Neutrosophic Sets and Systems, Vol. 67, 2024
University of New Mexico

Pythagorean Neutrosophic Triplet Groups

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Abstract: It is a well-known fact that groups are the only algebraic structures having a single binary operation that is mathematically so perfect that it is impossible to introduce a richer structure within it. The main purpose of this study is to introduce the notion of the Pythagorean neutrosophic triplet (PNT) which is the generalization of neutrosophic triplet (NT). The PNT is an algebraic structure of three ordered pairs that satisfy several properties under the binary operation (B-Operation) $"*"$. Furthermore, we used the PNTs to introduce the novel concept of a Pythagorean neutrosophic triplet group (PNTG). The algebraic structure (AS) of PNTG is different from the neutrosophic triplet group (NTG). We discussed some properties, related results, and particular examples of these novel concepts. We further studied Pythagorean neutro-homomorphism, Pythagorean neutro-isomorphism, etc., for PNTGs. Moreover, we discussed the main distinctions between the neutrosophic triplet group (NTG) and the PNTG.

Keywords: Neutrosophic triplet, Pythagorean neutrosophic triplet, Neutrosophic triplet group, Pythagorean neutrosophic triplet group.

1. Introduction

Neutrosophy is a novel tributary of philosophy associated with the origin, nature, and scope of neutralities. Smarandache [21] defined the notion of neutrosophic set (NS) and neutrosophic logic (NL) in 1995. Each proposition in NL is almost to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. The NS is the extension of fuzzy sets (FSs) [25], classical sets [12], intuitionistic fuzzy sets (IFSs) [6], Pythagorean fuzzy sets (PFSs) [16], and interval-valued fuzzy sets, etc., while as NL is the generalization of fuzzy logic, intuitionistic fuzzy logic, ete. The theory of neutrosophic set [22] is utilized to discuss problems involving imprecision, uncertainty, indeterminacy, incompleteness, inconsistency, and falsity. Smarandache and Kandasamy used neutrosophic theory to investigate neutrosophic ASs in [11], [12], [13] by embedding an indeterminacy I'' into the AS. They combine indeterminacy with the elements of the AS under the binary operation $*$ and say it neutrosophic element (NE), and the novel AS is known as neutrosophic AS. Moreover, they further developed several NASs such as neutrosophic fields, neutrosophic groups, neutrosophic N-groups, neutrosophic bigroups, neutrosophic vector spaces, neutrosophic bisemigroups, neutrosophic semigroups, neutrosophic N-semigroup, neutrosophic groupoids, neutrosophic bigroupoids, neutrosophic biloops, neutrosophic loops, and neutrosophic N-loop, and so on.

In algebraic structures, the groups [8], [9], [24] are so significant that they contribute the role of backbone in practically all the theory of ASs. In the study of algebra, Groups are the most foundational and well-off AS under certain binary operations 👎. Groups give concrete foundations for many ASs, such as rings, fields, vector spaces, and so on. Many other fields, including physics, chemistry, combinatorics, biology, and others, use groups to explore symmetries and other behavior among their elements. Group action is the most crucial feature of a group. In daily life problems, matrix groups, permutation groups, lie groups, transformation groups, and other forms of groups are widely employed as a mathematical tools. In this regard, generalized groups [8] are very significant. For the first time, Smarandache and Ali [20] gave the notion of neutrosophic triplet (NT). The newly defined notion of NTs is highly dependable on the binary operation $*$. Furthermore, they utilized the concept of NTs to introduce an NTG. Regarding to the structural and foundational properties, the NTG is distinct from the classical group. The NT has a strong structure than Molaei's Generalized Group [15]. Zhang et al. [26] defined a new congruence relation based on commutative NTG. They induced quotient structure by neutrosophic triplet subgroup and proved neutron-homomorphism basic theorem. Zhang et al. [27] gave the notions of neutrosophic triplet subgroups, strong neutrosophic triplet subgroups and weak commutative neutrosophic triplet groups. Further, they established quotient structures on strong neutrosophic triplet subgroups. Hu and Zhang studied the relationships among various neutrosophic extended triplet cyclic associative semihypergroups [10]. The main properties of strong pure neutrosophic extended triplet cyclic associative semihypergroups are obtained. A lot of researchers have been dealing with neutrosophic triplet metric space, neutrosophic triplet vector space, neutrosophic triplet inner product, and neutrosophic triplet normed space in ([17], [18], [19]). The concept of neutrosophic extended triplet was given by by Smarandache in [23]. The neutrosophic extended triplet is the generalization of neutrosophic triplet. Li et al. [14] studied neutrosophic extended triplet group based on neutrosophic quadruple numbers. They proved some significant results with respect to neutrosophic quadruple numbers. Bal et al. [7] defined the neutrosophic image, neutrosophic inverse-image, neutrosophic kernel, and the NET subgroup. The notion of the neutrosophic triplet coset and its relation with the classical coset were discussed. Furthermore, the neutrosophic triplet normal subgroups, and neutrosophic triplet quotient groups were studied. Zhou and Xin proposed the notion of ideals on neutrosophic extended triplet groups and discussed their different properties [29]. The concept of singular neutrosophic extended triplet group was given by Zhang et al. in [28]. They discussed different properties and proved some significant results.

The concept of neutrosophic ring was defined in [11] which is the generalization of classical rings. Agboola et al. [1,2] introduced the notion of refined neutrosophic ring which is the extension of neutrosophic ring. Ahmad et al. [3] solved the imperfect duplets problem in refined neutrosophic rings. Ali et al. [4] proposed the concepts of zero divisor, neutrosophic triplet subring, neutrosophic triplet ideal, nilpotent integral neutrosophic domain, and neutrosophic triplet ring homomorphism. Moreover, they gave the idea of a neutrosophic triplet field.

