# The refined indefinite neutrosophic integral

**Yaser Ahmad Alhasan[1,*], Mohammed Mustafa Ahmed[2], Raja Abdullah Abdulfatah[3] and Suliman Sheen[4]**

[1,*]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, KSA; y.alhasan@psau.edu.sa
[2]Department of mathematics, Faculty of science, Islamic University of Madinah, Madinah 42351, Saudi Arabia; mohamemedm086@gmail.com
[3]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.; r.abdulfatah@psau.edu.sa
[4]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.; S.almleh@psau.edu.sa

*Corresponding author: y.alhasan@psau.edu.sa

**Abstract**: This work presented the refined neutrosophic indefinite integral, where the substitution method for calculating integrals in the refined neutrosophic field that contain two part of indeterminacy $(I_1, I_2)$ was presented. We also proved a theorem through which we were able to find most of the integrals for the refined neutrosophic functions.

**Keywords:** refined neutrosophic indefinite integral; substitution; indeterminacy.

## 1. Introduction and Preliminaries

To describe a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction, Smarandache suggested the neutrosophic Logic as an alternative to the current logics. Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1 I_1, b_2 I_2, \dots, b_n I_n)$ where $a, b_1, b_2, \dots, b_n \in R$ or $C$ [1]

Agboola introduced the concept of refined neutrosophic algebraic structures [2]. Also, the refined neutrosophic rings $I$ was studied in paper [3], where it assumed that $I$ splits into two indeterminacies $I_1$ [contradiction (true (T) and false (F))] and $I_2$ [ignorance (true (T) or false (F))]. It then follows logically that: [3]

$$I_1 I_1 = {I_1}^2 = I_1 \qquad (1)$$

$$I_2 I_2 = {I_2}^2 = I_2 \qquad (2)$$

$$I_1 I_2 = I_2 I_1 = I_1 \qquad (3)$$

In addition, there are many papers presenting studies on refined neutrosophic numbers [4-5-6-7-8].
Smarandache discussed neutrosophic indefinite integral (Refined Indeterminacy) [11]
Let $g: \mathbb{R} \to \mathbb{R} \cup \{I_1\} \cup \{I_2\} \cup \{I_3\}$, where $I_1, I_2$, and $I_3$ are types of sub indeterminacies,

$$g(x) = 7x - 2I_1 + x^2 I_2 + 4x^3 I_3$$

then:

$$F(x) = \int [7x - 2I_1 + x^2 I_2 + 4x^3 I_3] dx$$

$$=\frac{7x^2}{2}-2xI_1+\frac{x^3}{3}I_2+x^4I_3+a+bI_1+cI_2+dI_3$$

where $a$ and $b$ are real constants.

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [9-10].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part. This is the reason for studying neutrosophic integration and methods of its integration in this paper.

This paper dealt with several topics, in the first part of which introduction and preliminaries were presented, and in the main discussion part the refined neutrosophic indefinite integral that contain two part of indeterminacy $(I_1, I_2)$. In the last part, a conclusion to the paper is given.

## 2. Main Discussion

### The refined neutrosophic indefinite integral

**Definition 1**

Let $f: R(I_1,I_2) \to R(I_1,I_2)$, to evaluate $\int f(x,I_1,I_2)dx$

put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

by substitution, we get:

$$\int f(x,I_1,I_2)dx = \int f(u)\acute{g}(u)du$$

then we can directly integral it.

