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Single-Valued Quadripartitioned Neutrosophic *d*-Ideal of *d*-Algebra

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Abstract: The conception of single-valued quadripartitioned neutrosophic *d*-ideal (SVQN-*d*-I) of single-valued quadripartitioned neutrosophic *d*-algebra (SVQN-*d*-A) as an expansion of neutrosophic *d*-Ideal and neutrosophic *d*-Algebra has been attempted to be introduced in this article. Additionally, we identify various characteristics of them. Additionally, SVQN-*d*-I and SVQN-*d*-A examples have been provided.

Keywords: NS; SVNS; SVQNS; *d*-Algebra; *d*-Ideal; SVQN-*d*-I; SVQN-*d*-A.

1. Introduction: In the year 1996, Imai & Iseki [34] grounded the notion of BCI-Algebra as an extension of BCK-Algebra [35], and studied several properties of them. Neggers & Kim [41] later extended the framework of BCK-Algebra by incorporating the concept of d-Algebra (*d*-A). Neggers et al. [40] utilised the principle of ideal theory to *d*-A in 1999 and proposed the thought of *d*-Ideal (*d*-I) of *d*-A. Abdullah and Hasan [1] grounded the notion of semi *d*-I of *d*-A in 2013. In order to convey the membership of an expression in mathematics, Zadeh [48] originally suggested the term fuzzy set (FS) in 1965. Later, by extending the ideas of FS theory, Atanassov [3] devised the intuitionistic FS (IFS) concept. The concept of fuzzy *d*-I of *d*-A was first proposed by Jun et al. [37] in 2000. Subsequently, Jun et al. [36] presented the intuitionistic fuzzy *d*-A and utilized the concept of *d*-A on IFS. The idea of intuitionistic fuzzy *d*-I of *d*-A was developed by Hasan [29]. Hasan [30] further studied the concept of semi *d*-I of *d*-A in the context of IFS theory. Afterwards, Hasan and Saqban [33] defined the concept of doubt intuitionistic fuzzy semi *d*-I of *d*-A in 2020. Later, Hasan [31] also presented the idea of intuitionistic fuzzy *d*-Filter in 2020. In 2021, Hasan [32] developed the idea of direct product of intuitionistic fuzzy topological *d*-A (IF-*d*-A). Smarandache [45] introduced the concept of the neutrosophic set (NS) as a logical development of IFS theory. Later, as a modification of NS, Wang et al. [47] invented the idea of single-valued NS (SVNS) in 2010. Till now, many mathematicians around the globe gives their contribution [2, 5-9, 11-28, 38-39, 42-44, 46] in the area of NS and its extensions. Following that, in 2021, Das and Hasan [10] presented the idea of neutrosophic d-I of neutrosophic d-A. In 2016, Chatterjee et al. [4] improved upon the basic idea of NS and proposed the notion of a single-valued quadripartitioned neutrosophic set (SVQNS). Subsequently, single-valued quadripartitioned neutrosophic topological space has been investigated by Das et al. [6].

We obtain the concept of SVQN-d-I of SVQN-d-A in this article as a generalisation of neutrosophic d-I and neutrosophic d-A. Additionally, we identify various characteristics of them. Furthermore, we provide a few examples that demonstrate SVQN-d-I and SVQN-d-A.

Research gap: There hasn't been any research on SVQN-d-A or SVQN-d-I published in the most recent publications.

Motivation: In order to close the investigation gap, we describe the principles of SVQN-d-A and SVQN-d-I and provide some first findings.

The following sections make up the remaining portion of our paper:

The definitions and preliminary information on d-A, d-I, fuzzy d-A and fuzzy d-I are reviewed in section 2. The idea of SVQN-d-A and SVQN-d-I are introduced in section 3, along with some propositions, theorems and other information regarding SVQN-d-I of d-A. The conclusion of the work we have done for this article is covered in section 4.

2. Some Relevant Results:

We offer some current definitions and findings in this section that are highly helpful in developing the article's primary findings.

Let us consider a universal set Z, and 0 be a constant in it. Consider a binary operation ‘ * ’ on Z. Then, the pair (Z, *) is referred to as a d-A [41] if the following axiom holds:

(i) $y * y = 0$, for all $y \in Z$;

(ii) $0 * y = 0$, for all $y \in Z$;

(iii) $y * u = 0$ and $u * y = 0 \Rightarrow y = u$, for all $y, u \in Z$.

We will consider $y \leq u$ if and only if $y * u = 0$.

Let us consider a d-A (Z, *). Then, (Z, *) is referred [41] to as

(i) bounded d-A if $\exists$ an element $r \in Z$ such that $i * r = 0, \forall i \in Z$, i.e. $i \leq r, \forall i \in Z$.

(ii) commutative d-A iff $i * (i * r) = r * (r * i), \forall i, r \in Z$.

Let us consider a d-A (Z, *) with binary operator ‘ * ’. Then, S ($\subseteq Z$) is said to be a d-sub-A [41] of (Z, *) iff $\tilde{\eta}, \tilde{\alpha} \in S \Rightarrow \tilde{\eta} * \tilde{\alpha} \in S$.

Let ‘ * ’ be a binary operator on a d-A (W, *). Then, Z ($\subseteq W$) is referred to as a [41] d-I if the following holds:

(i) $\tilde{\alpha} * \tilde{\eta} \in Z, \tilde{\eta} \in Z \Rightarrow \tilde{\eta} \in Z$;

(ii) $\tilde{\alpha} \in Z, \tilde{\eta} \in W \Rightarrow \tilde{\alpha} * \tilde{\eta} \in Z$.

Assume that (W, *) be a d-A with binary operator ‘ * ’. Suppose that $Z = \{(i, T_Z(i)) : i \in W\}$ be a FS over W. Then, Z is referred to as a [37] fuzzy d-A (F-d-A) iff $T_Z(i * r) \geq \min\{T_Z(i), T_Z(r)\}, \forall i, r \in W$.

Suppose that $\tilde{\tilde{A}} = \{(\tilde{\tilde{o}}, T_{\tilde{\tilde{A}}}(\tilde{\tilde{o}})) : \tilde{\tilde{o}} \in W\}$ be a FS defined over a *d*-A *W* satisfying the following conditions:

(*i*) $T_{\tilde{\tilde{A}}}(\tilde{\tilde{o}}) \geq \min \{T_{\tilde{\tilde{A}}}(\tilde{\tilde{o}} * \tilde{\eta}), T_{\tilde{\tilde{A}}}(\tilde{\eta})\}$;

(*ii*) $T_{\tilde{\tilde{A}}}(\tilde{\tilde{o}} * \tilde{\eta}) \geq T_{\tilde{\tilde{A}}}(\tilde{\tilde{o}}), \forall \tilde{\tilde{o}}, \tilde{\eta} \in W$.