In this paper, we define the notion of the PNT which is the generalization of NT. We utilize the PNTs to introduce the novel concept of a PNTG. The algebraic structure of PNTG is different from the neutrosophic triplet group (NTG). We discuss some properties, related results, and particular examples of these novel concepts. We further study Pythagorean neutro-homomorphism, Pythagorean neutro-isomorphism, etc., for PNTGs. Moreover, we study the main distinctions between the NTG and the PNTG.

3. Neutrosophic Triplet Groups

In this section, we will recall the notions of NTGs.

Definition [20] Let $\aleph$ be a set together with a B-Operation $*$. Let ℓ be an element of $\aleph$. If there exist a neutral of ℓ characterized by $\overset{+}{\aleph}(\ell)$, and an opposite of ℓ characterized by $\overset{-}{\aleph}(\ell)$, with $\overset{+}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\ell)$ are the elements of $\aleph$, such that:

$$\ell * \overset{+}{\aleph}(\ell) = \overset{+}{\aleph}(\ell) * \ell = \ell,$$

and

$$\ell * \overset{-}{\aleph}(\ell) = \overset{-}{\aleph}(\ell) * \ell = \overset{+}{\aleph}(\ell).$$

Then, $\aleph$ is said to be an NT set.

The elements ℓ, $\overset{+}{\aleph}(\ell)$, and $\overset{-}{\aleph}(\ell)$ are collectively called as NT. The NT set is denoted by $(\ell, \overset{+}{\aleph}(\ell), \overset{-}{\aleph}(\ell))$, where ℓ is the first coordinate of NT, and by $\overset{+}{\aleph}(\ell)$, we mean neutral of ℓ.

Note that $\overset{+}{\aleph}(\ell)$ is not same as the classical algebraic unitary element. For the same element ℓ in N, the neutral "$\overset{+}{\aleph}(\ell)$" and opposites "$\overset{-}{\aleph}(\ell)$" of ℓ are not unique.

Definition [20] Let $(\aleph, *)$ be a NTS. Then, $\aleph$ is said to be a NTG, if the following properties are hold.

(i). If $(\aleph, *)$ is well-defined, i.e. for any ℓ, $\wp \in \aleph$, one has $\ell * \wp \in \aleph$.

(ii). If $(\aleph, *)$ is associative, i.e. $(\ell * \wp) * \hbar = \ell * (\wp * \hbar)$ for all ℓ, $\wp$, $\hbar \in \aleph$.

Definition [20] Let $(\aleph_1, *)$ and $(\aleph_2, \#)$ be two NTGs. Let $F : \aleph_1 \rightarrow \aleph_2$ be a mapping. Then, F is called neutro-homomorphism if for all ℓ, $\wp \in \aleph_1$, we have

(i). $F(\ell * \wp) = F(\ell) \# F(\wp)$,

(ii). $F(\overset{+}{\aleph}(\ell)) = \overset{+}{\aleph}(F(\ell))$,

and

(iii). $F(\overset{-}{\aleph}(\ell)) = \overset{-}{\aleph}(F(\ell))$.

4. Pythagorean neutrosophic triplet (PNT)

Definition Let $P_{\aleph}$ be a set together with a B-Operation $*$. Let ℓ_1 and ℓ_2 be any two elements of $P_{\aleph}$. If there exists neutrals of "ℓ_1" and "ℓ_2" known $\overset{+}{\aleph}(\ell_1)$ and $\overset{+}{\aleph}(\ell_2)$, are not same as the classical algebraic unitary elements, and an opposite of "ℓ_1" and "ℓ_2" called $\overset{-}{\aleph}(\ell_1)$ and $\overset{-}{\aleph}(\ell_2)$, with $\overset{+}{\aleph}(\ell_1)$, $\overset{+}{\aleph}(\ell_2)$, $\overset{-}{\aleph}(\ell_1)$, and $\overset{-}{\aleph}(\ell_2)$ are elements of $P_{\aleph}$, such that:

$$\ell_1 * \overset{+}{\aleph}(\ell_1) = \overset{+}{\aleph}(\ell_1) * \ell_1 = \ell_1,\ \ell_1 * \overset{-}{\aleph}(\ell_1) = \overset{-}{\aleph}(\ell_1) * \ell_1 = \overset{+}{\aleph}(\ell_1),$$
$$\ell_2 * \overset{+}{\aleph}(\ell_2) = \overset{+}{\aleph}(\ell_2) * \ell_2 = \ell_2,\ \ell_2 * \overset{-}{\aleph}(\ell_2) = \overset{-}{\aleph}(\ell_2) * \ell_2 = \overset{+}{\aleph}(\ell_2),$$

and

$$\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1).$$

Then, $P_{\aleph}$ is said to be a PNT set.

Definition The second component of the PNT is $(\wp_1, \wp_2)$, represented as $\overset{+}{\aleph}(\cdot)$, and $\overset{+}{\aleph}(\bullet)$, if there exist other elements (ℓ_1, ℓ_2), $(\hbar_1, \hbar_2)$ in PNT such that $\ell_1 * \wp_1 = \wp_1 * \ell_1 = \ell_1$, $\ell_1 * \hbar_1 = \hbar_1 * \ell_1 = \wp_1$, and $\ell_2 * \wp_2 = \wp_2 * \ell_2 = \ell_2$, $\ell_2 * \hbar_2 = \hbar_2 * \ell_2 = \wp_2$, and $\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1)$. The formed PNT is $[(\ell_1, \ell_2), (\wp_1, \wp_2), (\hbar_1, \hbar_2)]$.

Definition The element $(\hbar_1, \hbar_2)$ is the third component, represented as $\overset{-}{\aleph}(\cdot)$, and $\overset{-}{\aleph}(\bullet)$, of a PNT, if there exist other elements (ℓ_1, ℓ_2), $(\wp_1, \wp_2)$ in PNT such that $\ell_1 * \wp_1 = \wp_1 * \ell_1 = \ell_1$, $\ell_1 * \hbar_1 = \hbar_1 * \ell_1 = \wp_1$, and $\ell_2 * \wp_2 = \wp_2 * \ell_2 = \ell_2$, $\ell_2 * \hbar_2 = \hbar_2 * \ell_2 = \wp_2$, and $\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1)$. The formed PNT is $[(\ell_1, \ell_2), (\wp_1, \wp_2), (\hbar_1, \hbar_2)]$.

