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Cubic Spherical Neutrosophic Sets and Selection of Electric Truck Using Cosine Similarity Measure

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Abstract. The concepts of cubic spherical neutrosophic sets (CSNSs), introduced and investigated by Gomathi et al. [5], offer a geometric representation of collection of neutrosophic sets (NSs), enhancing their ability to capture uncertainty. The formulation characterizes information using points on a sphere with a defined center and radius, providing a more precise depiction of fuzziness inherent in uncertain data. The cubic spherical neutrosophic Archimedean triangular norms(ATN) and conorms (ATCN), expanding the model's capabilities to handle uncertainty. These algebraic operators enable the aggregation and combination of uncertain information, offering a more comprehensive approach to decision-making. The research further presents a method for solving multiple-criteria decision-making problems within the cubic spherical neutrosophic context, leveraging the newly integrated norms and conorms. The algorithm utilizes the cosine similarity measure of cubic spherical neutrosophic sets, exemplified through an application involving the selection of the most effective electric truck. This extended framework provides decision-makers with enhanced tools to navigate complex decision landscapes amidst uncertainty, facilitating more informed and robust choices across diverse domains.

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1. Introduction

In 1998, Smarandache [27] early proposed NSs, which are a generalization of fuzzy sets and intuitionistic fuzzy sets. The membership, indeterminacy and non-membership mappings used to describe NSs are such that the sum of their non-negative mapping values is less than three. NSs are frequently utilized in several fields, resulting in decision-making, clustering algorithms, distance measurement, entropy measurement, pattern recognition and medical diagnostics. As a generalization of NSs, numerous sets are Proposed including single-valued NSs [29], interval-valued NSs [28], neutrosophic hesitant FSs [33], bipolar NSs [3], spherical NSs [16], simplified NSs [35], multi-valued NSs [16], and probability multi-valued NSs [21]. In 1954, Menger [15] proposed the first description of TNs, and then Schweizer and Sklar [24,25] revised them to become those that are currently in use. Several investigations [2, 7, 11–13, 36] study the properties of above said norms including continuous, nilpotent, Archimedean, stringent and others.

The area of operations research that is categorized as $MCDM$ focuses on the explicit evaluation of multiple conflicting criteria while making decisions (in daily life and situations like businesses, governments and medicals). Contradictory standards are frequently present while examining possibilities. The cost of the truck is typically among the main criteria and the quality measure is commonly another, simply compared to the cost. When purchasing a truck, we may prioritize cost, towing capacity, loading capability, security and fuel consumption. Typically, the truck with the lowest price also has the maximum towing and loading capabilities. Whenever managing a portfolio, managers want to maximize profits while minimizing risks; yet, the stocks with the highest return potential often have the highest risk of dropping money. Customer happiness and service costs are fundamentally opposing factors in the service sector. People frequently implicitly consider several factors when making daily decisions and they may be satisfied with the results of those judgments if they are solely based on intuition. On the other hand, it's crucial to properly outline the problem when the stakes are high.

Research Gap and Motivation

The study of cubic spherical neutrosophic sets and their associated arithmetic operators present a novel avenue for handling uncertainty and indeterminacy in decision-making processes. However, despite its potential, there remains a notable research gap and several motivating factors for further exploration:

- **Lack of Comprehensive Frameworks:** Existing research on neutrosophic sets and their operators primarily focuses on conventional models, often overlooking the complexities inherent in real-world decision-making scenarios. The introduction of CSNS and its arithmetic operators offers a more comprehensive framework for addressing uncertainty, yet further exploration is needed to fully understand its implications and applicability across diverse domains.

- **Limited Applications and Case Studies:** While the concept of CSNS shows promise, there is a scarcity of practical applications and case studies demonstrating its effectiveness in real-world contexts. The absence of empirical validation hinders the wider adoption and understanding of CSNS-based methodologies, highlighting the need for empirical studies and practical implementations.
- **Potential for Methodological Enhancements:** The development of CSNS-based arithmetic operators opens avenues for further methodological enhancements and refinements. Exploring alternative aggregation techniques, refining parameter estimation methodologies and investigating the scalability of CSNS-based models are areas ripe for exploration and innovation.

The motivation behind this study stems from the need to address the challenges posed by uncertainty and indeterminacy in decision-making processes. Traditional decision-making models often struggle to accommodate the complexities and nuances inherent in real-world scenarios, leading to suboptimal outcomes and missed opportunities. The introduction of CSNS and its associated arithmetic operators offers a promising avenue for overcoming these challenges.

The motivation for studying CSNS lies in its potential to provide a more comprehensive and nuanced representation of uncertain information. By incorporating a spherical representation with a radius r and a triple at its center, CSNS allows decision-makers to capture degrees of membership, indeterminacy and non-participation in a more intuitive and meaningful manner. This in turn, facilitates more informed and robust decision-making processes across various domains.

Contribution

The study makes several significant contributions to the field of decision-making under uncertainty and indeterminacy:

- **Introduction of Cubic Spherical Neutrosophic Sets (CSNS):** The study introduces CSNS as a novel framework for representing uncertainty and indeterminacy in decision-making processes. By extending the concept of Neutrosophic Sets (NS) to include a spherical representation with a radius r and a triple at its center, CSNS offers a more comprehensive and nuanced approach to modeling uncertain information.
- **Development of Arithmetic Operators:** The research proposes two new arithmetic operators specifically tailored for CSNS: Weighted Arithmetic Cubic Spherical Neutrosophic Aggregation Operators and Weighted Geometric Cubic Spherical Neutrosophic Aggregation Operators. These operators address limitations of existing neutrosophic operators and provide more reliable and effective aggregation techniques for handling uncertain data.

- **Methodological Advancement in MCDM:** The study presents an innovative Multiple-Criteria Decision-Making (MCDM) method for selecting the best electric truck based on CSNS and its arithmetic operators. By leveraging CSNS-based aggregation techniques, the proposed method offers a systematic and robust approach to decision-making in complex, uncertain environments.
- **Practical Implications and Future Directions:** The research not only advances theoretical understanding but also holds practical implications for various domains. The introduction of CSNS and its associated arithmetic operators has the potential to enhance decision-making processes in diverse fields such as engineering, finance, healthcare and environmental management. Furthermore, the study opens avenues for future research, including empirical validation, comparative analysis with existing methodologies and exploration of alternative arithmetic operators.

This study presents the concept of algebraic operations between $CSNSs$ using TNs and $TCNs$. Moreover, some weighted aggregation operators that transform input values represented by $CSNVs$ to a single output value using these algebraic operations are proposed. Finally, a cubic spherical CSM depending on the radius is given for evaluating the level of similarity between $CSNVs$. In addition, we propose a technique for converting a set of NVs into a $CSNVs$.

2. Preliminaries

Definition 2.1. [8] A mapping $\Gamma : [0,1] \times [0,1] \to [0,1]$ is a neutrosophic TN if it satisfies the following:
(i) $\Gamma(\alpha, (1,0,0)) = \alpha$ and $\Gamma(\alpha, (0,1,1)) = 0$,
(ii) $\Gamma(\alpha, \beta) = \Gamma(\beta, \alpha)$,
(iii) $\Gamma(\alpha, \Gamma(\beta, \gamma)) = \Gamma(\beta, \Gamma(\alpha, \gamma))$,
(iv) $\Gamma(\alpha, \beta) \leq \Gamma(\alpha', \beta')$ where $\alpha \leq \alpha'$ and $\beta \leq \beta'$,
for all $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2), \gamma = (\gamma_1, \gamma_2) \in [0,1]$.

Definition 2.2. [8] A mapping $\Gamma^* : [0,1] \times [0,1] \to [0,1]$ is a neutrosophic TCN if it satisfies the following:
(i) $\Gamma^*(\alpha, (0,1,1)) = \alpha$ and $\Gamma^*(\alpha, (1,0,0)) = 0$,
(ii) $\Gamma^*(\alpha, \beta) = \Gamma^*(\beta, \alpha)$,
(iii) $\Gamma^*(\alpha, \Gamma^*(\beta, \gamma)) = \Gamma^*(\beta, \Gamma^*(\alpha, \gamma))$,
(iv) $\Gamma^*(\alpha, \beta) \leq \Gamma^*(\alpha', \beta')$ where $\alpha \leq \alpha'$ and $\beta \leq \beta'$,
for all $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2), \gamma = (\gamma_1, \gamma_2) \in [0,1]$.

Definition 2.3. [9] A monotonically strictly decreasing mapping $*: [0,1] \rightarrow [0,\infty)$ defined by $*(1) = 0$ is called an AFG of a t-norm Γ if $\Gamma(\alpha,\beta) = *^{-1}(*(\alpha) + *(\beta))$ for any $(\alpha,\beta) \in [0,1] \times [0,1]$.

Definition 2.4. [9,14] If Γ (resp. Γ^*) is neutrosophic TN (resp. TCN) on $[0,1]$ is said to be dual with respect to $\mathcal{S}$, if $\Gamma(\alpha,\beta) = \mathcal{S}(\Gamma^*(\mathcal{S}(\alpha),\mathcal{S}(\beta)))$ (resp. $\Gamma^*(\alpha,\beta) = \mathcal{S}(\Gamma(\mathcal{S}(\alpha),\mathcal{S}(\beta)))$) for any $\alpha,\beta \in [0,1]$.

Definition 2.5. [8,18] If Γ (resp. Γ^*) is neutrosophic TN (resp. TCN) on $[0,1]$, then the dual t-conorm $\mathcal{T}^*$ is defined as $\mathcal{T}^*(\alpha,\beta) = 1 - \Gamma(1-\alpha, 1-\beta)$, for any $\alpha,\beta \in [0,1]$.
Clearly Γ is an ATN if it is continuous and $\Gamma(\alpha,\alpha) < \alpha$ for all $\alpha \in (0,1)$ and Γ^* is an $ATCN$ if it is continuous $\Gamma^*(\alpha,\alpha) > \alpha$ for any $\alpha \in (0,1)$.

Definition 2.6. [8] A mapping $\mathcal{N} : [0,1] \times [0,1] \rightarrow [0,1]$ is a neutrosophic negator, if it satisfies three conditions:
(i) $\mathcal{N}(\alpha,(0,1,1)) = 1$ for any $\alpha \in [0,1]$,
(ii) $\mathcal{N}(\alpha,(1,0,0)) = 0$ for any $\alpha \in [0,1]$,
(iii) $\mathcal{N}(\alpha,\beta) \geq \mathcal{N}(\alpha',\beta')$ where $\alpha \leq \alpha'$ and $\beta \leq \beta'$.