Then, $\tilde{\tilde{A}}$ is referred to as a fuzzy *d*-I [37] (F-*d*-I).

The notion of SVQNS was grounded by Chatterjee et al. [4] as follows:

An SVQNS *P* over an universal set *W* is defined as follows:

$\tilde{\tilde{A}} = \{(\tilde{\eta}, T_{\tilde{\tilde{A}}}(\tilde{\eta}), C_{\tilde{\tilde{A}}}(\tilde{\eta}), U_{\tilde{\tilde{A}}}(\tilde{\eta}), F_{\tilde{\tilde{A}}}(\tilde{\eta})) : \tilde{\eta} \in W\}$.

Here, $T_{\tilde{\tilde{A}}}(\tilde{\eta})$, $C_{\tilde{\tilde{A}}}(\tilde{\eta})$, $U_{\tilde{\tilde{A}}}(\tilde{\eta})$ and $F_{\tilde{\tilde{A}}}(\tilde{\eta})$ ($\in[0, 1]$) denotes the degree of truth, contradiction, unknown and falsity membership value of each $\tilde{\eta}\in W$ respectively. So, $0 \leq T_{\tilde{\tilde{A}}}(\tilde{\eta}) + C_{\tilde{\tilde{A}}}(\tilde{\eta}) + U_{\tilde{\tilde{A}}}(\tilde{\eta}) + F_{\tilde{\tilde{A}}}(\tilde{\eta}) \leq 4, \forall \tilde{\eta}\in W$.

Assume that $\tilde{\tilde{A}} = \{(\hat{a}, T_{\tilde{\tilde{A}}}(\hat{a}), C_{\tilde{\tilde{A}}}(\hat{a}), U_{\tilde{\tilde{A}}}(\hat{a}), F_{\tilde{\tilde{A}}}(\hat{a})) : \hat{a} \in W\}$ and $\ddot{E} = \{(\hat{a}, T_{\ddot{E}}(\hat{a}), C_{\ddot{E}}(\hat{a}), U_{\ddot{E}}(\hat{a}), F_{\ddot{E}}(\hat{a})) : \hat{a} \in W\}$ be two SVQNSs over a fixed set *W*. Then,

(*i*) $\tilde{\tilde{A}} \subseteq \ddot{E}$ iff $T_{\tilde{\tilde{A}}}(\hat{a}) \leq T_{\ddot{E}}(\hat{a})$, $C_{\tilde{\tilde{A}}}(\hat{a}) \leq C_{\ddot{E}}(\hat{a})$, $U_{\tilde{\tilde{A}}}(\hat{a}) \geq U_{\ddot{E}}(\hat{a})$, $F_{\tilde{\tilde{A}}}(\hat{a}) \geq F_{\ddot{E}}(\hat{a}), \forall \hat{a} \in W$.

(*ii*) $\tilde{\tilde{A}} \cap \ddot{E} = \{(\hat{a}, \min \{T_{\tilde{\tilde{A}}}(\hat{a}), T_{\ddot{E}}(\hat{a})\}, \min \{C_{\tilde{\tilde{A}}}(\hat{a}), C_{\ddot{E}}(\hat{a})\}, \max \{U_{\tilde{\tilde{A}}}(\hat{a}), U_{\ddot{E}}(\hat{a})\}, \max \{F_{\tilde{\tilde{A}}}(\hat{a}), F_{\ddot{E}}(\hat{a})\}) : \hat{a} \in W\}$.

(*iii*) $\tilde{\tilde{A}} \cup \ddot{E} = \{(\hat{a}, \max \{T_{\tilde{\tilde{A}}}(\hat{a}), T_{\ddot{E}}(\hat{a})\}, \max \{C_{\tilde{\tilde{A}}}(\hat{a}), C_{\ddot{E}}(\hat{a})\}, \min \{U_{\tilde{\tilde{A}}}(\hat{a}), U_{\ddot{E}}(\hat{a})\}, \min \{F_{\tilde{\tilde{A}}}(\hat{a}), F_{\ddot{E}}(\hat{a})\}) : \hat{a} \in W\}$.

(*iv*) $\tilde{\tilde{A}}^c = \{(\hat{a}, F_{\tilde{\tilde{A}}}(\hat{a}), U_{\tilde{\tilde{A}}}(\hat{a}), C_{\tilde{\tilde{A}}}(\hat{a}), T_{\tilde{\tilde{A}}}(\hat{a})) : \hat{a} \in W\}$ and $\ddot{E}^c = \{(\hat{a}, F_{\ddot{E}}(\hat{a}), U_{\ddot{E}}(\hat{a}), C_{\ddot{E}}(\hat{a}), T_{\ddot{E}}(\hat{a})) : \hat{a} \in W\}$.

3. Single-Valued Quadripartitioned Neutrosophic *d*-Ideal:

We define the concept of single-valued quadripartitioned neutrosophic *d*-I of *d*-A in this section and provide a number of intriguing results regarding it.

Definition 3.1. Assume that $\hat{O} = \{(i, T_{\hat{o}}(i), C_{\hat{o}}(i), U_{\hat{o}}(i), F_{\hat{o}}(i)): i\in Z\}$ be an SVQNS defined over a *d*-A *Z*, which satisfies the following conditions:

i $T_{\hat{o}}(i * r) \geq \min \{T_{\hat{o}}(i), T_{\hat{o}}(r)\}$, for all $i, r \in Z$;

ii $C_{\hat{o}}(i * r) \geq \min \{C_{\hat{o}}(i), C_{\hat{o}}(r)\}$, for all $i, r \in Z$;

iii $U_{\hat{o}}(i * r) \leq \max \{U_{\hat{o}}(i), U_{\hat{o}}(r)\}$, for all $i, r \in Z$;

iv $F_{\hat{o}}(i * r) \leq \max \{F_{\hat{o}}(i), F_{\hat{o}}(r)\}$, for all $i, r \in Z$.

Then, the SVQNS $\hat{O}$ is referred to as an SVQN-*d*-A of *d*-A *Z*.

Theorem 3.1. For any SVQN-*d*-A $\hat{O} = \{(\tilde{\eta}, T_{\hat{o}}(\tilde{\eta}), C_{\hat{o}}(\tilde{\eta}), U_{\hat{o}}(\tilde{\eta}), F_{\hat{o}}(\tilde{\eta})): \tilde{\eta}\in Z\}$ of a *d*-A (*Z*, *),

i $T_{\hat{o}}(0) \geq T_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

ii $C_{\hat{o}}(0) \geq C_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

iii $U_{\hat{o}}(0) \leq U_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

iv $F_{\hat{o}}(0) \leq F_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$.

