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An Example of Two-Fold Fuzzy Algebras Based On Neutrosophic Real Numbers

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Abstract: This paper is dedicated to defining and studying for the first time a two-fold algebra over the neutrosophic real number ring by merging the fuzzy set mapping with the algebraic operations of the neutrosophic real number ring.

Also, we study the elementary algebraic properties of the defined two-fold algebra through its algebraic operations and substructures such as homomorphisms and ideals.

Keywords: two-fold algebraic structure, neutrosophic ring, fuzzy set, two-fold fuzzy neutrosophic real number

Introduction

The concept of fuzzy algebraic structure is considered as a direct application of fuzzy sets and fuzzy mappings [1-2, 4, 6-8], where a fuzzy mapping with truth and falsity value is used to build many algebraic structures.

Also, the concept of neutrosophic set was used by many different authors to generalize classical algebraic structures by using logical conditions instead of algebraic elements [13], where we can see neutrosophic rings, neutrosophic matrices, and neutrosophic mappings [5, 9-12].

Recently, Smarandache in [14] has defined two-fold neutrosophic algebras as novel algebraic structures, and this new concept has been used in [16] to define two-fold fuzzy algebra by combining the standard fuzzy number theoretical system defined in [15], with the concept of two-fold algebraic structure, and many interesting theorems and examples were illustrated about this topic.

On the other hand, Hatip et.al [17], have combined real vector spaces, complex vector spaces, and algebraic modules with a fuzzy well-defined mapping to define and study two-fold fuzzy vector spaces and two-fold fuzzy modules, where they studied many elementary properties of these new structures.

This has prompted us to define and study for the first time a two-fold algebra over the neutrosophic real number ring by merging the fuzzy set mapping with the algebraic operations of the neutrosophic real number ring.

For more details about two-fold structures, and their properties, see [14, 16-17].

Main discussion

Definition:

Let $f: \mathbb{R} \to [0,1]$ with: $\begin{cases} f(0) = 0 \\ f(1) = 1 \end{cases}$, then f is called a fuzzy mapping.

We use this definition of fuzzy mappings, that is because the property $\begin{matrix} f(0) = 0 \\ f(1) = 1 \end{matrix}$ is very useful in algebraic structures and operations.

Example:

To understand the concept of fuzzy mapping, we will illustrate two different fuzzy mappings defined on the real field R.

Define: $f, g, h: \mathbb{R} \to [0,1]$ such that:

$$f(x)=\begin{cases} x^2 \;\; if -1<x\leq 1 \\ \frac{1}{|x|} \;\; if \;\; x>1 \;\; or \;\; x<-1, \\ 0.4 \;\; if \;\; x=-1 \end{cases} \quad g(x)=\begin{cases} |x^5| \;\; if -1<x\leq 1 \\ \frac{1}{|x^5|} \;\; if \;\; x>1 \;\; or \;\; x<-1 \\ 0.6 \;\; if \;\; x=-1 \end{cases}$$

We can see that f and g lie in the closed interval [0,1], with f$(0)=g(0)=0, f(1)=g(1)=1$.

Definition:

Let $\mathbb{R}(I)=\{x+yI \;\; ; x,y\in\mathbb{R} \;\; , I^2=I\}$ be the ring of neutrosophic real numbers , and $f:\mathbb{R}\to[0,1]$ with: $\begin{cases} f(0)=0 \\ f(1)=1 \end{cases}$ be a fuzzy mapping

We define $f_I:\mathbb{R}(I)\to[0,1] : f_I(x+yI)=\max(f(x),f(y))$

And $\mathbb{R}_{f_I}(I)=\left\{(x+yI)_{f_I(x+yI)} \;\; ; x,y\in\mathbb{R}\right\}$ is called the two- fold neutrosophic real numbers fuzzy algebra (NRNFA).

Definition:

The algebraic operations on $\mathbb{R}_{f_I}(I)$ are defined as:

*: $\mathbb{R}_{f_I}(I)\times\mathbb{R}_{f_I}(I)\to$ such that: $\mathbb{R}_{f_I}(I)$

○: $\mathbb{R}_{f_I}(I)\times\mathbb{R}_{f_I}(I)\to$ $\mathbb{R}_{f_I}(I)$

$(x+yI)_{f_I(x+yI)} * (z+tI)_{f_I(z+tI)} = [(x+z)+(y+t)I]_{f_I[(x+z)+(y+t)I]}$

$(x+yI)_{f_I(x+yI)} \circ (z+tI)_{f_I(z+tI)} = [\;\; x\cdot z+I(xt+yz+yt)]_{f_I[\;\; x\cdot z+I(xt+yz+yt)]}$

Theorem 1:

1] *,○ are commutative operations.