Example Consider $(Z_{10}, *)$, where "*" is defined as $\ell * \wp = 3\ell\wp (\mathrm{mod}\, 10)$ and $Z_{10} = \{0, 1, 2, 3, ..., 9\}$ then,

*	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	3	6	9	2	5	8	1	4	7
2	0	6	2	8	4	0	6	2	8	4
3	0	9	8	7	6	5	4	3	2	1
4	0	2	4	6	8	0	2	4	6	8
5	0	5	0	5	0	5	0	5	0	5
6	0	8	6	4	2	0	8	6	4	2
7	0	1	2	3	4	5	6	7	8	9
8	0	4	8	2	6	0	4	8	2	6
9	0	7	4	1	8	5	2	9	6	3

1 and 9 produce an PNT because $\overset{+}{\aleph}(1)=7,$ as $1*7=21\equiv 1(\text{mod}10).$ Also $\overset{-}{\aleph}(1)=9,$ as $1*9=27\equiv 7(\text{mod}10).$ Now $\overset{+}{\aleph}(9)=7,$ as $9*7=189\equiv 9(\text{mod}10).$ Also $\overset{-}{\aleph}(9)=1,$ as $9*1=27\equiv 7(\text{mod}10).$ Thus, $[(1,9),(7,7),(9,1)]$ is a PNT because $1*9=27\equiv 7(\text{mod}10)=\overset{+}{\aleph}(1).$

Similarly, $[(4,6),(2,2),(6,4)]$ is a PNT. But 2 and 4 does not give rise to a PNT. Since $[(2,4),(2,2),(2,6)]$ does not implies that $2*4=\overset{+}{\aleph}(2),$ as $2*4=24\equiv 4(\text{mod}10)\neq\overset{+}{\aleph}(2).$ The trivial PNT is denoted by $[(0,0),(0,0),(0,0)],$ as $\overset{+}{\aleph}(0)=0$, $\overset{-}{\aleph}(0)=0$ and $0*0=0=\overset{-}{\aleph}(0).$

Theorem If $[(\ell,\wp),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp))]$ form a PNT, then $[(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp)),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\ell,\wp)]$ also form a Pythagorean neutrosophic.

Proof Of course $\overset{-}{\aleph}(\ell)*\ell=\overset{+}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp)*\wp=\overset{+}{\aleph}(\wp).$ We have to prove that $\overset{-}{\aleph}(\ell)*\overset{+}{\aleph}(\ell)=\overset{-}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp)*\overset{+}{\aleph}(\wp)=\overset{-}{\aleph}(\wp).$ Multiply by ℓ and $\wp$ to the L.H.S, we have

$$\overset{-}{\aleph}(\ell)*\overset{+}{\aleph}(\ell)=\overset{-}{\aleph}(\ell)\text{ and }\overset{-}{\aleph}(\wp)*\overset{+}{\aleph}(\wp)=\overset{-}{\aleph}(\wp)$$

$$\ell*\overset{-}{\aleph}(\ell)*\overset{+}{\aleph}(\ell)=\ell*\overset{-}{\aleph}(\ell)\text{ and }\wp*\overset{-}{\aleph}(\wp)*\overset{+}{\aleph}(\wp)=\wp*\overset{-}{\aleph}(\wp)$$

or

$$[\ell*\overset{-}{\aleph}(\ell)]*\overset{+}{\aleph}(\ell)=\overset{+}{\aleph}(\ell)\text{ and }[\wp*\overset{-}{\aleph}(\wp)]*\overset{+}{\aleph}(\wp)=\overset{+}{\aleph}(\wp)$$

or

$$\overset{+}{\aleph}(\ell)*\overset{+}{\aleph}(\ell)=\overset{+}{\aleph}(\ell)\text{ and }\overset{+}{\aleph}(\wp)*\overset{+}{\aleph}(\wp)=\overset{+}{\aleph}(\wp)$$

Again multiply by ℓ and $\wp$ to the L.H.S, we have:

$$\ell * \overset{+}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \ell * \overset{+}{\aleph}(\ell) \text{ and } \wp * \overset{+}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \wp * \overset{+}{\aleph}(\wp)$$

or

$$[\ell * \overset{+}{\aleph}(\ell)] * \overset{+}{\aleph}(\ell) = \ell \text{ and } [\wp * \overset{+}{\aleph}(\wp)] * \overset{+}{\aleph}(\wp) = \wp$$

or

$$\ell * \overset{+}{\aleph}(\ell) = \ell \text{ and } \wp * \overset{+}{\aleph}(\wp) = \wp$$

Pythagorean neutrosophic triplet group (PNTG)

Definition If $(P_{\aleph},*)$ be a PNT set. Then, $P_{\aleph}$ is said be a PNTG with the properties.

(i). If $(P_{\aleph},*)$ is well-defined, i.e. for any $\ell, \wp \in P_{\aleph}$, one has $\ell * \wp \in \aleph$.

(ii). If $(P_{\aleph},*)$ is associative i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$ for all ℓ_1, ℓ_2, $\wp_1$, $\wp_2$, $\hbar_1$, $\hbar_2 \in P_{\aleph}$,.

In general, the PNTG is different from the classical algebraic group.

The Pythagorean neutrosophic neutrals is considered as the classical unitary element and the Pythagorean neutrosophic opposites is considered as the classical inverse elements.