Proof. Suppose that $\hat{O} = \{(\tilde{\eta}, T_{\hat{o}}(\tilde{\eta}), C_{\hat{o}}(\tilde{\eta}), U_{\hat{o}}(\tilde{\eta}), F_{\hat{o}}(\tilde{\eta})): \tilde{\eta}\in Z\}$ be an SVQN-*d*-A of a *d*-A (*Z*, *). Assume that $\tilde{\eta} \in Z$. Then, by Definition 2.1 and Definition 3.1, we have

i $T_{\hat{o}}(0) = T_{\hat{o}}(\tilde{\eta} * \tilde{\eta}) \geq \min \{T_{\hat{o}}(\tilde{\eta}), T_{\hat{o}}(\tilde{\eta})\} = T_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

ii $C_{\hat{o}}(0) = C_{\hat{o}}(\tilde{\eta} * \tilde{\eta}) \geq \min \{C_{\hat{o}}(\tilde{\eta}), C_{\hat{o}}(\tilde{\eta})\} = C_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

iii $U_{\hat{o}}(0) = U_{\hat{o}}(\tilde{\eta} * \tilde{\eta}) \leq \max \{U_{\hat{o}}(\tilde{\eta}), U_{\hat{o}}(\tilde{\eta})\} = U_{\hat{o}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$;

iv $F_{\hat{O}}(0) = F_{\hat{O}}(\tilde{\eta} * \tilde{\eta}) \le \max\{F_{\hat{O}}(\tilde{\eta}), F_{\hat{O}}(\tilde{\eta})\} = F_{\hat{O}}(\tilde{\eta}), \forall \tilde{\eta} \in Z$.

Theorem 3.2. Let $\{\Omega_k: k\in\Delta\}$ be a collection of SVQN-*d*-As of Z. Then, their intersection $\bigcap_{k\in\Delta}\Omega_k$ is also an SVQN-*d*-A of Z.

Proof. Suppose that $\{\Omega_k: k\in\Delta\}$ be a family of SVQN-*d*-As of Z. We have, $\bigcap_{k\in\Delta}\Omega_k = \{(\hat{e}, \wedge T_{\Omega_k}(\hat{e}), \wedge C_{\Omega_k}(\hat{e}), \vee U_{\Omega_k}(\hat{e}), \vee F_{\Omega_k}(\hat{e})) : \hat{e} \in Z\}$. Suppose that $\hat{e}, \hat{a} \in Z$. Then, we have

i $\wedge T_{\Omega_k}(\hat{e} * \hat{a}) \ge \wedge \min\{T_{\Omega_k}(\hat{e}), T_{\Omega_k}(\hat{a})\} = \min\{\wedge T_{\Omega_k}(\hat{e}), \wedge T_{\Omega_k}(\hat{a})\}$
$\Rightarrow \wedge T_{\Omega_k}(\hat{e} * \hat{a}) \ge \min\{\wedge T_{\Omega_k}(\hat{e}), \wedge T_{\Omega_k}(\hat{a})\}$;

ii $\wedge C_{\Omega_k}(\hat{e} * \hat{a}) \ge \wedge \min\{C_{\Omega_k}(\hat{e}), C_{\Omega_k}(\hat{a})\} = \min\{\wedge C_{\Omega_k}(\hat{e}), \wedge C_{\Omega_k}(\hat{a})\}$
$\Rightarrow \wedge C_{\Omega_k}(\hat{e} * \hat{a}) \ge \min\{\wedge C_{\Omega_k}(\hat{e}), \wedge C_{\Omega_k}(\hat{a})\}$;

iii $\vee U_{\Omega_k}(\hat{e} * \hat{a}) \le \vee \max\{U_{\Omega_k}(\hat{e}), U_{\Omega_k}(\hat{a})\} = \max\{\vee U_{\Omega_k}(\hat{e}), \wedge U_{\Omega_k}(\hat{a})\}$
$\Rightarrow \vee U_{\Omega_k}(\hat{e} * \hat{a}) \le \max\{\vee U_{\Omega_k}(\hat{e}), \wedge U_{\Omega_k}(\hat{a})\}$;

iv $\vee F_{\Omega_k}(\hat{e} * \hat{a}) \le \vee \max\{F_{\Omega_k}(\hat{e}), F_{\Omega_k}(\hat{a})\} = \max\{\vee F_{\Omega_k}(\hat{e}), \wedge F_{\Omega_k}(\hat{a})\}$
$\Rightarrow \vee F_{\Omega_k}(\hat{e} * \hat{a}) \le \max\{\vee F_{\Omega_k}(\hat{e}), \wedge F_{\Omega_k}(\hat{a})\}$;

Hence, $\bigcap_{k\in\Delta}\Omega_k$ is an SVQN-*d*-A of Z.

Theorem 3.3. Assume that $\hat{O} = \{(b, T_{\hat{O}}(b), C_{\hat{O}}(b), U_{\hat{O}}(b), F_{\hat{O}}(b)) : b\in Z\}$ be an SVQN-*d*-A of a *d*-A Z. Then, the sets $Z_T = \{b \in Z: T_{\hat{O}}(b)= T_{\hat{O}}(0)\}$, $Z_C = \{b \in Z: C_{\hat{O}}(b)= C_{\hat{O}}(0)\}$, $Z_U = \{b \in Z: U_{\hat{O}}(b)= U_{\hat{O}}(0)\}$ and $Z_F = \{b \in Z: F_{\hat{O}}(b)= F_{\hat{O}}(0)\}$ are *d*-Sub-As of Z.

Proof. Suppose that $\hat{O} = \{(b, T_{\hat{O}}(b), C_{\hat{O}}(b), U_{\hat{O}}(b), F_{\hat{O}}(b)): b\in Z\}$ be an SVQN-*d*-A of a *d*-A $(Z, *)$. Given $Z_T = \{b \in Z : T_{\hat{O}}(b) = T_{\hat{O}}(0)\}$, $Z_C = \{b \in Z : C_{\hat{O}}(b) = C_{\hat{O}}(0)\}$, $Z_U = \{b \in Z : U_{\hat{O}}(b) = U_{\hat{O}}(0)\}$, and $Z_F = \{b \in Z : F_{\hat{O}}(b) = F_{\hat{O}}(0)\}$.