**Theorem 1**

If $\int f(x,I_1,I_2)dx = \varphi(x,I_1,I_2)$, then:

$$\int f\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$$
$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)\varphi\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

where $C$ is an indeterminate real constant (i.e. constant of the form $a+bI_1+cI_2$, where $a,b,c$ are real numbers, while $I_1, I_2 =$ indeterminacy) and $\dot{a}_2 \neq 0$ , $\dot{a}_2 \neq -\dot{c}_2$ and $\dot{a}_2 \neq -\dot{b}_2-\dot{c}_2$

Proof:

put: $(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2 = u \quad \Rightarrow \quad (\dot{a}+\dot{b}I_1+\dot{c}I_2)dx = du$

$$\Rightarrow \quad dx = \frac{1}{\dot{a}+\dot{b}I_1+\dot{c}I_2}\ du$$

$$\Rightarrow dx = \left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)du$$

$$\int f\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx = \int f(u)\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)du$$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)\varphi(u)+C$$

back to the variable $x$, we get:

$$\int f\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$$
$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)\varphi\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

**Using the previous theorem, we get on:**

1) $\int \left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)^n dx$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)\frac{\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)^n}{n+1}+C$$

2) $\int \frac{1}{(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2}dx$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)ln\left|(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right|+C$$

3) $\int e^{(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2}dx$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)e^{(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2}+C$$

4) $\int \frac{1}{\sqrt{(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2}}dx$

$$=2\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)\sqrt{(\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2}+C$$

5) $\int cos\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)sin\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

6) $\int sin\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=-\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)cos\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

7) $\int sec^2\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=\left(\frac{1}{a}-\frac{b}{a(a+b)}I\right)tan\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

8) $\int csc^2\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=-\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)cot\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

9) $\int sec\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)\ tan\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)sec\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

10) $\int csc\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)\ cot\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)dx$

$$=-\left(\frac{1}{\dot{a}}+\left[\frac{-\dot{a}\dot{b}}{\dot{a}(\dot{a}+\dot{c})(\dot{a}+\dot{b}+\dot{c})}\right]I_1-\left[\frac{\dot{c}}{\dot{a}(\dot{a}+\dot{c})}\right]I_2\right)csc\left((\dot{a}+\dot{b}I_1+\dot{c}I_2)x+r+sI_1+tI_2\right)+C$$

Example 1

1) $$\int\left((2-4I_1+3I_2)x+7\right)^8dx=\left(\frac{1}{2}+\left[\frac{8}{(2)(5)(1)}\right]I_1-\left[\frac{3}{(2)(5)}\right]I_2\right)\frac{\left((2-4I_1+3I_2)x+7\right)^9}{9}+C$$

$$=\left(\frac{1}{2}+\frac{4}{5}I_1-\frac{3}{10}I_2\right)\frac{\left((2-4I_1+3I_2)x+7\right)^9}{9}+C$$

2) $\int\frac{1}{(7-5I_1+6I_2)x+2I_1+I_2}dx$

$$=\left(\frac{1}{7}+\left[\frac{35}{(7)(13)(8)}\right]I_1-\left[\frac{6}{(7)(13)}\right]I_2\right)ln|(7-5I_1+6I_2)x+2I_1+I_2|+C$$

$$=\left(\frac{1}{7}+\frac{5}{104}I_1-\frac{6}{91}I_2\right)ln|(7-5I_1+6I_2)x+2I_1+I_2|+C$$

3) $$\int e^{(1+I_2)x-6I_2}\,dx=\left(1+\left[\frac{0}{(1)(2)(2)}\right]I_1-\left[\frac{1}{(1)(2)}\right]I_2\right)e^{(1+I_2)x-6I_2}+C$$

$$=\left(1-\frac{1}{2}I_2\right)e^{(1+I_2)x-6I_2}+C$$

4) $\int cos\left((3+3I_1+9I_2)x+7I_1\right)dx$

$$=\left(\frac{1}{3}+\left[\frac{-9}{(3)(12)(15)}\right]I_1-\left[\frac{9}{(3)(12)}\right]I_2\right)sin\left((3+3I_1+9I_2)x+7I_1\right)+C$$

$$=\left(\frac{1}{3}-\frac{1}{6}I_1-\frac{1}{4}I_2\right)sin\left((3+3I_1+9I_2)x+7I_1\right)+C$$