Definition 2.7. [5] A cubic spherical neutrosophic set $(CSNS)$ $\mathbb{C}_{csR}$ in $\mathbb{X}$ is defined by $\mathbb{C}_{csR} = \{< x, \frac{cs\mathbb{T}_C}{x}, \frac{cs\mathbb{I}_C}{x}, \frac{cs\mathbb{F}_C}{x}; csR >: x \in \mathbb{X}\}$, where $cs\mathbb{T}_C, cs\mathbb{I}_C, cs\mathbb{F}_C : \mathbb{X} \rightarrow [0,1]$ are mappings satisfies the condition $cs\mathbb{T}_C + cs\mathbb{I}_C + cs\mathbb{F}_C \leq 3$ and $csR > 0$ denote the radius of the sphere centred at the point $(\frac{cs\mathbb{T}_C}{x}, \frac{cs\mathbb{I}_C}{x}, \frac{cs\mathbb{F}_C}{x})$ in the cube.
If a collection $\{< cs\mathbb{T}_{\epsilon,1}, cs\mathbb{I}_{\epsilon,1}, cs\mathbb{F}_{\epsilon,1} >, < cs\mathbb{T}_{\epsilon,2}, cs\mathbb{I}_{\epsilon,2}, cs\mathbb{F}_{\epsilon,2} >, ..., < cs\mathbb{T}_{\epsilon,k_\epsilon}, cs\mathbb{I}_{\epsilon,k_\epsilon}, cs\mathbb{F}_{\epsilon,k_\epsilon} > \}$ of $CSNSs$ is assigned for any x_ϵ in $\mathbb{X}$. Then $\mathbb{U}_{csR} = \{< x_\epsilon, \frac{cs\mathbb{T}_U}{x_\epsilon}, \frac{cs\mathbb{I}_U}{x_\epsilon}, \frac{cs\mathbb{F}_U}{x_\epsilon}; csR_\epsilon >: x_\epsilon \in \mathbb{X}\}$ is a $CSNS$ in $\mathbb{X}$ where $< \frac{cs\mathbb{T}_U}{x_\epsilon}, \frac{cs\mathbb{I}_U}{x_\epsilon}, \frac{cs\mathbb{F}_U}{x_\epsilon} >=< \frac{\sum_{\eta=1}^{k_\epsilon} cs\mathbb{T}_{\epsilon,j}}{k_\epsilon}, \frac{\sum_{\eta=1}^{k_\epsilon} cs\mathbb{I}_{\epsilon,j}}{k_\epsilon}, \frac{\sum_{\eta=1}^{k_\epsilon} cs\mathbb{F}_{\epsilon,j}}{k_\epsilon} >$ and $csR_\epsilon = min\{max_{1\leq j\leq k_\epsilon} \sqrt{(\frac{cs\mathbb{T}_U}{x_\epsilon} - cs\mathbb{T}_{\epsilon,j})^2 + (\frac{cs\mathbb{I}_U}{x_\epsilon} - cs\mathbb{I}_{\epsilon,j})^2 + (\frac{cs\mathbb{F}_U}{x_\epsilon} - cs\mathbb{F}_{\epsilon,j})^2}, 1\}$.

Let $X = \{x,y\}$ and $\lambda_1, \lambda_2 \in NS(X)$ such that

$$\lambda_1 = \{\langle x, 0.88, 0.33, 0.22\rangle, \langle x, 0.77, 0.44, 0.11\rangle, \langle x, 0.55, 0.44, 0.22\rangle, \langle x, 0.66, 0.55, 0.33\rangle\}$$

$$\lambda_2 = \{\langle y, 0.66, 0.22, 0.11\rangle, \langle y, 0.88, 0.11, 0.22\rangle, \langle y, 0.88, 0.33, 0.11\rangle, \langle y, 0.99, 0.44, 0.22\rangle\}.$$

The CSNSs are $\lambda_{(R_1)} = \{\langle x, 0.72, 0.44, 0.22; 0.20\rangle : x \in X\}$ and
$\lambda_{(R_2)} = \{\langle y, 0.85, 0.28, 0.17; 0.22\rangle : y \in X\}$.

Definition 2.8. [5] Let $U_{csR} = \{< x, \frac{cs\mathbb{T}_U}{x}, \frac{cs\mathbb{I}_U}{x}, \frac{cs\mathbb{F}_U}{x}; csR >: x \in \mathbb{X}\}$ and $V_{csS} = \{< x, \frac{cs\mathbb{T}_V}{x}, \frac{cs\mathbb{I}_V}{x}, \frac{cs\mathbb{F}_V}{x}; csS >: x \in \mathbb{X}\}$ be $CSNSs$ in $\mathbb{X}$ and $\dagger \in \{min, max\}$. Then

(1) $U_{csR} \subset V_{csS}$ iff $csR \leq csS$ and $\frac{cs\mathbb{T}_U}{x} \leq \frac{cs\mathbb{T}_V}{x}$, $\frac{cs\mathbb{I}_U}{x} \geq \frac{cs\mathbb{I}_V}{x}$ and $\frac{cs\mathbb{F}_U}{x} \geq \frac{cs\mathbb{F}_V}{x}$ for any $x \in \mathbb{X}$,

(2) $U_{csR} = V_s$ iff $csR = s$ and $\frac{cs\mathbb{T}_U}{x} = \frac{cs\mathbb{T}_V}{x}$, $\frac{cs\mathbb{I}_U}{x} = \frac{cs\mathbb{I}_V}{x}$ and $\frac{cs\mathbb{F}_U}{x} = \frac{cs\mathbb{F}_V}{x}$ for any $x \in \mathbb{X}$,

(3) $U^c_{csR} = \{< x, \frac{cs\mathbb{F}_U}{x}, \frac{cs\mathbb{I}_U}{x}, \frac{cs\mathbb{T}_U}{x}; csR >: x \in \mathbb{X}\}$,

(4) $U_{csR} \cup_\dagger V_s = \{< x, max(\frac{cs\mathbb{T}_U}{x}, \frac{cs\mathbb{T}_V}{x}), min(\frac{cs\mathbb{I}_U}{x}, \frac{cs\mathbb{I}_V}{x}), min(\frac{cs\mathbb{F}_U}{x}, \frac{cs\mathbb{F}_V}{x});$
$\dagger\,(csR, csS) >: x \in \mathbb{X}\}$,

(5) $U_{csR} \cap_\dagger V_{csS} = \{< x, min(\frac{cs\mathbb{T}_U}{x}, \frac{cs\mathbb{T}_V}{x}), max(\frac{cs\mathbb{I}_U}{x}, \frac{cs\mathbb{I}_V}{x}), max(\frac{cs\mathbb{F}_U}{x}, \frac{cs\mathbb{F}_V}{x});$
$\dagger\,(csR, csS) >: x \in \mathbb{X}\}$.

The acronyms used in the current research are listed below.

Table 1. Acronyms

Abbreviation	Description
min	Minimum
max	Maximum
NSs	Neutrosophic Sets
NVs	Neutrosophic Values
$CSNSs$	Cubic Spherical Neutrosophic Sets
$CSNVs$	Cubic Spherical Neutrosophic Values
$MCDM$	Multiple-Criteria Decision-Making
CSM	Cosine Similarity Measure
ATN	Archimedean T-Norm
$ATCN$	Archimedean t-Conorm
AFG	Additive Functional Generator

3. Cubic Spherical Neutrosophic t-norm and t-conorm

Definition 3.1. Let $u =< cs\mathbb{T}_u, cs\mathbb{I}_u, cs\mathbb{F}_u; csR_u >$ and $v =< cs\mathbb{T}_v, cs\mathbb{I}_v, cs\mathbb{F}_u; csR_v >$ be two $CSNVs$ in $\mathbb{X}$ and $\dagger \in \{min, max\}$. The following are some set operations that can be defined between $CSNVs$:

(1) $u \oplus_\dagger v =< cs\mathbb{T}_u + cs\mathbb{T}_v - cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u cs\mathbb{F}_v; \dagger(csR_u, csR_v) >$,

(2) $u \otimes_\dagger v =< cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u + cs\mathbb{I}_v - cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u + cs\mathbb{F}_v - cs\mathbb{F}_u cs\mathbb{F}_v; \dagger(csR_u, csR_v) >$.

From Definition 3.1, by using TN and TCN, we may extend.
If Γ and Γ^* are dual TN and TCN with respect to the cubic spherical neutrosophic complement $\mathcal{S}$, respectively and $\mathcal{Q}$ is a TN or TCN. Then we can define the following algebraic operations among $CSNVs$.

(1) $u \oplus_Q v =< \Gamma^*(cs\mathbb{T}_u, cs\mathbb{T}_v), \Gamma(cs\mathbb{I}_u, cs\mathbb{I}_v), \Gamma(cs\mathbb{F}_u, cs\mathbb{F}_v); \mathcal{Q}(csR_u, csR_v) >$,

(2) $u \otimes_Q v =< \Gamma(cs\mathbb{T}_u, cs\mathbb{T}_v), \Gamma^*(cs\mathbb{I}_u, cs\mathbb{I}_v), \Gamma^*(cs\mathbb{F}_u, cs\mathbb{F}_v); \mathcal{Q}(csR_u, csR_v) >$.

If $\star : [0,1] \to [0,\infty)$ is the AFG of a continuous Archimedean ATN and $*(t) = \star(1-t)$ and $\wp : [0,1] \to [0,\infty)$ is the AFG of a continuous TCN. Then we can define the following algebraic operations among $CSNVs$ and $m > 0$.

(1) $u \oplus_\wp v = < *^{-1}(*(cs\mathbb{T}_u) + *(cs\mathbb{T}_v)), \star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v)),$
$\star^{-1}(\star(cs\mathbb{F}_u) + \star(cs\mathbb{F}_v)); \wp^{-1}(\wp(csR_u) + \wp(csR_v)) >,$

(2) $u \otimes_\wp v = < \star^{-1}(\star(cs\mathbb{T}_u) + \star(cs\mathbb{T}_v)), *^{-1}(*(cs\mathbb{I}_u) + *(cs\mathbb{I}_v)),$
$*^{-1}(*(cs\mathbb{F}_u) + *(cs\mathbb{F}_v)); \wp^{-1}(\wp(csR_u) + \wp(csR_v)) >,$

(3) $m_\wp u = < *^{-1}(m * (cs\mathbb{T}_u)), \star^{-1}(m \star (cs\mathbb{I}_u)), \star^{-1}(m \star (cs\mathbb{F}_u)); \wp^{-1}(m\wp(csR_u)) >,$

(4) $u^{m_\wp} = < \star^{-1}(m \star (cs\mathbb{T}_u)), *^{-1}(m * (cs\mathbb{I}_u)), *^{-1}(m * (cs\mathbb{F}_u)); \wp^{-1}(m\wp(csR_u)) > .$

Definition 3.2. Let $\star, *, \wp, \rho : [0,1] \to [0,\infty)$ be mappings such that $\star(t) = -log\ t$, $*(t) = -log\ (1-t)$, $\rho(t) = -log\ t$, and $\wp(t) = -log\ (1-t)$ and $m > 0$. Then we can define the following algebraic operations among $CSNVs$.