2] *,○ are associative operations.

3] * has $o_\circ$ as an identity, ○ has 1_1 as an identity.

4] ○ is distributive with respect to *.

5] Every $(x+yI)_{f_I(x+yI)}$ has an inverse with respect to (*).

6] $(x+yI)_{f_I(x+yI)}$ has an inverse with respect to (○) if and only if :$x\neq 0$. $x+y\neq 0$

Definition:

Let $P = A_0 + A_1 I$ be an ideal of ℝ(I), then we define the corresponding two-fold ideal as follows:

$$P_F = \{(x + yI)_{f_I(x+yI)} \in \mathbb{R}_{f_I}(I) \;;\; x + yI \in P\}$$

Definition:

Let $P = A_0 + A_1 I$ be an AH-ideal of ℝ(I), then we define the corresponding two-fold AH-ideal as follows:

$$P_F = \{(x + yI)_{f_I(x+yI)} \in \mathbb{R}_{f_I}(I) \;;\; x \in A_0 \;.\, y \in A_1\}$$

Theorem 2:

Let $P_F = (A_0 + A_1 I)_F$ be a two-fold ideals of $\mathbb{R}_{f_I}(I)$, then:

$$\begin{cases} B_{f_I(B)} * C_{f_I(C)} \in P_F \\ r_{f_I(r)} \circ B_{f_I(B)} \in P_F \end{cases} \qquad ; where \;\; B.C \in P_F \;\; .r \in \mathbb{R}(\mathrm{I})$$

Definition:

Let $\varphi: \mathbb{R}(\mathrm{I}) \to \mathbb{R}(\mathrm{I})$ be a ring homomorphism, we define:

$$\varphi_I\left(B_{f_I(B)}\right) = (\varphi(B))_{f_I(\varphi(B))} \qquad ; \;\; \varphi_I : \;\; \mathbb{R}_{f_I}(I) \to \mathbb{R}_{f_I}(I)$$

The mapping φ_I is called two-fold homomorphism.

If φ is an isomorphism, then φ_I is called two-fold isomorphism.

Theorem 3:

Let φ_I be two-fold homomorphism, then:

1] $\varphi_I(o_\circ) = o_\circ \;\; . \;\; \varphi_I(1_1) = 1_1$

2] $\varphi_I\left(B_{f_I(B)} * C_{f_I(C)}\right) = \varphi_I(B) * \varphi_I(C)$

3] $\varphi_I(B \circ \mathrm{C}) = \varphi_I(B) \circ \varphi_I(C)$

4] $\varphi_I(-B) = -\varphi_I(B)$

5] $\varphi_I\left(\frac{1}{B}\right) = \frac{1}{\varphi_I(B)} \qquad ; \quad B \;\; is \;\; invertible.$

6] $k_{er}(\varphi_I)$ is an ideal of $\mathbb{R}_{f_I}(I)$.

7] $I_m(\varphi_I)$ is a subring of $\mathbb{R}_{f_I}(I)$.

8] If P_F is an ideal of $\mathbb{R}(I)$, then $\varphi_I(P_F)$ is an ideal.

9] If P_F is an AH-ideal of $\mathbb{R}(I)$, then $\varphi_I(P_F)$ is an AH-ideal.

Definition:

Let φ_I: $\mathbb{R}_{f_I}(I) \to \mathbb{R}_{f_I}(I)$ $.\Psi_I$: $\mathbb{R}_{f_I}(I) \to \mathbb{R}_{f_I}(I)$, we define:

$\varphi_I \times \Psi_I$: $\mathbb{R}_{f_I}(I) \to \mathbb{R}_{f_I}(I)$ such that: $\varphi_I \times \Psi_I\left(B_{f_I(B)}\right) = \varphi_I(\Psi_I\left(B_{f_I(B)}\right))$

Theorem 4:

Let $\varphi_I.\Psi_I$: $\mathbb{R}_{f_I}(I) \to \mathbb{R}_{f_I}(I)$ be two-fold homomorphisms, then:

1] $\varphi_I \times \Psi_I$ is two-fold homomorphism.