Example Consider $(Z_5,*)$, where $*$ is defined as $\ell * \wp = \ell (\text{mod}\, 5)$. Then, $(Z_5,*)$ is a PNTG with the given table.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4

It is associative, i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$. Now take L. H. S to prove the R. H. S, so

$$\begin{aligned}(\ell_1 * \wp_1) * \hbar_1 &= \ell_1 * \hbar_1 \\ &= \ell_1\end{aligned}$$

also,

$$\begin{aligned}\ell_1 * (\wp_1 * \hbar_1) &= \ell_1 * \wp_1 \\ &= \ell_1.\end{aligned}$$

From (1) and (2), we have

$$(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1).$$

Similarly,

$$\begin{aligned}(\ell_2 * \wp_2) * \hbar_2 &= \ell_2 * \hbar_2 \\ &= \ell_2\end{aligned}$$

also,

$$\begin{aligned}\ell_2 * (\wp_2 * \hbar_2) &= \ell_2 * \wp_2 \\ &= \ell_2.\end{aligned}$$

From (1) and (2), we have

$$(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2).$$

The Pythagorean triplets are:

$$\left\{\begin{array}{l}[(0,1),(0,1),(0,1)]: \text{ such that } 0*1 = \overset{+}{\aleph}(0) \\ [(1,2),(1,2),(1,2)]: \text{ such that } 1*2 = \overset{+}{\aleph}(1) \\ [(4,3),(4,3),(4,3)]: \text{ such that } 4*3 = \overset{+}{\aleph}(4)\end{array}\right\}.$$

Thus, all the elements of Z_5 give rise to a PNT.

Example Consider $(Z_3,*)$, where $*$ is defined as $\ell * \wp = \ell + \wp + 3(\bmod 5)$. Then, $(Z_3,*)$ is a PNTG under the B-Operation $*$ with the below table.

*	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

It is associative, i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$.

Also, the Pythagorean triplets are:

$$\left\{\begin{array}{l}[(0,1),(0,0),(0,2)]: \text{ such that } 0*1 = \overset{+}{\aleph}(0) \\ [(1,2),(0,1),(2,2)]: \text{ such that } 1*2 = \overset{+}{\aleph}(1)\end{array}\right\}$$

Thus, all the elements of Z_3 give rise to a PNT.

Remark Let $(P_{\aleph},*)$ be a PNTG under $*$ and let $[(\ell,\wp),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp))]$ such that $\ell*\wp=\overset{+}{\aleph}(\ell)$, be a PNT. Then, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$, and also ne $\overset{+}{\aleph}(\ell)$, $\overset{+}{\aleph}(\wp)$ depends on the elements ℓ and $\wp$, and the B-Operation $*$.

We consider the below Example to prove the above remark.

In Example 2, consider the PNT, $[(1,2),(1,2),(1,2)]$: such that $1*2=\overset{+}{\aleph}(1)$, so the $\overset{+}{\aleph}(1)=0,1,2,3,4$. Similarly, $\overset{+}{\aleph}(\wp)=0,1,2,3,4$. Thus, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$.

Remark Let $(P_{\aleph},*)$ be a PNTG under $*$ and let $[(\ell,\wp),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp))]$ such that $\ell*\wp=\overset{+}{\aleph}(\ell)$, be a PNT. Then, $\overset{-}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp)$ is not unique in $P_{\aleph}$, and also $\overset{-}{\aleph}(\ell)$, $\overset{-}{\aleph}(\wp)$ depends on the elements ℓ and $\wp$, and the B-Operation $*$.

We consider the below Example to prove the above remark.

In Example 2, consider the PNT, $[(2,3),(2,3),(2,3)]$: such that $2*3=\overset{-}{\aleph}(2)$, so $\overset{-}{\aleph}(2)=0,1,2,3,4$. Similarly, $\overset{+}{\aleph}(\wp)=0,1,2,3,4$. Thus, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$.

Theorem All PNTG is a NTG.

Proof Let $P_{\aleph}$ be a PNTG. Then it satisfies the given conditions.

(i). If $(P_{\aleph},*)$ is well-defined, i.e. for any $\ell,\wp\in P_{\aleph}$, one has $\ell*\wp\in\aleph$.

(ii). If $(P_{\aleph},*)$ is associative i.e. $(\ell_1*\wp_1)*\hbar_1=\ell_1*(\wp_1*\hbar_1)$ and $(\ell_2*\wp_2)*\hbar_2=\ell_2*(\wp_2*\hbar_2)$ for all ℓ_1, ℓ_2, $\wp_1$, $\wp_2$, $\hbar_1$, $\hbar_2\in P_{\aleph}$,.

From conditions (i) and (ii), we have that $P_{\aleph}$ is a NT.

Example In example 1, $(Z_{10},*)$ is a NT but not a PNTG because the PNT $[(2,4),(2,2),(2,6)]$ does not implies that $2*4=\overset{+}{\aleph}(2)$, as $2*4=24\equiv4(\mathrm{mod}10)\neq\overset{+}{\aleph}(2)$. Hence, not all the elements of Z_{10} give rise to a PNT.

Definition Let $(P_{\aleph},*)$ be a PNTG. Then, $P_{\aleph}$ is called a commutative PNTG if for all $\ell,\wp\in P_{\aleph}$, we have $\ell*\wp=\wp*\ell$.

Example In example 2, $(Z_5,*)$ is a PNTG but not a commutative PNTG because $0*1\neq1*0$. Similarly, $1*2\neq2*1$.

Theorem The idempotent elements ℓ and $\wp$ give rise to PNT under the operation $*$ such that $\ell * \wp = \ell$.

