based MADM strategy under the PNS setting. Das et a1. [32] proposed the tangent similarity measure based MADM strategy in the PNS setting. Pramanik [33] presented the ARAS strategy in the PNS environment. Later on, Majumder et al. [34] established a MADM strategy based on the hyperbolic tangent similarity measure to determine the most significant environmental risks during the COVID-19 pandemic. Das and Tripathy [36] introduced the concept of topology on PNSs. Afterwards, Das et al. [35] grounded the idea of Q-algebra on PNSs, and introduced pentapartitioned neutrosophic Q-algebra and pentapartitioned neutrosophic Q-Ideal.

Research gap: Several studies of PNS, NS, single-valued NS [37], SVNS-Set and their applications have been depicted in [38]. However, no studies have been reported on the combination of PNS and soft set to deal with uncertainty. This research is important as the combination of PNS and soft set is more powerful in dealing with real problems with uncertainty over the existing NS.

Motivation: Having realizing the advantage and to address the research gap, we initiate to combine the PNS and SS which we call the notions of SVPNS-Set.

This article's primary goal is to define the concept of SVPNS-Set and outline its various properties.

This article's remaining portion is structured as follows:

Section-2 goes over the fundamental definitions and characteristics of NS, SVNS-Set and SVPNS. The concept of SVPNS-Set and its characteristics are introduced in Section-3. In section-4, we finally wrap up the paper by outlining some potential areas for further research.

2. Some Preliminary Results:

This section includes some fundamental definitions and findings on SVNS-Set and SVPN-Set that are pertinent to the article's main findings.

Definition 2.1.[15] Suppose that P be a collection of parameters. Assume that NS(Û) be the family of all NSs defined over a fixed set Û. Then, for any $S \subseteq P$, a pair (N, S) is referred to as a SVNS-Set over Û, where N is a function from S to NS(Û).

An SVNS-Set (N, S) is defined as follows:

$$(N, S)=\{(b, \{(l, Ý_{N(b)}(l), Ī_{N(b)}(l), Ŕ_{N(b)}(l)): l \in Û\}): b \in P, l \in Û\},$$

where $Ý_{N(b)}(l)$, $Ī_{N(b)}(l)$, $Ŕ_{N(b)}(l)$ are the degrees of truth membership function, indeterminacy membership function and false membership function of each $c \in Û$ with respect to the parameter $b \in P$.

Example 2.1. Suppose that Û={$\wp*_1$, $\wp*_2$, $\wp*_3$} be a set of three mobiles, and S={b_1(looks), b_2(RAM), b_3(cost)} be a family of parameters with respect to which the nature of mobile will be described. Assume that $K(b_1)$ = {($\wp*_1$, 0.6, 0.5, 0.5), ($\wp*_2$, 0.3, 0.8, 0.5), ($\wp*_3$, 0.5, 0.3, 0.4)}, $K(b_2)$ = {($\wp*_1$, 0.7, 0.4, 0.6), ($\wp*_2$, 0.6, 0.5, 0.4), ($\wp*_3$, 0.7, 0.3, 0.3)}, $K(b_3)$ = {($\wp*_1$, 0.8, 0.5, 0.4), ($\wp*_2$, 0.7, 0.8, 0.5), ($\wp*_3$, 0.5, 0.3, 0.6)} be three single-valued NSs over Û. Then, (K, S) = {(b_1, $K(b_1)$), (b_2, $K(b_2)$), (b_3, $K(b_3)$)} is an SVNS-Set over Û with respect to the set S.

Definition 2.2.[15] The complement (K^c, S) of an SVNS-Set (K, S) is defined as follows:

$$(K^l, S) = \{(b, \{(l, 1-Ý_{K(b)}(l), 1-Ī_{K(b)}(l), 1-Ŕ_{K(b)}(l)): l \in Û\}): b \in S\}.$$

Definition 2.3.[15] Assume that (K_1, S) and (K_2, S) are any two SVNS-Sets over Û. Then, (K_1, S) is referred to as a SVNS-Set of (K_2, S) if and only if $Ý_{K_1(§)}(\wp*) \leq Ý_{K_2(§)}(\wp*)$, $Ī_{K_1(§)}(\wp*) \geq Ī_{K_2(§)}(\wp*)$ and $Ŕ_{K_1(§)}(\wp*) \geq Ŕ_{K_2(§)}(\wp*)$, for all $§ \in S$ and $\wp* \in Û$. One may write, (K_1, S) $\subseteq$ (K_2, S). Then, (K_2, S) is referred to as single-valued neutrosophic soft super-set of (K_1, S).

Definition 2.4.[15] Suppose that (K_1, S) and (K_2, S) are any two SVNS-Sets over Û. Then, union of (K_1, S) and (K_2, S) is denoted by (K, S), where $K=K_1 \cup K_2$ is defined as follows:

$$(K, S) = \{(§, \{(l, Ý_{K(§)}(l), Ī_{K(§)}(l), Ŕ_{K(§)}(l)): l \in Û\}): § \in S\},$$

where $Ý_{K(§)}(l) = \max\{Ý_{K_1(§)}(l), Ý_{K_2(§)}(l)\}$, $Ī_{K(§)}(l) = \min\{Ī_{K_1(§)}(l), Ī_{K_2(§)}(l)\}$ and $Ŕ_{K(§)}(l) = \min\{Ŕ_{K_1(§)}(l), Ŕ_{K_2(§)}(l)\}$.

Definition 2.5.[15] Suppose that (K_1, S) and (K_2, S) are any two SVNS-Sets over Û. Then, intersection of (K_1, S) and (K_2, S) is denoted by (K, S), where $K=K_1 \cap K_2$ is defined as follows:

$$(K, S) = \{(§, \{(l, Ý_{K(§)}(l), Ī_{K(§)}(l), Ŕ_{K(§)}(l)): l \in Û\}): § \in S\},$$

where $Ý_{K(§)}(l) = \min\{Ý_{K_1(§)}(l), Ý_{K_2(§)}(l)\}$, $Ī_{K(§)}(l) = \max\{Ī_{K_1(§)}(l), Ī_{K_2(§)}(l)\}$, and $Ŕ_{K(§)}(l) = \max\{Ŕ_{K_1(§)}(l), Ŕ_{K_2(§)}(l)\}$.

Definition 2.6.[15] An SVNS-Set (N, S) over a fixed set Û is referred to as a null SVNS-Set if $Ý_{N(g)}(l)=0$, $Ī_{N(g)}(l)=1$, $Ŕ_{N(g)}(l)=1$, $\forall l \in Û$ with respect to the parameter $g \in S$. The null SVNS-Set may be denoted by $0_{(N, S)}$.

Definition 2.7.[15] An SVNS-S (N, S) over a fixed set Û is called an absolute SVNS-Set if $Ý_{N(g)}(l)=1$, $Ī_{N(g)}(l)=0$, $Ŕ_{N(g)}(l)=0$, $\forall l \in Û$ with respect to the parameter $g \in S$. The absolute SVNS-Set may be denoted by $1_{(N, S)}$. Clearly, $1^c_{(N, S)} = 0_{(N, S)}$ and $0^c_{(N, S)} = 1_{(N, S)}$.