Let $b, r \in Z_T$. Therefore, $T_{\hat{O}}(b)=T_{\hat{O}}(0)$, $T_{\hat{O}}(r)=T_{\hat{O}}(0)$. By Definition 3.1, $T_{\hat{O}}(b * r) \ge \min\{T_{\hat{O}}(b), T_{\hat{O}}(r)\} = \min\{T_{\hat{O}}(0), T_{\hat{O}}(0)\} = T_{\hat{O}}(0)$. This implies, $T_{\hat{O}}(b * r) \ge T_{\hat{O}}(0)$, for all $b, r \in Z_T$. Now, by Theorem 3.1, we have $T_{\hat{O}}(0) \ge T_{\hat{O}}(b * r)$. Therefore, $T_{\hat{O}}(b * r) = T_{\hat{O}}(0)$. This implies, $b * r \in Z_T$. Hence, $b, r\in Z_T \Rightarrow b * r \in Z_T$. Therefore, the set $Z_T = \{b \in Z : T_{\hat{O}}(b)=T_{\hat{O}}(0)\}$ is a *d*-Sub-A of Z.

Assume that $b, r \in Z_C$. Therefore, $C_{\hat{O}}(b)=C_{\hat{O}}(0)$, $C_{\hat{O}}(r)=C_{\hat{O}}(0)$. By using the Definition 3.1, we have $C_{\hat{O}}(b * r) \ge \min\{C_{\hat{O}}(b), C_{\hat{O}}(r)\} = \min\{C_{\hat{O}}(0), C_{\hat{O}}(0)\} = C_{\hat{O}}(0)$. This shows that, $C_{\hat{O}}(b * r) \ge C_{\hat{O}}(0)$. By Theorem 3.1, $C_{\hat{O}}(0) \ge C_{\hat{O}}(b * r)$. Therefore, $C_{\hat{O}}(b * r) = C_{\hat{O}}(0)$, which implies $b * r \in Z_C$. Hence, $b, r \in Z_C \Rightarrow b * r \in Z_C$. Therefore, the set $Z_C = \{b \in Z: C_{\hat{O}}(b)=C_{\hat{O}}(0)\}$ is a *d*-Sub-A of Z.

Let $b, r \in Z_U$. Therefore, $U_{\hat{O}}(b)=U_{\hat{O}}(0)$, $U_{\hat{O}}(r)=U_{\hat{O}}(0)$. By using the Definition 3.1, we have $U_{\hat{O}}(b * r) \le \max\{U_{\hat{O}}(b), U_{\hat{O}}(r)\} = \max\{U_{\hat{O}}(0), U_{\hat{O}}(0)\} = U_{\hat{O}}(0)$. Therefore, $U_{\hat{O}}(b * r) \le U_{\hat{O}}(0)$. By Theorem 3.1, $U_{\hat{O}}(0) \le U_{\hat{O}}(b * r)$. Hence, $U_{\hat{O}}(b * r) = U_{\hat{O}}(0)$. This shows that, $b * r \in Z_U$. Therefore, $b * r \in Z_U$ whenever $b, r \in Z_U$. Hence, the set $Z_U = \{b \in Z: U_{\hat{O}}(b)= U_{\hat{O}}(0)\}$ is a *d*-Sub-A of Z.

Let $b, r \in Z_F$. Therefore, $F_{\hat{O}}(b)=F_{\hat{O}}(0)$, $F_{\hat{O}}(r)=F_{\hat{O}}(0)$. Then, by using Definition 3.1, we have $F_{\hat{O}}(b * r) \le \max\{F_{\hat{O}}(b), F_{\hat{O}}(r)\} = \max\{F_{\hat{O}}(0), F_{\hat{O}}(0)\} = F_{\hat{O}}(0)$. Therefore, $F_{\hat{O}}(b * r) \le F_{\hat{O}}(0)$. By Theorem 3.1, $F_{\hat{O}}(0) \le F_{\hat{O}}(b * r)$. Hence, $F_{\hat{O}}(b * r)=F_{\hat{O}}(0)$. This shows that, $b * r \in Z_F$. Therefore, $b * r \in Z_F$ whenever $b, r \in Z_F$. Hence, the set $Z_F = \{b \in Z : F_{\hat{O}}(b)=F_{\hat{O}}(0)\}$ is a *d*-Sub-A of Z.

Definition 3.2. Suppose that $\hat{O} = \{(\hat{a}, T_{\hat{O}}(\hat{a}), C_{\hat{O}}(\hat{a}), U_{\hat{O}}(\hat{a}), F_{\hat{O}}(\hat{a})): \hat{a}\in Z\}$ be an SVQNS over a *d*-A Z. Then, the *T*-level α-cut, *C*-level α-cut, *U*-level α-cut, *F*-level α-cut of Ω are defined as follows:

i $Z(T_{\hat{O}}, \alpha) = \{\hat{a}\in Z: T_{\hat{O}}(\hat{a}) \ge \alpha\}$;

ii $Z(C_{\hat{O}}, \alpha) = \{\hat{a}\in Z: C_{\hat{O}}(\hat{a}) \ge \alpha\}$;

iii $Z(U_{\hat{O}}, \alpha) = \{\hat{a}\in Z: U_{\hat{O}}(\hat{a}) \le \alpha\}$;

iv $Z(F_{\hat{O}}, \alpha) = \{\hat{a}\in Z: F_{\hat{O}}(\hat{a}) \le \alpha\}$.

Theorem 3.4. If $\Omega = \{(\hat{e}, T_\Omega(\hat{e}), C_\Omega(\hat{e}), U_\Omega(\hat{e}), F_\Omega(\hat{e})): \hat{e}\in Z\}$ be an SVQN-*d*-A of an *d*-A *Z*, then, the *T*-level α-cut, *C*-level α-cut, *U*-level α-cut and *F*-level α-cut of Ω are *d*-sub-As of *Z*, for any $\alpha\in[0, 1]$.

Proof. Suppose that $\Omega = \{(\hat{e}, T_\Omega(\hat{e}), C_\Omega(\hat{e}), U_\Omega(\hat{e}), F_\Omega(\hat{e})): \hat{e}\in Z\}$ be an SVQN-*d*-A of a *d*-A *Z*. Then, the *T*-level α-cut of Ω is $Z(T_\Omega, \alpha) = \{\hat{e}\in Z: T_\Omega(\hat{e}) \ge \alpha\}$, *C*-level α-cut of Ω is $Z(C_\Omega, \alpha) = \{\hat{e}\in Z: C_\Omega(\hat{e}) \ge \alpha\}$, *U*-level α-cut of Ω is $Z(U_\Omega, \alpha) = \{\hat{e}\in Z: U_\Omega(\hat{e}) \le \alpha\}$ and *F*-level α-cut of Ω is $Z(F_\Omega, \alpha) = \{\hat{e}\in Z: F_\Omega(\hat{e}) \le \alpha\}$.