(1) $u \oplus_\wp v = < cs\mathbb{T}_u + cs\mathbb{T}_v - cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u cs\mathbb{F}_v; csR_u csR_v >,$

(2) $u \oplus_\rho v = < cs\mathbb{T}_u + cs\mathbb{T}_v - cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u cs\mathbb{F}_v; csR_u + csR_v - csR_u csR_v >,$

(3) $u \otimes_\wp v = < cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u + cs\mathbb{I}_v - cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u + cs\mathbb{F}_v - cs\mathbb{F}_u cs\mathbb{F}_v; csR_u csR_v >,$

(4) $u \otimes_\rho v = < cs\mathbb{T}_u cs\mathbb{T}_v, cs\mathbb{I}_u + cs\mathbb{I}_v - cs\mathbb{I}_u cs\mathbb{I}_v, cs\mathbb{F}_u + cs\mathbb{F}_v - cs\mathbb{F}_u cs\mathbb{F}_v;$
$csR_u + csR_v - csR_u csR_v >,$

(5) $m_\wp u = < 1 - (1 - cs\mathbb{T}_u)^m, cs\mathbb{I}_u^m, cs\mathbb{F}_u^m; csR_u^m >,$

(6) $m_\rho u = < 1 - (1 - cs\mathbb{T}_u)^m, cs\mathbb{I}_u^m, cs\mathbb{F}_u^m; 1 - (1 - csR_u)^m >,$

(7) $u^{m_\wp} = < cs\mathbb{T}_u^m, 1 - (1 - cs\mathbb{I}_u)^m, 1 - (1 - cs\mathbb{F}_u)^m; csR_u^m >,$

(8) $u^{m_\rho} = < cs\mathbb{T}_u^m, 1 - (1 - cs\mathbb{I}_u)^m, 1 - (1 - cs\mathbb{F}_u)^m; 1 - (1 - csR_u)^m > .$

Theorem 3.1. *Let* $u =< cs\mathbb{T}_u, cs\mathbb{I}_u, cs\mathbb{F}_u; csR_u >$, $v =< cs\mathbb{T}_v, cs\mathbb{I}_v, cs\mathbb{F}_v; csR_v >$, *and* $w =< cs\mathbb{T}_w, cs\mathbb{I}_w, cs\mathbb{F}_w; csR_w >$ *be* $CSNVs$ *and let* $m, n > 0$. *If* $\star : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *and* $*(t) = \star(1-t)$ *and* $\wp : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *or* $ATCN$. *Then*

(1) $u \oplus_\wp v = v \oplus_\wp u,$

(2) $u \otimes_\wp v = v \otimes_\wp u,$

(3) $(u \oplus_\wp v) \oplus_\wp w = u \oplus_\rho (v \oplus_\wp w),$

(4) $(u \otimes_\wp v) \otimes_\wp w = u \otimes_\wp (v \otimes_\wp w),$

(5) $m_\wp(u \oplus_\wp v) = m_\wp u \oplus_\wp m_\wp v,$

(6) $(m_\wp \oplus n_\wp)u = m_\wp u \oplus_\wp n_\wp u,$

(7) $(u \otimes_\wp v)^{m_\wp} = u^{m_\wp} \otimes_\wp v^{m_\wp},$

(8) $u^{m_\wp} \otimes_\wp u^{n_\wp} = u^{m_\wp + n_\wp}.$

Proof: (1) and (2) are trivial.

(3).
$$\begin{aligned}
(u \oplus_{\wp} v) \oplus_{\wp} w &= < \ast^{-1}(\ast(cs\mathbb{T}_u) + \ast(cs\mathbb{T}_v)), \star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v)), \star^{-1}(\star(cs\mathbb{F}_u) + \star(cs\mathbb{F}_v)); \\
&\quad \wp^{-1}(\wp(csR_u) + \wp(csR_v)) > \oplus_{\wp} < cs\mathbb{T}_w, cs\mathbb{I}_w, cs\mathbb{F}_w; csR_w > \\
&= < \ast^{-1}(\ast(\ast^{-1}(\ast(cs\mathbb{T}_u) + \ast(cs\mathbb{T}_v)) + \ast(cs\mathbb{T}_w))), \\
&\quad \star^{-1}(\star(\star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v)) + \star(cs\mathbb{I}_w))), \\
&\quad \star^{-1}(\star(\star^{-1}(\star(cs\mathbb{F}_u) + \star(cs\mathbb{F}_v)) + \star(cs\mathbb{F}_w))); \\
&\quad \wp^{-1}(\wp(\wp^{-1}(\wp(csR_u) + \wp(csR_v)) + \wp(csR_w))) > \\
&= < \ast^{-1}(\ast(cs\mathbb{T}_u) + \ast(cs\mathbb{T}_v) + \ast(cs\mathbb{T}_w)), \star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v) + \star(cs\mathbb{I}_w)), \\
&\quad \star^{-1}(\star(cs\mathbb{F}_u) + \star(cs\mathbb{F}_v) + \star(cs\mathbb{F}_w)); \wp^{-1}(\wp(csR_u) + \wp(csR_v) + \wp(csR_w)) > \\
&= < \ast^{-1}(\ast(cs\mathbb{T}_u) + \ast(\ast^{-1}(\ast(cs\mathbb{T}_v) + \ast(cs\mathbb{T}_w)))), \\
&\quad \star^{-1}(\star(cs\mathbb{I}_u) + \star(\star^{-1}(\star(cs\mathbb{I}_v) + \star(cs\mathbb{I}_w)))), \\
&\quad \star^{-1}(\star(cs\mathbb{F}_u) + \star(\star^{-1}(\star(cs\mathbb{F}_v) + \star(cs\mathbb{F}_w)))); \\
&\quad \wp^{-1}(\wp(csR_u) + \wp(\wp^{-1}(\wp(csR_v) + \wp(csR_w)))) > \\
&= < cs\mathbb{T}_u, cs\mathbb{I}_u; csR_u > \oplus_{\wp} < \ast^{-1}(\ast(cs\mathbb{T}_v) + \ast(cs\mathbb{T}_w)), \star^{-1}(\ast(cs\mathbb{I}_v) + \star(cs\mathbb{I}_w)), \\
&\quad \star^{-1}(\star(cs\mathbb{F}_v) + \star(cs\mathbb{F}_w)); \wp^{-1}(\wp(csR_v) + \wp(csR_w)) > \\
&\quad = u \oplus_{\wp} (v \oplus_{\wp} w).
\end{aligned}$$

(4).
$$\begin{aligned}
(u \otimes_{\wp} v) \otimes_{\wp} w &= < \star^{-1}(\star(cs\mathbb{T}_u) + \star(cs\mathbb{T}_v)), \ast^{-1}(\ast(cs\mathbb{I}_u) + \ast(cs\mathbb{I}_v)), \ast^{-1}(\ast(cs\mathbb{F}_u) + \ast(cs\mathbb{F}_v)); \\
&\quad \wp^{-1}(\wp(csR_u) + \wp(csR_v)) > \otimes_{\wp} < cs\mathbb{T}_w, cs\mathbb{I}_w, cs\mathbb{F}_w; csR_w > \\
&= < \star^{-1}(\star(\star^{-1}(\star(cs\mathbb{T}_u) + \star(cs\mathbb{T}_v)) + \ast(cs\mathbb{T}_w))), \\
&\quad \ast^{-1}(\ast(\ast^{-1}(\ast(cs\mathbb{I}_u) + \ast(cs\mathbb{I}_v)) + \ast(cs\mathbb{I}_w))), \\
&\quad \ast^{-1}(\ast(\ast^{-1}(\ast(cs\mathbb{F}_u) + \ast(cs\mathbb{F}_v)) + \ast(cs\mathbb{F}_w))); \\
&\quad \wp^{-1}(\wp(\wp^{-1}(\wp(csR_u) + \wp(csR_v)) + \wp(csR_w))) > \\
&= < \star^{-1}(\star(cs\mathbb{T}_u) + \star(cs\mathbb{T}_v) + \star(cs\mathbb{T}_w)), \ast^{-1}(\ast(cs\mathbb{I}_u) + \ast(cs\mathbb{I}_v) + \ast(cs\mathbb{I}_w)), \\
&\quad \ast^{-1}(\ast(cs\mathbb{F}_u) + \ast(cs\mathbb{F}_v) + \ast(cs\mathbb{F}_w)); \wp^{-1}(\wp(csR_u) + \wp(csR_v) + \wp(csR_w)) > \\
&= < \star^{-1}(\star(cs\mathbb{T}_u) + \star(\star^{-1}(\star(cs\mathbb{T}_v) + \star(cs\mathbb{T}_w)))), \\
&\quad \ast^{-1}(\ast(cs\mathbb{I}_u) + \ast(\ast^{-1}(\ast(cs\mathbb{I}_v) + \ast(cs\mathbb{I}_w)))), \\
&\quad \ast^{-1}(\ast(cs\mathbb{F}_u) + \ast(\ast^{-1}(\ast(cs\mathbb{F}_v) + \ast(cs\mathbb{F}_w)))); \\
&\quad \wp^{-1}(\wp(csR_u) + \wp(\wp^{-1}(\wp(csR_v) + \wp(csR_w)))) >
\end{aligned}$$

$$= < cs\mathbb{T}_u, cs\mathbb{I}_u; csR_u > \otimes_\wp < \star^{-1}(\star(cs\mathbb{T}_v) + \star(cs\mathbb{T}_w)),$$
$$*^{-1}(*(cs\mathbb{I}_v) + *(cs\mathbb{I}_w)), *^{-1}(*(cs\mathbb{F}_v) + *(cs\mathbb{F}_w));$$
$$\wp^{-1}(\wp(csR_v) + \wp(csR_w)) >$$
$$= u \otimes_\wp (v \otimes_\wp w).$$

(5). $$m_\wp(u \oplus_\wp v) = m_\wp < *^{-1}(*(cs\mathbb{T}_u) + *(cs\mathbb{T}_v)), \star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v)),$$
$$\star^{-1}(\star(cs\mathbb{T}_u) + \star(cs\mathbb{T}_v)); \wp^{-1}(\wp(csR_u) + \wp(csR_v)) >$$
$$= < *^{-1}(m * (*^{-1}(*(cs\mathbb{T}_u) + *(cs\mathbb{T}_v)))), \star^{-1}(m * (\star^{-1}(\star(cs\mathbb{I}_u) + \star(cs\mathbb{I}_v)))),$$
$$\star^{-1}(m * (\star^{-1}(\star(cs\mathbb{F}_u) + \star(cs\mathbb{F}_v)))); \wp^{-1}(m * (\wp^{-1}(\wp(csR_u) + \wp(csR_v)))) >$$
$$= < *^{-1}(m * (cs\mathbb{T}_u) + m * (cs\mathbb{T}_v)), \star^{-1}(m \star (cs\mathbb{I}_u) + m \star (cs\mathbb{I}_v)),$$
$$\star^{-1}(m \star (cs\mathbb{F}_u) + m \star (cs\mathbb{F}_v)), \wp^{-1}(m\wp(csR_u) + m\wp(csR_v)) >$$
$$= < *^{-1}(h = *(*^{-1}(m * (cs\mathbb{T}_u))) + *(*^{-1}(m * (cs\mathbb{T}_v)))),$$
$$\star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{I}_u))) + \star(\star^{-1}(m * (cs\mathbb{I}_v)))),$$
$$\star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{F}_u))) + \star(\star^{-1}(m * (cs\mathbb{F}_v))));$$
$$\wp^{-1}(\wp(\wp^{-1}(m\wp(csR_u))) + \wp(\wp^{-1}(m\wp(csR_v)))) >$$
$$= < *^{-1}(*(cs\mathbb{T}_{mu}) + *(cs\mathbb{T}_{mv})), \star^{-1}(\star(cs\mathbb{I}_{mu}) + \star(cs\mathbb{I}_{mv})),$$
$$\star^{-1}(\star(cs\mathbb{F}_{mu}) + \star(cs\mathbb{F}_{mv})); \wp^{-1}(\wp(csR_{mu}) + \wp(csR_{mv})) >$$
$$= m_\wp u \oplus_\wp m_\wp v.$$