5) $\int sec^2\big((-4+3I_1+8I_2)x-2I_2\big)\,dx$

$$=\left(\frac{-1}{4}+\left[\frac{12}{(-4)(4)(7)}\right]I_1-\left[\frac{8}{(-4)(4)}\right]I_2\right)tan\big((-4+3I_1+8I_2)x-2I_2\big)+C$$

$$=\left(\frac{-1}{4}-\frac{3}{28}I_1+\frac{1}{2}I_2\right)tan\big((-4+3I_1+8I_2)x-2I_2\big)+C$$

6) $\int csc\big((1-3I_1+I_2)x\big)\;\; cot\big((1-3I_1+I_2)x\big)\,dx$

$$=-\left(1+\left[\frac{3}{(1)(2)(-1)}\right]I_1-\left[\frac{1}{(1)(2)}\right]I_2\right)csc\big((1-3I_1+I_2)x\big)+C$$

$$=-\left(1-\frac{3}{2}I_1-\frac{1}{2}I_2\right)csc\big((1-3I_1+I_2)x\big)+C$$

$$=\left(-1+\frac{3}{2}I_1+\frac{1}{2}I_2\right)csc\big((1-3I_1+I_2)x\big)+C$$

7) $\int \frac{1}{\sqrt{(10-I_1+8I_2)x+9-4I_1}}\,dx$

$$=2\left(\frac{1}{10}+\left[\frac{10}{(10)(18)(17)}\right]I_1-\left[\frac{8}{(10)(18)}\right]I_2\right)\sqrt{(10-I_1+8I_2)x+9-4I_1}+C$$

$$=2\left(\frac{1}{10}+\frac{1}{306}I_1-\frac{2}{45}I_2\right)\sqrt{(10-I_1+8I_2)x+9-4I_1}+C$$

$$=\left(\frac{1}{5}+\frac{2}{306}I_1-\frac{4}{45}I_2\right)\sqrt{(10-I_1+8I_2)x+9-4I_1}+C$$

**Theorem 2**

Let $f: R(I_1, I_2) \to R(I_1, I_2)$, then:

$$\int \frac{\acute{f}(x, I_1, I_2)}{f(x, I_1, I_2)}\,dx = ln|f(x, I_1, I_2)| + C$$

Proof:

put: $f(x, I_1, I_2) = u \qquad \Rightarrow \acute{f}(x, I_1, I_2)dx = du$

$$\Rightarrow dx = \frac{1}{\acute{f}(x, I_1, I_2)}\,du$$

$$\Rightarrow dx = \frac{1}{\acute{u}}\,du$$

$$\int \frac{\acute{f}(x, I_1, I_2)}{f(x, I_1, I_2)}\,dx = \int \frac{\acute{u}}{u}\frac{1}{\acute{u}}\,du = \int \frac{1}{u}\,du = ln|u| + C$$

back to the $f(x, I_1, I_2)$, we get:

$$\int \frac{\acute{f}(x, I_1, I_2)}{f(x, I_1, I_2)}\,dx = ln|f(x, I_1, I_2)| + C$$

**Example 2**

1) $\int \frac{(1-2I_1+3I_2)x^3}{(3-6I_1+9I_2)x^4+4I_1+I_2}\,dx = \frac{1}{3}ln|(3-6I_1+9I_2)x^4+4I_1+I_2|+C$

2) $\int \frac{(4-I_1+I_2)e^{(4-I_1+I_2)x+5}}{e^{(4-I_1+I_2)x+5}-10I_2}dx = ln\left|e^{(4-I_1+I_2)x+5}-10I_2\right|+C$

3) $\int tan\big((1+I_1+I_2)x+7I_1\big)\,dx = \int \frac{sin\big((1+I_1+I_2)x+7I_1\big)}{cos\big((1+I_1+I_2)x+7I_1\big)}dx$

$$= -\left(1+\left[\frac{-1}{(1)(2)(3)}\right]I_1-\left[\frac{1}{(1)(2)}\right]I_2\right)ln\left|\cos\big((1+I_1+I_2)x+7I_1\big)\right|+C$$

$$= \left(-1+\frac{1}{6}I_1+\frac{1}{2}I_2\right)ln\left|cos\big((1+I_1+I_2)x+7I_1\big)\right|+C$$