Definition 2.8.[29] A SVPNS D over a fixed set Û is defined as follows:

$$D = \{(\eth, \acute{Y}_D(\eth), \acute{C}_D(\eth), \acute{Z}_D(\eth), \acute{U}_D(\eth), \acute{R}_D(\eth)) : \eth \in \hat{U}\},$$

where $\acute{Y}_D: \hat{U} \rightarrow [0, 1]$, $\acute{C}_D: \hat{U} \rightarrow [0, 1]$, $\acute{Z}_D: \hat{U} \rightarrow [0, 1]$, $\acute{U}_D: \hat{U} \rightarrow [0, 1]$, $\acute{R}_D: \hat{U} \rightarrow [0, 1]$ are respectively referred to as the truth membership function, contradiction membership function, ignorance membership function, unknown membership function and false membership function, such that

$$0 \leq \acute{Y}_D(\eth) + \acute{C}_D(\eth) + \acute{Z}_D(\eth) + \acute{U}_D(\eth) + \acute{R}_D(\eth) \leq 5, \text{ for all } \eth \in \hat{U}.$$

Definition 2.9.[29] Assume that $X = \{(\eth, \acute{Y}_X(\eth), \acute{C}_X(\eth), \acute{Z}_X(\eth), \acute{U}_X(\eth), \acute{R}_X(\eth)) : \eth \in \hat{U}\}$ and $Y = \{(\eth, \acute{Y}_Y(\eth), \acute{C}_Y(\eth), \acute{Z}_Y(\eth), \acute{U}_Y(\eth), \acute{R}_Y(\eth)) : \eth \in \hat{U}\}$ are two SVPNSs over Û. Then, the following results hold:

(*i*) $X \subseteq Y$ if and only if $\acute{Y}_X(\eth) \leq \acute{Y}_Y(\eth)$, $\acute{C}_X(\eth) \leq \acute{C}_Y(\eth)$, $\acute{Z}_X(\eth) \geq \acute{Z}_Y(\eth)$, $\acute{U}_X(\eth) \geq \acute{U}_Y(\eth)$, $\acute{R}_X(\eth) \geq \acute{R}_Y(\eth)$, for all $\eth \in \hat{U}$.

(*ii*) $X \cup Y = \{(\eth, \max\{\acute{Y}_X(\eth), \acute{Y}_Y(\eth)\}, \max\{\acute{C}_X(\eth), \acute{C}_Y(\eth)\}, \min\{\acute{Z}_X(\eth), \acute{Z}_Y(\eth)\}, \min\{\acute{U}_X(\eth), \acute{U}_Y(\eth)\}, \min\{\acute{R}_X(\eth), \acute{R}_Y(\eth)\}) : \eth \in \hat{U}\}$.

(*iii*) $X \cap Y = \{(\eth, \min\{\acute{Y}_X(\eth), \acute{Y}_Y(\eth)\}, \min\{\acute{C}_X(\eth), \acute{C}_Y(\eth)\}, \max\{\acute{Z}_X(\eth), \acute{Z}_Y(\eth)\}, \max\{\acute{U}_X(\eth), \acute{U}_Y(\eth)\}, \max\{\acute{R}_X(\eth), \acute{R}_Y(\eth)\}) : \eth \in \hat{U}\}$.

(*iv*) $X^c = \{(\eth, \acute{Y}_X(\eth), \acute{C}_X(\eth), 1-\acute{Z}_X(\eth), \acute{U}_X(\eth), \acute{R}_X(\eth)) : \eth \in \hat{U}\}$.

3. Single-Valued Pentapartitioned Neutrosophic Soft Set:

In this section, we procure the notion SVPNS-Sets and study some operations on them. Then, we formulate some results on SVPNS-Sets.

Definition 3.1. Let Û is a non-empty fixed set and Q be a collection of parameters. Suppose that, SVPN-Set(Û) denotes the set of all SVPN-Sets defined over Û. Then, for any $S \subseteq Q$, a pair (P_N, S) is referred to as an SVPNS-Set over Û, where P_N is a mapping from S to SVPN-Set(Û).

An SVPNS-Set (P_N, S) is defined as follows:

$$(P_N, S) = \{(h, \{(l, \acute{Y}_{P\_N(h)}(l), \acute{C}_{P\_N(h)}(l), \acute{Z}_{P\_N(h)}(l), \acute{U}_{P\_N(h)}(l), \acute{R}_{P\_N(h)}(l)) : l \in \hat{U}\}) : h \in Q, l \in \hat{U}\},$$

where $\acute{Y}_{P\_N(h)}(l)$, $\acute{C}_{P\_N(h)}(l)$, $\acute{Z}_{P\_N(h)}(l)$, $\acute{U}_{P\_N(h)}(l)$, and $\acute{R}_{P\_N(h)}(l)$ are the truth, contradiction, ignorance, unknown, and falsity membership values of each u with respect to the parameter $h \in Q$.

Example 3.1. Suppose that $\hat{U}=\{t_1, t_2, t_3, t_4\}$ is a fixed set consisting of four different colleges, and $Q=\{h_1$(grade), h_2(infrastructure), h_3(semester fee), h_4(placement), h_5(laboratory)$\}$ is a set of parameters corresponding to different colleges. Let $P\_N(h_1)=\{(t_1,0.9,0.6,0.4,0.2,0.2), (t_2,0.8,0.4,0.5, 0.3,0.3,0.4)\}$, $P\_N(h_2)=\{(t_1,0.61,0.35,0.22,0.4,0.21), (t_2,0.55,0.4,0.18,0.24,0.32)$, $P\_N(h_3)=\{(t_1,0.92,0.45, 0.56, 0.41,0.32), (t_2,0.75,0.3,0.2,0.41,0.55)$, $P\_N(h_4)=\{(t_1,0.82,0.5,0.4,0.6,0.2), (t_2,0.65,0.47,0.6,0.2,0.1)$, $P\_N(h_5)=\{(t_1,0.82,0.25,0.54,0.55,0.23), (t_2,0.77,0.6,0.57,0.8,0.9)$. Then, $(P_N, Q) = \{(h_1, P\_N(h_1)), (h_2, P\_N(h_2)), (h_3, P\_N(h_3)), (h_4, P\_N(h_4)), (h_5, P\_N(h_5))\}$ is an SVPNS-Set over Û with respect to the set Q.

Definition 3.2. The complement of an SVPNS-Set (P_N, Q) is denoted by $(P_N, Q)^c = (P_{N^c}, Q)$ and is defined by $(P_{N^c}, Q) = \{(h, \{(l, 1-\acute{Y}_{P\_N(h)}(l), 1-\acute{C}_{P\_N(h)}(l), 1-\acute{Z}_{P\_N(h)}(l), 1-\acute{U}_{P\_N(h)}(l), 1-\acute{R}_{P\_N(h)}(l)) : l \in \hat{U}\}) : h \in Q\}$.

Definition 3.3. Suppose that (S_1, Q) and (S_2, Q) are any two SVPNS-Sets over Û. Then, (S_1, Q) is referred to as a single-valued pentapartitioned neutrosophic soft sub-set of (S_2, Q) if and only if $Ý_{S_1(§)}(\ell) \leq Ý_{S_2(§)}(\ell)$, $Ć_{S_1(§)}(\ell) \geq Ć_{S_2(§)}(\ell)$, $Ź_{S_1(§)}(\ell) \geq Ź_{S_2(§)}(\ell)$, $Ú_{S_1(§)}(\ell) \geq Ú_{S_2(§)}(\ell)$, and $Ŕ_{S_1(§)}(\ell) \geq Ŕ_{S_2(§)}(\ell)$, $\forall$ §∈Q and ℓ∈Û. We write (S_1, Q) ⊆ (S_2, Q). Then, (S_2, Q) is referred to as a single-valued pentapartitioned neutrosophic soft super-set of (S_1, Q).

Definition 3.4. Suppose that (S_1, Q) and (S_2, Q) be any two SVPNS-Sets over a fixed set Û. Then, intersection of (S_1, Q) and (S_2, Q) is denoted by (S, Q), where $S = S_1 \cap S_2$ is defined as follows:

$$(S, Q)=\{(§, \{(k, Ý_{S(§)}(\wp*), Ć_{S(§)}(\wp*), Ź_{S(§)}(\wp*), Ú_{S(§)}(\wp*), Ŕ_{S(§)}(\wp*)): \wp*\in Û\}): §\in Q\},$$

where $Ý_{S(§)}(\wp*) = \min\{Ý_{S_1(§)}(\wp*), Ý_{S_2(§)}(\wp*)\}$, $Ć_{S(§)}(\wp*) = \max\{Ć_{S_1(§)}(\wp*), Ć_{S_2(§)}(\wp*)\}$, $Ź_{S(§)}(\wp*) = \max\{Ź_{S_1(§)}(\wp*), Ź_{S_2(§)}(\wp*)\}$, $Ú_{S(§)}(\wp*) = \max\{Ú_{S_1(§)}(\wp*), Ú_{S_2(§)}(\wp*)\}$ and $Ŕ_{S(§)}(\wp*) = \max\{Ŕ_{S_1(§)}(\wp*), Ŕ_{S_2(§)}(\wp*)\}$.

