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An Outranking Approach for MCDM-Problems with Neutrosophic Multi-Sets

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Abstract: In this paper, we introduced a new outranking approach for multi-criteria decision making (MCDM) problems to handle uncertain situations in neutrosophic multi environment. Therefore, we give some outranking relations of neutrosophic multi sets. We also examined some desired properties of the outranking relations and developed a ranking method for MCDM problems. Moreover, we describe a numerical example to verify the practicality and effectiveness of the proposed method.

Keywords: Single valued neutrosophic sets, neutrosophic multi-sets, outranking relations, decision making.

1. Introduction

Fuzzy set theory, intuitionistic fuzzy set theory and neutrosophic set theory is introduced by Zadeh [59], Atanassov [1] and Smarandache [28] to handle the uncertain, incomplete, indeterminate and inconsistent information, respectively. The above set theories have been applied to many different areas including real decision making problems [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 26, 27, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 58]. Also, several generalizations of the set theories made such as fuzzy multi-set theory [34, 35, 48], intuitionistic fuzzy multi-set theory [16, 31, 36, 37, 57] and n-valued refined neutrosophic set theory [29].

Another generalization of above theories that is relevant for our work is single valued neutrosophic refined (multi) set theory introduced by Ye [53, 56] which contain a few different values. A single valued neutrosophic multi set theory has truth-membership sequence $\left(\mu_A^1(t),\mu_A^2(t),...,\mu_A^P(t)\right)$, indeterminacy membership sequence $\left(\nu_A^1(t),\nu_A^2(t),...,\nu_A^P(t)\right)$ and falsity-membership sequence $\left(\omega_A^1(t),\omega_A^2(t),...,\omega_A^P(t)\right)$ of element $t \in T$. Recently, the single valued neutrosophic multi set theory have attracted widely attention in [20, 33, 50, 51, 52, 54, 55]. The paper is organized as follows; In Section 2 we give some basic notions of neutrosophic sets and neutrosophic multi-sets. In Section 3, we first introduce outranking relations of neutrosophic multi-sets with proprieties. In Section 4, we propose an outranking approach for to solving the multi-criteria decision making problems based on neutrosophic multi-set information. In Section 5, we propose a selection example to validate the practicality. Finally, in Section 6, we conclude the paper.

2. Preliminaries

In this section, we present the basic definitions and results of neutrosophic set theory [28, 33] and neutrosophic multi (or refined) set theory [12, 53] that are useful for subsequent discussions.

Definition 1 [28] let T be a universe. A neutrosophic set A over T is defined by

$$A = \left\{ \left\langle t, \left(\mu_A(t), \nu_A(t), \omega_A(t) \right) \right\rangle, t \in T \right\}.$$

where $\mu_A(t)$, $\nu_A(t)$ and $\omega_A(t)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by

$$\mu_A(t) : T \to]^-0,1^+[, \nu_A(t) : T \to]^-0,1^+[, \omega_A(t) : T \to]^-0,1^+[$$

such that $^-0 \le \mu_A(t) + \nu_A(t) + \omega_A(t) \le 3^+$.

Definition 2 [33] Let T be a universe. An single valued neutrosophic set (SVN-set) over T is a neutrosophic set over T, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$\mu_A(t) : T \to [0,1], \nu_A(t) : T \to [0,1], \omega_A(t) : T \to [0,1]$$

such that $0 \le \mu_A(t) + \nu_A(t) + \omega_A(t) \le 3$.

Definition 3 [53] Let T be a universe. A neutrosophic multiset set (Nms) $\mathcal{A}$ on T can be defined as follows:

$$\mathcal{A} = \{ \prec t, \left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t) \right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t) \right), \left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots w_{\mathcal{A}}^p(t) \right) \succ : t \in T \}$$

Where,

$$\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t) : T \to [0,1],$$

$$v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t) : T \to [0,1],$$

and

$$w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots w_{\mathcal{A}}^p(t) : T \to [0,1]$$

such that

$$0 \le sup\mu_{\mathcal{A}}^i(t) + supv_{\mathcal{A}}^i(t) + supw_{\mathcal{A}}^i(t) \le 3$$

$(i = 1,2,\dots,P)$ and $\left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots, \mu_{\mathcal{A}}^p(t) \right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots, v_{\mathcal{A}}^p(t) \right) and \left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots, w_{\mathcal{A}}^p(t) \right)$ Is the truth-membership sequence, indeterminacy-membership sequence and falsity- membership sequence of the element u, respectively. Also, P is called the dimension (cardinality) of Nms $\mathcal{A}$, denoted $d(\mathcal{A})$. We arrange the truth- membership sequence in decreasing order but the corresponding indeterminacy- membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic multisets on T is denoted by NMS(T).

Definition 4 [12, 53, 56] Let $A, B \in NMS(T)$. Then,

(1) $\mathcal{A}$ is said to be Nm-subset of $\mathcal{B}$ is denoted by $\mathcal{A} \,\widetilde{\subseteq}\, \mathcal{B}$ if $\mu_{\mathcal{A}}^i(t) \le \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) \ge v_{\mathcal{B}}^i(t)$, $w_{\mathcal{A}}^i(t) \ge w_{\mathcal{B}}^i(t)$, $\forall\, t \in T$ and $i = 1,2,\dots P$.

(2) $\mathcal{A}$ is said to be neutrosophic equal of $\mathcal{B}$ is denoted by $\mathcal{A} = \mathcal{B}$ if $\mu_{\mathcal{A}}^i(t) = \mu_{\mathcal{B}}^i(t)$, $v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t)$, $w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t)$, $\forall\, t \in T$ and $i = 1,2,\dots P$.