Solution Let ℓ and $\wp$ be idempotent elements. Then, by definition, $\ell^2 = \ell$ and $\wp^2 = \wp,$ which clearly implies that $\overset{+}{\aleph}(\ell) = \ell,$ $\overset{-}{\aleph}(\ell) = \ell$ and $\overset{+}{\aleph}(\wp) = \wp,$ $\overset{-}{\aleph}(\wp) = \wp.$ Also, $\ell * \wp = \ell = \overset{+}{\aleph}(\ell)$ or $\wp * \ell = \wp = \overset{+}{\aleph}(\wp).$ Hence ℓ and $\wp$ give rise to a PNT, that is,

$[(\ell,\wp),(\ell,\wp),(\ell,\wp)] : \ell * \wp = \ell.$

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then

$(i).$ $\ell_1 * \ell_2 = \ell_1 * \ell_3$ and $\wp_1 * \wp_2 = \wp_1 * \wp_3$ if and only if $\overset{+}{\aleph}(\ell_1) * \ell_2 = \overset{+}{\aleph}(\ell_1) * \ell_3$ and $\overset{+}{\aleph}(\wp_1) * \wp_2 = \overset{+}{\aleph}(\wp_1) * \wp_3.$

$(ii).$ $\ell_2 * \ell_1 = \ell_3 * \ell_1$ and $\wp_2 * \wp_1 = \wp_3 * \wp_1$ if and only if $\overset{+}{\aleph}(\ell_2) * \ell_1 = \overset{+}{\aleph}(\ell_3) * \ell_1$ and $\overset{+}{\aleph}(\wp_2) * \wp_1 = \overset{+}{\aleph}(\wp_3) * \wp_1.$

Proof $(i).$ Suppose that $\ell_1 * \ell_2 = \ell_1 * \ell_3$ and $\wp_1 * \wp_2 = \wp_1 * \wp_3.$ Since $P_{\aleph}$ is a PNTG, so $\overset{-}{\aleph}(\ell_1), \overset{-}{\aleph}(\wp_1) \in P_{\aleph}.$ Multiply $\overset{-}{\aleph}(\ell_1)$ to the L.H.S with $\ell_1 * \ell_2 = \ell_1 * \ell_3,$ we get:

$$\begin{aligned}
\overset{-}{\aleph}(\ell_1) * \ell_1 * \ell_2 &= \overset{-}{\aleph}(\ell_1) * \ell_1 * \ell_3 \\
[\overset{-}{\aleph}(\ell_1) * \ell_1] * \ell_2 &= [\overset{-}{\aleph}(\ell_1) * \ell_1] * \ell_3 \\
\overset{+}{\aleph}(\ell_1) * \ell_2 &= \overset{+}{\aleph}(\ell_1) * \ell_3.
\end{aligned}$$

Similarly, multiply $\overset{-}{\aleph}(\wp_1)$ to the left side with $\wp_1 * \wp_2 = \wp_1 * \wp_3,$ we get:

$$\begin{aligned}
\overset{-}{\aleph}(\wp_1) * \wp_1 * \wp_2 &= \overset{-}{\aleph}(\wp_1) * \wp_1 * \wp_3 \\
[\overset{-}{\aleph}(\wp_1) * \wp_1] * \wp_2 &= [\overset{-}{\aleph}(\wp_1) * \wp_1] * \wp_3 \\
\overset{+}{\aleph}(\wp_1) * \wp_2 &= \overset{+}{\aleph}(\wp_1) * \wp_3.
\end{aligned}$$

$(ii).$ The proof is similar to $(i).$

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then

(i). $\overset{-}{\aleph}(\ell_1)*\ell_2=\overset{-}{\aleph}(\ell_1)*\ell_3$ and $\overset{-}{\aleph}(\wp_1)*\wp_2=\overset{-}{\aleph}(\wp_1)*\wp_3$ then $\overset{+}{\aleph}(\ell_1)*\ell_2=\overset{+}{\aleph}(\ell_1)*\ell_3$ and $\overset{+}{\aleph}(\wp_1)*\wp_2=\overset{+}{\aleph}(\wp_1)*\wp_3$.

(ii). $\ell_2*\overset{-}{\aleph}(\ell_1)=\ell_3*\overset{-}{\aleph}(\ell_1)$ and $\wp_2*\overset{-}{\aleph}(\wp_1)=\wp_3*\overset{-}{\aleph}(\wp_1)$ then $\ell_2*\overset{+}{\aleph}(\ell_1)=\ell_3*\overset{+}{\aleph}(\ell_1)$ and $\wp_2*\overset{+}{\aleph}(\wp_1)=\wp_3*\overset{+}{\aleph}(\wp_1)$.

Proof (i). Suppose that $\overset{-}{\aleph}(\ell_1)*\ell_2=\overset{-}{\aleph}(\ell_1)*\ell_3$ and $\overset{-}{\aleph}(\wp_1)*\wp_2=\overset{-}{\aleph}(\wp_1)*\wp_3$. Since $P_{\aleph}$ is a PNTG with respect to $*$, so ℓ_1, $\wp_1\in P_{\aleph}$. Multiply ℓ_1 to the left side with $\overset{-}{\aleph}(\ell_1)*\ell_2=\overset{-}{\aleph}(\ell_1)*\ell_3$, we get:

$$\begin{aligned}\ell_1*\overset{-}{\aleph}(\ell_1)*\ell_2 &= \ell_1*\overset{-}{\aleph}(\ell_1)*\ell_3\\ [\ell_1*\overset{-}{\aleph}(\ell_1)]*\ell_2 &= [\ell_1*\overset{-}{\aleph}(\ell_1)]*\ell_3\\ \overset{+}{\aleph}(\ell_1)*\ell_2 &= \overset{+}{\aleph}(\ell_1)*\ell_3.\end{aligned}$$

Similarly, multiply $\wp_1$ to the left side with $\overset{-}{\aleph}(\wp_1)*\wp_2=\overset{-}{\aleph}(\wp_1)*\wp_3$, we get:

$$\begin{aligned}\wp_1*\overset{-}{\aleph}(\wp_1)*\wp_2 &= \wp_1*\overset{-}{\aleph}(\wp_1)*\wp_3\\ [\wp_1*\overset{-}{\aleph}(\wp_1)]*\wp_2 &= [\wp_1*\overset{-}{\aleph}(\wp_1)]*\wp_3\\ \overset{+}{\aleph}(\wp_1)*\wp_2 &= \overset{+}{\aleph}(\wp_1)*\wp_3.\end{aligned}$$

(ii). The proof is same as (i).