Definition 3.5. Suppose that (S_1, Q) and (S_2, Q) are any two SVPNS-Sets over a fixed set Û. Then, union of (S_1, Q) and (S_2, Q) is denoted by (S, Q), where $S = S_1 \cup S_2$ is defined as follows:

$$(S, Q) = \{(§, \{(\wp*, Ý_{S(§)}(\wp*), Ć_{S(§)}(\wp*), Ź_{S(§)}(\wp*), Ú_{S(§)}(\wp*), Ŕ_{S(§)}(\wp*)): \wp*\in Û\}): §\in Q\},$$

where $Ý_{S(§)}(\wp*) = \max\{Ý_{S_1(§)}(\wp*), Ý_{S_2(§)}(\wp*)\}$, $Ć_{S(§)}(\wp*) = \min\{Ć_{S_1(§)}(\wp*), Ć_{S_2(§)}(\wp*)\}$, $Ź_{S(§)}(\wp*) = \min\{Ź_{S_1(§)}(\wp*), Ź_{S_2(§)}(\wp*)\}$, $Ú_{S(§)}(\wp*) = \min\{Ú_{S_1(§)}(\wp*), Ú_{S_1(§)}(\wp*)\}$ and $Ŕ_{S(§)}(\wp*) = \min\{Ŕ_{S_1(§)}(\wp*), Ŕ_{S_1(§)}(\wp*)\}$.

Definition 3.6. An SVPNS-Set (S, Q) over a fixed set Û is called a null SVPNS-Set if and only if $Ý_{S(§)}(\wp*) = 0$, $Ć_{S(§)}(\wp*) = 1$, $Ź_{S(§)}(\wp*) = 1$, $Ú_{S(§)}(\wp*) = 1$ and $Ŕ_{S(§)}(\wp*) = 1$, $\forall \wp*\in$Û with respect to the parameter §∈Q. The null SVPNS-Set is denoted by $0_{(S, Q)}$.

Definition 3.7. An SVPNS-Set (S, Q) over a fixed set Û is referred to as an absolute SVPNS-Set if and only if $Ý_{S(§)}(\wp*) = 1$, $Ć_{S(§)}(\wp*) = 0$, $Ź_{S(§)}(\wp*) = 0$, $Ú_{S(§)}(\wp*) = 0$ and $Ŕ_{S(§)}(\wp*) = 0$, $\forall \wp*\in$Û with respect to the parameter § ∈Q. The absolute SVPNS-Set is denoted by $1_{(S, Q)}$.

Clearly, $1^c_{(S, Q)} = 0_{(S, Q)}$ and $0^c_{(S, Q)} = 1_{(S, Q)}$.

Theorem 3.1. Suppose that (K, A) and (L, A) are two SVPNS-Sets over the same universe Û. Then, the following results hold:

(*i*) (K, A) ∪ (K, A) = (K, A) & (K, A) ∩ (K, A) = (K, A);

(*ii*) (K, A) ∪ (L, A) = (L, A) ∪ (K, A) & (K, A) ∩ (L, A) = (L, A) ∩ (K, A);

(*iii*) (K, A) ∪ $0_{(K, A)}$ = (K, A) & (K, A) ∩ $0_{(K, A)}$ = $0_{(K, A)}$;

(*iv*) (K, A) ∪ $1_{(K, A)}$ = $1_{(K, A)}$ & (K, A) ∩ $1_{(K, A)}$ = (K, A);

(*v*) $[(K, A)^c]^c$ = (K, A).

Proof. (*i*) Assume that (K, A) = $\{(§, \{(\wp*, Ý_{K(§)}(\wp*), Ć_{K(§)}(\wp*), Ź_{K(§)}(\wp*), Ú_{K(§)}(\wp*), Ŕ_{K(§)}(\wp*)): \wp*\in Û\}): §\in C\}$ is an SVPNS-Set over a universe Û.

Now, (K, A) ∪ (K, A)

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), $Ý_{K(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗)}, min {$Ú_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗)}, min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= (K, A).

Therefore, (K, A) ∪ (K, A) = (K, A).

Now, (K, A) ∩ (K, A)

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), $Ý_{K(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗)}, max {$Ú_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗)}, max {$Ŕ_{K(Ω)}$(℘∗), $Ŕ_{K(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= (K, A).

Therefore, (K, A) ∩ (K, A) = (K, A).

(*ii*) Assume that (K, C)={(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} and (L, C)= {(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C} is any two SVPNS-Sets over the same universe Û.

Now, (K, A) ∪ (L, A)

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, max{ $Ý_{K(§)}$ (℘∗), $Ý_{L(§)}$ (℘∗)}, min{ $Ć_{K(§)}$ (℘∗), $Ć_{L(§)}$ (℘∗)}, min{ $Ź_{K(§)}$ (℘∗), $Ź_{L(§)}$ (℘∗)}, min{$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, min{$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, max{ $Ý_{L(§)}$ (℘∗), $Ý_{K(§)}$ (℘∗)}, min{ $Ć_{L(§)}$ (℘∗), $Ć_{K(§)}$ (℘∗)}, min{ $Ź_{L(§)}$ (℘∗), $Ź_{K(§)}$ (℘∗)}, min{$Ú_{L(§)}$(℘∗), $Ú_{K(§)}$(℘∗)}, min{$Ŕ_{L(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

= (L, A) ∪ (K, A).

Therefore, (K, A) ∪ (L, A) = (L, A) ∪ (K, A).

Further, we have

(K, A) ∩ (L, A)

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, min{ $Ý_{K(§)}$ (℘∗), $Ý_{L(§)}$ (℘∗)}, max{ $Ć_{K(§)}$ (℘∗), $Ć_{L(§)}$ (℘∗)}, max{ $Ź_{K(§)}$ (℘∗), $Ź_{L(§)}$ (℘∗)}, max{$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, max{$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, min{$Ý_{L(§)}$ (℘∗), $Ý_{K(§)}$ (℘∗)}, max{$Ć_{L(§)}$ (℘∗), $Ć_{K(§)}$ (℘∗)}, max{$Ź_{L(§)}$ (℘∗), $Ź_{K(§)}$ (℘∗)}, max{$Ú_{L(§)}$(℘∗), $Ú_{K(§)}$(℘∗)}, max{$Ŕ_{L(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}.

= (*L*, *A*) ∩ (*K*, *A*).

Therefore, (*K*, *A*) ∩ (*L*, *A*) = (*L*, *A*) ∩ (*K*, *A*).

(*iii*) Assume that (*K*, *A*) = {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} is an SVPNS-Set over a universe Û.

Now, (*K*, *A*) ∪ $0_{(K, A)}$

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, 0, 1, 1, 1, 1):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max{$Ý_{K(§)}$(℘∗), 0}, min{$Ć_{K(§)}$(℘∗), 1}, min{$Ź_{K(§)}$(℘∗), 1}, min{$Ú_{K(§)}$(℘∗), 1}, min{$Ŕ_{K(§)}$(℘∗), 1}):℘∗∈Û}): §∈*C*}.

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*}.

= (*K*, *A*).

Therefore, (*K*, *A*) ∪ $0_{(K, A)}$ = (*K*, *A*).

Further, we have

(*K*, *A*) ∩ $0_{(K, A)}$

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, 0, 1, 1, 1, 1):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, min{$Ý_{K(§)}$(℘∗), 0}, max{$Ć_{K(§)}$(℘∗), 1}, max{$Ź_{K(§)}$(℘∗), 1}, max{$Ú_{K(§)}$(℘∗), 1}, max{$Ŕ_{K(§)}$(℘∗), 1}):℘∗∈Û}): §∈*C*}.

= {(§, {(℘∗, 0, 1, 1, 1, 1):℘∗∈Û}): §∈*C*}.

= $0_{(K, A)}$.

Therefore, (*K*, *A*) ∩ $0_{(K, A)}$ = $0_{(K, A)}$.

(*iv*) Let (*K*, *A*) = {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} be an SVPNS-Set over a universe Û.

Now, (*K*, *A*) ∪ $1_{(K, A)}$

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, 1, 0, 0, 0, 0):℘∗∈Û}): §∈*C*}.

