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Neutrosophic Spherical Sets in MCDM

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Abstract:

Decision-making plays a crucial role in achieving success across various scenarios, especially when confronted with complex issues inundated with abundant facts and information. Employing multi-criteria decision-making (MCDM) methods and techniques becomes particularly indispensable in tackling such formidable challenges. This study introduces novel neutrosophic SWGM and SWAM accuracy functions, which enhance traditional aggregation operators. Furthermore, it introduces the CODAS technique tailored for addressing Multiple Attribute Group Decision Making problems utilizing the newly defined operators. To exemplify the proposed methodology, a supplier selection problem is examined.

Keywords: Neutrosophic Spherical Set (NSS); Decision Matrix (D-Mx); Negative Ideal Solution (NIS); Positive Ideal Solution (PIS); Spherical Weighted Arithmetic Mean (SWAM); Spherical Weighted Geometric Mean (SWGM); Score function (SF); Accuracy Function (AF).

1. Introduction

Multi-Criteria Decision-Making (MCDM) is a structured approach that considers multiple criteria and attributes to assess and pinpoint the best option or resolution among a set of competing alternatives. Decision-makers are often tasked with navigating conflicting objectives or standards when selecting from available options in diverse real-life contexts. MCDM serves as a tool to assist decision-makers in achieving the most advantageous decision by carefully weighing and addressing these considerations.

The MCDM consists of criteria, a set of alternatives, and expert evaluations of the alternatives for each criterion. These sections evaluate the specialized knowledge and score the options based on suitability. These days, a vast array of MCDM approaches have been developed and applied in many different kinds of industries [1], the transportation industry [2], economics [3], health [4], energy planning [5], manufacturing [6], construction [7], supplier selection [8], and more.

A recent development in MCDM is the distance-based methodology known as Combinative Distance-based Assessment (CODAS). This methodology compares the Euclidean distance (ED) with the Taxicab distance (TD) to determine which alternatives are preferred.

Uncertainty stands as one of the pivotal factors impacting the decision-making process. Employing a Fuzzy Set (FS) offers a means to surmount this uncertainty. As a development of classical set theory, Lotfi A. Zadeh invented FSs [9] in 1965. The idea behind FSs is to represent and manipulate uncertainty more flexibly and realistically, especially in situations where traditional binary logic may not be suitable. Fuzzy MCDM techniques aim to resolve the uncertainty associated with decision-making problems [10].

Atanassov presented a broader version of fuzzy sets called intuitionistic fuzzy sets, which provide a more comprehensive treatment of ambiguity and uncertainty [11], and are formally known as Intuitionistic Fuzzy Sets (IFS). IFSs extend the capabilities of traditional fuzzy sets by encompassing notions of non-membership and hesitancy. Employing an intuitionistic fuzzy Multi-Criteria Decision-Making (MCDM) approach, Karagoz S, Deveci M, Simic V, Aydin N, and Bolukbas [12] evaluated various choices for the selection of a designated dismantling center location.

An approach utilizing CODAS technique, grounded on intuitionistic fuzzy [13] Multiple Criteria Decision Making (MCDM), is proposed to aid in waste management. The method involves employing the intuitionistic fuzzy weighted averaging operator to amalgamate the diverse viewpoints of decision-makers into a unified consensus.

Expanding on the notion of Intuitionistic Fuzzy Sets (IFS), a mathematical concept called Interval-Valued Intuitionistic Fuzzy Sets (IVIFS) incorporates intervals to represent degrees of membership and non-membership. Roy, Das, Kar, and Pamuèar (2019) extended the CODAS approach with IVIFS, offering a framework for assessing Multiple Criteria Decision Making (MCDM) challenges where only partial weight information is available. Additionally, Peng and Garg [15] introduced methodologies for addressing emergency decision-making using similarity measures, CODAS, and weighted distance-based approximation within interval-valued fuzzy soft sets.

The Pythagorean Fuzzy Set (PFS) is the foundation of a recently introduced novel structure intended to handle uncertainty in practical decision-making situations. When awarding membership, nonstandard FSs, IFSs, and IVFSs permit a degree of commitment smaller than one. A class of nonstandard Pythagorean fuzzy subsets is introduced in [16], where the membership grades are pairs (a, b) that meet the condition that $a^2 + b^2 \leq 1$. PFS is far more effective at modeling such uncertainty than an IFS.

Peng, Xindong, and Ma, Xueling investigated an algorithm for resolving MCDM problems based on CODAS and created a novel approach for handling MCDM difficulties in a Pythagorean fuzzy environment [17]. Zhang, X., Xu, Z outline some new Pythagorean Fuzzy Set (PFS) operating regulations and discuss their beneficial characteristics [18]. To successfully address the MCDM difficulties involving PFSs, it is also suggested that an enhanced strategy for order preference be similar to the optimum solution method.

The Neutrosophic Set (NS) theory extends classical sets, FSs, and IFSs that aim to manage unclear, incomplete, and contrasting facts. This approach, which resolved indeterminacy using a new type of set and allowed for a more refined representation of uncertain particulars, was given by Florentin Smarandache [19-20].