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)]:\ell_1*\wp_1=\overset{+}{\aleph}(\ell_1)$, be a PNT then $\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)=\overset{+}{\aleph}(\ell_1*\wp_1)$.

Proof Consider L.H.S, $\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)$. Now, multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\begin{aligned}\ell_1*\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)*\wp_1 &= [\ell_1*\overset{+}{\aleph}(\ell_1)]*[\overset{+}{\aleph}(\wp_1)*\wp_1]\\ &= \ell_1*\wp_1.\end{aligned}$$

Now consider R.H.S, we have $\overset{+}{\aleph}(\ell_1*\wp_1)$. Again multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\ell_1*\overset{+}{\aleph}(\ell_1*\wp_1)*\wp_1=[\ell_1*\wp_1]*\overset{+}{\aleph}(\ell_1*\wp_1),$$

as $*$ is associative, so we have

$$\ell_1 * \overset{+}{\aleph}(\ell_1 * \wp_1) * \wp_1 = \ell_1 * \wp_1.$$

Hence proved.

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)]: \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\ell_1 * \wp_1).$

Proof Consider L.H.S, $\overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1)$. Now multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\begin{aligned}\ell_1 * \overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1) * \wp_1 &= [\ell_1 * \overset{-}{\aleph}(\ell_1)] * [\overset{-}{\aleph}(\wp_1) * \wp_1] \\ &= \overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) \\ &= \overset{+}{\aleph}(\ell_1 * \wp_1), \text{ By above theorem.}\end{aligned}$$

Now consider R.H.S, we have $\overset{-}{\aleph}(\ell_1 * \wp_1)$. Again multiply to the L.S with ℓ_1 and to the R.S with $\wp_1$, we get:

$$\ell_1 * \overset{-}{\aleph}(\ell_1 * \wp_1) * \wp_1 = [\ell_1 * \wp_1] * \overset{-}{\aleph}(\ell_1 * \wp_1),$$

as $*$ is associative, so we have

$$\ell_1 * \overset{-}{\aleph}(\ell_1 * \wp_1) * \wp_1 = \overset{+}{\aleph}(\ell_1 * \wp_1).$$

Hence proved.

Theorem Let $(P_{\aleph},*)$ be a commutative PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)]: \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then

(i). $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1).$

(ii). $\overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1) = \overset{-}{\aleph}(\wp_1) * \overset{-}{\aleph}(\ell_1).$

Proof Consider the L.H.S $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1)$. According to Theorem 7, we have

$$\begin{aligned}\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) &= \overset{+}{\aleph}(\ell_1 * \wp_1) \\ &= \overset{+}{\aleph}(\wp_1 * \ell_1),\end{aligned}$$

as $P_{\aleph}$ is commutative, so

$$\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1),$$

again by Theorem 7.

Thus $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1).$

(ii). The proof is same as (i).

Definition Let $(P_{\aleph},*)$ be a PNTG with respect to $*$, and let $P_{\aleph}' \subseteq P_{\aleph}$. Then, $P_{\aleph}'$ is said to be a PNT subgroup of $P_{\aleph}$ if $P_{\aleph}'$ itself a PNTG with respect to $*$.

Example Consider $(Z_5,*)$, where $*$ is defined as $\ell * \wp = \ell(\text{mod}\,5)$. Then, $(Z_5,*)$ is a PNTG under the binary operation $*$, and $P_{\aleph}' = \{0,1,2,3\}$ be a subset of Z_5. Then, clearly $P_{\aleph}'$ is a PNT subgroup of Z_5.

Proposition Let $(P_{\aleph},*)$ be a PNTG and $P_{\aleph}' \subseteq P_{\aleph}$. Then $P_{\aleph}'$ is a PNT subgroup of N $\Leftrightarrow$ the following properties are satisfied.

(i). $\ell_1 * \wp_1 \in P_{\aleph}'$ for all ℓ_1, $\wp_1 \in P_{\aleph}'$.

(ii). $\overset{+}{\aleph}(\ell_1)$, $\overset{+}{\aleph}(\wp_1) \in P_{\aleph}'$ for all ℓ_1, $\wp_1 \in P_{\aleph}'$.

(iii). $\overset{-}{\aleph}(\ell_1)$, $\overset{-}{\aleph}(\wp_1) \in P_{\aleph}'$ for all ℓ_1, $\wp_1 \in P_{\aleph}'$.

Proof It is easy to prove.

Definition Let $P_{\aleph}$ be a PNTG and let $\ell \in P_{\aleph}$. The smallest positive integer $n \geq 1$ such that $\ell^n = \overset{+}{\aleph}(\ell)$ is called PNT order. It is denoted by $pnto(\ell)$.

Example Consider $(Z_5,*)$, where $*$ is defined as $\ell * \wp = \ell(\text{mod}\,5)$ then

$$pnto(1) = 1,\ pnto(2) = 1,\ pnto(3) = 1,\ pnto(4) = 1.$$

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $\alpha \in P_{\aleph}$. Then $\overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \overset{+}{\aleph}(\alpha)$. In general $(\overset{+}{\aleph}(\alpha))^n = \overset{+}{\aleph}(\alpha)$, where $n \geq 1$.

Proof Consider $\overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \overset{+}{\aleph}(\alpha)$. Multiply α to the left side, we get;

$$\alpha * \overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \alpha * \overset{+}{\aleph}(\alpha)$$
$$[\alpha * \overset{+}{\aleph}(\alpha)] * \overset{+}{\aleph}(\alpha) = [\alpha * \overset{+}{\aleph}(\alpha)]$$
$$\alpha * \overset{+}{\aleph}(\alpha) = \alpha$$
$$\alpha = \alpha$$

Similarly, we can easily see that $(\overset{+}{\aleph}(\alpha))^n = \overset{+}{\aleph}(\alpha)$ for a nonzero positive integer n.

Definition Let $P_\aleph$ be a PNTG and $\alpha \in P_\aleph$. Then, $P_\aleph$ is called Pythagorean neutro-cyclic triplet group if $P_\aleph = \langle \alpha \rangle$. We say that α is a generator part of the PNT.