Let $\hat{e}, \tilde{\eta}\in Z(T_\Omega, \alpha)$. So, $T_\Omega(\hat{e}) \ge \alpha$, $T_\Omega(\tilde{\eta}) \ge \alpha$. Now, we have $T_\Omega(\hat{e} * \tilde{\eta}) \ge \min\{T_\Omega(q), T_\Omega(\tilde{\eta})\} \ge \min\{\alpha, \alpha\} \ge \alpha$. This implies, $\hat{e} * \tilde{\eta} \in Z(T_\Omega, \alpha)$. Therefore, $\hat{e} * \tilde{\eta} \in Z(T_\Omega, \alpha)$, whenever $\hat{e}, \tilde{\eta} \in Z(T_\Omega, \alpha)$. Hence, *T*-level α-cut of Ω i.e., $Z(T_\Omega, \alpha)$ is a *d*-Sub-A of *Z*.

Let $\hat{e}, \tilde{\eta}\in Z(C_\Omega, \alpha)$. So, $C_\Omega(\hat{e}) \ge \alpha$, $C_\Omega(\tilde{\eta}) \ge \alpha$. Now, we have $C_\Omega(\hat{e} * \tilde{\eta}) \ge \min\{C_\Omega(\hat{e}), C_\Omega(\tilde{\eta})\} \ge \min\{\alpha, \alpha\} \ge \alpha$. This implies, $\hat{e} * \tilde{\eta} \in Z(C_\Omega, \alpha)$. Therefore, $\hat{e} * \tilde{\eta} \in Z(C_\Omega, \alpha)$, whenever $\hat{e}, \tilde{\eta} \in Z(C_\Omega, \alpha)$. Hence, *C*-level α-cut of Ω i.e., $Z(C_\Omega, \alpha)$ is a *d*-Sub-A of *Z*.

Let $\hat{e}, \tilde{\eta}\in Z(U_\Omega,\alpha)$. So, $U_\Omega(\hat{e}) \le \alpha$, $U_\Omega(\tilde{\eta}) \le \alpha$. Now, we have $U_\Omega(\hat{e} * \tilde{\eta}) \le \max\{U_\Omega(\hat{e}), U_\Omega(\tilde{\eta})\} \le \max\{\alpha, \alpha\} \le \alpha$. This implies, $\hat{e} * \tilde{\eta} \in Z(U_\Omega, \alpha)$. Therefore, $\hat{e} * \tilde{\eta} \in Z(U_\Omega, \alpha)$, whenever $\hat{e}, \tilde{\eta} \in Z(U_\Omega, \alpha)$. Hence, *U*-level α-cut of Ω i.e., $Z(U_\Omega, \alpha)$ is a *d*-Sub-A of *Z*.

Let $\hat{e}, \tilde{\eta}\in Z(F_\Omega,\alpha)$. So, $F_\Omega(\hat{e}) \le \alpha$, $F_\Omega(\tilde{\eta}) \le \alpha$. Now, we have $F_\Omega(\hat{e} * \tilde{\eta}) \le \max\{F_\Omega(\hat{e}), F_\Omega(\tilde{\eta})\} \le \max\{\alpha, \alpha\} \le \alpha$. This implies, $\hat{e} * \tilde{\eta} \in Z(F_\Omega, \alpha)$. Therefore, $\hat{e} * \tilde{\eta} \in Z(F_\Omega, \alpha)$, whenever $\hat{e}, \tilde{\eta} \in Z(F_\Omega, \alpha)$. Hence, *F*-level α-cut of Ω i.e., $Z(F_\Omega, \alpha)$ is a *d*-Sub-A of *Z*.

Definition 3.3. Suppose that $\tilde{\hat{O}} = \{(\tilde{\eta}, T_{\check{O}}(\tilde{\eta}), C_{\check{O}}(\tilde{\eta}), U_{\check{O}}(\tilde{\eta}), F_{\check{O}}(\tilde{\eta})): \tilde{\eta}\in W\}$ be an SVQNS of a *d*-A *W*. Then, $\tilde{\hat{O}}$ is referred to as SVQN-*d*-I if the following conditions hold:

i $T_{\check{O}}(\tilde{\eta}) \ge \min\{T_{\check{O}}(\tilde{\eta} * \tilde{\alpha}), T_{\check{O}}(\tilde{\alpha})\}$ & $T_{\check{O}}(\tilde{\eta} * \tilde{\alpha}) \ge T_{\check{O}}(\tilde{\eta})$, $\forall\ \tilde{\eta}, \tilde{\alpha}\in\hat{O}$;

ii $C_{\check{O}}(\tilde{\eta}) \ge \min\{C_{\check{O}}(\tilde{\eta} * \tilde{\alpha}), C_{\check{O}}(\tilde{\alpha})\}$ & $C_{\check{O}}(\tilde{\eta} * \tilde{\alpha}) \ge C_{\check{O}}(\tilde{\eta})$, $\forall\ \tilde{\eta}, \tilde{\alpha}\in\hat{O}$;

iii $U_{\check{O}}(\tilde{\eta}) \le \max\{U_{\check{O}}(\tilde{\eta} * \tilde{\alpha}), U_{\check{O}}(\tilde{\alpha})\}$ & $U_{\check{O}}(\tilde{\eta} * \tilde{\alpha}) \ge U_{\check{O}}(\tilde{\eta})$, $\forall\ \tilde{\eta}, \tilde{\alpha}\in\hat{O}$;

iv $F_{\check{O}}(\tilde{\eta}) \le \max\{F_{\check{O}}(\tilde{\eta} * \tilde{\alpha}), F_{\check{O}}(\tilde{\alpha})\}$ & $F_{\check{O}}(\tilde{\eta} * \tilde{\alpha}) \ge F_{\check{O}}(\tilde{\eta})$, $\forall\ \tilde{\eta}, \tilde{\alpha}\in\hat{O}$.

Theorem 3.5. Let us consider a *d*-A (*W*, *) with a binary operator ‘ * ’. Suppose that $\tilde{\hat{O}} = \{(\hat{e}, T_{\check{O}}(\hat{e}), C_{\check{O}}(\hat{e}), I_{\check{O}}(\hat{e}), F_{\check{O}}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of *W*. Then, $T_{\check{O}}(0) \ge T_{\check{O}}(\hat{e})$, $C_{\check{O}}(0) \ge C_{\check{O}}(\hat{e})$, $U_{\check{O}}(0) \le U_{\check{O}}(\hat{e})$, $F_{\check{O}}(0) \le F_{\check{O}}(\hat{e})$, $\forall\hat{e} \in W$.