= {(§, {(℘∗, max{$Ý_{K(§)}$(℘∗), 1}, min{$Ć_{K(§)}$(℘∗), 0}, min{$Ź_{K(§)}$(℘∗), 0}, min{$Ú_{K(§)}$(℘∗), 0}, min{$Ŕ_{K(§)}$(℘∗), 0}):℘∗∈Û}): §∈*C*}.

= {(§, {(℘∗, 1, 0, 0, 0, 0):℘∗∈Û}): §∈*C*}.

= $1_{(K, A)}$.

Therefore, (*K*, *A*) ∪ $1_{(K, A)}$ = $1_{(K, A)}$.

Further, we have

$(K, A) \cap 1_{(K, A)}$

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, 1, 0, 0, 0, 0):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, min{$Ý_{K(§)}$(℘∗), 1}, max{$Ć_{K(§)}$(℘∗), 0}, max{$Ź_{K(§)}$(℘∗), 0}, max{$Ú_{K(§)}$(℘∗), 0}, max{$Ŕ_{K(§)}$(℘∗), 0}):℘∗∈Û}): §∈C}.

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

= (K, A).

Therefore, $(K, A) \cap 1_{(K, A)} = (K, A)$.

(*v*) Assume that (K, A) = {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} is an SVPNS-Set over a universe Û. Then, $(K, A)^c$ = {(§, {(℘∗, 1-$Ý_{K(§)}$(℘∗), 1-$Ć_{K(§)}$(℘∗), 1-$Ź_{K(§)}$(℘∗), 1-$Ú_{K(§)}$(℘∗), 1-$Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}.

Now, we have

$((K, A)^c)^c$

= {(§, {(℘∗, 1-(1-$Ý_{K(§)}$(℘∗)), 1-(1-$Ć_{K(§)}$(℘∗)), 1-(1-$Ź_{K(§)}$(℘∗)), 1-(1-$Ú_{K(§)}$(℘∗)), 1-(1-$Ŕ_{K(§)}$(℘∗))):℘∗∈Û}): §∈C}

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C}

= (K, C)

Therefore, $((K, A)^c)^c = (K, A)$.

Theorem 3.2. Suppose that (K, A), (L, A) and (M, A) are three SVPNS-Sets over the same universe Û. Then, the following results hold:

(*i*) $(K, A) \cup [(L, A) \cup (M, A)] = [(K, A) \cup (L, A)] \cup (M, A)$.

(*ii*) $(K, A) \cap [(L, A) \cap (M, A)] = [(K, A) \cap (L, A)] \cap (M, A)$.

(*iii*) $(K, A) \cup [(L, A) \cap (M, A)] = [(K, A) \cup (L, A)] \cap [(K, A) \cup (M, A)]$.

(*iv*) $(K, A) \cap [(L, A) \cup (M, A)] = [(K, A) \cap (L, A)] \cup [(K, A) \cap (M, A)]$.

Proof. (*i*) Suppose that (K, A), (L, A) and (M, A) are any three SVPNS-Sets over the same universe Û. Now, we have

$(K, A) \cup [(L, A) \cup (M, A)]$

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ [{(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, $Ý_{M(§)}$(℘∗), $Ć_{M(§)}$(℘∗), $Ź_{M(§)}$(℘∗), $Ú_{M(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)):℘∗∈Û}): §∈C}]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, max {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, min {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, min {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, min {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, min {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈C}

= {(§, {(℘∗, max {Ý$_{K(§)}$(℘∗), Ý$_{L(§)}$(℘∗), Ý$_{M(§)}$(℘∗)}, min {Ć$_{K(§)}$(℘∗), Ć$_{L(§)}$(℘∗), Ć$_{M(§)}$(℘∗)}, min {Ź$_{K(§)}$(℘∗), Ź$_{L(§)}$ (℘∗), Ź$_{M(§)}$ (℘∗)}, min {Ú$_{K(§)}$(℘∗), Ú$_{L(§)}$ (℘∗), Ú$_{M(§)}$ (℘∗)}, min {Ŕ$_{K(§)}$(℘∗), Ŕ$_{L(§)}$ (℘∗), Ŕ$_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max {Ý$_{K(§)}$(℘∗), Ý$_{L(§)}$(℘∗)}, min {Ć$_{K(§)}$(℘∗), Ć$_{L(§)}$(℘∗)}, min {Ź$_{K(§)}$(℘∗), Ź$_{L(§)}$(℘∗)}, min {Ú$_{K(§)}$(℘∗), Ú$_{L(§)}$(℘∗)}, min {Ŕ$_{K(§)}$(℘∗), Ŕ$_{L(§)}$(℘∗)}):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, Ý$_{M(§)}$(℘∗), Ć$_{M(§)}$(℘∗), Ź$_{M(§)}$(℘∗), Ú$_{M(§)}$(℘∗), Ŕ$_{M(§)}$(℘∗)):℘∗∈Û}): §∈*C*}

= [(*K*, *C*) ∪ (*L*, *C*)] ∪ (*M*, *C*)

Therefore, (*K*, *A*) ∪ [(*L*, *A*) ∪ (*M*, *A*)] = [(*K*, *A*) ∪ (*L*, *A*)] ∪ (*M*, *A*).

(*ii*) Assume that (*K*, *A*), (*L*, *A*) and (*M*, *A*) are any three SVPNS-Sets over the same universe Û.

Now, we have

(*K*, *A*) ∩ [(*L*, *A*) ∩ (*M*, *A*)]

= {(§, {(℘∗, Ý$_{K(§)}$(℘∗), Ć$_{K(§)}$(℘∗), Ź$_{K(§)}$(℘∗), Ú$_{K(§)}$(℘∗), Ŕ$_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∩ [{(§, {(℘∗, Ý$_{L(§)}$(℘∗), Ć$_{L(§)}$(℘∗), Ź$_{L(§)}$(℘∗), Ú$_{L(§)}$(℘∗), Ŕ$_{L(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, Ý$_{M(§)}$(℘∗), Ć$_{M(§)}$(℘∗), Ź$_{M(§)}$(℘∗), Ú$_{M(§)}$(℘∗), Ŕ$_{M(§)}$(℘∗)):℘∗∈Û}): §∈*C*}]

= {(§, {(℘∗, Ý$_{K(§)}$(℘∗), Ć$_{K(§)}$(℘∗), Ź$_{K(§)}$(℘∗), Ú$_{K(§)}$(℘∗), Ŕ$_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, min {Ý$_{L(§)}$(℘∗), Ý$_{M(§)}$(℘∗)}, max {Ć$_{L(§)}$(℘∗), Ć$_{M(§)}$(℘∗)}, max {Ź$_{L(§)}$(℘∗), Ź$_{M(§)}$(℘∗)}, max {Ú$_{L(§)}$(℘∗), Ú$_{M(§)}$(℘∗)}, max {Ŕ$_{L(§)}$(℘∗), Ŕ$_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, min {Ý$_{K(§)}$(℘∗), Ý$_{L(§)}$(℘∗), Ý$_{M(§)}$(℘∗)}, max {Ć$_{K(§)}$(℘∗), Ć$_{L(§)}$(℘∗), Ć$_{M(§)}$(℘∗)}, max {Ź$_{K(§)}$(℘∗), Ź$_{L(§)}$ (℘∗), Ź$_{M(§)}$ (℘∗)}, max {Ú$_{K(§)}$(℘∗), Ú$_{L(§)}$ (℘∗), Ú$_{M(§)}$ (℘∗)}, max {Ŕ$_{K(§)}$(℘∗), Ŕ$_{L(§)}$ (℘∗), Ŕ$_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, min {Ý$_{K(§)}$(℘∗), Ý$_{L(§)}$(℘∗)}, max {Ć$_{K(§)}$(℘∗), Ć$_{L(§)}$(℘∗)}, max {Ź$_{K(§)}$(℘∗), Ź$_{L(§)}$(℘∗)}, max {Ú$_{K(§)}$(℘∗), Ú$_{L(§)}$(℘∗)}, max {Ŕ$_{K(§)}$(℘∗), Ŕ$_{L(§)}$(℘∗)}):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, Ý$_{M(§)}$(℘∗), Ć$_{M(§)}$(℘∗), Ź$_{M(§)}$(℘∗), Ú$_{M(§)}$(℘∗), Ŕ$_{M(§)}$(℘∗)):℘∗∈Û}): §∈*C*}

= [(*K*, *A*) ∩ (*L*, *A*)] ∩ (*M*, *A*)

Therefore, (*K*, *A*) ∩ [(*L*, *A*) ∩ (*M*, *A*)] = [(*K*, *A*) ∩ (*L*, *A*)] ∩ (*M*, *A*).