2] if $\varphi_I.\Psi_I$ are two isomorphisms, then $\varphi_I \times \Psi_I$ is an isomorphism.

Definition:

Let P_F be an ideal of $\mathbb{R}_{f_I}(I)$, then:

$\mathbb{R}_{f_I}(I)/P_F = \left\{B_{f_I(B)} \circ P_F \quad ; B \in \mathbb{R}(I)\right\}$ is called the factor of P_F.

Theorem 5:

Let P_F be an ideal of $\mathbb{R}_{f_I}(I)$, then:

$(\mathbb{R}_I(I)/P_F\ ,\ *',\ \circ')$ is a ring, with:

$*': \mathbb{R}_I(I)/P_F \times \mathbb{R}_I(I)/P_F \to \mathbb{R}_I(I)/P_F$

$\circ': \mathbb{R}_I(I)/P_F \times \mathbb{R}_I(I)/P_F \to \mathbb{R}_I(I)/P_F$

$$\begin{cases} \left(B_{f_I(B)} \circ P_F\right) *' \left(C_{f_I(C)} \circ P_F\right) = (B * C) \circ P_F \\ \left(B_{f_I(B)} \circ P_F\right) \circ' \left(C_{f_I(C)} \circ P_F\right) = (B \circ C) \circ P_F \end{cases}$$

Theorem 6:

Let $\mathbb{R}_I(I)/P_F$ be the two- fold factor ring of P_F, then:

If S/P_F is an ideal of $\mathbb{R}_I(I)/P_F$, then S is an ideal of $\mathbb{R}_I(I)$ and contains (P).

Theorem 7:

Let φ_I be two-fold homomorphism, then:

$\mathbb{R}_{f_I}(I)/k_{er}(\ \varphi_I) \cong I_m(\ \varphi_I)$

Proof of theorem 1:

Let $B = b_0 + b_1 I\ .C = c_0 + c_1 I\ .D = d_0 + d_1 I \ \in \mathbb{R}(I)$, then:

1] $B_{f_I(B)} * C_{f_I(C)} = (B+C)_{f_I(B+c)} = (C+B)_{f_I(C+B)} = C_{f_I(C)} * \ B_{f_I(B)}$

$B_{f_I(B)} \circ C_{f_I(C)} = (B \cdot C)_{f_I(Bc)} = (CB)_{f_I(CB)} = C_{f_I(C)} \circ \ B_{f_I(B)}$

2] $B_{f_I(B)} * (C_{f_I(C)} * D_{f_I(D)}) = B_{f_I(B)} * (C+D)_{f_I(C+D)} = (B+C+D)_{f_I(B+C+D)} = (B+C)_{f_I(B+C)} * D_{f_I(D)} = (B_{f_I(B)} * C_{f_I(C)}) * D_{f_I(D)}$

$B_{f_I(B)} \circ (C_{f_I(C)} \circ D_{f_I(D)}) = B_{f_I(B)} \circ (CD)_{f_I(CD)} = (BCD)_{f_I(BCD)} = (BC)_{f_I(BC)} * D_{f_I(D)}$

$= (B_{f_I(B)} \circ C_{f_I(C)}) \circ D_{f_I(D)}$

3] $B_{f_I(B)} * o_\circ = (B + \circ)_{f_I(B+\circ)} = B_{f_I(B)}$

$B_{f_I(B)} \circ 1_1 = (B \cdot 1)_{f_I(B\cdot 1)} = B_{f_I(B)}$

4] $B_{f_I(B)} \circ (C_{f_I(C)} * D_{f_I(D)}) = B_{f_I(B)} \circ (C+D)_{f_I(C+D)} = (BC+BD)_{f_I(BC+BD)} = (BC)_{f_I(BC)} * (BD)_{f_I(BD)} = (B_{f_I(B)} \circ C_{f_I(C)}) * (B_{f_I(B)} \circ D_{f_I(D)})$

5] The inverse of $(x+yI)_{f_I(x+yI)}$ for (*) is: $(-x-yI)_{f_I(-x-yI)}$

6] The inverse of $(x+yI)_{f_I(x+yI)}$ for (○) is: $\left(\frac{1}{x} + I(\frac{1}{x+y} - \frac{1}{x})\right)_{f_I\left(\frac{1}{x}+I(\frac{1}{x+y}-\frac{1}{x})\right)}$