(6). $$(m_\wp + n_\wp)u = < *^{-1}((m+n) * (cs\mathbb{T}_u)), \star^{-1}((m+n) \star (cs\mathbb{I}_u)), \star^{-1}((m+n) \star (cs\mathbb{F}_u))^{-1}$$
$$((m+n)\wp(csR_u)) >$$
$$= < *^{-1}(m * (cs\mathbb{T}_u) + n * (cs\mathbb{T}_u)), \star^{-1}(m \star (cs\mathbb{I}_u) + n \star (cs\mathbb{I}_u)),$$
$$\star^{-1}(m \star (cs\mathbb{F}_u) + n \star (cs\mathbb{F}_u)); \wp^{-1}(m\wp(csR_u) + n\wp(csR_u)) >$$
$$= < *^{-1}(*(*^{-1}(m * (cs\mathbb{T}_u))) + *(*^{-1}(n * (cs\mathbb{T}_u)))), \star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{I}_u))) +$$
$$\star(\star^{-1}(n \star (cs\mathbb{I}_u)))), \star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{F}_u))) +$$
$$\star(\star^{-1}(n \star (cs\mathbb{F}_u)))); \wp^{-1}(\wp(\wp^{-1}(m\wp(csR_u))) + \wp(\wp^{-1}(n\wp(csR_u)))) >$$
$$= < *^{-1}(*(cs\mathbb{T}_{m_\wp u}) + (*(cs\mathbb{T}_{n_\wp u}))), \star^{-1}(\star(cs\mathbb{I}_{m_\wp u}) + (\star(cs\mathbb{I}_{n_\wp u}))),$$
$$\star^{-1}(\star(cs\mathbb{F}_{m_\wp u}) + (\star(cs\mathbb{F}_{n_\wp u}))); \wp^{-1}(\wp(csR_{m_\wp u}) + (\wp(csR_{n_\wp u}))) >$$
$$= m_\wp u \oplus_\wp n_\wp u.$$

(7). $$(u \otimes_\wp v)^{m_\wp} = < \star^{-1}(m \star (cs\mathbb{T}_{u\otimes_\wp v})), *^{-1}(m * (cs\mathbb{I}_{u\otimes_\wp v})), *^{-1}(m * (cs\mathbb{F}_{u\otimes_\wp v})); \wp^{-1}(m\wp(csR_{u\otimes_\wp v})) >$$
$$= < \star^{-1}(m \star (\star^{-1}(\star(cs\mathbb{T}_u)))), *^{-1}(m * (*^{-1}(*(cs\mathbb{I}_u)))), *^{-1}(m * (*^{-1}(*(cs\mathbb{F}_u))));$$
$$\wp^{-1}(m\wp(\wp^{-1}(\wp(csR_u)))) >$$

$$= < \star^{-1}(m \star (cs\mathbb{T}_u) + m \star (cs\mathbb{T}_v)), *^{-1}(m * (cs\mathbb{I}_u) + m * (cs\mathbb{I}_v)),$$
$$*^{-1}\,(m * (cs\mathbb{F}_u) + m * (cs\mathbb{F}_v)); \wp^{-1}(m\wp(csR_u) + m\wp(csR_v)) >$$
$$= < \star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{T}_u))) + \star(\star^{-1}(m \star (cs\mathbb{T}_v)))),$$
$$*^{-1}\,(*(*^{-1}(m * (cs\mathbb{I}_u))) + *(*^{-1}(m * (cs\mathbb{I}_v)))),$$
$$*^{-1}\,(*(*^{-1}(m * (cs\mathbb{F}_u))) + *(*^{-1}(m * (cs\mathbb{F}_v))));$$
$$\wp^{-1}(\wp(\wp^{-1}(m\wp(csR_u))) + \wp(\wp^{-1}(m\wp(csR_v)))) >$$
$$= < \star^{-1}(\star(cs\mathbb{T}_{u^{m_\wp}}) + \star(cs\mathbb{T}_{v^{m_\wp}})), *^{-1}(*(cs\mathbb{I}_{u^{m_\wp}}) + *(cs\mathbb{I}_{v^{m_\wp}})),$$
$$*^{-1}\,(*(cs\mathbb{F}_{u^{m_\wp}}) + *(cs\mathbb{F}_{v^{m_\wp}})); \wp^{-1}(\wp(csR_{u^{m_\wp}}) + \wp(csR_{v^{m_\wp}})) >$$
$$= u^{m_\wp} \otimes v^{m_\wp}.$$

(8). $$u_{csR}^{m_\wp + n_\wp} = < \star^{-1}((m+n) * (cs\mathbb{T}_u)), *^{-1}((m+n) * (cs\mathbb{I}_u)), *^{-1}((m+n) * (cs\mathbb{F}_u));$$
$$\wp^{-1}((m+n)\wp(csR_u)) >$$
$$= < \star^{-1}(m \star (cs\mathbb{T}_u) + n \star (cs\mathbb{T}_u)), *^{-1}(m * (cs\mathbb{I}_u) + n * (cs\mathbb{I}_u)),$$
$$*^{-1}\,(m * (cs\mathbb{F}_u) + n * (cs\mathbb{F}_u)); \wp^{-1}(m\wp(csR_u) + n\wp(csR_u)) >$$
$$= < \star^{-1}(\star(\star^{-1}(m \star (cs\mathbb{T}_u))) + \star(\star^{-1}(n \star (cs\mathbb{T}_u)))), *^{-1}(*(*^{-1}(m * (cs\mathbb{I}_u)))+$$
$$*\,(*^{-1}(n * (cs\mathbb{I}_u)))), *^{-1}(*(*^{-1}(m * (cs\mathbb{F}_u))) + *(*^{-1}(n * (cs\mathbb{F}_u))));$$
$$\wp^{-1}(\wp(\wp^{-1}(m\wp(csR_u))) + \wp(\wp^{-1}(n\wp(csR_u)))) >$$
$$= < \star^{-1}(\star(cs\mathbb{T}_{u^m}) + \star(cs\mathbb{T}_{u^n})), *^{-1}(*(cs\mathbb{I}_{u^m}) + *(cs\mathbb{I}_{u^n})),$$
$$*^{-1}\,(*(cs\mathbb{F}_{u^m}) + *(cs\mathbb{F}_{u^n})); \wp^{-1}(\wp(csR_{u^m}) + \wp(csR_{u^n})) >$$
$$= u_{csR}^{m_\wp} \otimes_\wp u_{csR}^{n_\wp}.$$

4. Weighted Arithmetic Cubic Spherical Neutrosophic Aggregation Operators

Definition 4.1. Consider the collection $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ of $CSNVs$. If $\star : [0,1] \to [0,\infty)$ is the AFG of a continuous ATN and $*(t) = \star(1-t)$ and $\wp : [0,1] \to [0,\infty)$ is the AFG of a continuous ATN or $ATCN$. Then a weighted arithmetic cubic spherical neutrosophic aggregation operator is defined and denoted by $\mathbb{CSNWA}_\wp(u_1, u_2, \ldots, u_k) = (\wp) \bigoplus_{\epsilon=1}^{k} \omega_\epsilon u_\epsilon$, where $0 \le \omega_\epsilon \le 1$ for any $\epsilon = 1, 2, 3, \ldots, k$ subject to the condition $\sum_{\epsilon=1}^{k} \omega_\epsilon = 1$.

Theorem 4.1. *Consider* $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ *of* $CSNVs$. *If* $h : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *and* $*(t) = \star(1-t)$ *and* $\wp : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *or* $ATCN$. *Then* $\mathbb{CSNWA}_\wp(u_1, u_2, \ldots, u_k) = < *^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon * (cs\mathbb{T}_{u_\epsilon}))$, $\star^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon \star (cs\mathbb{I}_{u_\epsilon}))$, $\star^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon \star (cs\mathbb{F}_{u_\epsilon}))$; $\wp^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon \wp(csR_{u_\epsilon})) >$ *where* $0 \le \omega_\epsilon \le 1$ *for all* $\epsilon = 1, 2, 3, \ldots, k$ *subject to the condition* $\sum_{\epsilon=1}^{k} \omega_\epsilon = 1$.

Proof. *Clearly $\mathbb{CSNWA}_{\wp}(u_1, u_2, \ldots, u_k)$ is a CSNV. The second part can be seen to be true by using mathematical induction. If $k = 2$, we have*

$$\begin{aligned}
\mathbb{CSNWA}_{\wp}(u_1, u_2, \ldots, u_k) =& \omega_{1_\wp} u_1 \oplus_\wp \omega_{2_\wp} u_2 \\
=& < *^{-1}(*(cs\mathbb{T}_{\omega_{1_\wp}} u_1) + *(cs\mathbb{T}_{\omega_{2_\wp}} u_2)), \star^{-1}(\star(cs\mathbb{I}_{\omega_{1_\wp}} u_1) + \star(cs\mathbb{I}_{\omega_{2_\wp}} u_2)), \\
& \star^{-1}(\star(cs\mathbb{F}_{\omega_{1_\wp}} u_1) + \star(cs\mathbb{F}_{\omega_{2_\wp}} u_2)); \wp^{-1}(\wp(csR_{\omega_{1_\wp}} u_1) + \wp(csR_{\omega_{2_\wp}} u_2)) > \\
=& < *^{-1}(*((*^{-1}(\omega_1 h(cs\mathbb{T}_{u_1}))) + *(*^{-1}(\omega_2 * (cs\mathbb{T}_{u_2})))), \\
& \star^{-1}(\star((\star^{-1}(\omega_1 h(cs\mathbb{I}_{u_1}))) + \star(\star^{-1}(\omega_2 * (cs\mathbb{I}_{u_2})))), \star^{-1}(\star((\star^{-1}(\omega_1 * (cs\mathbb{F}_{u_1}))) \\
& + \star(\star^{-1}(\omega_2 * (cs\mathbb{F}_{u_2})))); \wp^{-1}(\wp((\wp^{-1}(\omega_1 h(csR_{u_1}))) + \wp(\wp^{-1}(\omega_1 h(csR_{u_2})))) > \\
=& < *^{-1}(\omega_1 * (cs\mathbb{T}_{u_1}) + \omega_2 h(cs\mathbb{T}_{u_2})), \star^{-1}(\omega_1 \star (cs\mathbb{I}_{u_1}) + \omega_2 \star (cs\mathbb{I}_{u_2})), \\
& \star^{-1}(\omega_1 \star (cs\mathbb{F}_{u_1}) + \omega_2 \star (cs\mathbb{F}_{u_2})); \wp^{-1}(\omega_1 \wp(csR_{u_1}) + \omega_2 \wp(csR_{u_2})) > \\
=& < *^{-1}(\sum_{\eta=1}^{2} \omega_\eta * (cs\mathbb{T}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{2} \omega_\eta * (cs\mathbb{I}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{2} \omega_\eta \star (cs\mathbb{F}_{u_\eta})); \\
& \wp^{-1}(\sum_{\eta=1}^{2} \omega_\eta \wp(csR_{u_\eta})) > .
\end{aligned}$$

$$\begin{aligned}
\mathbb{CSNWA}_{\wp}(u_1, u_2, ..., u_{k-1}) =& < *^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \star (cs\mathbb{T}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta * (cs\mathbb{I}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \star (cs\mathbb{F}_{u_\eta})); \\
& \wp^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \wp(csR_{u_\eta})) >
\end{aligned}$$