Smarandache illustrates in [21-22] that offsets and off-uninorms have applicability within digital image processing, particularly for tasks like image segmentation and edge detection. Furthermore, the paper offers algorithms and examples to elucidate these concepts.

One specific type of NS is a single-valued set which has been proposed to manage with incomplete information. [23] offers a novel method for solving multi-attribute group decision-making issues by applying the order choose by similarity technique to a single-valued neutrosophic environment. Additionally, create the TOPSIS technique for MADM in a streamlined neutrosophic setting.

Broumi, Je, and Smarandache are set to enhance the TOPSIS method [24] to accommodate interval neutrosophic uncertain linguistic information. They will introduce an extended version of the TOPSIS method tailored for resolving multiple attribute decision-making dilemmas where attribute values are expressed as interval neutrosophic uncertain linguistic variables and attribute weights remain unspecified. Broumi introduced the innovative concept of the Neutrosophic Inverse Soft Expert Set (NISES) in [25], which finds application within the Failure Mode and Effect Analysis (FMEA) framework.

H. Garg presents novel applications for combining Single-Valued Neutrosophic (SVN) data, which are applied to solve problems related to MCDM [26]. Gundogdu, F. Kutlu, and Kahraman, C. presented the idea of generalized three-dimensional Spherical Fuzzy Fets (SFSs) with a few critical distinctions from previous FSs [27]. The spherical vague distances, established with examples, provide the basis of the new kind of FS. An illustrated example of spherical SF TOPSIS, a MCDM approach, is shown.

To evaluate the obstacles to the growth of clean energy, a proposed technique based on MCDM approaches in a SFS has been mentioned in [28]. Additionally, CODAS outperformed the other approaches when the outcomes of the MOORA, COPRAS, and CODAS procedures were compared. Biswas, Chatterjee, and Majumder [29] apply a SFS to rank the statements. After calculating scores, they utilize an MCDA based on the SFS to determine the statements' relative ranking according to the judgments of a selection panel. The LOPCOW (modified SF LOgarithmic Percentage Change-driven Objective Weighting) approach is employed.

Smarandache introduced the concepts of Neutrosophic Two-Fold Algebra [30-31] along with its corresponding Neutrosophic Two-Fold Law, and explored their extensions into Fuzzy Two-Fold Algebras and Laws. Additionally, they discovered nine novel topologies while enhancing and revisiting seven previously established ones [32]. Smarandache demonstrated that the Super Hyper Function [33] serves as a broader framework encompassing classical Function, Super Function, and Hyper Function. They also pioneered the Super Hyper Soft Set and its variations, including the Fuzzy and Fuzzy Extension Super Hyper Soft Set, [34] while establishing that the Super Hyper Soft Set comprises multiple Hyper Soft Sets.

In this study, we create a novel notion, the Neutrosophic Spherical Set (NSS), by fusing the ideas of spherical measure and neutrosophic logic. The spherical fuzzy distances established in the literature are the foundation for the new class of Neutrosophic sets. The presentation includes the proofs for addition, subtraction, and multiplication arithmetic operations. Accuracy functions, scoring, and aggregation operations are constructed. An exemplary example of Spherical Neutrosophic CODAS, a MCDM process, is shown.

2. Preliminaries

Definition 2.1 [19]

Consider M be the universe. A NS $\tilde{K}$ in M is characterized by a truth $T_{\tilde{K}}$, indeterminacy $I_{\tilde{K}}$ and a falsity $F_{\tilde{K}}$ membership functions

$$\tilde{K}=\left\{\left\langle \tilde{k},\left(T_{\tilde{K}}\left(\tilde{k}\right)\right),\left(I_{\tilde{K}}\left(\tilde{k}\right)\right),\left(F_{\tilde{K}}\left(\tilde{k}\right)\right)\right\rangle : \tilde{k}\in M, T_K, I_K, F_K \in]^{-}0,1^{+}[\right\}$$

then

$$0^{-}\le\left(T_{\tilde{K}}\left(\tilde{k}\right)\right)+\left(I_{\tilde{K}}\left(\tilde{k}\right)\right)+\left(F_{\tilde{K}}\left(\tilde{k}\right)\right)\le 3^{+}$$

Definition 2.2 [27]

A SFS $\tilde{S}$ of the universe of discourse Z is given by $\tilde{S}=\{<s,(T_{\tilde{S}}(s),I_{\tilde{S}}(s),F_{\tilde{S}}(s))>| s\in Z\}$

Where $T_{\tilde{S}}(s):Z\rightarrow[0,1]$, $I_{\tilde{S}}(s):Z\rightarrow[0,1]$, $F_{\tilde{S}}(s):Z\rightarrow[0,1]$ and

$$0\le T_{\tilde{S}}^{2}(s)+I_{\tilde{S}}^{2}(s)+F_{\tilde{S}}^{2}(s)\le 1 \quad \forall s\in Z$$

For each s, the numbers $T_{\tilde{S}}(s), I_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are membership, non-membership and hesitancy of s to $\tilde{A}$, respectively.