Proof of theorem (2):

Let $.C \ \in P \ .r \in \mathbb{R}(I)$, then:

$$\begin{cases} B+C \in P \\ r \cdot B \in P \end{cases}$$

So that: $\begin{cases} (B+C)_{f_I(B+C)} = B_{f_I(B)} * C_{f_I(C)} \in P_F \\ r_{f_I(r)} \circ B_{f_I(B)} = (r \cdot B)_{f_I(r\cdot B)} \ \in P_F \end{cases}$

Proof of theorem (3):

1]$\begin{cases} \varphi_I(o_\circ) = (\varphi(o))_{f_I(\varphi(o))} = o_\circ \\ \varphi_I(1_1) = (\varphi(1))_{f_I(\varphi(1))} = 1_1 \end{cases}$

2] $\varphi_I(B_{f_I(B)} * C_{f_I(C)}) = \ \varphi_I(B+C)_{f_I(B+c)} = (\varphi(B) + \varphi(C))_{f_I(\varphi(B)+\varphi(C))} = \varphi(B)_{f_I(\varphi(B))} * \varphi(C)_{f_I(\varphi(C))} = \ \varphi_I(\ B_{f_I(B)}) * \ \varphi_I(\ C_{f_I(C)})$

3] $\varphi_I(B_{f_I(B)} \circ C_{f_I(C)}) = \ \varphi_I(BC)_{f_I(Bc)} = (\varphi(B)\varphi(C))_{f_I(\varphi(B)\varphi(C))} = \varphi(B)_{f_I(\varphi(B))} \circ \varphi(C)_{f_I(\varphi(C))} = \varphi \ (\ B_{f_I(B)}) \circ \varphi(\ C_{f_I(C)})$

4] $\varphi_I((-B)_{f_I(-B)}) = \ (-\varphi(B))_{f_I(-\varphi(B))} = \ -\varphi(B)_{f_I(\varphi(B))} = -\varphi_I(B_{f_I(B)})$

5] It can be proved by the same.

6] $k_{er}(\ \varphi_I) = (k_{er}\ \ \varphi)_{f_I} = \ \{B_{f_I(B)}\ \ ;\ \ B \in k_{er}\ \ \varphi\}$, which is an ideal of $\mathbb{R}_{f_I}(I)$, that is because $(k_{er}\ \ \varphi)$ is an ideal of $\mathbb{R}(I)$.

7] $I_m(\ \varphi_I) = (I_m\ \ \varphi)_{f_I} = \ \{B_{f_I(B)}:\ B \in I_m\ \ \varphi\}$, which is a subring of $\mathbb{R}_{f_I}(I)$, that is because $(I_m\ \ \varphi)$ is a subring of $\mathbb{R}(I)$.

8] $\varphi_I(P_F) = (\varphi(P))_{f_I} = \ \left\{(\varphi(B))_{f_I(\varphi(B))}:\ B \in P\right\}$, and it is an ideal of $\mathbb{R}_{f_I}(I)$ because $\varphi(P)$ is an ideal.

9] It can be proved by the same.

Proof of theorem (4):

1] $\varphi_I \times \Psi_I\left(B_{f_I(B)}\right) = \varphi_I(\Psi(B))_{f_I(\Psi(B))} = (\varphi\Psi(\mathrm{B}))_{f_I(\varphi\Psi(\mathrm{B}))}$

Thus $\varphi_I \times \Psi_I\left(B_{f_I(B)} * C_{f_I(C)}\right) = (\varphi\Psi(B+C))_{f_I(\varphi\Psi(B+C))} = (\varphi\Psi(\mathrm{B}) + \varphi\Psi(\mathrm{C}))_{f_I(\varphi\Psi(\mathrm{B})+\varphi\Psi(\mathrm{C}))} = \ [\varphi_I * \Psi_I(B)] \circ [\varphi_I * \Psi_I(C)]$.

2] It holds directly from the definition.