(3) The complement of $\mathcal{A}$ denoted by $\mathcal{A}^{\tilde{c}}$ and is defined by

$$\mathcal{A}^{\tilde{c}} = \prec t, \left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots, w_{\mathcal{A}}^p(t) \right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t) \right), \left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t) \right) \succ : t \in T \}$$

(4) If $\mu^i_{\mathcal{A}}(t) = 0$ and $v^i_{\mathcal{A}}(t) = w^i_{\mathcal{A}}(t) = 1$ for all $t \in T$ and $i = 1,2, \dots P$, then $\mathcal{A}$ is called null ns-set and denoted by Φ.

(5) If $\mu^i_{\mathcal{A}}(t) = 1$ and $v^i_{\mathcal{A}}(t) = w^i_{\mathcal{A}}(t) = 0$ for all $t \in T$ and $i = 1,2, \dots P$, then $\mathcal{A}$ is called universal ns-set and denoted by $\tilde{T}$.

(6) The union of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{\cup} \mathcal{B} = \mathcal{C}$ and is defined by

$$\mathcal{C} = \{\prec t, \left(\mu^1_{\mathcal{C}}(t), \mu^2_{\mathcal{C}}(t), \dots \mu^p_{\mathcal{C}}(t)\right), \left(v^1_{\mathcal{C}}(t), v^2_{\mathcal{C}}(t), \dots v^p_{\mathcal{C}}(t)\right), \left(w^1_{\mathcal{C}}(t), w^2_{\mathcal{C}}(t), \dots w^p_{\mathcal{C}}(t)\right) \succ : t \in T\}$$

Where $\mu^i_{\mathcal{C}} = \mu^i_{\mathcal{A}}(t) \vee \mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{C}} = v^i_{\mathcal{A}}(t) \wedge v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{C}} = w^i_{\mathcal{A}}(t) \wedge w^i_{\mathcal{B}}(t)$, $\forall t \in T$ and $i = 1,2, \dots P$.

(7) The intersection of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{\cap} \mathcal{B} = \mathcal{D}$ and is defined by

$$\mathcal{D} = \{\prec t, \left(\mu^1_{\mathcal{D}}(t), \mu^2_{\mathcal{D}}(t), \dots \mu^p_{\mathcal{D}}(t)\right), \left(v^1_{\mathcal{D}}(t), v^2_{\mathcal{D}}(t), \dots v^p_{\mathcal{D}}(t)\right), \left(w^1_{\mathcal{D}}(t), w^2_{\mathcal{D}}(t), \dots w^p_{\mathcal{D}}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{D}} = \mu^i_{\mathcal{A}}(t) \vee \mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{D}} = v^i_{\mathcal{A}}(t) \wedge v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{D}} = w^i_{\mathcal{A}}(t) \wedge w^i_{\mathcal{B}}(t)$, $\forall\, t \in T$ and $i = 1,2, \dots P$.

(8) The addition of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{+} \mathcal{B} = \mathcal{U}_1$ and is defined by

$$\mathcal{U}_1 = \{\prec t, \left(\mu^1_{\mathcal{U}_1}(t), \mu^2_{\mathcal{U}_1}(t), \dots \mu^p_{\mathcal{U}_1}(t)\right), \left(v^1_{\mathcal{U}_1}(t), v^2_{\mathcal{U}_1}(t), \dots v^p_{\mathcal{U}_1}(t)\right), \left(w^1_{\mathcal{U}_1}(t), w^2_{\mathcal{U}_1}(t), \dots w^p_{\mathcal{U}_1}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{U}_1} = \mu^i_{\mathcal{A}}(t) + \mu^i_{\mathcal{B}}(t) - \mu^i_{\mathcal{A}}(t).\mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{U}_1} = v^i_{\mathcal{A}}(t).v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{U}_1} = w^i_{\mathcal{A}}(t).w^i_{\mathcal{B}}(t)$ $\forall\, t \in T$ and $i = 1,2, \dots P$.

(9) The multiplication of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{x} \mathcal{B} = \mathcal{U}_2$ and is defined by

$$\mathcal{U}_2 = \{\prec t, \left(\mu^1_{\mathcal{U}_2}(t), \mu^2_{\mathcal{U}_2}(t), \dots \mu^p_{\mathcal{U}_2}(t)\right), \left(v^1_{\mathcal{U}_2}(t), v^2_{\mathcal{U}_2}(t), \dots v^p_{\mathcal{U}_2}(t)\right), \left(w^1_{\mathcal{U}_2}(t), w^2_{\mathcal{U}_2}(t), \dots w^p_{\mathcal{U}_2}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{U}_2} = \mu^i_{\mathcal{A}}(t).\mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{U}_2} = v^i_{\mathcal{A}}(t) + v^i_{\mathcal{B}}(t) - v^i_{\mathcal{A}}(t).v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{U}_2} = w^i_{\mathcal{A}}(t) + w^i_{\mathcal{B}}(t) w^i_{\mathcal{A}}(t).w^i_{\mathcal{B}}(t)$ $\forall\, t \in T$ and $i = 1,2, \dots P$.

Here $\vee, \wedge, +, ., -$ denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.