$$\begin{aligned}
\mathbb{CSNWA}_{\wp}(u_1, u_2, \ldots, u_k) =& \mathbb{CSNWA}_{\wp}(u_1, u_2, \ldots, u_{k-1}) \oplus_\wp \omega_{k_\wp} u_k \\
=& < *^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta * (cs\mathbb{T}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \star (cs\mathbb{I}_{u_\eta})), \star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \star (cs\mathbb{F}_{u_\eta})); \\
& \wp^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta \wp(csR_{u_\eta})) > \oplus_\wp < *^{-1}(*(cs\mathbb{T}_{\omega_{k_\wp}} u_1)), \star^{-1}(\star(cs\mathbb{I}_{\omega_{k_\wp}} u_1)), \\
& \star^{-1}(\star(cs\mathbb{F}_{\omega_{k_\wp}} u_1)); \wp^{-1}(\wp(cs\mathbb{I}_{\omega_{k_\wp}} u_1)) > \\
=& < *^{-1}(*((*^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta * (cs\mathbb{T}_{u_\eta}))) + *(*^{-1}(\omega_k * (cs\mathbb{T}_{u_k})))), \\
& \star^{-1}(\star((\star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta * (cs\mathbb{I}_{u_\eta}))) + \star(\star^{-1}(\omega_k \star (cs\mathbb{I}_{u_k})))), \\
& \star^{-1}(\star((\star^{-1}(\sum_{\eta=1}^{k-1} \omega_\eta * (cs\mathbb{F}_{u_\eta}))) + \star(\star^{-1}(\omega_k \star (cs\mathbb{F}_{u_k}))));
\end{aligned}$$

$$\wp^{-1}(\wp((\wp^{-1}(\sum_{\eta=1}^{k-1}\omega_\eta\wp(csR_{u_\eta}))) + \wp(\wp^{-1}(\omega_k\wp(csR_{u_k})))) >$$

$$= < *^{-1}(\sum_{\eta=1}^{k-1}\omega_\eta * (cs\mathbb{T}_{u_\eta}) + \omega_k * (cs\mathbb{T}_{u_k})), \star^{-1}(\sum_{\eta=1}^{k-1}\omega_\eta \star (cs\mathbb{I}_{u_\eta}) + \omega_k \star (cs\mathbb{I}_{u_k})),$$

$$\star^{-1}(\sum_{\eta=1}^{k-1}\omega_\eta \star (cs\mathbb{F}_{u_\eta}) + \omega_k \star (cs\mathbb{F}_{u_k})); \wp^{-1}(\sum_{\eta=1}^{k-1}\omega_\eta\wp(csR_{u_\eta}) + \omega_k\wp(csR_{u_k}))$$

$$= < *^{-1}(\sum_{\epsilon=1}^{k}\omega_\epsilon * (cs\mathbb{T}_{u_\epsilon})), \star^{-1}(\sum_{\epsilon=1}^{k}\omega_\epsilon \star (cs\mathbb{I}_{u_\epsilon})), \star^{-1}(\sum_{\epsilon=1}^{k}\omega_\epsilon \star (cs\mathbb{F}_{u_\epsilon}));$$

$$\wp^{-1}(\sum_{\epsilon=1}^{k}\omega_\eta\wp(csR_{u_k})) > .$$

This completes the proof.

Definition 4.2. Let $\star, *, \wp, \rho : [0,1] \to [0,\infty)$ be mappings such that $\star(t) = -log\ t$, $*(t) = -log\ (1-t), \wp(t) = -log\ t$ and $\rho(t) = -log\ (1-t)$. The CSNWA aggregating operators listed below can be considered specific cases of definition 4.1.
$\mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k) =< 1-\prod_{\epsilon=1}^k(1-cs\mathbb{T}_{u_\epsilon})^{\omega_\epsilon}, \prod_{\epsilon=1}^k cs\mathbb{I}_{u_\epsilon}^{\omega_\epsilon}, \prod_{\epsilon=1}^k cs\mathbb{F}_{u_\epsilon}^{\omega_\epsilon}; \prod_{\epsilon=1}^k csR_{u_\epsilon}^{\omega_\epsilon} >$ and $\mathbb{CSNWA}^A_\rho(u_1, u_2, \ldots, u_k) =< 1-\prod_{\epsilon=1}^k(1-cs\mathbb{T}_{u_\epsilon})^{\omega_\epsilon}, \prod_{\epsilon=1}^k cs\mathbb{I}_{u_\epsilon}^{\omega_\epsilon}, \prod_{\epsilon=1}^k cs\mathbb{F}_{u_\epsilon}^{\omega_\epsilon}; 1-\prod_{\epsilon=1}^k(1-csR_{u_\epsilon})^{\omega_\epsilon} >$.

It can be easily prove that the $\mathbb{CSNWA}$ operator has the following properties.

1. Idempotency property: If all u_η $(\eta = 1, 2, \ldots, k)$ are equal, that is, $u_\eta = u$ for any η, then $\mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k) = u$.

2. Boundary property: Let $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ be a collection of $CSNVs$ in $\mathbb{X}$, and $u^- = min_\eta u_\eta$, $u^+ = max_\eta u_\eta$. Then $u^- \leq \mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k) \leq u^+$.

3. Monotonicity property: Let u_η $(\eta = 1, 2, \ldots, k)$ and u'_η $(\eta = 1, 2, \ldots, k)$ be two $CSNVs$ in $\mathbb{X}$. If $u_\eta \leq u'_\eta$, then $\mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k) \leq \mathbb{CSNWA}^A_\wp(u'_1, u'_2, \ldots, u'_k)$.

4. Permutation property: Let $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ be a collection of $CSNVs$ in $\mathbb{X}$, Then $\mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k) = \mathbb{CSNWA}^A_\wp(u'_1, u'_2, \ldots, u'_k)$, where $\mathbb{CSNWA}^A_\wp(u_1, u_2, \ldots, u_k)$ is a permutation of $\mathbb{CSNWA}^A_\wp(u'_1, u'_2, \ldots, u'_k)$.

5. **Weighted Geometric Cubic Spherical Neutrosophic Aggregation Operators**

Definition 5.1. Consider the collection $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ of $CSNVs$. If $\star : [0,1] \to [0,\infty)$ is the AFG of a continuous ATN and $*(t) = \star(1-t)$ and $\wp : [0,1] \to [0,\infty)$ is the AFG of a continuous ATN or $ATCN$. Then a weighted geometric cubic spherical neutrosophic aggregation operator is defined and denoted by

$\mathbb{CSNWG}_{\wp}(u_1, u_2, \ldots, u_k) = (\wp)\bigotimes_{\epsilon=1}^{k} \omega_\epsilon u_\epsilon$, where $0 \leq \omega_\epsilon \leq 1$ for all $\epsilon = 1, 2, 3, \ldots, k$ subject to the condition $\sum_{\epsilon=1}^{k} \omega_\epsilon = 1$.

Theorem 5.1. *Consider the collection* $\{u_\epsilon =< cs\mathbb{T}_{u_\epsilon}, cs\mathbb{I}_{u_\epsilon}, cs\mathbb{F}_{u_\epsilon}; csR_{u_\epsilon} >: \epsilon = 1, 2, 3, \ldots, k\}$ *of* $CSNVs$*. If* $\star : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *and* $*(t) = \star(1-t)$ *and* $\wp : [0,1] \to [0,\infty)$ *is the* AFG *of a continuous* ATN *or* $ATCN$*. Then we have*
$\mathbb{CSNWG}_{\wp}(u_1, u_2, \ldots, u_k) =< \star^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon f(cs\mathbb{T}_{u_\epsilon}))$,
$*^{-1}\left(\sum_{\epsilon=1}^{k} \omega_\epsilon * (cs\mathbb{I}_{u_\epsilon})\right), *^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon * (cs\mathbb{F}_{u_\epsilon})); \wp^{-1}(\sum_{\epsilon=1}^{k} \omega_\epsilon \wp(csR_{u_\epsilon})) >$
where $0 \leq \omega_\epsilon \leq 1$ *for any* $\epsilon = 1, 2, 3, \ldots, k$ *subject to the condition* $\sum_{\epsilon=1}^{k} \omega_\epsilon = 1$.
Proof. *Straightforward to Theorem 4.1.*

Definition 5.2. Let $\star, *, \wp, \rho : [0,1] \to [0,\infty)$ be mappings such that $\star(t) = -log\, t$, $*(t) = -log\,(1-t)$, $\wp(t) = -log\, t$ and $\rho(t) = -log\,(1-t)$. The CSNWA aggregating operators listed below can be considered specific cases of definition 5.1:
$\mathbb{CSNWG}^{a}_{\wp}(u_1, u_2, \ldots, u_k) =$
$< \prod_{\epsilon=1}^{k} cs\mathbb{T}_{u_\epsilon}^{\omega_\epsilon}, 1 - \prod_{\epsilon=1}^{k}(1 - cs\mathbb{I}_{u_\epsilon})^{\omega_\epsilon}, 1 - \prod_{\epsilon=1}^{k}(1 - cs\mathbb{F}_{u_\epsilon})^{\omega_\epsilon}; \prod_{\epsilon=1}^{k} csR_{u_\epsilon}^{\omega_\epsilon} >$
$\mathbb{CSNWG}^{a}_{\rho}(u_1, u_2, \ldots, u_k) =$
$< \prod_{\epsilon=1}^{k} cs\mathbb{T}_{u_\epsilon}^{\omega_\epsilon}, 1 - \prod_{\epsilon=1}^{k}(1 - cs\mathbb{I}_{u_\epsilon})^{\omega_\epsilon}, 1 - \prod_{\epsilon=1}^{k}(1 - cs\mathbb{F}_{u_\epsilon})^{\omega_\epsilon}; 1 - \prod_{\epsilon=1}^{k}(1 - csR_{u_\epsilon})^{\omega_\epsilon} >$

6. An Application of Cubic Spherical Neutrosophic Values

For $CSNVs$, we define a similarity measure in this section. Then, in a cubic spherical neutrosophic fuzzy environment, we provide an $MCDM$ technique employing this similarity measure and the suggested aggregation operators. Then, using the suggested approach, we resolve a real-world decision problem from the literature involving picking the optimal solar cell.

6.1. *A similarity Measure for Cubic Spherical Neutrosophic Values*

In an uncertain context, similarity measures are crucial tools for figuring out how similar things are to one another. Due to their ability to handle ambiguity and the fact that they have attracted many research efforts based on similarity measures within neutrosophic research, more and more researchers have begun to explore NSs. The following is the similarity measure for $CSNVs$.

Definition 6.1. If $u =< cs\mathbb{T}_u, cs\mathbb{I}_u, cs\mathbb{F}_u; csR_u >$ and $v =< cs\mathbb{T}_v, cs\mathbb{I}_v, cs\mathbb{F}_u; csR_u >$ be $CSNVs$ in $\mathbb{X}$. Then the cubic spherical cosine similarity measure is defined and denoted by $cs\mathbb{CSM}(u,v) = \frac{cs\mathbb{T}_u cs\mathbb{T}_v + cs\mathbb{I}_u cs\mathbb{I}_v + cs\mathbb{F}_u cs\mathbb{F}_v}{\sqrt{cs\mathbb{T}_u^2 + cs\mathbb{I}_u^2 + cs\mathbb{F}_u^2}\sqrt{cs\mathbb{T}_v^2 + cs\mathbb{I}_v^2 + cs\mathbb{F}_v^2}} \times \frac{|csR_u - csR_v|}{\max\{csR_u, csR_v\}}$.

6.2. *A MCDM Method*

In the cubic spherical neutrosophic environment, an $MCDM$ method is suggested in this section. The suggested approach is used to solve an $MCDM$ problem that has been taken from the literature to demonstrate its effectiveness in the next subsection. Following are the steps of the suggested method that we can present:

Step 1: Suppose there are k alternatives that $A = \{A_1, A_2, \ldots, A_k\}$ expert has evaluated in light of a list with j criteria as $C = \{c_1, c_2, \ldots, c_\eta\}$.