3. Neutrosophic spherical set

The squared sum of the parameters in NSSs can range 0 and $\sqrt{3}$, it is possible to define each of them individually between 0 and 1 independently. In this section, the explanation of NSS and overview of spherical distance measurement, arithmetic operation and aggregation and de-neutrosophication processes are provided.

Definition 3.1. NSS $\tilde{S}$ of the universe of discourse Z is given by

$$\tilde{S}=\{<s,(T_{\tilde{S}}(s),I_{\tilde{S}}(s),F_{\tilde{S}}(s))>| s\in Z\} \tag{1}$$

Where,

$$T_{\tilde{S}}(s):Z\rightarrow[0,1],\quad I_{\tilde{S}}(s):Z\rightarrow[0,1],\ F_{\tilde{S}}(s):Z\rightarrow[0,1]$$

and

$$0\le T_{\tilde{S}}^{2}(s)+I_{\tilde{S}}^{2}(s)+F_{\tilde{S}}^{2}(s)\le\sqrt{3} \quad \forall s\in Z \tag{2}$$

For each s, the numbers $T_{\tilde{S}}(s), I_{\tilde{S}}(s)$ *and* $F_{\tilde{S}}(s)$ are the degree of Membership, Non-Membership, and Hesitant Membership of s to $\tilde{S}$, respectively**Error! Reference source not found.**.

Definition 3.2. Basic Operators

$$\tilde{A}\oplus\tilde{B}=\left\{\left(T_{\tilde{A}}^{2}+T_{\tilde{B}}^{2}-T_{\tilde{A}}^{2}T_{\tilde{B}}^{2}\right)^{\frac{1}{2}},\left(I_{\tilde{A}}^{2}+I_{\tilde{B}}^{2}-I_{\tilde{A}}^{2}I_{\tilde{B}}^{2}\right)^{\frac{1}{2}},\left(F_{\tilde{A}}^{2}+F_{\tilde{B}}^{2}-F_{\tilde{A}}^{2}F_{\tilde{B}}^{2}\right)^{\frac{1}{2}}\right\} \quad (3)$$

$$\tilde{A}\otimes\tilde{B}=\left\{\left(T_{\tilde{A}}T_{\tilde{B}}\right),\left(I_{\tilde{A}}I_{\tilde{B}}\right),\left(F_{\tilde{A}}F_{\tilde{B}}\right)\right\} \quad (4)$$

$$\lambda\bullet\tilde{A}=\left\{\left(1-\left(1-T_{\tilde{A}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-I_{\tilde{A}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-F_{\tilde{A}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\right\} \quad (5)$$

$$\tilde{A}^{\lambda}=\left\{T_{\tilde{A}}^{\lambda},I_{\tilde{A}}^{\lambda},F_{\tilde{A}}^{\lambda}\right\}\lambda>0 \quad (6)$$

Definition 3.3. For these NSS $\tilde{M}=\left(T_{\tilde{M}},I_{\tilde{M}},F_{\tilde{M}}\right)$ and $\tilde{N}=\left(T_{\tilde{N}},I_{\tilde{N}},F_{\tilde{N}}\right)$, the following applies to $\lambda,\lambda_1,\lambda_2>0$.

1. $\tilde{M}\oplus\tilde{N}=\tilde{N}\oplus\tilde{M}$ (7)
2. $\tilde{M}\otimes\tilde{N}=\tilde{M}\otimes\tilde{N}$ (8)
3. $\lambda\left(\tilde{M}\oplus\tilde{N}\right)=\lambda\tilde{M}\oplus\lambda\tilde{N}$ (9)
4. $\lambda_1\tilde{M}\oplus\lambda_2\tilde{M}=\left(\lambda_1+\lambda_2\right)\tilde{M}$ (10)
5. $\left(\tilde{M}\otimes\tilde{N}\right)^{\lambda}=\tilde{M}^{\lambda}\otimes\tilde{N}^{\lambda}$ (11)
6. $\tilde{M}^{\lambda_1}\otimes\tilde{M}^{\lambda_2}=\tilde{M}^{\lambda_1+\lambda_2}$ (12)

Proof:

According to Definition 3.2, we will prove equations (7-9 and 11) since equation (10 and 12) are comparable to the corresponding proofs of equations (9 and 11),

1. $\tilde{M}\oplus\tilde{N}=\tilde{N}\oplus\tilde{M}$

$$\tilde{M}\oplus\tilde{N}=\left\{\left(T_{\tilde{M}}^{2}+T_{\tilde{N}}^{2}-T_{\tilde{M}}^{2}T_{\tilde{N}}^{2}\right)^{\frac{1}{2}},\left(I_{\tilde{M}}^{2}+I_{\tilde{N}}^{2}-I_{\tilde{M}}^{2}I_{\tilde{N}}^{2}\right)^{\frac{1}{2}},\left(F_{\tilde{M}}^{2}+F_{\tilde{N}}^{2}-F_{\tilde{M}}^{2}F_{\tilde{N}}^{2}\right)^{\frac{1}{2}}\right\}$$

$$\tilde{N}\oplus\tilde{M}=\left\{\left(T_{\tilde{N}}^{2}+T_{\tilde{M}}^{2}-T_{\tilde{N}}^{2}T_{\tilde{M}}^{2}\right)^{\frac{1}{2}},\left(I_{\tilde{N}}^{2}+I_{\tilde{M}}^{2}-I_{\tilde{N}}^{2}I_{\tilde{M}}^{2}\right)^{\frac{1}{2}},\left(F_{\tilde{N}}^{2}+F_{\tilde{M}}^{2}-F_{\tilde{N}}^{2}F_{\tilde{M}}^{2}\right)^{\frac{1}{2}}\right\}$$

Hence 1 is proved.