Definition 5 [13] Let

$$\mathcal{A} = \{\prec t, \left(\mu^1_{\mathcal{A}}(t), \mu^2_{\mathcal{A}}(t), \dots \mu^p_{\mathcal{A}}(t)\right), \left(v^1_{\mathcal{A}}(t), v^2_{\mathcal{A}}(t), \dots v^p_{\mathcal{A}}(t)\right), \left(w^1_{\mathcal{A}}(t), w^2_{\mathcal{A}}(t), \dots w^p_{\mathcal{A}}(t)\right) \succ : t \in T\}$$

and

$$\mathcal{B} = \{\prec t, \left(\mu^1_{\mathcal{B}}(t), \mu^2_{\mathcal{B}}(t), \dots \mu^p_{\mathcal{B}}(t)\right), \left(v^1_{\mathcal{B}}(t), v^2_{\mathcal{B}}(t), \dots v^p_{\mathcal{B}}(t)\right), \left(w^1_{\mathcal{A}}(t), w^2_{\mathcal{A}}(t), \dots w^p_{\mathcal{A}}(t)\right) \succ : t \in T\}$$

and be two NMSs, then the normalized hamming distance between $\mathcal{A}$ and $\mathcal{B}$ can be defined as follows:

$$d_{NHD}(\mathcal{A}, \mathcal{B}\ ) = \frac{1}{3n.P} \sum_{j=1}^{P} \sum_{i=1}^{n} \left(\left|\mu^j_{\mathcal{A}}(t_i) - \mu^j_{\mathcal{B}}(t_i)\right| + \left|v^j_{\mathcal{A}}(t_i) - v^j_{\mathcal{B}}(t_i)\right| + \left|w^j_{\mathcal{A}}(t_i) - w^j_{\mathcal{B}}(t_i)\right| \right).$$

3. The Outranking Relations of Neutrosophic Multi-Sets

In this section, the binary relations between two neutrosophic refined sets that are based on ELECTRE by extending the studies in [22]. Some of it is quoted from [13, 22, 35, 49].

Definition 6 *Let* $\mathcal{A} = \{\prec t, \left(\mu^i_{\mathcal{A}}(t), v^i_{\mathcal{A}}(t), w^i_{\mathcal{A}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ *and* $\mathcal{B} = \{\prec t, \left(\mu^i_{\mathcal{B}}(t), v^i_{\mathcal{B}}(t), w^i_{\mathcal{B}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ *be two NMS on* T. *Then, the strong dominance relation, weak dominance relation, and indifference relation of NMS can be defined as follows:*

1. If $\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) < v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) < w^i_{\mathcal{B}}(t)$ or $\mu^i_{\mathcal{A}}(t) > \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \forall t \in T$ and $i = 1,2,3, \dots, p$. Then $\mathcal{A}$ strongly dominates $\mathcal{B}$ ($\mathcal{B}$ is strongly dominated by $\mathcal{A}$), denoted by $\mathcal{A} \succ_s \mathcal{B}$.
2. If $\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) \geq v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) < w^i_{\mathcal{B}}(t)$ or $\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) < v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) \geq w^i_{\mathcal{B}}(t), \forall t \in T$ and $i = 1,2,3, \dots, p$. Then $\mathcal{A}$ weakly dominates $\mathcal{B}$ ($\mathcal{B}$ is weakly dominated by $\mathcal{A}$), denoted by$\mathcal{A} \succ_w \mathcal{B}$.
3. If $\mu^i_{\mathcal{A}}(t) = \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \ \forall t \in T$ and $i = 1,2,3, \dots, p$. Then $\mathcal{A}$ indifferent to $\mathcal{B}$, denoted by$\mathcal{A} \sim_l \mathcal{B}$.
4. If none of the relations mentioned above exist between $\mathcal{A}$ and $\mathcal{B}$ for any $t \in T$, then $\mathcal{A}$ and $\mathcal{B}$ are incomparable, denoted by $\mathcal{A} \perp \mathcal{B}$.

Proposition 7 *Let* $\mathcal{A} = \{\prec t, \left(\mu^i_{\mathcal{A}}(t), v^i_{\mathcal{A}}(t), w^i_{\mathcal{A}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ *and* $\mathcal{B} = \{\prec t, \left(\mu^i_{\mathcal{B}}(t), v^i_{\mathcal{B}}(t), w^i_{\mathcal{B}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ be two NMS on T, then the following properties can be obtained:

1. $If \ \mathcal{B} \subset \mathcal{A}, then \ \mathcal{A} \succ_s \mathcal{B};$
2. $If \ \mathcal{A} \succ_s \mathcal{B}, then \ If \ \mathcal{B} \subseteq \mathcal{A};$
3. $\mathcal{A} \sim_l \mathcal{B} \ if \ and \ only \ if \ \mathcal{A} = \mathcal{B}.$

Proof:

1. $If \ \mathcal{B} \subset \mathcal{A}$, then $\mu^i_{\mathcal{B}}(t) \leq \mu^i_{\mathcal{A}}(t), v^i_{\mathcal{B}}(t) \geq v^i_{\mathcal{A}}(t), w^i_{\mathcal{B}}(t) \geq w^i_{\mathcal{A}}(t), \forall t \in T$ and $i = 1,2,3, \dots, p$. $\mathcal{A} \succ_s \mathcal{B}$ is definitely validated according to the strong dominance relation in Definition 6.

2. $\mathcal{A} \succ_s \mathcal{B}$ then based on Definition 6, $\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) < v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) < w^i_{\mathcal{B}}(t)$ *or* $\mu^i_{\mathcal{A}}(t) > \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \forall t \in T$ *and* $i = 1,2,3, \dots, p$. are realized. Then we have $\mathcal{B} \subseteq \mathcal{A}$.

3. *Necessity*: $\mathcal{A} \sim_l \mathcal{B} \Rightarrow \mathcal{A} = \mathcal{B}$. According to the indifference relation in Definition 6 it is known that $\mu^i_{\mathcal{A}}(t) = \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \ \forall t \in T$ and $i = 1,2,3, \dots, p$. Clearly $\mathcal{A} \subseteq \mathcal{A}$ and $\mathcal{B} \subseteq \mathcal{A}$ are achieved, then $\mathcal{A} = \mathcal{B}$.