(*iii*) Suppose that (*K*, *A*), (*L*, *A*) and (*M*, *A*) are any three SVPNS-Sets over the same universe Û.

Now, we have

(*K*, *A*) ∪ [(*L*, *A*) ∩ (*M*, *A*)]

= {(§, {(℘∗, Ý$_{K(§)}$(℘∗), Ć$_{K(§)}$(℘∗), Ź$_{K(§)}$(℘∗), Ú$_{K(§)}$(℘∗), Ŕ$_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ [{(§, {(℘∗, Ý$_{L(§)}$(℘∗), Ć$_{L(§)}$(℘∗), Ź$_{L(§)}$(℘∗), Ú$_{L(§)}$(℘∗), Ŕ$_{L(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, Ý$_{M(§)}$(℘∗), Ć$_{M(§)}$(℘∗), Ź$_{M(§)}$(℘∗), Ú$_{M(§)}$(℘∗), Ŕ$_{M(§)}$(℘∗)):℘∗∈Û}): §∈*C*}]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, min {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, max {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, max {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, max {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, max {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), min {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, min { $Ć_{K(§)}$(℘∗), max{$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, min { $Ź_{K(§)}$(℘∗), max {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, min {$Ú_{K(§)}$(℘∗), max {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, min {$Ŕ_{K(§)}$(℘∗), max {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}):℘∗∈Û}): §∈*C*}

Further, we have

[(*K*, *A*) ∪ (*L*, *A*)]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, min {$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}):℘∗∈Û}): §∈*C*},

and [(*K*, *A*) ∪ (*M*, *A*)]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈*C*} ∪ {(§, {(℘∗, $Ý_{M(§)}$(℘∗), $Ć_{M(§)}$(℘∗), $Ź_{M(§)}$(℘∗), $Ú_{M(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, min {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}.

Now, [(*K*, *A*) ∪ (*L*, *A*)] ∩ [(*K*, *A*) ∪ (*M*, *A*)]

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, min { $Ú_{K(§)}$ (℘∗), $Ú_{L(§)}$ (℘∗)}, min { $Ŕ_{K(§)}$ (℘∗), $Ŕ_{L(§)}$ (℘∗)}):℘∗∈Û}): §∈*C*} ∩ {(§, {(℘∗, max{ $Ý_{K(§)}$ (℘∗), $Ý_{M(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, min {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, min {max {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, max {$Ý_{K(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, max {min {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, min {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, max {min {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, min {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, max {min {$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, min {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, max {min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}, min {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}}):℘∗∈Û}): §∈*C*}

= {(§, {(℘∗, max {$Ý_{K(§)}$(℘∗), min {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, min { $Ć_{K(§)}$(℘∗), max {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, min {$Ź_{K(§)}$(℘∗), max {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, min {$Ú_{K(§)}$(℘∗), max {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, min {$Ŕ_{K(§)}$(℘∗), max {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}):℘∗∈Û}): §∈*C*}

= (*K*, *A*) ∪ [(*L*, *A*) ∩ (*M*, *A*)].

Therefore, (*K*, *A*) ∪ [(*L*, *A*) ∩ (*M*, *A*)] = [(*K*, *A*) ∪ (*L*, *A*)] ∩ [(*K*, *A*) ∪ (*M*, *A*)].

(*iv*) Assume that (*K*, *A*), (*L*, *A*) and (*M*, *A*) are any three SVPNS-Sets over the same universe Û.

Now, we have

(*K*, *A*) ∩ [(*L*, *A*) ∪ (*M*, *A*)]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗))℘∗∈Û}): §∈C} ∩ [{(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, $Ý_{M(§)}$(℘∗), $Ć_{M(§)}$(℘∗), $Ź_{M(§)}$(℘∗), $Ú_{M(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)):℘∗∈Û}): §∈C}]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, max {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, min {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, min {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, min {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, min {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈C}

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), max {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, max { $Ć_{K(§)}$(℘∗), min {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, max { $Ź_{K(§)}$(℘∗), min {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, max {$Ú_{K(§)}$(℘∗), min {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, max {$Ŕ_{K(§)}$(℘∗), min {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}):℘∗∈Û}): §∈C}

Further, we have

[(K, A) ∩ (L, A)]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, $Ý_{L(§)}$(℘∗), $Ć_{L(§)}$(℘∗), $Ź_{L(§)}$(℘∗), $Ú_{L(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)):℘∗∈Û}): §∈C}

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, max {$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, max {$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}):℘∗∈Û}): §∈C},

and [(K, A) ∩ (M, A)]

= {(§, {(℘∗, $Ý_{K(§)}$(℘∗), $Ć_{K(§)}$(℘∗), $Ź_{K(§)}$(℘∗), $Ú_{K(§)}$(℘∗), $Ŕ_{K(§)}$(℘∗)):℘∗∈Û}): §∈C} ∩ {(§, {(℘∗, $Ý_{M(§)}$(℘∗), $Ć_{M(§)}$(℘∗), $Ź_{M(§)}$(℘∗), $Ú_{M(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)):℘∗∈Û}): §∈C}

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, max {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, max {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈C}.

Now, [(K, A) ∩ (L, A)] ∪ [(K, A) ∩ (M, A)]

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, max { $Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, max { $Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}):℘∗∈Û}): §∈C} ∪ {(§, {(℘∗, min { $Ý_{K(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}, max {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}, max {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}):℘∗∈Û}): §∈C}

= {(§, {(℘∗, max {min {$Ý_{K(§)}$(℘∗), $Ý_{L(§)}$(℘∗)}, min {$Ý_{K(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, min {max {$Ć_{K(§)}$(℘∗), $Ć_{L(§)}$(℘∗)}, max {$Ć_{K(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, min {max {$Ź_{K(§)}$(℘∗), $Ź_{L(§)}$(℘∗)}, max {$Ź_{K(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, min {max {$Ú_{K(§)}$(℘∗), $Ú_{L(§)}$(℘∗)}, max {$Ú_{K(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, min {max {$Ŕ_{K(§)}$(℘∗), $Ŕ_{L(§)}$(℘∗)}, max {$Ŕ_{K(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}}):℘∗∈Û}): §∈C}

= {(§, {(℘∗, min {$Ý_{K(§)}$(℘∗), max {$Ý_{L(§)}$(℘∗), $Ý_{M(§)}$(℘∗)}}, max { $Ć_{K(§)}$(℘∗), min {$Ć_{L(§)}$(℘∗), $Ć_{M(§)}$(℘∗)}}, max { $Ź_{K(§)}$(℘∗), min {$Ź_{L(§)}$(℘∗), $Ź_{M(§)}$(℘∗)}}, max {$Ú_{K(§)}$(℘∗), min {$Ú_{L(§)}$(℘∗), $Ú_{M(§)}$(℘∗)}}, max {$Ŕ_{K(§)}$(℘∗), min {$Ŕ_{L(§)}$(℘∗), $Ŕ_{M(§)}$(℘∗)}}):℘∗∈Û}): §∈C}

= (K, C) ∩ [(L, C) ∪ (M, C)].

Therefore, (K, A) ∩ [(L, A) ∪ (M, A)] = [(K, A) ∩ (L, A)] ∪ [(K, A) ∩ (M, A)].

Definition 3.8. Assume that (K, Y) and (L, Z) are any two SVPNS-Sets over the same universe Û. Then, the operation 'AND' or 'Meet' is defined as follows:

$$(K, Y) \wedge (L, Z) = (P, Y\times Z),$$

where the corresponding truth, contradiction, ignorance, unknown and false membership values of (P, $Y\times Z$) are measured by $Ý_{P(u,v)}(\wp*) = \min\{Ý_{K(u)}(\wp*), Ý_{L(v)}(\wp*)\}$, $Ć_{P(u,v)}(\wp*) = \min\{Ć_{K(u)}(\wp*), Ć_{L(v)}(\wp*))$, $Ź_{P(u,v)}(\wp*) = \frac{(Ź_{K(u)}(\wp*)+Ź_{L(v)}(\wp*))}{2}$, $Ú_{P(u,v)}(\wp*) = \frac{(Ú_{K(u)}(\wp*)+ Ú_{L(v)}(\wp*))}{2}$, $Ŕ_{P(u,v)}(\wp*) = \max\{Ŕ_{K(u)}(\wp*), Ŕ_{L(v)}(\wp*)\}$, for all $u\in K$, $v\in L$, $\wp*\in$ Û.