Step 2: For each criterion, the expert chooses the weight vector and converts the assessment results of the alternatives into $CSNVs$.

Step 3: If there are any cost criteria based on their values, the complement operation is used.

Step 4: Evaluation findings for each choice that are expressed as "$CSNVs$ are transformed using suggested weighted aggregation operations.

Step 5: The variation in $cs\mathbb{CSM}$ between each alternative's aggregate value and the ideal alternative's positive value $< 1, 0, 0; 1 >$ is determined.

Step 6: The alternative with the greatest similarity value is taken to be the best.

6.3. *Selection of Electric truck using CSNVs*

In recent years, there has been an enormous increase in demand for electric vehicles. As a result of its success in the global and regional markets across many demography, several renowned heavy-duty vehicle manufacturers started investing in developing electric heavy commercial vehicles as a sustainable solution that will replace conventional heavy commercial vehicles for its key advantage of zero air pollution. In India, the demand for electric trucks is constantly growing and the government works on several policies and agendas to meet the increasing demand and to further boost the sales of electric commercial heavy vehicles across all categories. The automobile manufacturers are competing to capture the growing Indian market by launching a range of electric commercial vehicles from electric auto rickshaw that has a payload capacity of a few hundred kilograms to full-scale electric trucks that pull tonnes of load.

At Truck Junction, more than 326 electric commercial vehicles are available. When compared to a conventional commercial vehicle that has a mileage of 8-14 kmpl, an electric vehicle has a range of up to 300km per charge which varies depending on the payload. The typical charging time of an electric vehicle ranges between 4-6 hours. The payload capacity of a typical electric truck ranges between 3.5 tons to 12 tons. When comparing a conventional vehicle with an electric vehicle from the construction point of view, the major upgrade is the replacement of an IC engine with motors. These are BLDC motors which have an efficiency of around 96-98 percent whereas a conventional IC engine can have a maximum efficiency of 36 percent.

Further, the battery packs are mounted under the frame with the motors either placed at center and the wheels are connected via transmission rods or each wheel has a dedicated motor with an integrated gearbox for high torque application. This gives a strategic advantage for an electric truck in terms of distribution of weight evenly across the length of the vehicle and allows more loading space.

Step 1: Consider that the company's engineering, project and purchasing divisions each have three experts ($cs\mathbb{T}_1$, $cs\mathbb{T}_2$ and $cs\mathbb{T}_3$). The set of 6 suppliers, $\mathbb{ET}_1 - \mathbb{ET}_6$, were selected by the three experts from the departments based on seven distinct criteria, including estimated cost ($\mathbb{C}_1$), delivery efficiency ($\mathbb{C}_2$), product flexibility ($\mathbb{C}_3$), reputation and management level ($\mathbb{C}_4$) and eco-design ($\mathbb{C}_5$).

It is essential for the company to ensure that its suppliers care about the environment and follow green guidelines in how they run their business. A supplier is more appealing to a company if they are more environmentally friendly. Furthermore, it may be beneficial to forge a long-term partnership with eco-friendly providers. The alternative ratings on the linguistic scale [LS] used by the decision-makers are shown in Table 2 together with their particular interpretations.

Linguistics Term	Symbolic representation	$< cs\mathbb{T}_U, cs\mathbb{I}_U, cs\mathbb{F}_U > \times 10^{-1}$
No influence	$\emptyset_1$	$< 1, 8, 9 >$
Low influence	$\emptyset_2$	$< 4, 6, 7 >$
Medium influence	$\emptyset_3$	$< 5, 4, 5 >$
High influence	$\emptyset_4$	$< 8, 2, 2 >$
Very high influence	$\emptyset_5$	$< 9, 1, 1 >$

TABLE 2. LS for the calculation of DM's priorities.

Step 2: Expert recommendations over suppliers according to the each criteria is shown in Table 3.

DM's	$\mathbb{T}_1$					$\mathbb{T}_2$					$\mathbb{T}_3$				
$\mathbb{A}s$	$cs\mathbb{C}_1$	$cs\mathbb{C}_2$	$cs\mathbb{C}_3$	$cs\mathbb{C}_4$	$cs\mathbb{C}_5$	$cs\mathbb{C}_1$	$cs\mathbb{C}_2$	$cs\mathbb{C}_3$	$cs\mathbb{C}_4$	$cs\mathbb{C}_5$	$cs\mathbb{C}_1$	$cs\mathbb{C}_2$	$cs\mathbb{C}_3$	$cs\mathbb{C}_4$	$cs\mathbb{C}_5$
$\mathbb{ET}_1$	$\emptyset_4$	$\emptyset_1$	$\emptyset_3$	$\emptyset_2$	$\emptyset_5$	$\emptyset_3$	$\emptyset_2$	$\emptyset_1$	$\emptyset_4$	$\emptyset_3$	$\emptyset_5$	$\emptyset_1$	$\emptyset_5$	$\emptyset_4$	$\emptyset_5$
$\mathbb{ET}_2$	$\emptyset_1$	$\emptyset_4$	$\emptyset_2$	$\emptyset_3$	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	$\emptyset_4$	$\emptyset_1$	$\emptyset_2$	$\emptyset_4$	$\emptyset_3$	$\emptyset_4$	$\emptyset_3$	$\emptyset_4$
$\mathbb{ET}_3$	$\emptyset_3$	$\emptyset_4$	$\emptyset_5$	$\emptyset_2$	$\emptyset_5$	$\emptyset_5$	$\emptyset_2$	$\emptyset_2$	$\emptyset_3$	$\emptyset_5$	$\emptyset_3$	$\emptyset_4$	$\emptyset_2$	$\emptyset_2$	$\emptyset_3$
$\mathbb{ET}_4$	$\emptyset_1$	$\emptyset_5$	$\emptyset_1$	$\emptyset_3$	$\emptyset_1$	$\emptyset_1$	$\emptyset_3$	$\emptyset_5$	$\emptyset_1$	$\emptyset_1$	$\emptyset_2$	$\emptyset_1$	$\emptyset_3$	$\emptyset_5$	$\emptyset_2$
$\mathbb{ET}_5$	$\emptyset_5$	$\emptyset_3$	$\emptyset_3$	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	$\emptyset_4$	$\emptyset_4$	$\emptyset_5$	$\emptyset_3$	$\emptyset_3$	$\emptyset_5$	$\emptyset_1$	$\emptyset_1$	$\emptyset_3$
$\mathbb{ET}_6$	$\emptyset_2$	$\emptyset_3$	$\emptyset_1$	$\emptyset_5$	$\emptyset_4$	$\emptyset_1$	$\emptyset_5$	$\emptyset_3$	$\emptyset_2$	$\emptyset_1$	$\emptyset_2$	$\emptyset_4$	$\emptyset_5$	$\emptyset_3$	$\emptyset_2$

TABLE 3. Expert recommendations over suppliers according to the criteria

Step 3: Table 8 in Appendix A, transforms linguistic evaluations into NVs. The main variable pairwise comparison matrix for each decision maker is shown in Table 9. We take the complement of these values since C_1 and C_4 are the cost criteria. Thus, we obtain the cubic spherical neutrosophic group normalized matrix illustrated in Table 4. The NVs in this decision matrix $[DM]$ must be transformed into $CSNVs$. In this approach, The decision matrix in Table 9 is used to determine the greatest radius values.

Sup	$cs\mathbb{C}_1 \times 10^{-1}$	$cs\mathbb{C}_2 \times 10^{-1}$	$cs\mathbb{C}_3 \times 10^{-1}$	$cs\mathbb{C}_4 \times 10^{-1}$	$cs\mathbb{C}_5 \times 10^{-1}$
$\mathbb{ET}_1$	$< 2,2,7;4 >$	$< 2,7,8;3 >$	$< 5,4,5;7 >$	$< 3,3,7;5 >$	$< 8,2,2;4 >$
$\mathbb{ET}_2$	$< 6,5,4;7 >$	$< 6,3,4;3 >$	$< 7,3,3;5 >$	$< 6,5,4;5 >$	$< 4,5,6;7 >$
$\mathbb{ET}_3$	$< 3,3,6;4 >$	$< 7,3,3;5 >$	$< 5,4,5;6 >$	$< 6,5,4;2 >$	$< 8,2,2;4 >$
$\mathbb{ET}_4$	$< 8,7,2;3 >$	$< 5,4,5;7 >$	$< 5,4,5;7 >$	$< 5,4,5;7 >$	$< 2,7,8;3 >$
$\mathbb{ET}_5$	$< 3,3,6;4 >$	$< 7,2,2;4 >$	$< 5,5,5;6 >$	$< 6,6,4;9 >$	$< 5,5,5;2 >$
$\mathbb{ET}_6$	$< 8,7,3;3 >$	$< 7,2,2;4 >$	$< 5,4,5;7 >$	$< 4,4,6;5 >$	$< 4,5,6;7 >$

Table 4. The CSN decision matrix

Step 4: Applying the aggregation operations $\mathbb{CSNWA}^A_\wp$, $\mathbb{CSNWA}^A_\rho$, $\mathbb{CSNWG}^A_\wp$ and $\mathbb{CSNWG}^A_\rho$ defined via $*(t) = -log\,t, h(t) = -log\,(1-t), \wp(t) = -log\,t$ and $\rho(t) = -log\,(1-t)$, the decision matrix expressed with $CSNVs$ for all decisions is aggregated. Table 5 presents the aggregated cubic spherical neutrosophic decision matrix using $CSNVs$.

Sup	$\mathbb{CSNWA}^A_\wp \times 10^{-1}$	$\mathbb{CSNWA}^A_\rho \times 10^{-1}$	$\mathbb{CSNWG}^A_\wp \times 10^{-1}$	$\mathbb{CSNWG}^A_\rho \times 10^{-1}$
$\mathbb{ET}_1$	$< 4,4,6;4 >$	$< 4,4,6;4 >$	$< 3,5,7;4 >$	$< 3,5,7;4 >$
$\mathbb{ET}_2$	$< 6,4,4;5 >$	$< 6,4,4;5 >$	$< 6,4,4;5 >$	$< 6,4,4;5 >$
$\mathbb{ET}_3$	$< 6,3,4;5 >$	$< 6,3,4;5 >$	$< 6,3,4;5 >$	$< 6,3,4;5 >$
$\mathbb{ET}_4$	$< 6,5,4;5 >$	$< 6,5,4;6 >$	$< 5,6,5;5 >$	$< 5,6,5;6 >$
$\mathbb{ET}_5$	$< 6,3,4;4 >$	$< 6,3,4;5 >$	$< 5,4,4;4 >$	$< 5,4,4;5 >$
$\mathbb{ET}_6$	$< 7,4,3;4 >$	$< 7,4,3;5 >$	$< 6,4,4;4 >$	$< 6,4,4;5 >$

Table 5. Aggregated values of CSNSs

Step 5: Each aggregated $CSNV$ and positive ideal alternative are compared using the CSM established in Definition 6.1 to determine exactly related or equivalent they are to each other. The evaluation test between alternatives and the ideal positive alternative's results are shown in Table 6.