Sufficiency: $\mathcal{A} = \mathcal{B} \Rightarrow \mathcal{A} \sim_l \mathcal{B}$. If $\mathcal{A} = \mathcal{B}$, then it is know that $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$, which means

$\mu^i_{\mathcal{B}}(t) \leq \mu^i_{\mathcal{A}}(t), v^i_{\mathcal{B}}(t) \geq v^i_{\mathcal{A}}(t), w^i_{\mathcal{B}}(t) \geq w^i_{\mathcal{A}}(t) \ or \ \mu^i_{\mathcal{A}}(t) = \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \ \forall t \in T$ and $i = 1,2,3, \dots, p$. are obtained. Due to the indifference relation in Definition 6, $\mathcal{A} \sim_l \mathcal{B}$ is definitely obtained.

Proposition 8 *Let* $\mathcal{A} = \{\prec t, \left(\mu^i_{\mathcal{A}}(t), v^i_{\mathcal{A}}(t), w^i_{\mathcal{A}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$, $\mathcal{B} = \{\prec t, \left(\mu^i_{\mathcal{B}}(t), v^i_{\mathcal{B}}(t), w^i_{\mathcal{B}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ and $C = \{\prec t, \left(\mu^i_C(t), v^i_C(t), w^i_C(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ be three NMS on T, if $\mathcal{A} \succ_s \mathcal{B} \ and \ \mathcal{B} \succ_s C$, then $\mathcal{A} \succ_s C$.

Proof: According to the strong dominance relation in Definition 6, if $\mathcal{A} \succ_s \mathcal{B}$, then $\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) < v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) < w^i_{\mathcal{B}}(t)$ or $\mu^i_{\mathcal{A}}(t) > \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) = v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) = w^i_{\mathcal{B}}(t), \forall t \in T$ and $i = 1,2,3, \dots, p$.

if $\mathcal{B} \succ_s C$, then $\mu^i_{\mathcal{B}}(t) \geq \mu^i_C(t), v^i_{\mathcal{B}}(t) < v^i_C(t), w^i_{\mathcal{B}}(t) < w^i_C(t)$ *or* $\mu^i_{\mathcal{B}}(t) > \mu^i_C(t), v^i_{\mathcal{B}}(t) = v^i_C(t), w^i_{\mathcal{B}}(t) = w^i_C(t), \forall t \in T$ and $i = 1,2,3, \dots, p$.

Therefore the further derivations are: If

$$\mu^i_{\mathcal{A}}(t) \geq \mu^i_{\mathcal{B}}(t), v^i_{\mathcal{A}}(t) < v^i_{\mathcal{B}}(t), w^i_{\mathcal{A}}(t) < w^i_{\mathcal{B}}(t), \ \dots\dots(1)$$

$$\mu_{\mathcal{B}}^{i}(t) \geq \mu_{C}^{i}(t),\ \ v_{\mathcal{B}}^{i}(t) < v_{C}^{i}(t),\ \ w_{\mathcal{B}}^{i}(t) < w_{C}^{i}(t), \ldots\ (2)$$

from (1) and (2)

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_{s} C$ is realized. If

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{\mathcal{B}}^{i}(t),\ \ldots (3)$$

$$\mu_{\mathcal{B}}^{i}(t) > \mu_{C}^{i}(t),\ \ v_{\mathcal{B}}^{i}(t) = v_{C}^{i}(t),\ \ w_{\mathcal{B}}^{i}(t) = w_{C}^{i}(t), \ldots\ (4)$$

from (3) and (4)

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_{s} C$ is achieved. If

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{\mathcal{B}}^{i}(t),\ \ldots (5)$$

$$\mu_{\mathcal{B}}^{i}(t) \geq \mu_{C}^{i}(t),\ \ v_{\mathcal{B}}^{i}(t) < v_{C}^{i}(t),\ \ w_{\mathcal{B}}^{i}(t) < w_{C}^{i}(t), \ldots\ (6)$$

from (5) and (6)

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_{s} C$ is obtained. If

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{\mathcal{B}}^{i}(t),\ \ldots (7)$$

$$\mu_{\mathcal{B}}^{i}(t) > \mu_{C}^{i}(t),\ \ v_{\mathcal{B}}^{i}(t) = v_{C}^{i}(t),\ \ w_{\mathcal{B}}^{i}(t) = w_{C}^{i}(t), \ldots (8)$$

from (7) and (8)

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_{s} C$ is realized. Therefore, if $\mathcal{A} \succ_{s} \mathcal{B}\ and\ \mathcal{B} \succ_{s} C,$ then $\mathcal{A} \succ_{s} C.$

Proposition 9 *Let* $\mathcal{A} = \{\prec t, \left(\mu_{\mathcal{A}}^{i}(t), v_{\mathcal{A}}^{i}(t), w_{\mathcal{A}}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$, $\mathcal{B} = \{\prec t, \left(\mu_{\mathcal{B}}^{i}(t), v_{\mathcal{B}}^{i}(t), w_{\mathcal{B}}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$ and $C = \{\prec t, \left(\mu_{C}^{i}(t), v_{C}^{i}(t), w_{C}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$ be three NMS on $T,$ if $\mathcal{A} \sim_{l} \mathcal{B}\ and\ \mathcal{B} \sim_{l} C,$ then $\mathcal{A} \sim_{l} C.$

Proof: Clearly, if $\mathcal{A} \sim_{l} \mathcal{B}\ and\ \mathcal{B} \sim_{l} C,$ then $\mathcal{A} \sim_{l} C$ is surely validated.