Proof. Let $\tilde{\hat{O}} = \{(\hat{e}, T_{\check{O}}(\hat{e}), C_{\check{O}}(\hat{e}), U_{\check{O}}(\hat{e}), F_{\check{O}}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of *W*. Now,

since $T_{\check{O}}(\hat{e} * \hat{e}) \ge T_{\check{O}}(\hat{e})$, so $T_{\check{O}}(0) \ge T_{\check{O}}(\hat{e})$;

since $C_{\check{O}}(\hat{e} * \hat{e}) \ge C_{\check{O}}(\hat{e})$, so $C_{\check{O}}(0) \ge C_{\check{O}}(\hat{e})$;

since $U_{\check{O}}(\hat{e} * \hat{e}) \le U_{\check{O}}(\hat{e})$, so $U_{\check{O}}(0) \le U_{\check{O}}(\hat{e})$;

since $F_{\check{O}}(\hat{e} * \hat{e}) \le F_{\check{O}}(\hat{e})$, so $F_{\check{O}}(0) \le F_{\check{O}}(\hat{e})$.

Theorem 3.6. Suppose that (*W*, *) be a *d*-A with a binary operator ‘ * ’. Assume that $\hat{O} = \{(\hat{e}, T_{\hat{O}}(\hat{e}), C_{\hat{O}}(\hat{e}), U_{\hat{O}}(\hat{e}), F_{\hat{O}}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of *W*. If $\hat{e} * \hat{a} \le s$, then $T_{\hat{O}}(\hat{e}) \ge \min\{T_{\hat{O}}(\hat{a}), T_{\hat{O}}(s)\}$, $C_{\hat{O}}(\hat{e}) \ge \min\{C_{\hat{O}}(\hat{a}), C_{\hat{O}}(s)\}$, $U_{\hat{O}}(\hat{e}) \le \max\{U_{\hat{O}}(\hat{a}), U_{\hat{O}}(s)\}$ and $F_{\hat{O}}(\hat{e}) \le \max\{F_{\hat{O}}(\hat{a}), F_{\hat{O}}(s)\}$.

Proof. Assume that $\hat{O} = \{(\hat{e}, T_{\hat{O}}(\hat{e}), C_{\hat{O}}(\hat{e}), U_{\hat{O}}(\hat{e}), F_{\hat{O}}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-Ideal of *W*. Let us consider three elements $\hat{e}, \hat{a}, \tilde{\eta}$ ($\in W$) such that $\hat{e} * \hat{a} \le \tilde{\eta}$. By Definition 2.1, we have $(\hat{e} * \hat{a}) * \tilde{\eta} = 0$.

Now, we have

i. $T_{\hat{O}}(\hat{e}) \ge \min\{T_{\hat{O}}(\hat{e} * \hat{a}), T_{\hat{O}}(\hat{a})\} \ge \min\{\min\{T_{\hat{O}}((\hat{e} * \hat{a}) * \tilde{\eta}), T_{\hat{O}}(\tilde{\eta})\}, T_{\hat{O}}(\hat{a})\} = \min\{\min\{T_{\hat{O}}(0), T_{\hat{O}}(\tilde{\eta})\}, T_{\hat{O}}(\hat{a})\} \ge \min\{T_{\hat{O}}(\tilde{\eta}), T_{\hat{O}}(\hat{a})\}$.

Therefore, $T\hat{o}(\hat{e}) \geq \min\{T\hat{o}(\hat{a}), T\hat{o}(\tilde{\eta})\}$.

ii. $C\hat{o}(\hat{e}) \geq \min\{C\hat{o}(\hat{e} * \hat{a}), C\hat{o}(\hat{a})\} \geq \min\{\min\{C\hat{o}((\hat{e} * \hat{a}) * \tilde{\eta}), C\hat{o}(\tilde{\eta})\}, C\hat{o}(\hat{a})\} = \min\{\min\{C\hat{o}(0), C\hat{o}(\tilde{\eta})\}, C\hat{o}(\hat{a})\} \geq \min\{C\hat{o}(\tilde{\eta}), C\hat{o}(\hat{a})\}$.

Therefore, $C\hat{o}(\hat{e}) \geq \min\{C\hat{o}(\hat{a}), C\hat{o}(\tilde{\eta})\}$.

iii. $U\hat{o}(\hat{e}) \leq \max\{U\hat{o}(\hat{e} * \hat{a}), U\hat{o}(\hat{a})\} \leq \max\{\max\{U\hat{o}((\hat{e} * \hat{a}) * \tilde{\eta}), U\hat{o}(\tilde{\eta})\}, U\hat{o}(\hat{a})\} = \max\{\max\{U\hat{o}(0), U\hat{o}(\tilde{\eta})\}, U\hat{o}(\hat{a})\} \leq \max\{U\hat{o}(\tilde{\eta}), U\hat{o}(\hat{a})\}$.

Therefore, $U\hat{o}(\hat{e}) \leq \max\{U\hat{o}(\hat{a}), U\hat{o}(\tilde{\eta})\}$.

iv. $F\hat{o}(\hat{e}) \leq \max\{F\hat{o}(\hat{e} * \hat{a}), F\hat{o}(\hat{a})\} \leq \max\{\max\{F\hat{o}((\hat{e} * \hat{a}) * \tilde{\eta}), F\hat{o}(\tilde{\eta})\}, F\hat{o}(\hat{a})\} = \max\{\max\{F\hat{o}(0), F\hat{o}(\tilde{\eta})\}, F\hat{o}(\hat{a})\} \leq \max\{F\hat{o}(\tilde{\eta}), F\hat{o}(\hat{a})\}$.

Therefore, $F\hat{o}(\hat{e}) \leq \max\{F\hat{o}(\hat{a}), F\hat{o}(\tilde{\eta})\}$.

Theorem 3.7. Let us consider an SVQN-*d*-I $\tilde{\check{O}} = \{(\hat{e}, T\check{o}(\hat{e}), C\check{o}(\hat{e}), U\check{o}(\hat{e}), F\check{o}(\hat{e})) : \hat{e} \in W\}$ of W, and let $\hat{e}, \hat{a} \in W$. If $\hat{e} \leq \hat{a}$, then $T\check{o}(\hat{e}) \geq T\check{o}(\hat{a})$, $C\check{o}(\hat{e}) \geq C\check{o}(\hat{a})$, $U\check{o}(\hat{e}) \leq U\check{o}(\hat{a})$ and $F\check{o}(\hat{e}) \leq F\check{o}(\hat{a})$.

Proof. Let $\tilde{\check{O}} = \{(\hat{e}, T\check{o}(\hat{e}), C\check{o}(\hat{e}), U\check{o}(\hat{e}), F\check{o}(\hat{e})) : \hat{e} \in W\}$ be an SVQN-*d*-I of W, and let $\hat{e}, \hat{a} \in W$ such that $\hat{e} \leq \hat{a}$. By Definition 2.1, we have $\hat{e} * \hat{a} = 0$.