Example 3.2. Assume that (K, E) and (L, F) are two SVPNS-Sets over the common universe Û.

The tabulated representation of SVPNS-Set (K, E) is given as follows:

Û	Grade	Infrastructures	Faculties
t_1	[0.88,0.45,0.66,0.2,0.1]	[0.73,0.2,0.88,0.44,0.64]	[0.89,0.82,0.45,0.25,0.75]
t_2	[0.85,0.27,0.58,0.26,0.72]	[0.65,0.14,0.52,0.36,0.52]	[0.78,0.98,0.65,0.43,0.64]
t_3	[0.76,0.44,0.77,0.65,0.25]	[0.41,0.25,0.69,0.57,0.74]	[0.98,0.65,0.55,0.35,0.39]
t_4	[0.82,0.65,0.14,0.86,0.37]	[0.66,0.65,0.14,.34,0.46]	[0.78,0.35,0.48,0.65,0.66]

The tabulated representation of SVPNS-Set (L, F) is given as follows:

Û	Semester Fee	Faculties	Students facilities
t_1	[0.85,0.5,0.47,0.65,0.2]	[0.88,0.54,0.47,0.58,0.22]	[0.46,0.48,0.35,0.14,0.36]
t_2	[0.77,0.52,0.19,0.77,0.25]	[0.65,0.42,0.75,0.65,0.69]	[0.78,0.65,0.87,0.69,0.49]
t_3	[0.73,0.2,0.88,0.44,0.64]	[0.95,0.25,0.85,0.45,0.19]	[0.98,0.36,0.97,0.54,0.63]
t_4	[0.69,0.68,0.61,0.25,0.96]	[0.58,0.45,0.64,0.85,0.47]	[0.87,0.69,0.55,0.45,0.47]

The tabulated representation of SVPNS-Set (K, E) $\wedge$ (L, F) is as follows:

Û	(Grade, Semester Fee)	(Grade, Faculties)	(Grade, Students facilities)
t_1	[0.85,0.45,0.565,0.425,0.2]	[0.88,0.45,0.565,0.39,0.22]	[0.46,0.45,0.505,0.17,0.36]
t_2	[0.77,0.27,0.385,0.515,0.72]	[0.65,0.42,0.76,0.65,0.72]	[0.78,0.27,0.725,0.475,0.72]
t_3	[0.73,0.2,0.825,0.545,0.64]	[0.76,0.25,0.81,0.55,0.25]	[0.76,0.47,0.87,0.595,0.63]
t_4	[0.69,0.65,0.375,0.555,0.96]	[0.58,0.45,0.39,0.855,0.47]	[0.82,0.65,0.345,0.655,0.47]

Û	(Infrastructures, Semester Fee)	(Infrastructures, Faculties)	(Infrastructures, Students facilities)

t_1	[0.73,0.2,0.675,0.545,0.64]	[0.73,0.2,0.675,0.51,0.64]	[0.46,0.2,0.615,0.29,0.64]
t_2	[0.65,0.14,0.355,0.565,0.52]	[0.65,0.14,0.635,0.505,0.69]	[0.65,0.14,0.695,0.525,0.52]
t_3	[0.41,0.2,0.785,0.505,0.74]	[0.41,0.25,0.77,0.51,0.74]	[0.41,0.25,0.83,0.555,0.74]
t_4	[0.66,0.65,0.375,0.295,0.96]	[0.58,0.45,0.39,0.595,0.47]	[0.66,0.65,0.345,0.395,0.47]

Û	(Faculties, Semester Fee)	(Faculties, Faculties)	(Faculties, Students facilities)
t_1	[0.85,0.5,0.46,0.45,0.75]	[0.88,0.54,0.46,0.415,0.75]	[0.46,0.48,0.4,0.195,0.75]
t_2	[0.77,0.52,0.42,0.6,0.64]	[0.65,0.42,0.7,0.54,0.69]	[0.78,0.65,0.76,0.56,0.64]
t_3	[0.73,0.2,0.715,0.395,0.64]	[0.95,0.25,0.7,0.4,0.39]	[0.98,0.36,0.76,0.445,0.63]
t_4	[0.69,0.35,0.545,0.45,0.96]	[0.58,0.35,0.56,0.75,0.66]	[0.78,0.35,0.515,0.55,0.66]

Definition 3.9. Let us consider two SVPNS-Sets (*K*, *Y*) and (*L*, *Z*) over the same universe Û. Then, the operation 'OR' or 'Join' operation is defined by

$$(K, Y) \vee (L, Z) = (§, Y\times Z),$$

where the corresponding truth membership value, contradiction membership value, ignorance membership value, unknown membership value and false membership value of (§, *Y*×*Z*) are measured by $Ý_{Q(u,v)}(℘*) = \max\{Ý_{K(u)}(℘*), Ý_{L(v)}(℘*)\}$, $Ć_{Q(u,v)}(℘*) = \max\{Ć_{K(u)}(℘*), Ć_{L(v)}(℘*)\}$, $Ź_{Q(u,v)}(℘*) = \frac{(Ź_{K(u)}(℘*)+ Ź_{L(v)}(℘*))}{2}$, $Ú_{Q(u,v)}(℘*) = \frac{(Ú_{K(u)}(℘*)+ Ú_{L(v)}(℘*))}{2}$, $Ŕ_{Q(u,v)}(℘*) = \min\{Ŕ_{K(u)}(℘*), Ŕ_{L(v)}(℘*)\}$, for all $u\in K$, $v\in L$, $℘*\in Û$.

Example 3.3. Assume that (*K*, *E*) and (*L*, *F*) are two SVPNS-Sets over the same universe Û as shown in Example 3.2. Then, the tabulated representation of 'OR' or 'Join' operation between (*K*, *E*) and (*L*, *F*) is as follows:

Û	(Grade, Semester Fee)	(Grade, Faculties)	(Grade, Students facilities)
t_1	[0.88,0.5,0.565,0.425,0.1]	[0.88,0.54,0.565,0.39,0.1]	[0.88,0.48,0.505,0.17,0.1]
t_2	[0.85,0.52,0.385,0.515,0.25]	[0.85,0.42,0.665,0.455,0.69]	[0.85,0.65,0.725,0.475,0.49]
t_3	[0.76,0.44,0.825,0.545,0.25]	[0.95,0.44,0.81,0.55,0.19]	[0.98,0.44,0.87,0.595,0.25]
t_4	[0.82,0.68,0.375,0.555,0.37]	[0.82,0.65,0.39,0.855,0.37]	[0.87,0.69,0.345,0.655,0.37]

Û	(Infrastructures, Semester Fee)	(Infrastructures, Faculties)	(Infrastructures, Students facilities)
t_1	[0.87,0.69,0.715,0.445,0.47]	[0.88,0.54,0.675,0.51,0.22]	[0.73,0.48,0.615,0.29,0.36]
t_2	[0.77,0.52,0.355,0.565,0.25]	[0.65,0.42,0.635,0.505,0.52]	[0.78,0.65,0.695,0.525,0.49]

t_3	[0.73,0.25,0.785,0.505,0.64]	[0.95,0.25,0.77,0.51,0.19]	[0.98,0.36,0.83,0.555,0.63]
t_4	[0.69,0.68,0.375,0.295,0.46]	[0.66,0.65,0.39,0.595,0.46]	[0.87,0.69,0.345,0.395,0.46]

Û	(Faculties, Semester Fee)	(Faculties, Faculties)	(Faculties, Students facilities)
t_1	[0.89,0.82,0.46,0.45,0.2]	[0.89,0.82,0.46,0.415,0.22]	[0.89,0.82,0.4,0.195,0.36]
t_2	[0.78,0.98,0.42,0.6,0.25]	[0.78,0.98,0.7,0.54,0.64]	[0.78,0.98,0.76,0.56,0.49]
t_3	[0.98,0.65,0.715,0.395,0.39]	[0.98,0.65,0.7,0.4,0.19]	[0.98,0.65,0.76,0.445,0.39]
t_4	[0.78,0.68,0.545,0.45,0.66]	[0.78,0.45,0.56,0.75,0.47]	[0.87,0.69,0.515,0.55,0.47]

Theorem 3.3. Let us consider two SVPNS-Sets (K, Y) and (L, Z) defined over the same universe Û. Then, the following results hold:

(*i*) $[(K, Y) \vee (L, Z)]^c = (K, Y)^c \wedge (L, Z)^c$;

(*ii*) $[(K, Y) \wedge (L, Z)]^c = (K, Y)^c \vee (L, Z)^c$.