Proposition 10 *Let* $\mathcal{A} = \{\prec t, \left(\mu_{\mathcal{A}}^{i}(t), v_{\mathcal{A}}^{i}(t), w_{\mathcal{A}}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$, $\mathcal{B} = \{\prec t, \left(\mu_{\mathcal{B}}^{i}(t), v_{\mathcal{B}}^{i}(t), w_{\mathcal{B}}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$ and $C = \{\prec t, \left(\mu_{C}^{i}(t), v_{C}^{i}(t), w_{C}^{i}(t)\right) \succ: t \in T, (i = 1,2,3, \ldots, p)\}$ be three NMS on $T = \{t_1, t_2, \ldots, t_n\},$ then the following results can be obtained.

1.
$1 - irreflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \nsucc_{s} \mathcal{A};$
$2 - asymmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \succ_{s} \mathcal{B} \Rightarrow \mathcal{B} \nsucc_{s} \mathcal{A};$
$3 - transitivity: \forall \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \succ_{s} \mathcal{B}, \mathcal{B} \succ_{s} C, then\ \mathcal{A} \succ C.$

2.
$4 - irreflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \nsucc_{w} \mathcal{A};$
$5 - asymmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \succ_{w} \mathcal{B} \Rightarrow \mathcal{B} \nsucc_{w} \mathcal{A};$
$6 - non-transitivity: \exists \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \succ_{s} \mathcal{B}, \mathcal{B} \succ_{s} C, then\ \mathcal{A} \succ C.$

3.
$7 - reflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \sim_{l} \mathcal{A};$
$8 - symmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \sim_{l} \mathcal{B} \Rightarrow \mathcal{B} \sim_{l} \mathcal{A};$
$9 - transitivity: \exists \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \sim_{l} \mathcal{B}, \mathcal{B} \sim_{l} C, then\ \mathcal{A} \sim_{l} C.$

Example 11 1,2,4,5 and 6 are exemplified as follows.

1. If $\mathcal{A} = \langle (0.8,0.5, \ldots, 0.6), (0.3,0.1, \ldots, 0.5), (0.2,0.3, \ldots, 0.4) \rangle$ is a NMSs, then $\mathcal{A} \nsucc_{s} \mathcal{A}$ can be obtained.
2. If $\mathcal{A} = \langle (0.5,0.7, \ldots, 0.6), (0.2,0.3, \ldots, 0.4), (0.1,0.3, \ldots, 0.2) \rangle$ and $\mathcal{B} = \langle (0.4,0.6, \ldots, 0.5), (0.3,0.4, \ldots, 0.5), (0.2,0.5, \ldots, 0.3) \rangle$ are two NMSs, then $\mathcal{A} \succ_{s} \mathcal{B}, but\ \mathcal{B} \nsucc_{s} \mathcal{A}$ is realized.

3. If $\mathcal{A} = \langle(0.7,0.4,\dots,0.5),(0.4,0.2,\dots,0.6),(0.3,0.3,\dots,0.2)\rangle$ is a NMSs, then $\mathcal{A} \nsucc_w \mathcal{A}$ can be obtained.
4. If $\mathcal{A} = \langle(0.5,0.7,\dots,0.6),(0.5,0.6,\dots,0.4),(0.1,0.3,\dots,0.2)\rangle$ and $\mathcal{B} = \langle(0.3,0.5,\dots,0.6),(0.2,0.3,\dots,0.1),(0.2,0.5,\dots,0.3)\rangle$ are two NMSs, then $\mathcal{A} \succ_w \mathcal{B}, however\ \mathcal{B} \nsucc_w \mathcal{A}$.
5. If $\mathcal{A} = \langle(0.5,0.7,\dots,0.6),(0.3,0.2,\dots,0.4),(0.1,0.3,\dots,0.2)\rangle$,
6. $\mathcal{B} = \langle(0.5,0.6,\dots,0.4),(0.5,0.4,\dots,0.6),(0.2,0.5,\dots,0.3)\rangle$ and $C = \langle(0.4,0.3,\dots,0.2),(0.6,0.5,\dots,0.7),(0.3,0.6,\dots,0.8)\rangle$ are three NMSs, then $\mathcal{A} \succ_w \mathcal{B}\ and\ \mathcal{B} \succ_w C$ are obtained, $\mathcal{A} \succ_w C$.

Proposition 11 [22] Let t_1 and t_2 be two actions, the performances for actions t_1 and t_2 be in the form of NMSs, and $P = s \cup w \cup l$ mean that "t_1 is at least as good as t_2", then four situations may arise:

1. t_1Pt_2 and not t_2Pt_1, that is $t_1 \succ_s t_2$ or $t_1 \succ_w t_2$;
2. t_2Pt_1 and not t_1Pt_2, that is $t_2 \succ_s t_1$ or $t_2 \succ_w t_1$;
3. $t_1Pt_2\ and\ t_2Pt_1$, that is $t_1 \sim_l t_2$;
4. not t_1Pt_2 and not t_2Pt_1, that is $t_1 \perp t_2$.

4. An outranking approach for MCDM with simplified neutrosophic multi-set information

In this section, we introduced an approach for a MCDM problem with neutrosophic multi-set information. Some of it is quoted from [22, 35, 49].

Definition 12 [15] Let $X = (x_1, x_2, \dots, x_n)$ be a set of alternatives, $C = (c_1, c_2, \dots, c_n)$ be the set of criteria, $w = (w_1, w, \dots, w_n)^T$ be the weight vector of the criterions $C_j (j = 1,2,\dots,n)$ such that $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$ and $Z_{ij} = \langle(\mu_{ij}^1\mu_{ij}^2,\dots,\mu_{ij}^n),(v_{ij}^1v_{ij}^2,\dots,v_{ij}^n),(w_{ij}^1w_{ij}^2,\dots,w_{ij}^n)\rangle$ be the decision matrix in which the rating values of the alternatives in for NMSs. Then,

$$
\left[Z_{ij}\right]_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array}
\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\
\begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{pmatrix} \end{array}
$$

is called an NMS-multi-criteria decision making matrix of the decision maker.