Step 6: The ranking of electric trucks:

The selection of electric trucks using cubic spherical neutrosophic sets offers a comprehensive framework for decision-making in the procurement process. By aggregating expert recommendations and criteria evaluations into cubic spherical neutrosophic decision matrices, we can

Methods / $cs\mathbb{CSM}$	$cs\mathbb{CSM}$ (A_1, A^+)	$cs\mathbb{CSM}$ (A_2, A^+)	$cs\mathbb{CSM}$ (A_3, A^+)	$cs\mathbb{CSM}$ (A_4, A^+)	$cs\mathbb{CSM}$ (A_5, A^+)	$cs\mathbb{CSM}$ (A_6, A^+)
$\mathbb{CSNWA}^A_\wp$	0.512	0.700	0.786	0.624	0.761	0.788
$\mathbb{CSNWA}^A_\rho$	0.512	0.700	0.786	0.624	0.761	0.788
$\mathbb{CSNWG}^A_\wp$	0.327	0.677	0.714	0.492	0.686	0.732
$\mathbb{CSNWG}^A_\rho$	0.327	0.677	0.714	0.492	0.686	0.732

TABLE 6. Cosine similarity scores

Methods	Ranking	Best EV
$\mathbb{CSNWAAO} - \mathbb{CD}$ [5]	$\mathbb{ET}_6 > \mathbb{ET}_5 > \mathbb{ET}_3 > \mathbb{ET}_2 > \mathbb{ET}_4 > \mathbb{ET}_1$	$\mathbb{ET}_6$
$\mathbb{CSNWGAO} - \mathbb{CD}$ [5]	$\mathbb{ET}_6 > \mathbb{ET}_3 > \mathbb{ET}_5 > \mathbb{ET}_2 > \mathbb{ET}_1 > \mathbb{ET}_4$	$\mathbb{ET}_6$
$\mathbb{CSNWA}^A_\wp$	$\mathbb{ET}_6 > \mathbb{ET}_3 > \mathbb{ET}_5 > \mathbb{ET}_2 > \mathbb{ET}_4 > \mathbb{ET}_1$	$\mathbb{ET}_6$
$\mathbb{CSNWA}^A_\rho$	$\mathbb{ET}_6 > \mathbb{ET}_3 > \mathbb{ET}_5 > \mathbb{ET}_2 > \mathbb{ET}_4 > \mathbb{ET}_1$	$\mathbb{ET}_6$
$\mathbb{CSNWG}^A_\wp$	$\mathbb{ET}_6 > \mathbb{ET}_3 > \mathbb{ET}_5 > \mathbb{ET}_2 > \mathbb{ET}_4 > \mathbb{ET}_1$	$\mathbb{ET}_6$
$\mathbb{CSNWG}^A_\rho$	$\mathbb{ET}_6 > \mathbb{ET}_3 > \mathbb{ET}_5 > \mathbb{ET}_2 > \mathbb{ET}_4 > \mathbb{ET}_1$	$\mathbb{ET}_6$

TABLE 7. Overall ranking of Electric trucks

effectively rank electric trucks based on various criteria. The final Table 7 presents the overall ranking of electric trucks using different aggregation methods, providing valuable insights for decision-makers in selecting the most suitable electric truck for their needs.

Comparison Analysis

We compared the results of proposed methods with the existing CSNS methods and their visualization represents the overall raking of electric trucks are presented : The cubic spherical

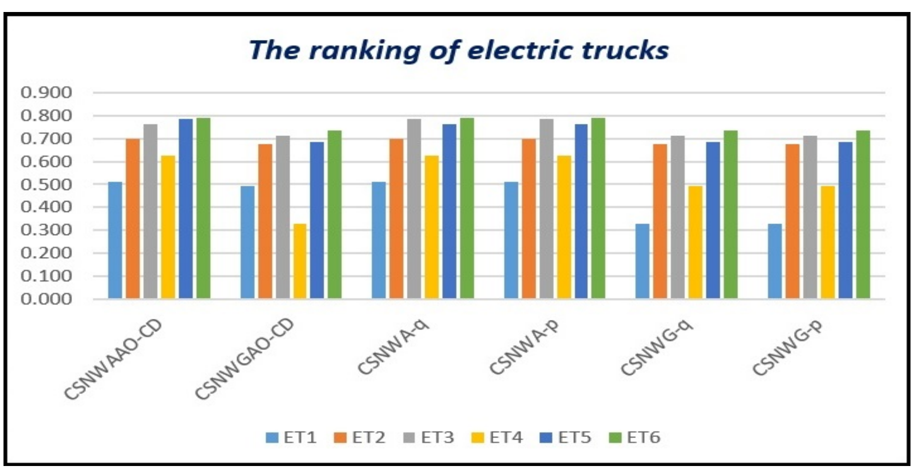

FIGURE 1. Comparison of Ranking of Electric Trucks

neutrosophic set utilizes a spherical framework to portray uncertainty among true, false, and neutral functions, addressing vagueness comprehensively. This method surpasses traditional averaging, providing a collective opinion representation. In Multi-Criteria Decision Making (MCDM), the decision maker's engagement determines criteria weights and preferences; a substantial influence is recommended for sphere representation. Recognizing CSNS limitations is crucial for effective MCDM utilization. Addressing constraints enhances CSNS applicability and reliability in decision-making contexts.

7. Comparison of Cubic Spherical Neutrosophic Sets (CSNS) with Traditional Neutrosophic Sets (NS)

To compare cubic spherical neutrosophic sets (CSNS) with traditional neutrosophic sets (NS), we consider several aspects:

(1) **Representation of Uncertainty:**
Neutrosophic sets and CSNS handle uncertainty through three parameters: truth membership (T), indeterminacy membership (I) and falsity membership (F). However, CSNS extend this representation by adding a fourth parameter, the radius (r), which captures the degree of neutrality or neutrality in the information provided.

(2) **Geometric Interpretation:**
CSNS provide a geometric interpretation of uncertainty in a hypersphere, where the center represents T, I and F and the radius represents the degree of neutrality (r). This geometric interpretation allows for a more intuitive understanding of uncertainty.

(3) **Aggregation Operators:**
While neutrosophic sets have aggregation operators for combining uncertain information, CSNS introduce weighted geometric aggregation operators tailored specifically to handle uncertainty represented in hyperspheres. These operators take into account T, I, F and r, providing a comprehensive way to combine uncertain information.

(4) **Handling Neutrality:**
CSNS explicitly account for neutrality through the radius parameter, which represents the degree to which an element is neither true nor false nor indeterminate. This allows for a more nuanced handling of neutrality compared to traditional neutrosophic sets.

(5) **Archimedean Triangular Norms and Conorms:**
Archimedean triangular norms and conorms are commonly used in fuzzy logic and fuzzy set theory to model conjunction and disjunction operations. CSNATN and CSNATCN extend these operations to CSNS, allowing for the combination of uncertain information in a way that respects the geometric structure of hyperspheres.

8. Conclusion

The main aim of this study is to introduce the concept of a $CSNS$, defined as a sphere with radius r and a triple at its center, representing membership, indeterminacy, and non-participation. $CSNSs$ extend the idea of NSs by depicting these degrees through spheres. Arithmetic operators such as $CSNWA$ and $CSNWG$ are crucial for integrating neutrosophic data. To overcome limitations of existing operators like $NWAO$ and $NWGO$, we propose "weighted arithmetic cubic spherical neutrosophic aggregation operators" and "weighted geometric cubic spherical neutrosophic aggregation operators," offering improved reliability and effectiveness. An $MCDM$ method is developed for selecting the best electric truck based on these operators. Future work aims to enhance other arithmetic operators like Dombi, Hamacher, and Einstein through this framework.

Appendix A

Expert	$\mathbb{A}$	$cs\mathbb{C}_1 \times 10^{-1}$	$cs\mathbb{C}_2 \times 10^{-1}$	$cs\mathbb{C}_3 \times 10^{-1}$	$cs\mathbb{C}_4 \times 10^{-1}$	$cs\mathbb{C}_5 \times 10^{-1}$
$\mathbb{T}_1$	$\mathbb{ET}_1$	$<8,2,2>$	$<1,8,9>$	$<5,4,5>$	$<4,6,7>$	$<9,1,1>$
	$\mathbb{ET}_2$	$<1,8,9>$	$<8,2,2>$	$<4,6,7>$	$<5,4,5>$	$<1,8,9>$
	$\mathbb{ET}_3$	$<5,4,5>$	$<8,2,2>$	$<9,1,1>$	$<4,6,7>$	$<9,1,1>$
	$\mathbb{ET}_4$	$<1,8,9>$	$<9,1,1>$	$<1,8,9>$	$<5,4,5>$	$<1,8,9>$
	$\mathbb{ET}_5$	$<9,1,1>$	$<5,4,5>$	$<5,4,5>$	$<1,8,9>$	$<4,6,7>$
	$\mathbb{ET}_6$	$<4,6,7>$	$<5,4,5>$	$<1,8,9>$	$<9,1,1>$	$<8,2,2>$
$\mathbb{T}_2$	$\mathbb{ET}_1$	$<5,4,5>$	$<4,6,7>$	$<1,8,9>$	$<8,2,2>$	$<5,4,5>$
	$\mathbb{ET}_2$	$<4,6,7>$	$<5,4,5>$	$<8,2,2>$	$<1,8,9>$	$<4,6,7>$
	$\mathbb{ET}_3$	$<9,1,1>$	$<4,6,7>$	$<4,6,7>$	$<5,4,5>$	$<9,1,1>$
	$\mathbb{ET}_4$	$<1,8,9>$	$<5,4,5>$	$<9,1,1>$	$<1,8,9>$	$<1,8,9>$
	$\mathbb{ET}_5$	$<5,4,5>$	$<8,2,2>$	$<8,2,2>$	$<9,1,1>$	$<5,4,5>$
	$\mathbb{ET}_6$	$<1,8,9>$	$<9,1,1>$	$<5,4,5>$	$<4,6,7>$	$<1,8,9>$
$\mathbb{T}_3$	$\mathbb{ET}_1$	$<9,1,1>$	$<1,8,9>$	$<9,1,1>$	$<8,2,2>$	$<9,1,1>$
	$\mathbb{ET}_2$	$<8,2,2>$	$<5,4,5>$	$<8,2,2>$	$<5,4,5>$	$<8,2,2>$
	$\mathbb{ET}_3$	$<5,4,5>$	$<8,2,2>$	$<4,6,7>$	$<4,6,7>$	$<5,4,5>$
	$\mathbb{ET}_4$	$<4,6,7>$	$<1,8,9>$	$<5,4,5>$	$<9,1,1>$	$<4,6,7>$
	$\mathbb{ET}_5$	$<5,4,5>$	$<9,1,1>$	$<1,8,9>$	$<1,8,9>$	$<5,4,5>$
	$\mathbb{ET}_6$	$<4,6,7>$	$<8,2,2>$	$<9,1,1>$	$<5,4,5>$	$<4,6,7>$