2. $\tilde{M}\otimes\tilde{N}=\tilde{M}\otimes\tilde{N}$

$$\tilde{M}\otimes\tilde{N}=\left\{\left(T_{\tilde{M}}T_{\tilde{N}}\right),\left(I_{\tilde{M}}I_{\tilde{N}}\right),\left(F_{\tilde{M}}F_{\tilde{N}}\right)\right\}$$

$$\tilde{N}\otimes\tilde{M}=\left\{\left(T_{\tilde{N}}T_{\tilde{M}}\right),\left(I_{\tilde{N}}I_{\tilde{M}}\right),\left(F_{\tilde{N}}F_{\tilde{M}}\right)\right\}$$

Hence 2 is proved.

3. $\lambda\left(\tilde{M}\oplus\tilde{N}\right)=\lambda\tilde{M}\oplus\lambda\tilde{N}$

$$\lambda\left(\tilde{M}\oplus\tilde{N}\right)=\lambda\left\{\left(T_{\tilde{M}}{}^{2}+T_{\tilde{N}}{}^{2}-T_{\tilde{M}}{}^{2}T_{\tilde{N}}{}^{2}\right)^{\frac{1}{2}},\left(I_{\tilde{M}}{}^{2}+I_{\tilde{N}}{}^{2}-I_{\tilde{M}}{}^{2}I_{\tilde{N}}{}^{2}\right)^{\frac{1}{2}},\left(F_{\tilde{M}}{}^{2}+F_{\tilde{N}}{}^{2}-F_{\tilde{M}}{}^{2}F_{\tilde{N}}{}^{2}\right)^{\frac{1}{2}}\right\}$$

$$=\begin{Bmatrix}\left(1-\left(1-\left(T_{\tilde{M}}{}^{2}+T_{\tilde{N}}{}^{2}-T_{\tilde{M}}{}^{2}T_{\tilde{N}}{}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-\left(I_{\tilde{M}}{}^{2}+I_{\tilde{N}}{}^{2}-I_{\tilde{M}}{}^{2}I_{\tilde{N}}{}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}},\\ \left(1-\left(1-\left(F_{\tilde{M}}{}^{2}+F_{\tilde{N}}{}^{2}-F_{\tilde{M}}{}^{2}F_{\tilde{N}}{}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}}\end{Bmatrix}$$

$$\lambda\tilde{M}\oplus\lambda\tilde{N}=\left\{\left(1-\left(1-T_{\tilde{M}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-I_{\tilde{M}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-F_{\tilde{M}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\right\}$$
$$\oplus\left\{\left(1-\left(1-T_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-I_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-F_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\right\}$$

$$=\begin{Bmatrix}\left(1-\left(1-T_{\tilde{M}}^{2}\right)^{\lambda}+1-\left(1-T_{\tilde{N}}^{2}\right)^{\lambda}-\left(1-\left(1-T_{\tilde{M}}^{2}\right)^{\lambda}\right)\left(1-\left(1-T_{\tilde{N}}^{2}\right)^{\lambda}\right)\right)^{\frac{1}{2}},\\ \left(1-\left(1-I_{\tilde{M}}^{2}\right)^{\lambda}+1-\left(1-I_{\tilde{N}}^{2}\right)^{\lambda}-\left(1-\left(1-I_{\tilde{M}}^{2}\right)^{\lambda}\right)\left(1-\left(1-I_{\tilde{N}}^{2}\right)^{\lambda}\right)\right)^{\frac{1}{2}},\\ \left(1-\left(1-F_{\tilde{M}}^{2}\right)^{\lambda}+1-\left(1-F_{\tilde{N}}^{2}\right)^{\lambda}-\left(1-\left(1-F_{\tilde{M}}^{2}\right)^{\lambda}\right)\left(1-\left(1-F_{\tilde{N}}^{2}\right)^{\lambda}\right)\right)^{\frac{1}{2}}\end{Bmatrix}$$

$$=\left\{\left(1-\left(1-T_{\tilde{M}}^{2}\right)^{\lambda}\left(1-T_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-I_{\tilde{M}}^{2}\right)^{\lambda}\left(1-I_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-F_{\tilde{M}}^{2}\right)^{\lambda}\left(1-F_{\tilde{N}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\right\}$$

$$=\begin{Bmatrix}\left(1-\left(1-\left(T_{\tilde{M}}^{2}+T_{\tilde{N}}^{2}-T_{\tilde{M}}^{2}T_{\tilde{N}}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}},\left(1-\left(1-\left(I_{\tilde{M}}^{2}+I_{\tilde{N}}^{2}-I_{\tilde{M}}^{2}I_{\tilde{N}}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}},\\ \left(1-\left(1-\left(F_{\tilde{M}}^{2}+F_{\tilde{N}}^{2}-F_{\tilde{M}}^{2}F_{\tilde{N}}^{2}\right)\right)^{\lambda}\right)^{\frac{1}{2}}\end{Bmatrix}$$

Hence 3 is proved.