Definition 13 [22, 35] In multi-criteria decision making problems;

1. The cost-type criterion values can be transformed into benefit-type criterion values as follows:

$$
\alpha_{ij} = \begin{cases} Z_{ij} & \text{for benefit criterion } C_j, \\ (Z_{ij})^c & \text{for benefit criterion } C_j, \end{cases} \qquad (i = 1,2,\dots,m; j = 1,2,\dots,n) \tag{9}
$$

where $(Z_{ij})^c$ is complement of Z_{ij} as defined in Definition 4.

2. The concordance set of subscripts, which should satisfy the constraint $Z_{ij}PZ_{kj}$, is represented as:

$$O_{ik} = \{j:\ Z_{ij}PZ_{kj}\}\ (i,k = 1,2,\dots,m).$$

$Z_{ij}PZ_{kj}$ represents $Z_{ij} >_s Z_{kj}$ or $Z_{ij} >_w Z_{kj}$ or $Z_{ij} \backsim Z_{kj}$.

3. The concordance index h_{ik} between x_i and x_k is thus defined as follows:

$$
h_{ik} = \sum_{j \in O_{ik}} w_j \tag{10}
$$

Thus, the concordance matrix C is:

$$H = h_{ik} = \begin{pmatrix} - & h_{12} & \cdots & h_{1n} \\ h_{21} & - & \cdots & h_{2n} \\ \vdots & & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & - \end{pmatrix}$$

In *H*; h_{ik} $(i \neq k)$ denote the degree to which the evaluations of x_i are at least as good as those of the competitorx_k, and the degree to which x_i is inferior to x_k decreases with increasing h_{ik}.

4. The discordance set of subscripts for criteria is given as;

$$G_{ik} = J - O_{ik}.$$

5. The discordance index $G(x_i\,;\,x_k)$ is represented as:

$$G_{ik} = \frac{\max\limits_{j \in G_{ik}}\{d(Z_{ij}, Z_{kj})\}}{\max\limits_{j \in J}\{d(Z_{ij}, Z_{kj})\}} \tag{11}$$

here $d(Z_{ij}, Z_{kj})$ denotes the normalized Hamming distance between Z_{ij} and Z_{kj} as defined in Definition 5.

Thus, the discordance matrix *D* is:

$$g_{ik} = \begin{pmatrix} - & g_{12} & \cdots & g_{1n} \\ g_{21} & - & \cdots & g_{2n} \\ \vdots & & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & - \end{pmatrix}$$

In *G*; g_{ik} $(i \neq k)$ denote the degree to which the evaluations of x_i are at least as good as those of the competitorx_k, and the degree to which x_i is inferior to x_k decreases with increasing g_{ik}.

6. To rank all alternatives, the net dominance index of x_k

$$h_{ik} = \sum_{i=1, i\neq k}^{n} h_{ik} - \sum_{i=1, i\neq k}^{n} h_{ki} \tag{12}$$

and the net disadvantage index of x_k is

$$g_{ik} = \sum_{i=1, i\neq k}^{n} g_{ik} - \sum_{i=1, i\neq k}^{n} g_{ki} \tag{13}$$

In here, h_k is the sum of the concordance indices between x_k and x_k $(i \neq k)$ minus the sum of the concordance indices between x_k $(i \neq k)$ and x_k, and reflects the dominance degree of the alternative x_k among the relevant alternatives. Meanwhile, g_k reflects the disadvantage degree of the alternative x_k among the relevant alternatives. Therefore, x_k obtains a greater dominance over the other alternatives that are being compared as h_k increases and g_k decreases.

Definition 14 [35] The ranking rules of two alternatives are

i. If $h_i < h_k$ and $g_i > g_k$ then x_k is superior to x_i, as denoted by $x_k \succ x_i$;

ii. If $h_i = h_k$ and $g_i = g_k$ then x_k is indifferent to x_i, as denoted by $x_k \sim x_i$;

i. if the relation between x_k and x_i does not belong to (i) or (ii);then x_k and x_i are incomparable; as denoted by $x_k \perp x_i$.

Now, we give an algorithm to develop a new approach as

Algorithm:

Step 1 Give the decision-making matrix$\left[Z_{ij}\right]_{m\times n}$; for decision;

Step 2 Compute the weighted normalized matrix as;

$$\left[\gamma_{ij}\right]_{m\times n} = \alpha_{ij} w_j \qquad i = 1, 2, ..., m;\; j = 1, 2, ..., n.$$

where w_j is the weight of the j th criterion with $\sum_{j=1}^{n} w_j = 1$.

Step 3 Find the concordance set of subscripts;

Step 4 Find the discordance set of subscripts;

Step 5 Compute the concordance matrix $H = (h_{ik})_{n\times n}$

Step 6 Compute the discordance matrix $G = (g_{ik})_{n\times n}$

Step 7. Compute the net dominance index of each alternative h_i (i=1,2,3,...,m)

Step 8. Compute the net disadvantage index of each alternative g_i (i=1,2,...,m)

Step 9. Rank all alternatives and select the best alternative.

5 Illustrative examples

In this section, we introduced an example for a MCDM problem with neutrosophic refined information. Some of it is quoted from [22, 35, 49].