Proof. (*i*) Assume that $(K, Y) = \{(y, \{(\wp*, Ý_{K(y)}(\wp*), Ć_{K(y)}(\wp*), Ź_{K(y)}(\wp*), Ú_{K(y)}(\wp*), Ŕ_{K(y)}(\wp*)): \wp* \in Û\}): y \in Y\}$ and $(L, Z) = \{(y, \{(\wp*, Ý_{L(y)}(\wp*), Ć_{L(y)}(\wp*), Ź_{L(y)}(\wp*), Ú_{L(y)}(\wp*), Ŕ_{L(y)}(\wp*)): \wp* \in Û\}): y \in Z\}$ are any two SVPNS-Sets over the same universe Û. Suppose that $(§, Y \times Z) = (K, Y) \vee (L, Z)$, where $Q(u, v) = \{(\wp*, \max\{Ý_{K(u)}(\wp*), Ý_{L(v)}(\wp*)), \max\{Ć_{K(u)}(\wp*), Ć_{L(v)}(\wp*)\}, \frac{(Ź_{K(u)}(\wp*)+ Ź_{L(v)}(\wp*))}{2}, \frac{(Ú_{K(u)}(\wp*)+ Ú_{L(v)}(\wp*))}{2}, \min\{Ŕ_{K(u)}(\wp*), Ŕ_{L(v)}(\wp*)\}) : \wp* \in Û, u \in K, v \in L\}$.

We have, $[(K, Y) \vee (L, Z)]^c$

$= \{(\wp*, \min\{1-Ý_{K(u)}(\wp*), 1-Ý_{L(v)}(\wp*)\}, \min\{1-Ć_{K(u)}(\wp*), 1-Ć_{L(v)}(\wp*)\}, \frac{(1-Ź_{K(u)}(\wp*))+(1-Ź_{L(v)}(\wp*))}{2}, \frac{(1-Ú_{K(u)}(\wp*))+(1- Ú_{L(v)}(\wp*))}{2}, \max\{1-Ŕ_{K(u)}(\wp*), 1-Ŕ_{L(v)}(\wp*)\}): \wp* \in Û, u \in K, v \in L\}$.

Now, $(K, Y)^c \wedge (L, Z)^c$

$= \{(\wp*, Ý_{K(y)}(\wp*), Ć_{K(y)}(\wp*), Ź_{K(y)}(\wp*), Ú_{K(y)}(\wp*), Ŕ_{K(y)}(\wp*)): \wp* \in Û, u \in K, v \in L)\}^c \wedge \{(\wp*, Ý_{L(y)}(\wp*), Ć_{L(y)}(\wp*), Ź_{L(y)}(\wp*), Ú_{L(y)}(\wp*), Ŕ_{L(y)}(\wp*)): \wp* \in Û, u \in K, v \in L\}^c$.

$= \{(\wp*, 1-Ý_{K(y)}(\wp*), 1-Ć_{K(y)}(\wp*), 1-Ź_{K(y)}(\wp*), 1-Ú_{K(y)}(\wp*), 1-Ŕ_{K(y)}(\wp*)): \wp* \in Û, u \in K, v \in L\} \wedge \{(\wp*, 1-Ý_{L(y)}(\wp*), 1-Ć_{L(y)}(\wp*), 1-Ź_{L(y)}(\wp*), 1-Ú_{L(y)}(\wp*), 1- Ŕ_{L(y)}(\wp*)): \wp* \in Û, u \in K, v \in L\}$.

$= \{(\wp*, \min\{1-Ý_{K(u)}(\wp*), 1-Ý_{L(v)}(\wp*)\}, \min\{1-Ć_{K(u)}(\wp*), 1-Ć_{L(v)}(\wp*)\}, \frac{(1-Ź_{K(u)}(\wp*))+(1- Ź_{L(v)}(\wp*))}{2}, \frac{(1-Ú_{K(u)}(\wp*))+(1- Ú_{L(v)}(\wp*))}{2}, \max\{1-Ŕ_{K(u)}(\wp*), 1-Ŕ_{L(v)}(\wp*)\}): \wp* \in Û, u \in K, v \in L\}$.

Therefore, $[(K, Y) \vee (L, Z)]^c = (K, Y)^c \wedge (L, Z)^c$.

(*ii*) Assume that $(K, Y) = \{(\wp*, \acute{Y}_{K(y)}(\wp*), \acute{C}_{K(y)}(\wp*), \acute{Z}_{K(y)}(\wp*), \acute{U}_{K(y)}(\wp*), \acute{R}_{K(y)}(\wp*)): \wp*\in\hat{U}\}$ and $(L, Z) = \{(\wp*, \acute{Y}_{L(y)}(\wp*), \acute{C}_{L(y)}(\wp*), \acute{Z}_{L(y)}(\wp*), \acute{U}_{L(y)}(\wp*), \acute{R}_{L(y)}(\wp*)): \wp*\in\hat{U}\}$ be any two SVPNS-Sets over the same universe $\hat{U}$. Suppose that $(P, Y\times Z) = [(K, Y) \wedge (L, Z)]$, where $P(u, v) = \{(\wp*, \min\{\acute{Y}_{K(u)}(\wp*), \acute{Y}_{L(v)}(\wp*)\}, \min\{\acute{C}_{K(u)}(\wp*), \acute{C}_{L(v)}(\wp*)\}, \frac{(\acute{Z}_{K(u)}(\wp*)+\acute{Z}_{L(v)}(\wp*))}{2}, \frac{(\acute{U}_{K(u)}(\wp*)+\acute{U}_{L(v)}(\wp*))}{2}, \max\{\acute{R}_{K(u)}(\wp*), \acute{R}_{L(v)}(\wp*)): \wp*\in\hat{U}, u\in K, v\in L\}$.

Now, $[(K, Y) \wedge (L, Z)]^c$

$= \{(\wp*, \min\{1-\acute{Y}_{K(u)}(\wp*), 1-\acute{Y}_{L(v)}(\wp*)\}, \min\{1-\acute{C}_{K(u)}(\wp*), 1-\acute{C}_{L(v)}(\wp*)\}, \frac{(1-\acute{Z}_{K(u)}(\wp*))+(1-\acute{Z}_{L(v)}(\wp*))}{2}, \frac{(1-\acute{U}_{K(u)}(\wp*))+(1-\acute{U}_{L(v)}(\wp*))}{2}, \max\{1-\acute{R}_{K(u)}(\wp*), 1-\acute{R}_{L(v)}(\wp*)\}): \wp*\in\hat{U}, u\in K, v\in L\}^c$.

$= \{(\wp*, \max\{1-\acute{Y}_{K(u)}(\wp*), 1-\acute{Y}_{L(v)}(\wp*)\}, \max\{1-\acute{C}_{K(u)}(\wp*), 1-\acute{C}_{L(v)}(\wp*)\}, \frac{(1-\acute{Z}_{K(u)}(\wp*))+(1-\acute{Z}_{L(v)}(\wp*))}{2}, \frac{(1-\acute{U}_{K(u)}(\wp*))+(1-\acute{U}_{L(v)}(\wp*))}{2}, \min\{1-\acute{R}_{K(u)}(\wp*), 1-\acute{R}_{L(v)}(\wp*)\}): \wp*\in\hat{U}, u\in K, v\in L\}$.