Since

4. $\lambda_1\tilde{M}\oplus\lambda_2\tilde{M}=\left(\lambda_1+\lambda_2\right)\tilde{M}$

5. $\left(\tilde{M}\otimes\tilde{N}\right)^{\lambda}=\tilde{M}^{\lambda}\otimes\tilde{N}^{\lambda}$

$$\left(\tilde{M} \otimes \tilde{N}\right)^{\lambda} = \left\{\left(T_{\tilde{M}}T_{\tilde{N}}, I_{\tilde{M}}I_{\tilde{N}}, F_{\tilde{M}}F_{\tilde{N}}\right)^{\lambda}\right\}$$

$$= \left\{T_{\tilde{M}}{}^{\lambda}T_{\tilde{N}}{}^{\lambda}, I_{\tilde{M}}{}^{\lambda}I_{\tilde{N}}{}^{\lambda}, F_{\tilde{M}}{}^{\lambda}F_{\tilde{N}}{}^{\lambda}\right\}$$

$$\tilde{M}^{\lambda} \otimes \tilde{N}^{\lambda} = \left\{T_{\tilde{M}}{}^{\lambda}, I_{\tilde{M}}{}^{\lambda}, F_{\tilde{M}}{}^{\lambda}\right\} \otimes \left\{T_{\tilde{N}}{}^{\lambda}, I_{\tilde{N}}{}^{\lambda}, F_{\tilde{N}}{}^{\lambda}\right\}$$

$$= \left\{T_{\tilde{M}}{}^{\lambda}T_{\tilde{N}}{}^{\lambda}, I_{\tilde{M}}{}^{\lambda}I_{\tilde{N}}{}^{\lambda}, F_{\tilde{M}}{}^{\lambda}F_{\tilde{N}}{}^{\lambda}\right\}$$

Hence 5 is proved.

Definition 3.4. SWAM as, $z = \left(z_1, z_2, z_3, ...z_n\right);\ \ z_j \in [0,1]; \sum_{j=1}^{n} z_j \leq \sqrt{3}$ SWAM is defined as;

$$SWAM_z\left(\tilde{A}_1, \tilde{A}_2, ...\tilde{A}_n\right) = z_1\tilde{A}_1 + z_2\tilde{A}_2 + z_3\tilde{A}_3 + ... + z_n\tilde{A}_n$$

$$\left\{\left[1-\prod_{j=1}^{n}\left(1-T_{\tilde{A}}{}^{2}\right)^{z_j}\right]^{\frac{1}{2}}, \left[1-\prod_{j=1}^{n}\left(1-I_{\tilde{A}}{}^{2}\right)^{z_j}\right]^{\frac{1}{2}}, \left[1-\prod_{j=1}^{n}\left(1-F_{\tilde{A}}{}^{2}\right)^{z_j}\right]^{\frac{1}{2}}\right\} \tag{13}$$

Definition 3.5. SWGM as, $z = \left(z_1, z_2, z_3, ...z_n\right);\ \ z_j \in [0,1]; \sum_{j=1}^{n} z_j \leq \sqrt{3}$

SWGM is defined as; $SWGM_z\left(\tilde{A}_1, \tilde{A}_2, ...\tilde{A}_n\right) = \tilde{A}_1^{z_1} + \tilde{A}_2^{z_2} + \tilde{A}_3^{z_3} + ... + \tilde{A}_n^{z_n}$

$$\left\{\prod_{j=1}^{n} T_{\tilde{A}}{}^{z_j}, \prod_{j=1}^{n} I_{\tilde{A}}{}^{z_j}, \prod_{j=1}^{n} F_{\tilde{A}}{}^{z_j}\right\} \tag{14}$$

Definition 3.6. The SF and AF for NSS classification are defined by;

$$Score\left(\tilde{S}\right) = \left(T_{ijw} - F_{ijw}\right)^2 - \left(I_{ijw} - F_{ijw}\right)^2 \tag{15}$$

$$Accuracy\left(\tilde{S}\right) = T_{\tilde{S}}{}^{2} + I_{\tilde{S}}{}^{2} + F_{\tilde{S}}{}^{2} \tag{16}$$

Note that: $\tilde{S} < \tilde{T}$ iff

1. $Score\left(\tilde{S}\right) < Score\left(\tilde{T}\right) or$
2. $Score\left(\tilde{S}\right) = Score\left(\tilde{T}\right) and\ Accuracy\left(\tilde{S}\right) < Accuracy\left(\tilde{T}\right)$ (17)

4. Neutrosophic Spherical CODAS

A D-Mx with entries that represent the assessment scores of every choice in relation to every criterion in a neutrosophic environment can be used to represent an MCDM problem. Suppose that $S = \left\{s_1, s_2, s_3, ...s_m\right\}\left(m \geq 2\right)$ represents distinct collection of m possible options and

$K=\{K_1,K_2,K_3,....K_n\}$ be the weight vector derived from every requirement that meet $0\le z_j\le 1$ and $\sum_{j=1}^{n} z_j\le\sqrt{3}$.