Example 15 Assume that $X = (x_1, x_2, x_3, x_4)$ be a set of alternatives and $C = (c_1, c_2, c_3, c_4)$ be the set of criterions, $w = (0.1,0.3,0.2,0.4)^T$ be the weight vector of the criterions $C_j (j = 1,2, \dots, n)$. The four alternatives are to be evaluated under the above four criteria in the form of NMSs. Then,

Step 1. The decision matrix $\left[Z_{ij}\right]_{m\times n}$ is given as;

$$\left(\begin{array}{l}
\langle(0:1;\ 0:2;\ 0:4;\ 0:5);\ (0:6;\ 0:3;\ 0:5;\ 0:2);\ (0:2;\ 0:4;\ 0:5;\ 0:6)\rangle\\
\langle(0:3;\ 0:4;\ 0:6;\ 0:7);\ (0:2;\ 0:5;\ 0:1;\ 0:8);\ (0:3;\ 0:4;\ 0:6;\ 0:8)\rangle\\
\langle(0:1;\ 0:2;\ 0:5;\ 0:6);\ (0:1;\ 0:3;\ 0:5;\ 0:2);\ (0:1;\ 0:5;\ 0:7;\ 0:9)\rangle\\
\langle(0:2;\ 0:3;\ 0:4;\ 0:5);\ (0:3;\ 0:2;\ 0:4;\ 0:6);\ (0:2;\ 0:3;\ 0:5;\ 0:7)\rangle
\end{array}\right.$$

$$\begin{array}{l}
\langle(0:3;\ 0:5;\ 0:7;\ 0:8);\ (0:4;\ 0:3;\ 0:6;\ 0:2);\ (0:1;\ 0:3;\ 0:5;\ 0:2)\rangle\\
\langle(0:2;\ 0:3;\ 0:4;\ 0:5);\ (0:1;\ 0:4;\ 0:3;\ 0:6);\ (0:2;\ 0:3;\ 0:4;\ 0:5)\rangle\\
\langle(0:1;\ 0:2;\ 0:6;\ 0:7);\ (0:3;\ 0:2;\ 0:5;\ 0:4);\ (0:1;\ 0:2;\ 0:5;\ 0:6)\rangle\\
\langle(0:3;\ 0:4;\ 0:6;\ 0:8);\ (0:2;\ 0:1;\ 0:3;\ 0:6);\ (0:4;\ 0:3;\ 0:2;\ 0:5)\rangle
\end{array}$$

$$\begin{array}{l}
\langle(0:2;\ 0:4;\ 0:5;\ 0:6);\ (0:3;\ 0:5;\ 0:2;\ 0:6);\ (0:1;\ 0:2;\ 0:5;\ 0:6)\rangle\\
\langle(0:4;\ 0:5;\ 0:7;\ 0:8);\ (0:1;\ 0:6;\ 0:2;\ 0:3);\ (0:1;\ 0:4;\ 0:3;\ 0:6)\rangle\\
\langle(0:3;\ 0:6;\ 0:8;\ 0:9);\ (0:2;\ 0:4;\ 0:1;\ 0:5);\ (0:2;\ 0:1;\ 0:3;\ 0:6)\rangle\\
\langle(0:1;\ 0:2;\ 0:4;\ 0:6);\ (0:1;\ 0:3;\ 0:7;\ 0:4);\ (0:3;\ 0:4;\ 0:6;\ 0:7)\rangle
\end{array}$$

$$\left.\begin{array}{l}
\langle(0:1;\ 0:2;\ 0:4;\ 0:5);\ (0:2;\ 0:3;\ 0:5;\ 0:4);\ (0:1;\ 0:3;\ 0:7;\ 0:4)\rangle\\
\langle(0:3;\ 0:4;\ 0:5;\ 0:6);\ (0:3;\ 0:1;\ 0:2;\ 0:5);\ (0:3;\ 0:6;\ 0:8;\ 0:9)\rangle\\
\langle(0:1;\ 0:3;\ 0:4;\ 0:5);\ (0:1;\ 0:4;\ 0:6;\ 0:7);\ (0:1;\ 0:2;\ 0:6;\ 0:7)\rangle\\
\langle(0:2;\ 0:4;\ 0:5;\ 0:7);\ (0:2;\ 0:3;\ 0:5;\ 0:6);\ (0:3;\ 0:2;\ 0:4;\ 0:6)\rangle
\end{array}\right)$$

Step 2. The weighted normalized matrix $\left[\gamma_{ij}\right]_{m\times n}$ is computed as;

$$\left(\begin{array}{l}
(0:7943;\ 0:8513;\ 0:9124;\ 0:9330);\ (0:0875;\ 0:0350;\ 0:0669;\ 0:0220);\ (0:0220;\ 0:0104;\ 0:0669;\ 0:0875)\\
(0:6968;\ 0:7596;\ 0:8579;\ 0:8985);\ (0:0647;\ 0:1877;\ 0:0311;\ 0:3829);\ (0:1014;\ 0:1420;\ 0:2403;\ 0:3829)\\
(0:6309;\ 0:7247;\ 0:8705;\ 0:9028);\ (0:2080;\ 0:0688;\ 0:1294;\ 0:0436);\ (0:2080;\ 0:1294;\ 0:2140;\ 0:3690)\\
(0:5253;\ 0:6178;\ 0:6931;\ 0:7578);\ (0:1329;\ 0:0853;\ 0:1848;\ 0:3068);\ (0:0853;\ 0:1329;\ 0:2421;\ 0:3822)
\end{array}\right.$$