4) $\int \frac{1}{1+tan(5-2I_1-2I_2)x}dx = \int \frac{1}{1+\frac{sin(5-2I_1-2I_2)x}{cos(5-2I_1-2I_2)x}}dx$

$$= \frac{1}{2}\int \frac{2\,cos(5-2I_1-2I_2)x}{cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x}dx$$

$$= \frac{1}{2}\int \frac{cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x+cos(5-2I_1-2I_2)x-sin(5-2I_1-2I_2)x}{cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x}dx$$

$$= \frac{1}{2}\int dx+\frac{1}{2}\int \frac{cos(5-2I_1-2I_2)x-sin(5-2I_1-2I_2)\,x}{cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x}dx$$

$$= \frac{1}{2}x+\frac{1}{2}\left(\frac{1}{5}+\left[\frac{10}{(5)(3)(1)}\right]I_1-\left[\frac{-2}{(5)(3)}\right]I_2\right)ln|cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x|+C$$

$$= \frac{1}{2}x+\left(\frac{1}{10}+\frac{1}{3}I_1+\frac{1}{15}I_2\right)ln|cos(5-2I_1-2I_2)x+sin(5-2I_1-2I_2)x|+C$$

**Theorem 3**

Let $f: R(I_1,I_2) \to R(I_1,I_2)$, then:

$$\int \frac{\acute{f}(x,I_1,I_2)}{\sqrt{f(x,I_1,I_2)}}dx = 2\sqrt{f(x,I_1,I_2)}+C$$

Proof:

put: $f(x,I_1,I_2) = u \qquad \Rightarrow \acute{f}(x,I_1,I_2)dx = du$

$$\Rightarrow dx = \frac{1}{\acute{f}(x,I_1,I_2)}\ du$$

$$\Rightarrow dx = \frac{1}{\acute{u}}\ du$$

$$\int \frac{\acute{f}(x,I_1,I_2)}{\sqrt{f(x,I_1,I_2)}}dx = \int \frac{\acute{u}}{\sqrt{u}}\frac{1}{\acute{u}}du = \int \frac{1}{\sqrt{u}}du = 2\sqrt{u}+C$$

back to $f(x,I_1,I_2)$, we get:

$$\int \frac{\acute{f}(x,I_1,I_2)}{\sqrt{f(x,I_1,I_2)}}dx = 2\sqrt{f(x,I_1,I_2)}+C$$

**Example 3**

1) $$\int \frac{-(1+2I_1+2I_2)x+5I_2}{\sqrt{(2+4I_1+4I_2)x^2-10I_2x}}dx = -\sqrt{(2+4I_1+4I_2)x^2-10I_2x}+C$$

2) $$\int \frac{(5-3I_1+8I_2)x^2}{\sqrt{(5-3I_1+8I_2)x^3-2I_1+7I_2}}dx = \frac{2}{3}\sqrt{(5-3I_1+8I_2)x^3-2I_1+7I_2}+C$$

**Theorem 4**

$f: R(I_1, I_2) \to R(I_1, I_2)$, then:

$$\int [f(x, I_1, I_2)]^n \acute{f}(x, I_1, I_2) \ dx = \frac{[f(x, I_1, I_2)]^{n+1}}{n+1}+C$$

Proof:

put: $f(x, I_1, I_2) = u \qquad \Rightarrow \acute{f}(x, I_1, I_2)dx = du$

$$\Rightarrow dx = \frac{1}{\acute{f}(x, I_1, I_2)} \ du$$

$$\Rightarrow dx = \frac{1}{\acute{u}} \ du$$

$$\int [f(x, I_1, I_2)]^n \acute{f}(x, I_1, I_2) \ dx = \int u^n \acute{u}\frac{1}{\acute{u}}du = \int u^n du = \frac{u^{n+1}}{n+1}+C$$

back to $f(x, I_1, I_2)$, we get:

$$\int [f(x, I_1, I_2)]^n \acute{f}(x, I_1, I_2) \ dx = \frac{[f(x, I_1, I_2)]^{n+1}}{n+1}+C$$