Now, we have

$T\check{o}(\hat{e}) \geq \min\{T\check{o}(\hat{e} * \hat{a}), T\check{o}(\hat{a})\} = \min\{T\check{o}(0), T\check{o}(\hat{a})\}, T\check{o}(\hat{a})\} = T\check{o}(\hat{a})$.

$\Rightarrow T\check{o}(\hat{e}) \geq T\check{o}(\hat{a})$.

$C\check{o}(\hat{e}) \geq \min\{C\check{o}(\hat{e} * \hat{a}), C\check{o}(\hat{a})\} = \min\{C\check{o}(0), C\check{o}(\hat{a})\}, C\check{o}(\hat{a})\} = C\check{o}(\hat{a})$.

$\Rightarrow C\check{o}(\hat{e}) \geq C\check{o}(\hat{a})$.

$U\check{o}(\hat{e}) \leq \max\{U\check{o}(\hat{e} * \hat{a}), U\check{o}(\hat{a})\} = \max\{U\check{o}(0), U\check{o}(\hat{a})\}, U\check{o}(\hat{a})\} = U\check{o}(\hat{a})$.

$\Rightarrow U\check{o}(\hat{e}) \leq U\check{o}(\hat{a})$.

and $F\check{o}(\hat{e}) \leq \max\{F\check{o}(\hat{e} * \hat{a}), F\check{o}(\hat{a})\} = \max\{F\check{o}(0), F\check{o}(\hat{a})\}, F\check{o}(\hat{a})\} = F\check{o}(\hat{a})$.

$\Rightarrow F\check{o}(\hat{e}) \leq F\check{o}(\hat{a})$.

Theorem 3.8. If $\{\hat{O}_j : j \in \Delta\}$ be a collection of SVQN-*d*-Is of a *d*-A W, then $\bigcap_{j\in\Delta} \hat{O}_j$ is also an SVQN-*d*-I of W.

Proof. Suppose that $\{\hat{O}_j : j \in \Delta\}$ be a collection of SVQN-*d*-Is of a *d*-A W. Then, we have $\bigcap_{j\in\Delta} \hat{O}_j = \{(s, \wedge T_{\hat{O}_j}(s), \wedge C_{\hat{O}_j}(s), \vee U_{\hat{O}_j}(s), \vee F_{\hat{O}_j}(s)) : s \in W\}$.

Now, we have

$\wedge T_{\hat{O}_j}(s) \geq \wedge\{\min\{T_{\hat{O}_j}(s * r), T_{\hat{O}_j}(r)\}\} \geq \min\{\wedge T_{\hat{O}_j}(s * r), \wedge T_{\hat{O}_j}(r)\}$;

$\wedge C_{\hat{O}_j}(s) \geq \wedge\{\min\{C_{\hat{O}_j}(s * r), C_{\hat{O}_j}(r)\}\} \geq \min\{\wedge C_{\hat{O}_j}(s * r), \wedge C_{\hat{O}_j}(r)\}$;

$\vee U_{\hat{O}_j}(s) \leq \vee\{\max\{U_{\hat{O}_j}(s * r), U_{\hat{O}_j}(r)\}\} \leq \max\{\vee U_{\hat{O}_j}(s * r), \vee U_{\hat{O}_j}(r)\}$;

and $\vee F_{\hat{O}_j}(s) \leq \vee\{\max\{F_{\hat{O}_j}(s * r), F_{\hat{O}_j}(r)\}\} \leq \max\{\vee F_{\hat{O}_j}(s * r), \vee F_{\hat{O}_j}(r)\}$.

Since $T_{\hat{O}_j}(s * r) \geq T_{\hat{O}_j}(s)$, $C_{\hat{O}_j}(s * r) \geq C_{\hat{O}_j}(s)$, $U_{\hat{O}_j}(s * r) \leq U_{\hat{O}_j}(s)$, $F_{\hat{O}_j}(s * r) \leq F_{\hat{O}_j}(s)$, $\forall j \in \Delta$, so $\wedge T_{\hat{O}_j}(s * r) \geq \wedge T_{\hat{O}_j}(s)$, $\wedge C_{\hat{O}_j}(s * r) \geq \wedge C_{\hat{O}_j}(s)$, $\vee U_{\hat{O}_j}(s * r) \leq \vee U_{\hat{O}_j}(s)$, $\vee F_{\hat{O}_j}(s * r) \leq \vee F_{\hat{O}_j}(s)$. Hence, $\bigcap_{j\in\Delta} \hat{O}_j = \{(s, \wedge T_{\hat{O}_j}(s), \wedge C_{\hat{O}_j}(s), \vee U_{\hat{O}_j}(s), \vee F_{\hat{O}_j}(s)) : s \in W\}$ is an SVQN-*d*-I of W.

Theorem 3.9. Suppose that W be a *d*-A, and ' * ' be a binary operator defined on it. Suppose that $\tilde{\check{O}} = \{(\tilde{\alpha}, T\check{o}(\tilde{\alpha}), C\check{o}(\tilde{\alpha}), U\check{o}(\tilde{\alpha}), F\check{o}(\tilde{\alpha})) : \hat{e} \in W\}$ be an SVQN-*d*-I of W. Then, the FSs $\{(\hat{e}, T\check{o}(\hat{e})) : \hat{e} \in W\}$, $\{(\hat{e}, C\check{o}(\hat{e})) : \hat{e} \in W\}$, $\{(\hat{e}, 1\text{-}U\check{o}(\hat{e})) : \hat{e} \in W\}$ and $\{(\hat{e}, 1\text{-}F\check{o}(\hat{e})) : \hat{e} \in W\}$ are the F-*d*-Is of W.

Proof. Assume that $\tilde{\tilde{O}}$= $\{(\hat{e}, T\check{o}(\hat{e}), C\check{o}(\hat{e}), U\check{o}(\hat{e}), F\check{o}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of W. Therefore, $T\check{o}(\hat{e}) \geq \min\{T\check{o}(\hat{e} * \tilde{\eta}), T\check{o}(\tilde{\eta})\}$; $T\check{o}(\hat{e} * \tilde{\eta}) \geq T\check{o}(\hat{e})$; $C\check{o}(\hat{e}) \geq \min\{C\check{o}(\hat{e} * \tilde{\eta}), C\check{o}(\tilde{\eta})\}$; $C\check{o}(\hat{e} * \tilde{\eta}) \geq C\check{o}(\hat{e})$; $U\check{o}(\hat{e}) \leq \max\{U\check{o}(\hat{e} * \tilde{\eta}), U\check{o}(\tilde{\eta})\}$; $U\check{o}(\hat{e} * \tilde{\eta}) \leq U\check{o}(\hat{e})$; $F\check{o}(\hat{e}) \leq \max\{F\check{o}(\hat{e} * \tilde{\eta}), F\check{o}(\tilde{\eta})\}$; $F\check{o}(\hat{e} * \tilde{\eta}) \leq F\check{o}(\hat{e})$, $\forall \hat{e}, \tilde{\eta} \in W$.