$$\begin{array}{l}
(0:8865;\ 0:9330;\ 0:9649;\ 0:9779);\ (0:0498;\ 0:0350;\ 0:0875;\ 0:0620);\ (0:0104;\ 0:0350;\ 0:0669;\ 0:0220)\\
(0:6170;\ 0:6968;\ 0:7596;\ 0:8122);\ (0:0311;\ 0:1420;\ 0:1014;\ 0:2403);\ (0:0647;\ 0:1014;\ 0:1420;\ 0:1877)\\
(0:6309;\ 0:7247;\ 0:9028;\ 0:9311);\ (0:0188;\ 0:0436;\ 0:1294;\ 0:0971);\ (0:0208;\ 0:0436;\ 0:1294;\ 0:1674)\\
(0:6178;\ 0:6931;\ 0:8151;\ 0:9146);\ (0:0853;\ 0:0412;\ 0:1329;\ 0:3068);\ (0:1848;\ 0:1329;\ 0:0853;\ 0:2421)
\end{array}$$

$$\begin{array}{l}
(0:8513;\ 0:9124;\ 0:9330;\ 0:9502);\ (0:0350;\ 0:0669;\ 0:0720;\ 0:0875);\ (0:0104;\ 0:0220;\ 0:0669;\ 0:0875)\\
(0:7596;\ 0:8122;\ 0:8985;\ 0:9352);\ (0:0311;\ 0:0203;\ 0:0647;\ 0:1014);\ (0:0311;\ 0:1420;\ 0:1014;\ 0:2403)\\
(0:7860;\ 0:9028;\ 0:9563;\ 0:9791);\ (0:0436;\ 0:0971;\ 0:0208;\ 0:1294);\ (0:0436;\ 0:0208;\ 0:0688;\ 0:1674)\\
(0:3981;\ 0:5253;\ 0:6931;\ 0:8151);\ (0:0412;\ 0:1329;\ 0:3822;\ 0:1848);\ (0:0412;\ 0:1329;\ 0:3822;\ 0:6018)
\end{array}$$

$$\begin{matrix} (0:7943;\ 0:8513;\ 0:9124;\ 0:9330); & (0:0220;\ 0:0350;\ 0:0669;\ 0:0498); & (0:0104;\ 0:0350;\ 0:1134;\ 0:0498) \\ (0:6968;\ 0:7596;\ 0:8122;\ 0:8579); & (0:1014;\ 0:0311;\ 0:0647;\ 0:1877); & (0:1014;\ 0:2403;\ 0:2403;\ 0:4988) \\ (0:6309;\ 0:7860;\ 0:8325;\ 0:8705); & (0:0228;\ 0:0971;\ 0:1674;\ 0:2140); & (0:0208;\ 0:0436;\ 0:1674;\ 0:2140) \\ (0:5253;\ 0:6931;\ 0:7578;\ 0:8670); & (0:1853;\ 0:1329;\ 0:2421;\ 0:3068); & (0:0329;\ 0:0853;\ 0:1848;\ 0:3068) \end{matrix} \Bigg)$$

Step 3. The concordance set is found as;

$$O_{12} = \{\ \}; O_{21} = \{4\}; O_{31} = \{\ \}; O_{41} = \{\ \}; O_{13} = \{1,2\}; O_{23} = \{\ \};$$
$$O_{32} = \{\ \}; O_{42} = \{\ \}; O_{14} = \{4\}; O_{24} = \{1,3\}; O_{34} = \{1,2\}; O_{43} = \{\ \}.$$

Step 4. The discordance set is found as;

$$G_{12} = \{1,2,3,4\}; G_{21} = \{1,2,3\}; G_{31} = \{1,2,3,4\}; G_{41} = \{1,2,3,4\}; O_{13} = \{1,2\}; G_{23} = \{1,2,3,4\};$$
$$G_{32} = \{1,2,3,4\}; G_{42} = \{1,2,3,4\}; G_{14} = \{1,2,3\}; G_{24} = \{2,4\}; G_{34} = \{3,4\}; G_{43} = \{1,2,3,4\}.$$

where $\{\ \}$ denotes "empty".

Step 5. The concordance is computed as;

$$H = \begin{pmatrix} - & 0 & 0.4 & 0.4 \\ 0.4 & - & 0.4 & 0.3 \\ 0 & 0 & - & 0.4 \\ 0 & 0 & 0 & - \end{pmatrix}$$

Step 6. The discordance matrix is computed as;

$$G = \begin{pmatrix} - & 1 & 0.6612 & 1 \\ 0.9958 & - & 1 & 0.5778 \\ 1 & 1 & - & 1 \\ 1 & 1 & 1 & - \end{pmatrix}$$

Step 7. The net dominance index of each alternative h_i (i=1,2,3,4) is computed as;

$$h_1 = 0.4, h_2 = 1.1, h_3 = -0.4 \text{ and } h_4 = -1.1, \Rightarrow h_4 < h_3 < h_1 < h_2;$$

Step 8. The net disadvantage index of each alternative g_i (i=1,2,3,4) is computed as;

$$g_1 = -0.3346, g_2 = -0.428, g_3 = 0.3388 \text{ and } g_4 = 0.4242, \Rightarrow g_4 > g_3 > g_1 > g_2.$$

Step 9. The final ranking is and the best alte $x_2 \succ x_1 \succ x_3 \succ x_4$ rnative is x_2.

6. Conclusions

This paper developed a multi-criteria decision making method for neutrosophic multi-sets based on these given the outranking relations. In further research, we will develop different methods and compare the different methods on neutrosophic multi-sets. The contribution of this study is that the proposed approach is simple and convenient with regard to computing, and effective in decreasing the loss of evaluative information. More effective decision methods of this proposes a new outranking approach will be investigated in the near future and applied these concepts to engineering, game theory, multi-agent systems, decision-making and so on.

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