**Example 5**

1) $$\int x^2[(3+2I_1+2I_2)x^3]^{12}dx = \frac{1}{3}\int 3x^2[(3+2I_1+2I_2)x^3]^{12}dx$$

$$= \frac{1}{9+6I_1+6I_2}\frac{[(3+2I_1+2I_2)x^3]^{13}}{13}+C$$

$$= \left(\frac{1}{9}+\left[\frac{-54}{(9)(15)(21)}\right]I_1-\left[\frac{6}{(9)(15)}\right]I_2\right)\frac{[(3+2I_1+2I_2)x^3]^{13}}{13}+C$$

$$= \left(\frac{1}{9}-\frac{2}{105}I_1-\frac{2}{45}I_2\right)\frac{[(3+2I_1+2I_2)x^3]^{13}}{13}+C$$

2) $$\int \frac{1}{\sqrt{(2+I_1+I_2)x-I_1+2I_2}}\left(\sqrt{(2+I_1+I_2)x-I_1+2I_2}\right)^{14}dx$$

$$= 2\left(\frac{1}{2}-\left[\frac{1}{(3)(4)}\right]I_1-\left[\frac{1}{(2)(3)}\right]I_2\right)\frac{\left(\sqrt{(2+I_1+I_2)x-I_1+2I_2}\right)^{15}}{15}+C$$

$$= \left(1 - \frac{1}{6}I_1 - \frac{1}{3}I_2\right)\frac{\left(\sqrt{(2 + I_1 + I_2)x - I_1 + 2I_2}\right)^{15}}{15} + C$$

## 3. Conclusions

The integral is very important in our life, and is used especially for example in calculating areas whose shape is not familiar. This led us to study the refined neutrosophic indefinite integral that contain two parts of indeterminacy $(I_1, I_2)$. Where the method of integration by substitution are applied to the neutrosophic functions by presenting several theorems through which we were able to apply them to directly find the refined neutrosophic indefinite integral.

**Acknowledgments** "This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2024/R/1445)".

## References

[1] Smarandache, F., " (T,I,F)- Neutrosophic Structures", Neutrosophic Sets and Systems, vol 8, pp. 3-10, 2015.

[2] Agboola,A.A.A. "On Refined Neutrosophic Algebraic Structures", Neutrosophic Sets and Systems, vol 10, pp. 99-101, 2015.

[3] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[4] Smarandache, F.,"n-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, USA, vol 4, pp. 143-146, 2013.

[5] Vasantha Kandasamy,W.B; Smarandache,F. "Neutrosophic Rings" Hexis, Phoenix, Arizona, 2006, http://fs.gallup.unm.edu/NeutrosophicRings.pdf

[6] Zeina, M., Abobala, M., "On The Refined Neutrosophic Real Analysis Based on Refined Neutrosophic Algebraic AH-Isometry", Neutrosophic Sets and Systems, Volume 54, pp. 158-168, 2023.

[7] Agboola, A.A.A.; Akinola, A.D; Oyebola, O.Y., "Neutrosophic Rings I", Int. J. of Math. Comb., vol 4, pp.1-14, 2011.

[8] Celik, M., and Hatip, A., " On The Refined AH-Isometry And Its Applications In Refined Neutrosophic Surfaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[9] Alhasan,Y., "The neutrosophic integrals and integration methods", Neutrosophic Sets and Systems, Volume 43, pp. 290-301, 2021.

[10] Alhasan,Y., "The definite neutrosophic integrals and its applications", Neutrosophic Sets and Systems, Volume 49, pp. 277-293, 2022.

[11] Smarandache, F., "Neutrosophic Precalculus and Neutrosophic Calculus", book, 2015.

Received: Feb 10, 2024. Accepted: April 30, 2024