Step 1. Let DMs use the linguistic terms (LT) listed in Table 1 to complete the assessment matrices for decisions and criteria.

Table 1. Terms used in linguistics and their associated Spherical Neutrosophic Number

LT		(T, I, F)
Probably More Significant	PMS	(.9, .6, .2)
Extremely Significant	ES	(.8, .7, .2)
High Priority	HP	(.7, .6, .5)
Relatively Greater Significance	RGS	(.6, .7, .4)
Equally Important	EI	(.5, .8, .4)
Very Minimal Significance	VMS	(.4, .6, .7)
Low Priority	LP	(.5, .7, .6)
extremely low significant	ELS	(.5, .6 .6)
Definitely Not Important	DNI	(.2, .9, .6)

Step 2. Aggregate the outcomes reached by DM.

Aggregate the outcomes reached by DM using SWAM. Aggregate the DMs' Neutrosophic Spherical linguistic judgements of the selection criteria. Assemble and neutrosophic D-Mx based on DMs' views. Indicate the Alternative's evaluation value.

$S_i\left(i=1,2,...m\right)$ with respect to criterion $K_j\left(j=1,2...n\right)$ by $K_j\left(\tilde{S}_i\right)=\left(T_{ij},I_{ij},F_{ij}\right)$ and $D=\left(K_j\left(\tilde{S}_i\right)\right)_{m\times n}$ is a Neutrosophic Spherical Decision Matrix (NS D-Mx). D-Mx for MCDM problem using NSS, $D=\left(K_j\left(\tilde{S}_i\right)\right)_{m\times n}$ must be put together as shown in equation (18).

$$D=\left(K_j\left(\tilde{S}_i\right)\right)_{m\times n}=\begin{pmatrix}\left(\tilde{T}_{11},\tilde{I}_{11},\tilde{F}_{11}\right) & \left(\tilde{T}_{12},\tilde{I}_{12},\tilde{F}_{12}\right) & \cdots & \left(\tilde{T}_{1n},\tilde{I}_{1n},\tilde{F}_{1n}\right)\\ \left(\tilde{T}_{21},\tilde{I}_{21},\tilde{F}_{21}\right) & \left(\tilde{T}_{22},\tilde{I}_{22},\tilde{F}_{22}\right) & \cdots & \left(\tilde{T}_{2n},\tilde{I}_{2n},\tilde{F}_{2n}\right)\\ \vdots & \vdots & \cdots & \vdots\\ \left(\tilde{T}_{m1},\tilde{I}_{m1},\tilde{F}_{m1}\right) & \left(\tilde{T}_{m2},\tilde{I}_{m2},\tilde{F}_{m2}\right) & \cdots & \left(\tilde{T}_{mn},\tilde{I}_{mn},\tilde{F}_{mn}\right)\end{pmatrix} \quad (18)$$

Step 3. Build the weighted aggregated NS D-Mx. Following the determination of the alternative ratings and the weights assigned to the criteria, the aggregated weighted NS D-Mx is built using multiplication equation and then the aggregated weighted NS D-Mx can be defined as follows:

$$D=\left(K_j\left(\tilde{S}_{iz}\right)\right)_{m\times n}=\begin{pmatrix}\left(T_{11z},I_{11z},F_{11z}\right) & \left(T_{12z},I_{12z},F_{12z}\right) & \dots & \left(T_{1nz},I_{1nz},F_{1nz}\right)\\ \left(T_{21z},I_{21z},F_{21z}\right) & \left(T_{22z},I_{22z},F_{22z}\right) & \dots & \left(T_{2nz},I_{2nz},F_{2nz}\right)\\ \vdots & \vdots & \dots & \vdots\\ \left(T_{m1z},I_{m1z},F_{m1z}\right) & \left(T_{m2z},I_{m2z},F_{m2z}\right) & \dots & \left(T_{mnz},I_{mnz},F_{mnz}\right)\end{pmatrix} \tag{19}$$

Step 4. Utilising Eq. (20), deneutroscope the aggregated weighted D-Mx.

$$Score\left(K_j\left(\tilde{S}_{iz}\right)\right)=\left(T_{ijz}-F_{ijz}\right)^2-\left(I_{ijz}-F_{ijz}\right)^2 \tag{20}$$

Step 5. Find the NSPIS and NSNIS according to the SF acquired in Step 4.