5. Pythagorean neutro-homomorphism (PN-h)

In this section, we introduce PN-h and Pythagorean neutro-isomorphisms (PN-i) for the PNTGs.

Definition Let $(P_{\aleph_1}, *)$ and $(P_{\aleph_2}, \#)$ be two PNTGs. Let $F : P_{\aleph_1} \to P_{\aleph_2}$ be mapping. Then, F is called PN-h if for all ℓ, $\wp \in P_{\aleph_1}$, we have

(i). $F(\ell * \wp) = F(\ell) \ \# \ F(\wp)$.

(ii). $F(\overset{+}{\aleph}(\ell)) = \overset{+}{\aleph}(F(\ell))$.

(iii). $F(\overset{-}{\aleph}(\ell)) = \overset{-}{\aleph}(F(\ell))$.

Example Let $P_{\aleph_1}$ be a PNTGs with respect to $*$ modulo 10 in $(Z_{10}, *)$, where $*$ is defined as: $\ell * \wp = \ell(\text{mod}10)$, and $Z_{10} = \{0,1,2,3,...,9\}$. Let $F : P_{\aleph_1} \to P_{\aleph_1}$ be a mapping defined as:

$$F(0) = 0,\ F(1) = 1,\ F(2) = 2,\ F(3) = 3,\ F(4) = 4,\ F(5) = 5,\ F(6) = 6,$$
$$F(7) = 7,\ F(8) = 8,\ F(9) = 9.$$

Then, clearly F is a PN-h because conditions (i), (ii) and (iii) are satisfied easily.

Definition A PN-h is called PN-i if it is one--one and onto.

Example Let $P_{\aleph_1}$ be a PNTGs with respect to $*$ modulo 5 in $(Z_5, *)$, where $*$ is defined as: $\ell * \wp = \ell(\text{mod}5)$, and $Z_5 = \{0,1,2,3,4\}$. Let $F : P_{\aleph_1} \to P_{\aleph_1}$ be a mapping defined as:

$$F(0) = 0,\ F(1) = 1,\ F(2) = 2,\ F(3) = 3,\ F(4) = 4.$$

Then, clearly F is a PN-i.

Note that F further defines that,

(i). F is called Pythagorean neutro-endomorphism of $P_{\aleph_1}$, if $P_{\aleph_1} = P_{\aleph_2}$.

(ii). F is called Pythagorean neutro-epimorphism if F is onto.

(iii). F is called Pythagorean neutro-monomorphism if F is $(1-1)$.

(iv). A Pythagorean neutro-endomorphism F of a Pythagorean triplet group $P_{\aleph}$ is called a Pythagorean neutro-automorphism of G if F is $(1-1)$ and onto.

Definition Two given PNTGs $(P_{\aleph_1}, *)$ and $(P_{\aleph_2}, \#)$ are said to be Pythagorean neutro-isomorphic to each other if there exists a PN-i between $P_{\aleph_1}$ and $P_{\aleph_2}$.

They are written as $P_{\aleph_1} \cong P_{\aleph_2}$, and read as "the PNTG $P_{\aleph_1}$ is Pythagorean neutro-isomorphic to the PNTG $P_{\aleph_2}$ ".

Theorem The relation "$\cong$" of Pythagorean neutro-isomorphism over the set of all the Pythagorean neutrosophic triplet group is an equivalence relation.

Proof Let $P_{\aleph_1}$, $P_{\aleph_2}$ and $P_{\aleph_3}$ be three Pythagorean neutrosophic triplet groups. Then $P_{\aleph_1} \cong P_{\aleph_1}$ by the identity Pythagorean neutro-isomorphism.

For symmetry, suppose $P_{\aleph_1} \cong P_{\aleph_2}$. Then there exists a Pythagorean neutro-isomorphism $F : P_{\aleph_1} \to P_{\aleph_2}$. Since F is $(1-1)$ and onto Pythagorean neutro-homomorphism from $P_{\aleph_1}$ onto $P_{\aleph_2}$, therefore $F^{-1} : P_{\aleph_2} \to P_{\aleph_1}$ is a Pythagorean neutro-isomorphism. Thus $P_{\aleph_2} \cong P_{\aleph_1}$.

For transitivity, suppose, $P_{\aleph_1} \cong P_{\aleph_2}$ and $P_{\aleph_2} \cong P_{\aleph_3}$. Since $P_{\aleph_1}$ is Pythagorean neutro-isomorphism to $P_{\aleph_2}$ and $P_{\aleph_2}$ is Pythagorean neutro-isomorphism to $P_{\aleph_3}$, therefore, there exists, Pythagorean neutro-isomorphism F and g such that $F : P_{\aleph_1} \to P_{\aleph_2}$ and $g : P_{\aleph_2} \to P_{\aleph_3}$.

Since $g \circ F : P_{\aleph_1} \to P_{\aleph_3}$ is a Pythagorean neutro-isomorphism from $P_{\aleph_1}$ into $P_{\aleph_3}$. It proves that $P_{\aleph_1} \cong P_{\aleph_3}$.

Hence the relation "$\cong$" is an equivalence relation over the set of all Pythagorean neutrosophic triplet groups.

6. Distinctions and comparison

The distinctions between NT [20] and PNTG are:

(i). In the NT, any single element $\ell \in \aleph$, gives rise to a neutrosophic triplet, while in the PNTG, two elements ℓ_1, $\wp_1 \in P_{\aleph}$ give rise to a PNT with the extra condition, that is, $\ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1)$.

(ii). The structure of neutrosophic triplet is $(\ell, \wp, \hbar)$, where $\wp = \overset{+}{\aleph}(\ell)$, and $\hbar = \overset{-}{\aleph}(\ell)$, while the structure of PNT is

$$[(\ell_1, \wp_1), (\ell_2, \wp_2), (\hbar_1, \hbar_2)]: \text{ such that } \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$$

where $\ell_2 = \overset{+}{\aleph}(\ell_1)$, $\ell_3 = \overset{-}{\aleph}(\ell_1)$, $\wp_2 = \overset{+}{\aleph}(\wp_1)$, and $\wp_3 = \overset{-}{\aleph}(\wp_1)$.