Now, $(K, Y)^c \vee (L, Z)^c$

$= \{(\wp*, \acute{Y}_{K(y)}(\wp*), \acute{C}_{K(y)}(\wp*), \acute{Z}_{K(y)}(\wp*), \acute{U}_{K(y)}(\wp*), \acute{R}_{K(y)}(\wp*)): \wp*\in\hat{U}, u\in K, v\in L\}^c \vee \{(\wp*, \acute{Y}_{L(y)}(\wp*), \acute{C}_{L(y)}(\wp*), \acute{Z}_{L(y)}(\wp*), \acute{U}_{L(y)}(\wp*), \acute{R}_{L(y)}(\wp*)): \wp*\in\hat{U}, u\in K, v\in L\}^c$.

$= \{(\wp*, 1-\acute{Y}_{K(y)}(\wp*), 1-\acute{C}_{K(y)}(\wp*), 1-\acute{Z}_{K(y)}(\wp*), 1-\acute{U}_{K(y)}(\wp*), 1-\acute{R}_{K(y)}(\wp*)): \wp*\in\hat{U}, u\in K, v\in L\} \vee \{(\wp*, 1-\acute{Y}_{L(y)}(\wp*), 1-\acute{C}_{L(y)}(\wp*), 1-\acute{Z}_{L(y)}(\wp*), 1-\acute{U}_{L(y)}(\wp*), 1-\acute{R}_{L(y)}(\wp*)): \wp*\in\hat{U}, u\in K, v\in L\}$.

$= \{(\wp*, \max\{1-\acute{Y}_{K(u)}(\wp*), 1-\acute{Y}_{L(v)}(\wp*)\}, \max\{1-\acute{C}_{K(u)}(\wp*), 1-\acute{C}_{L(v)}(\wp*)\}, \frac{(1-\acute{Z}_{K(u)}(\wp*))+(1-\acute{Z}_{L(v)}(\wp*))}{2}, \frac{(1-\acute{U}_{K(u)}(\wp*))+(1-\acute{U}_{L(v)}(\wp*))}{2}, \min\{1-\acute{R}_{K(u)}(\wp*), 1-\acute{R}_{L(v)}(\wp*)\}): \wp*\in\hat{U}, u\in K, v\in L\}$.

Therefore, $[(K, Y) \wedge (L, Z)]^c = (K, Y)^c \vee (L, Z)^c$.

4. Conclusions:

In this article, we have extended the notion of single-valued neutrosophic soft set, and grounded the idea of SVPNS-Set. By introducing the notion of SVPNS-Set, we have formulated some results on them. It is hoped that, researchers of different branches of science can done many new investigations based on these notion of SVPNS-Set.

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References:

1. Molodtsov, D. (1999). Soft set theory-First results. Computer & Mathematics with Applications, 37, 19-31.
2. Ali, M.I., Feng, F., Liu, X., Min, W.K., & Shabir, M. (2009). On some new operations in soft set theory. Computer & Mathematics with Applications, 57(9), 1547-1553.
3. Maji, P.K., Biswas, R., & Roy, A.R. (2002). An application of soft sets in a decision making problem. Computer & Mathematics with Applications, 44, 1077-1083.
4. Maji, P.K., Biswas, R., & Roy, A.R. (2003). Soft set theory. Computer & Mathematics with Applications, 45, 555-562.
5. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. Rehoboth, American Research Press.
6. Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353.
7. Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-96.
8. Deli, I., Ali, M., & Smarandache, F. (2015). Bipolar neutrosophic sets and their applications based on multicriteria decision making problems. International Conference on Advanced Mechatronic Systems(ICAMechS), Beijing, 2015, 249-254, doi: 10.1109/ICAMechS.2015.7287068.
9. Biswas, P., Pramanik, S., & Giri, B.C. (2014). Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2, 102-110.
10. Biswas, P, Pramanik, S. & Giri, B.C. (2014). A new methodology for neutrosophic multi-attribute decision-making with unknown weight information. Neutrosophic Sets and Systems, 3, 42-50.
11. Pramanik, S., Dalapati, S., Alam, S., Smarandache, S., & Roy, T.K. (2018). NS-cross entropy based MAGDM under single valued neutrosophic set environment. Information, 9(2), 37.
12. Maji, P.K. (2012). A neutrosophic soft set approach to a decision making problem. Annals of Fuzzy Mathematics and Informatics, 3(2), 313-319.
13. Maji, P.K. (2013). Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics, 5(1), 157-168.
14. Dey, P.P., Pramanik, S., & Giri, B.C. (2015). Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. Critical Review, 11, 41-55.
15. Dey, P.P., Pramanik, S., & Giri, B.C. (2016). Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. Journal of New Results in Science, 10, 25-37.
16. Dey, P.P., Pramanik, S., & Giri, B.C. (2016). Neutrosophic soft multi-attribute decision making based on grey relational projection method. Neutrosophic Sets and Systems, 11, 98-106.

17. Das, S., Kumar, S., Kar, S., & Pal, T. (2019). Group decision making using neutrosophic soft matrix: An algorithmic approach. Journal of King Saud University-Computer and Information Sciences, 31(4), 459-468.
18. Karaaslan, F. (2015). Neutrosophic soft sets with applications in decision making. International Journal of Information Science and Intelligent System, 4(2), 1-20.
19. Jha, S., Kumar, R., Son, L.H., Chatterjee, J.M., Khari, M., Yadav, N., & Smarandache, F. (2019). Neutrosophic soft set decision making for stock trending analysis. Evolving Systems, 10, 621-627.
20. Pramanik, S., & Dalapati, S. (2016). GRA based multi criteria decision making in generalized neutrosophic soft set environment. Global Journal of Engineering Science and Research Management, 3(5), 153-169.
21. Pramanik, S., Dey, P.P., & Giri, B.C. (2015). TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. Neutrosophic Sets and Systems, 10, 88-95.
22. Mukherjee, A., & Das, R. (2020). Neutrosophic bipolar vague soft set and its application to decision making problems. Neutrosophic Sets and Systems, 32, 410-424.
23. Bera, T., & Mahapatra, N.K. (2017). Introduction to neutrosophic soft topological space. OPSEARCH, 54, 841-867.
24. Bera, T., & Mahapatra, N.K. (2018). On neutrosophic soft topological space. Neutrosophic Sets and Systems, 19, 3-15.
25. Aras, Ç.G., Ozturk, T.Y., & Bayramov, S. (2019). Separation axioms on neutrosophic soft topological spaces. Turkish Journal of Mathematics, 43, 498-510.
26. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. Neutrosophic Sets and Systems, 38, 235-243.
27. Mehmood, A., Nadeem, F., Park, C., Gorge, N., Humaira, K., Khan, M.R., & Abbas, N. (2020). Neutrosophic soft α-open set in neutrosophic soft topological spaces. Journal of Algorithms and Computation, 52(1), 37-66.
28. Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set, Neutrosophic Sets and Systems, 22, 168-170.
29. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. Neutrosophic Sets and Systems, 36, 184-192.
30. Das, S., Shil, B., & Pramanik, S. (2021). SVPNS-MADM strategy based on GRA in SVPNS Environment. *Neutrosophic Sets and Systems*, 47, 50-65.
31. Das, S., Shil, B., & Tripathy, B.C. (2021). Tangent similarity measure based MADM-strategy under SVPNS-environment. Neutrosophic Sets and Systems, 43, 93-104.

32. Pramanik, S. (2023). SVPNN-ARAS strategy for MCGDM under pentapartitioned neutrosophic number environment. *Serbian Journal of Management*, *18*(2), 405-420. doi: 10.5937/sjm18-44545.
33. Majumder, P., Paul, A., & Pramanik, S. (2023). Single-valued pentapartitioned neutrosophic weighted hyperbolic tangent similarity measure to determine the most significant environmental risks during the COVID-19 pandemic. Neutrosophic Sets and Systems, 57, 57-75.
34. Das, S., Das, R., Granados, C., & Mukherjee, A. (2021). Pentapartitioned neutrosophic Q-Ideals of Q-algebra. Neutrosophic Sets and Systems, 41, 52-63.
35. Das, S., & Tripathy, B.C. (2021). Pentapartitioned neutrosophic topological space. *Neutrosophic Sets and Systems*, 45, 121-132.

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