Regarding the NS-PIS:

$$S^*=\left\{K_j,\max_i\left\langle Score\left(K_j\left(S_{iz}\right)\right)\right\rangle \mid j=1,2,\dots n\right\} \tag{21}$$

$$S^*=\left\{\left\langle K_1,\left(T_1^*,I_1^*,F_1^*\right)\right\rangle,\left\langle K_2,\left(T_2^*,I_2^*,F_2^*\right)\right\rangle,\dots\left\langle K_n,\left(T_n^*,I_n^*,F_n^*\right)\right\rangle\right\}$$

Regarding the NS -NIS:

$$S^-=\left\{K_j,\min_i\left\langle Score\left(K_j\left(S_{iz}\right)\right)\right\rangle \mid j=1,2,\dots n\right\} \tag{22}$$

$$S^-=\left\{\left\langle K_1,\left(T_1^-,I_1^-,F_1^-\right)\right\rangle,\left\langle K_2,\left(T_2^-,I_2^-,F_2^-\right)\right\rangle,\dots\left\langle K_n,\left(T_n^-,I_n^-,F_n^-\right)\right\rangle\right\}$$

Step 6. The distances between alternative S_i, NS-PIS, and NS-NIS should be calculated, accordingly.

For the NS-NIS:

$$D\left(S_i,S^-\right)=\sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\left(T_{S_i}-T_{S^-}\right)^2+\left(I_{S_i}-I_{S^-}\right)^2+\left(F_{S_i}-F_{S^-}\right)^2\right)} \tag{23}$$

For the NS-NIS:

$$D\left(S_i,S^*\right)=\sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\left(T_{S_i}-T_{S^*}\right)^2+\left(I_{S_i}-I_{S^*}\right)^2+\left(F_{S_i}-F_{S^*}\right)^2\right)} \tag{24}$$

Step 7 Calculate the minimum and maximum distances to the NS-NIS and NS-PIS, respectively.

$$D_{\max}\left(S_i,S^-\right)=\max_{i\le i\le m}\left(S_i,S^-\right) \tag{25}$$

$$D_{\min}\left(S_i,S^*\right)=\min_{i\le i\le m}\left(S_i,S^*\right) \tag{26}$$

Step 8 Compute the revised proximity ratio in Equation (27).

$$\xi\left(S_i\right)=\frac{D\left(S_i,S^-\right)}{D_{\max}\left(S_i,S^-\right)}-\frac{D\left(S_i,S^*\right)}{D_{\min}\left(S_i,S^*\right)} \tag{27}$$

Equation (27) because the subtraction's second element is at least equal to its first element, the result is zero or negative. We altered this equality from Equation (28) so that we might get zero or a result.

$$\xi\left(S_i\right)=\frac{D\left(S_i,S^*\right)}{D_{\min}\left(S_i,S^*\right)}-\frac{D\left(S_i,S^-\right)}{D_{\max}\left(S_i,S^-\right)} \quad (28)$$

Step 9. Determine the best solution by rating the alternatives in the best possible order. We organize the alternatives according to the rising closeness ratio values since we wish to use Equation (28).

5.Illustrative Example

A supplier selection issue is devised and solved by employing our recommended technique. Four vendors of air conditioners were considered count $\left(S_1,S_2,S_3,S_4\right)$ and evaluated for their efficacy. The number of qualitative and quantitative aspects considered will determine how many different criteria are used to pick suppliers. In accordance with on the number of qualitative and quantitative factors are considered, the decision-making criteria for supplier selection may change. Several criteria and sub-criteria have been established using a comprehensive literature assessment. Four of these criteria are used in this exemplary example: price $\left(K_1\right)$, quality $\left(K_2\right)$, delivery $\left(K_3\right)$ and performance $\left(K_4\right)$. Three decision makers with experience in supply chain and logistics management (ĐM1, ĐM2, and ĐM3) take part in the procedure for evaluation. The weights of these DMs, which are, respectively, 0.4, 0.5 and 0.3, represent their various levels of experience.

First, the judgements made by the decision-makers are compiled using the language phrases listed in Table 1 with regard to the objective. A decision is rendered in Tables 2-4.

Table 2. Decisions of DM1

ĐM1	(K_1)	(K_2)	(K_3)	(K_4)
S_1	ES	HP	EI	RGS
S_2	PMS	EI	HP	EI
S_3	LP	RGS	ES	ELS
S_4	ELS	ES	LP	HP

Table 3. Decisions of DM2

ĐM2	(K_1)	(K_2)	(K_3)	(K_4)
S_1	PMS	HP	ES	PMS

S_2	VMS	ES	HP	EI
S_3	HP	RGS	RGS	RGS
S_4	ELS	EI	LP	LP

Table 4. Decisions of DM3

ĐM3	(K_1)	(K_2)	(K_3)	(K_4)
S_1	HP	ES	PMS	RGS
S_2	VMS	PMS	ES	VMS
S_3	VMS	ELS	HP	HP
S_4	LP	EI	ES	RGS

The significance levels of the DMs are considered when combining these judgements utilizing the SWAM and SWGM operators. The decision matrices shown in Tables 5 and 6 are obtained.