Clearly, the NTG has a weaker structure than the PNTG.

7. Conclusions

Inspiring from the neutrosophic triplet, we developed the notion of PNT which is a group of three ordered pairs that satisfy certain properties with some binary operation. The purpose of this paper is first to introduce the PNT and then used these PNTs to introduce the novel concept of a PNTG, which differs from a classical group in terms of structural features. The PNTG is completely different from the neutrosophic group. We discussed some properties, basic results, and particular examples of these novel concepts. We further studied PN-h, PN-i, etc., for NTs. Moreover, we discussed the main distinctions between the NTG and the PNTG.

Conflict of Interest

The authors declare that they have no conflict of interests.

Acknowledgments: This work is financially supported by the Higher Education Commission of Pakistan (Grant No: 7750/Federal/NRPU/R&D/HEC/2017).

Authorship Contributions: All authors contributed equally.

References

[1]. E.O. Adeleke, A.A.A. Agboola, F. Smarandache. Refined Neutrosophic Rings I. International Journal of Neutrosophic Science, 2(2) (2020), 77-81.

[2]. E.O. Adeleke, A.A.A. Agboola, F. Smarandache, Refined Neutrosophic Rings II. International Journal of Neutrosophic Science, 2(2) (2020), pp. 89-94.

[3]. K.D. Ahmad. On Imperfect Duplets In Some refined Neutrosophic Rings. *journal of neutrosophic and fuzzy systems, 2022*.

[4]. M. Ali, F. Smarandache, M. Khan. Study on the development of neutrosophic triplet ring and neutrosophic triplet field. *Mathematics*, *6*(4) (2018), 46.

[5]. A.M.D.J.S. Alkouri, A.R. Salleh. Complex intuitionistic fuzzy sets. In AIP conference proceedings, 1482(1) (2012), 464-470.

[6]. T.K. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets Syst 20 (1986), 87-96.

[7]. M. Bal, M.M. Shalla, N. Olgun. Neutrosophic triplet cosets and quotient groups. *Symmetry, 10*(4) (2018), 126.

[8]. D.S. Dummit, R.M. Foote. Abstract algebra. 3rd edn. John Viley & Sons Inc, New Jersey, 2004.

[9]. I.N. Herstein. Topics in algebra. Xerox college publishing, Lexington, 1975.

[10]. M. Hu, X. Zhang. On cyclic associative semihypergroups and neutrosophic extended triplet cyclic associative semihypergroups. *Mathematics*, *10*(4) (2022), 535.

[11] W.V. Kandasamy, F. Smarandache. Some neutrosophic algebraic structures and neutrosophic n-algebraic structures. Hexis, Frontigan, (2006), 219.

[12]. W.V. Kandasamy F. Smarandache. N-algebraic structures and s-n-algebraic structures. Hexis, Phoenix, (2006), 209.

[13]. W.V. Kandasamy F. Smarandache. Basic neutrosophic algebraic structures and their applications to fuzzy and neutrosophic models. Hexis, Frontigan, (2004), 149.

[14]. Q. Li Y. Ma, X. Zhang, J. Zhang. Neutrosophic extended triplet group based on neutrosophic quadruple numbers. *Symmetry*, *11*(5) (2019), 696.

[15]. M.R. Molaei. Generalized groups. Bul Inst Politehn Ia, si Sect I 45(49) (1999), 21-24.

[16]. X. Peng Y. Yang. Some results for Pythagorean fuzzy sets. International Journal of Intelligent Systems, 30(11) (2015), 1133-1160.

[17]. M. Sahin, K. Abdullah. Neutrosophic triplet normed space. Open Phys. 2017, 15, 697–704.

[18]. M. Şahin, A. Kargın. *Neutrosophic Triplet Inner Product*. Infinite Study, 2017.

[19]. M. Şahin, A. Kargın. Neutrosophic triplet v-generalized metric space. *Axioms*, *7*(3) (2018), 67.

[20]. F. Smarandache, M. Ali. Neutrosophic triplet group. Neural Computing and Applications, 29(7) (2018), 595-601.

[21]. F. Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

[22]. F. Smarandache. Neutrosophic set, a generalization of the intuitionistic fuzzy set, In: 2006 IEEE international conference on granular computing, 10-12 May 2006, 38--42. doi:10.1109/GRC.2006.1635754.

[23]. F. Samarandache. Neutrosophic Extenede Triplets. Special collection,Arizona State University: Tempe, AZ, USA, 2016.

[24]. D.B. Surowski. The uniqueness aspect of the fundamental theorem of finite Abelian groups. Amer Math Monthly 102 (1995), 162-163.

[25]. L.A. Zadeh. Fuzzy sets. Inform Control 8 (1965), 338--353.

[26]. X. Zhang, F. Smarandache, M. Ali, X. Liang. Commutative neutrosophic triplet group and neutro-homomorphism basic theorem. *Collected Papers. Volume IX: On Neutrosophic Theory and Its Applications in Algebra*, 2022, 215.

[27]. X. Zhang, Q. Hu, F. Smarandache, X. An. On neutrosophic triplet groups: Basic properties, NT-subgroups, and some notes. *Symmetry*, *10*(7) (2018), 289.

[28]. X. Zhang, X. Wang, F. Smarandache, T.G. Jaíyéolá, T. Lian. Singular neutrosophic extended triplet groups and generalized groups. *Cognitive Systems Research*, *57 (2019)*, 32-40.

[29]. X. Zhou, X.L. Xin. Ideals on neutrosophic extended triplet groups. *AIMS Mathematics*, *7*(3) (2022), 4767-4777.

Received: Feb 9, 2024. Accepted: Apr 29, 2024