Table 8. The matrix of pairwise comparisons for the primary DM evaluation

Expert	$\mathbb{A}$	$cs\mathbb{C}_1 \times 10^{-1}$	$cs\mathbb{C}_2 \times 10^{-1}$	$cs\mathbb{C}_3 \times 10^{-1}$	$cs\mathbb{C}_4 \times 10^{-1}$	$cs\mathbb{C}_5 \times 10^{-1}$
$\mathbb{T}_1$	$\mathbb{ET}_1$	$< 2,2,8 >$	$< 1,8,9 >$	$< 5,4,5 >$	$< 7,6,4 >$	$< 9,1,1 >$
	$\mathbb{ET}_2$	$< 9,8,1 >$	$< 8,2,2 >$	$< 4,6,7 >$	$< 5,4,5 >$	$< 1,8,9 >$
	$\mathbb{ET}_3$	$< 5,4,5 >$	$< 8,2,2 >$	$< 9,1,1 >$	$< 7,6,4 >$	$< 9,1,1 >$
	$\mathbb{ET}_4$	$< 9,8,1 >$	$< 9,1,1 >$	$< 1,8,9 >$	$< 5,4,5 >$	$< 1,8,9 >$
	$\mathbb{ET}_5$	$< 1,1,9 >$	$< 5,4,5 >$	$< 5,4,5 >$	$< 9,8,1 >$	$< 4,6,7 >$
	$\mathbb{ET}_6$	$< 7,6,4 >$	$< 5,4,5 >$	$< 1,8,9 >$	$< 1,1,9 >$	$< 8,2,2 >$
$\mathbb{T}_2$	$\mathbb{ET}_1$	$< 5,4,5 >$	$< 4,6,7 >$	$< 2,2,8 >$	$< 5,4,5 >$	$< 5,4,5 >$
	$\mathbb{ET}_2$	$< 7,6,4 >$	$< 5,4,5 >$	$< 8,2,2 >$	$< 9,8,1 >$	$< 4,6,7 >$
	$\mathbb{ET}_3$	$< 1,1,9 >$	$< 4,6,7 >$	$< 4,6,7 >$	$< 5,4,5 >$	$< 9,1,1 >$
	$\mathbb{ET}_4$	$< 9,8,1 >$	$< 5,4,5 >$	$< 9,1,1 >$	$< 9,8,1 >$	$< 1,8,9 >$
	$\mathbb{ET}_5$	$< 5,4,5 >$	$< 8,2,2 >$	$< 8,2,2 >$	$< 1,1,9 >$	$< 5,4,5 >$
	$\mathbb{ET}_6$	$< 9,8,1 >$	$< 9,1,1 >$	$< 5,4,5 >$	$< 7,6,4 >$	$< 1,8,9 >$
$\mathbb{T}_3$	$\mathbb{ET}_1$	$< 1,1,9 >$	$< 1,8,9 >$	$< 9,1,1 >$	$< 2,2,8 >$	$< 9,1,1 >$
	$\mathbb{ET}_2$	$< 2,2,8 >$	$< 5,4,5 >$	$< 8,2,2 >$	$< 5,4,5 >$	$< 8,2,2 >$
	$\mathbb{ET}_3$	$< 5,4,5 >$	$< 8,2,2 >$	$< 4,6,7 >$	$< 7,6,4 >$	$< 5,4,5 >$
	$\mathbb{ET}_4$	$< 7,6,4 >$	$< 1,8,9 >$	$< 5,4,5 >$	$< 1,1,9 >$	$< 4,6,7 >$
	$\mathbb{ET}_5$	$< 5,4,5 >$	$< 9,1,1 >$	$< 1,8,9 >$	$< 9,8,1 >$	$< 5,4,5 >$
	$\mathbb{ET}_6$	$< 7,6,4 >$	$< 8,2,2 >$	$< 9,1,1 >$	$< 5,4,5 >$	$< 4,6,7 >$

Table 9. Normalized decision matrix

References

[1] Ashraf, S.; Abdullah, S.; Smarandache, F.; Amin, N.u. Logarithmic hybrid aggregation operators based on single-valued neutrosophic sets and their applications in decision support systems. *Symmetry* **2019**, 11, 364. https://doi.org/10.3390/sym11030364

[2] Baczynski, W.M.; Jayaram, B. Fuzzy Implications. *Studies in Fuzziness and Soft Computing* **2008**, 231.

[3] Deli, I; Ali, M.; Smarandache, F. Bipolar neutrosophic sets and their applications based on multicriteria decision making problems. Advanced Mechatronic Systems. *(ICAMechs) International Conference* **2015**, 249-254. doi: 10.1109/ICAMechS.2015.7287068

[4] Garg, H.; Nancy. Multi-Criteria Decision-Making Method Based on Prioritized Muirhead Mean Aggregation Operator under Neutrosophic Set Environment. *Symmetry* **2018**, 10, 280. https://doi.org/10.3390/sym10070280

[5] Gomathi, S.; Krishnaprakash, S.; Karpagadevi, M.; Said Broumi. Cubic Spherical Neutrosophic Sets. *International Journal of Neutrosophic Science* **2023**, 21(4), 172-180.

[6] Hamid, M. T.; Riaz, M.; Naeem, K. A study on weighted aggregation operators for q-rung orthopair m-polar fuzzy set with utility to multistage decision analysis, *International Journal of Intelligent Systems* **2022**, 37(9), 6354-6387. https://doi.org/10.1002/int.22847

[7] Hu, B.; Kwong, C. K. On type-2 fuzzy sets and their t-norm operations. *Inf. Sci.* **2014**, 255, 5881. https://doi.org/10.1016/j.ins.2013.07.023

[8] Hu, Q.; Zhang, X. New Similarity Measures of Single-Valued Neutrosophic Multisets Based on the Decomposition Theorem and Its Application in Medical Diagnosis. *Symmetry* **2018**, 10, 466. https://doi.org/10.3390/sym10100466

[9] Klement, E. P.; Mesiar, R.; Pap, E. Triangular norms. Position paper I: basic analytical and algebraic properties, *Fuzzy Sets and Systems* **2004**, 143(1), 5-26. doi:10.1016/j.fss.2003.06.007

[10] Kumam, W.; Naeem, K.; Riaz, M.; Khan, M. J.; Kumam, P. Comparison measures for Pythagorean m-polar fuzzy sets and their applications to robotics and movie recommender system. *AIMS Mathematics* **2023**, 8(5), 10357-10378. doi: 10.3934/math.2023524

[11] Klement, E. P.; Mesiar, R.; Pap, E. Triangular norms. Position paper I: Basic analytical and algebraic properties. *Fuzzy Sets Syst.* **2004**, 143, 526. https://doi.org/10.1016/j.fss.2003.06.007

[12] Li, D. Type-2 triangular norms and their residual operators. *Inf. Sci.* **2015**, 317, 259277. https://doi.org/10.1016/j.ins.2016.01.092

[13] Liu, P. The aggregation operators based on archimedean t-conorm and t-norm for single-valued neutrosophic numbers and their application to decision making. *Int. J. Fuzzy Syst.* **2016**, 18, 849863.

[14] Logical Algebraic, Analytic and Probabilistic Aspects of Triangular Norms. New York, NY, USA: Elsevier; **2005** edited by E.P Klement and R. Mesiar.

[15] Menger, K. Statistical metrics. Natl. *Acad. Sci.* **1942**, 28, 535537. doi: 10.1073/pnas.28.12.535

[16] Molla, M. U.; Giri, B. C. Multi-criteria decision making problem with spherical neutrosophic sets. *OPSEARCH* **2022**. https://doi.org/10.1007/s12597-022-00593-1

[17] Naeem, K.; Divvaz, B. Information measures for MADM under m-polar neutrosophic environment. *Granular Computing* **2023**, 8, 597616. https://doi.org/10.1007/s41066- 022-00340-3

[18] Nguyen H. T; Walker R. A. A First Course in Fuzzy Logic. *Boca Raton, Fla, USA: CRC Press*, **1997**.

[19] Palanikumar, M.; Arulmozhi, K. MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators. *Neutrosophic Sets and Systems* **2022**, 51, 1.

[20] Peide Liu; Yanchang Chu; Yanwei Li; Yubao Chen. Some Generalized Neutrosophic Number Hamacher Aggregation Operators and Their Application to Group Decision Making. *International Journal of Fuzzy Systems* **2014**, 16(2), 242-255.

[21] Peng, H g.; Zhang, Hy.; Wang, Jq. Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems. *Neural Comput and Applic* **2018** 30, 563583. https://doi.org/10.1007/s00521-016-2702-0

[22] Rehman, Ateeq Ur; Muhammad Gulistan; Zahid Khan; Fuad S. Al-Duais. A Study of Neutrosophic Cubic Hesitant Fuzzy Hybrid Geometric Aggregation Operators and its Application to Multi Expert Decision Making System. *Neutrosophic Sets and Systems* **2022**, 50, 1. https://digitalrepository.unm.edu/nss_journal/vol50/iss1/5

[23] Riaz, M.; Naeem, K. and Afzal, D. A similarity measure under Pythagorean fuzzy soft environment with applications. *Comp. Appl. Math.* **2020**, 39, 269. https://doi.org/10.1007/s40314-020-01321-5

[24] Schweizer, B.; Sklar, A. Associative mappings and statistical triangle inequalities. *Publ. Math.* **1961**, 8, 169186.

[25] Schweizer, B.; Sklar, A. Probabilistic Metric Spaces. *North-Holl and: New York, NY, USA* **1983**.

[26] Siraj, A; Fatima, T., Afzal, D., Naeem, K. and Karaaslan, F. Pythagorean m-polar Fuzzy Neutrosophic Topology with Applications. *Neutrosophic Sets and Systems* **2022**, 48, 251-290. Retrieved from https://fs.unm.edu/NSS2/index.php/111/article/view/2103

[27] Smarandache, F. Neutrosophy: Neutrosophic Probability. Set and Logic: Analytic Synthesis and Synthetic Analysis. *American Research Press: Santa Fe, NM, USA* **1998**, 2538.

[28] Wang; Haibin; Florentin Smarandache; Yan-Qing Zhang; Rajshekhar Sunderraman. *Interval neutrosophic sets and logic; theory and applications in computing* **2005**. https://digitalrepository.unm.edu/math_fsp/38

[29] Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. "Single valued neutrosophic sets. Multispace Multistruct. *Neutrosophic Transdiscipl.* **2010**, 4, 410413.

[30] Wang, L.; Zhang, Hy.; Wang, Jq. Frank Choquet Bonferroni Mean Operators of Bipolar Neutrosophic Sets and Their Application to Multi-criteria Decision-Making Problems. *Int. J. Fuzzy Syst* **2018**. 20, 1328. https://doi.org/10.1007/s40815-017-0373-3

[31] Wu, Xh., Wang, Jq.; Peng, Jj. et al. Cross-Entropy and Prioritized Aggregation Operator with Simplified Neutrosophic Sets and Their Application in Multi-Criteria Decision-Making Problems. *Int. J. Fuzzy Syst.* **2016**, 18, 11041116. https://doi.org/10.1007/s40815-016-0180-2

[32] Yingying Zhang; Rui Yong; Jun Ye, Zhen Zhong; Shuang Zheng. Dombi Aggregation Operators of Linguistic Neutrosophic Numbers for Multiple Attribute Group Decision-Making Problems in Landslide Treatment Schemes. *Neutrosophic Sets and Systems* **2022**, 51, 724-743. doi: 10.5281/zenodo.7135407

[33] Ye, J. Single valued neutrosophic minimum spanning tree and its clustering method. *Journal of Intelligent Systems* **2014**, 23(3), 311324.

[34] Ye, Jun and Florentin Smarandache. Similarity measure of refined single-Valued neutrosophic sets and its multicriteria decision making method. *Neutrosophic Sets and Systems* **2016**, 12, 1. https://digitalrepository.unm.edu/nss_journal/vol12/iss1/5

[35] Ye, Jun. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent and Fuzzy Systems* **2014**, 26(5), 2459-2466. doi: 10.3233/IFS-130916

[36] Zhang, B. Notes on type-2 triangular norms and their residual operators. *Inf. Sci.* **2016**, 346, 338350. doi: https://doi.org/10.1016/j.ins.2016.01.092

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