Proof of theorem (5):

$$\left(B_{f_I(B)} \circ P_F\right) *' \left(C_{f_I(C)} \circ P_F\right) = (B * C) \circ P_F = (C * B) \circ P_F$$

$$= \left(C_{f_I(C)} \circ P_F\right) *' \left(B_{f_I(B)} \circ P_F\right)$$

$$\left(B_{f_I(B)} \circ P_F\right) \circ' \left(C_{f_I(C)} \circ P_F\right) = (B \circ C) \circ P_F = (C \circ \mathrm{B}) \circ P_F$$

$$= \left(C_{f_I(C)} \circ P_F\right) \circ' \left(B_{f_I(B)} \circ P_F\right)$$

$$\left(B_{f_I(B)} \circ P_F\right) *' \left[\left(C_{f_I(C)} \circ P_F\right) *' \left(D_{f_I(D)} \circ P_F\right)\right] = (B * C * D)_{f_I(B*C*D)} \circ P_F$$

$$= \left[\left(B_{f_I(B)} \circ P_F\right) *' \left(C_{f_I(C)} \circ P_F\right)\right] *' \left(D_{f_I(D)} \circ P_F\right)$$

$$\left(B_{f_I(B)} \circ P_F\right) \circ' \left[\left(C_{f_I(C)} \circ P_F\right) \circ' \left(D_{f_I(D)} \circ P_F\right)\right] = (B \circ C \circ D)_{f_I(BCD)} \circ P_F$$

$$= \left[\left(B_{f_I(B)} \circ P_F\right) \circ' \left(C_{f_I(C)} \circ P_F\right)\right] \circ' \left(D_{f_I(D)} \circ P_F\right)$$

$$\left(B_{f_I(B)} \circ P_F\right) *' \left(-B_{f_I(B)} \circ P_F\right) = o_\circ \circ P_F.$$

$\left(B_{f_I(B)} \circ P_F\right) \circ' \left[\left(C_{f_I(C)} \circ P_F\right) *' \left(D_{f_I(D)} \circ P_F\right)\right] = \left(B_{f_I(B)} \circ P_F\right) \circ' \left[(C * D)_{f_I(CD)} \circ P_F\right] =$

$\left[\left(B_{f_I(B)} \circ P_F\right) \circ' \left(C_{f_I(C)} \circ P_F\right)\right] *' \left[\left(B_{f_I(B)} \circ P_F\right) \circ' \left(D_{f_I(D)} \circ P_F\right)\right].$

$\left(B_{f_I(B)} \circ P_F\right) *' (o_\circ \circ P_F) = B_{f_I(B)} \circ P_F,$

$\left(B_{f_I(B)} \circ P_F\right) \circ' (1_1 \circ P_F) = B_{f_I(B)} \circ P_F.$

Thus $(\mathbb{R}_I(I)/P_F .*'.\circ')$ is a commutative ring with unity.

Proof of theorem (6):

Assume that S/P_F is an ideal of $\mathbb{R}_I(I)/P_F$, then:

$S \subseteq \mathbb{R}(I)$ is an ideal, with $P \subseteq S$.

Proof of theorem (7):

Since $\mathbb{R}(I)/k_{er}\ (\varphi) \cong I_m\ (\varphi)$, we can write:

$\mathbb{R}_{f_I}(I)/(k_{er}(\varphi))_{f_I} \cong (I_m(\varphi))_{f_I}$, thus:

$\mathbb{R}_{f_I}(I)/k_{er}(\ \varphi_I) \cong I_m(\ \varphi_I)$

Example:

Consider $f: \mathbb{R} \to [0.1]\ ;\begin{cases} f(0) = 0 \\ f(1) = 1 \\ f(x) = \frac{1}{|x|}\ ; |x| > 1 \\ f(x) = |x|\ ; 0 < |x| < 1 \end{cases}$

For $B = 3 + 2I \in \mathbb{R}(I)$. $B_{f_I(B)} = (3 + 2I)_{\frac{1}{2}}$.

For $C = 2 + 5I \in \mathbb{R}(I)$. $C_{f_I(C)} = (2 + 5I)_{\frac{1}{2}}$.

$B * C = (5 + 7I)_{\frac{1}{5}}$. $B \circ \mathrm{C} = (6 + 29I)_{\frac{1}{6}}$

$-B = (-3 - 2I)_{\frac{1}{2}}$. $-C = (-2 - 5I)_{\frac{1}{2}}$

Conclusion

In this paper, we have defined and study for the first time a two-fold algebra over neutrosophic real number ring by merging the fuzzy set mapping with the algebraic operations of the neutrosophic real number ring.

Also, we studied the elementary algebraic properties of the defined two-fold algebra through its algebraic operations and substructures such as homomorphisms and ideals.

In the future, we aim to generalize our study to other neutrosophic algebraic structures.

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