Since $T\check{o}(\hat{e}) \geq \min\{T\check{o}(\hat{e} * \tilde{\eta}), T\check{o}(\tilde{\eta})\}$ and $T\check{o}(\hat{e} * \tilde{\eta}) \geq T\check{o}(\hat{e})$, $\forall \hat{e}, \tilde{\eta}\in W$, so $\{(\hat{e}, T\check{o}(\hat{e})) : \hat{e}\in W\}$ is a F-*d*-I of W.

Similarly, it is very easy to shown that, the FS $\{(\hat{e}, C\check{o}(\hat{e})) : \hat{e}\in W\}$ is also a F-*d*-I of W.

Further, since $U\check{o}(\hat{e}) \leq \max\{U\check{o}(\hat{e} * \tilde{\eta}), U\check{o}(\tilde{\eta})\}$ and $U\check{o}(\hat{e} * \tilde{\eta}) \leq U\check{o}(\hat{e})$, $\forall \hat{e}, \tilde{\eta}\in W$, so $1\text{-}U\check{o}(\hat{e}) \geq \min\{1\text{-}U\check{o}(\hat{e} * \tilde{\eta}), 1\text{-}U\check{o}(\tilde{\eta})\}$, $1\text{-}U\check{o}(\hat{e} * \tilde{\eta}) \geq 1\text{-}U\check{o}(\hat{e})$. Hence, the FS $\{(\hat{e}, 1\text{-}U\check{o}(\hat{e})) : \hat{e}\in W\}$ is a F-*d*-I of W.

Similarly, it can be easily shown that, the FS $\{(\hat{e}, 1\text{-}F\check{o}(\hat{e})) : \hat{e}\in W\}$ is also a F-*d*-I of W.

Theorem 3.10. If $\tilde{\tilde{O}}$ = $\{(\hat{e}, T\check{o}(\hat{e}), C\check{o}(\hat{e}), U\check{o}(\hat{e}), F\check{o}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of a *d*-A W, then the sets (i) $T\check{o}(W) = \{\hat{e}\in W : T\check{o}(\hat{e})=T\check{o}(0)\}$, (ii) $C\check{o}(W) = \{\hat{e}\in W : C\check{o}(\hat{e})=C\check{o}(0)\}$, (iii) $U\check{o}(W) = \{\hat{e}\in W : U\check{o}(\hat{e})=U\check{o}(0)\}$, and (iv) $F\check{o}(W) = \{\hat{e}\in W : F\check{o}(\hat{e})=F\check{o}(0)\}$ are *d*-Is of W.

Proof. Assume that $\tilde{\tilde{O}}$ = $\{(\hat{e}, T\check{o}(\hat{e}), C\check{o}(\hat{e}), U\check{o}(\hat{e}), F\check{o}(\hat{e})) : \hat{e}\in W\}$ be an SVQN-*d*-I of W.

(i) Suppose that $\hat{e} * \tilde{\alpha} \in T\check{o}(W)$ and $\tilde{\alpha} \in T\check{o}(W)$. So, $T\check{o}(\hat{e} * \tilde{\alpha})=T\check{o}(0)$, and $T\check{o}(\tilde{\alpha})=T\check{o}(0)$. Since $\tilde{\tilde{O}}$ is an SVQN-*d*-I of W, so we have $T\check{o}(\hat{e}) \geq \min\{T\check{o}(\hat{e} * \tilde{\alpha}), T\check{o}(\tilde{\alpha})\} = \min\{T\check{o}(0), T\check{o}(0)\} = T\check{o}(0)$, which implies $T\check{o}(\hat{e}) \geq T\check{o}(0)$. We have, $T\check{o}(0) \geq T\check{o}(\hat{e})$ by using Theorem 3.1. Hence, $T\check{o}(\hat{e})=T\check{o}(0)$. This implies, $\hat{e}\in T\check{o}(W)$. Therefore, $\hat{e} * \tilde{\alpha}\in T\check{o}(W)$ and $\tilde{\alpha} \in T\check{o}(W)$ implies $\hat{e}\in T\check{o}(W)$.

Again, let $\hat{e} \in T\check{o}(W)$ and $\tilde{\alpha} \in W$. Therefore, $T\check{o}(\hat{e})=T\check{o}(0)$. Since $\tilde{\tilde{O}}$ is an SVQN-*d*-I of W, so $T\check{o}(\hat{e} * \tilde{\alpha}) \geq T\check{o}(\hat{e}) = T\check{o}(0)$. Therefore, $T\check{o}(\hat{e} * \tilde{\alpha}) \geq T\check{o}(0)$. We have, $T\check{o}(0) \geq T\check{o}(\hat{e} * \tilde{\alpha})$ by using Theorem 3.1. This implies, $T\check{o}(\hat{e} * \tilde{\alpha}) = T\check{o}(0)$ i.e., $\hat{e} * \tilde{\alpha} \in T\check{o}(W)$. Hence, $\hat{e} * \tilde{\alpha} \in T\check{o}(W)$, whenever $\hat{e}\in T\check{o}(W)$ and $\tilde{\alpha}\in W$. Therefore, the set $T\check{o}(W) = \{\hat{e}\in W : T\check{o}(\hat{e})=T\check{o}(0)\}$ is a *d*-I of W.

Similarly, it can be easily verified that, the sets (ii) $C\check{o}(W) = \{\hat{e}\in W : C\check{o}(\hat{e})=C\check{o}(0)\}$, (iii) $U\check{o}(W) = \{\hat{e}\in W : U\check{o}(\hat{e})=U\check{o}(0)\}$ and (iv) $F\check{o}(W) = \{\hat{e}\in W : F\check{o}(\hat{e})=F\check{o}(0)\}$ are the *d*-Is of W.

4. Conclusions:

In this paper, the concepts of SVQN-*d*-I of *d*-A are introduced. Also, we have looked into a number of SVQN-*d*-Is of *d*-A properties and relations. Furthermore, a number of intriguing findings on SVQN-*d*-I of *d*-A have been developed into theorems, remarks, and corollaries. It is hoped that numerous new studies such as single-valued quadripartitioned neutrosophic semi-*d*-I, doubt single-valued quadripartitioned neutrosophic semi-*d*-I, and single-valued quadripartitioned neutrosophic semi-*d*-Filter of *d*-A can be conducted in the future using the concept of SVQN-*d*-I as a foundation.

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