Table 5. NS D-Mx by using SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.873,0.682,0.340)	(0.773,0.673,0.487)	(0.821,0.764,0.311)	(0.825,0.707,0.364)
S_2	(0.743, 0.643,0.652)	(0.821,0.764,0.311)	(0.773,0.673,0.487)	(0.517,0.806,0.549)
S_3	(0.629,0.682,0.638)	(0.621,0.723,0.502)	(0.752,0.723,0.415)	(0.646,0.690,0.544)
S_4	(0.540,0.673,0.643)	(0.687,0.814,0.379)	(0.657,0.744,0.582)	(0.649,0.715,0.568)

Table 6. NS D-Mx by using SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.779,0.576,0.190)	(0.678,0.567,0.330)	(0.656,0.656,0.191)	(0.66,0.603,0.235)

S_2	(0.460,0.541,0.394)	(0.656,0.656,0.191)	(0.678,0.567,0.330)	(0.407,0.701,0.393)
S_3	(0.481,0.576,0.517)	(0.512,0.622,0.376)	(0.636,0.622,0.269)	(0.527,0.585,0.418)
S_4	(0.435,0.567,0.541)	(0.525,0.725,0.252)	(0.501,0.651,0.389)	(0.525,0.612,0.445)

Table 7 displays the important weights of the language phrases used to express the criteria determined by DMs.

Table 7. The weights assigned to each criterion

Criteria	DM1	DM2	DM3
(K_1)	LP	VMS	HP
(K_2)	RGS	EI	RGS
(K_3)	PMS	RGS	ES
(K_4)	HP	HP	VMS

The weight of each criterion is determined by the decision-makers' strategies for the criteria aggregated by the SWAM operator provided in Equation (13), which are shown in Table 8.

Table 8. Aggregation of Criteria weights according to SWAM operator

Criteria	Weights of each criterion
(K_1)	(0.576,0.682,0.672)
(K_2)	(0.605,0.791,0.434)
(K_3)	(0.834,0.715,0.330)
(K_4)	(0.751,0.691,0.535)

The aggregated weighted neutrosophic spherical choice matrices are constructed using Equation (4) once the weights assigned to the criteria and evaluations of the substitutions have been determined, as illustrated in Tables 9 and 10.

Table 9. Weighted NS D-Mx according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.503,0.465,0.228)	(0.468,0.532,0.211)	(0.685,0.547,0.103)	(0.620,0.488,0.195)
S_2	(0.428,0.439,0.438)	(0.497,0.604,0.135)	(0.645,0.481,0.161)	(0.388,0.557,0.293)
S_3	(0.362,0.465,0.429)	(0.376,0.571,0.218)	(0.627,0.517,0.137)	(0.485,0.477,0.291)
S_4	(0.311,0.459,0.432)	(0.415,0.643,0.165)	(0.548,0.532,0.192)	(0.487,0.494,0.304)

Table 10. Weighted NS D-Mx according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	(0.449,0.393,0.128)	(0.410,0.448,0.143)	(0.548,0.469,0.063)	(0.498,0.416,0.126)
S_2	(0.265,0.369,0.265)	(0.397,0.519,0.083)	(0.566,0.405,0.109)	(0.306,0.484,0.210)
S_3	(0.277,0.393,0.348)	(0.310,0.492,0.163)	(0.531,0.445,0.089)	(0.396,0.404,0.224)
S_4	(0.251,0.387,0.364)	(0.317,0.573,0.109)	(0.418,0.466,0.128)	(0.395,0.423,0.238)

SF are calculated using Equation (19) and Tables 11 and 12, which are based on Tables 9 and 10. PIS are represented by blue values, while NIS values are represented by yellow values.

Table 11. SF according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.0196191	-0.0370441	0.1424804	0.0945117
S_2	0.0001038	-0.0893889	0.1322302	-0.0605558
S_3	0.0031443	-0.1001252	0.0961946	0.0030986
S_4	0.0140321	-0.1663112	0.0111351	-0.0024082

Table 12. SF according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.033096	-0.0217872	0.0700743	0.0542487
S_2	-0.0108362	-0.0913385	0.1209456	-0.0660912
S_3	0.0029492	-0.0864905	0.0687862	-0.002763
S_4	0.0122932	-0.1718773	-0.0300535	-0.0095923

The NS-PIS and NS-NIS corresponding to the highest and worst scores are shown in Tables 13 and 14.

Table 13. NS-PIS and NS-NIS according to SWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Best)	(0.503,0.465,0.228)	(0.468,0.532,0.211)	(0.685,0.547,0.103)	(0.620,0.488,0.195)
S^- (Worst)	(0.428,0.439,0.438)	(0.415,0.643,0.165)	(0.548,0.532,0.192)	(0.388,0.557,0.293)

Table 14. NS-PIS and NS-NIS according to SWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Great)	(0.449,0.393,0.128)	(0.410,0.448,0.143)	(0.566,0.405,0.109)	(0.498,0.416,0.126)
S^- (Poor)	(0.265,0.369,0.265)	(0.317,0.573,0.109)	(0.418,0.466,0.128)	(0.306,0.484,0.210)

Based on Equations (23 and 24), the next step we can figure out how far apart option S_i is from both the NS-PIS and NS-NIS, respectively. Tables 15 and 16 provide their information.

Table 15. Distance to PIS and NIS according to SWAM operator

Alternatives	$D(S_i, S^*)$	$D(S_i, S^-)$
S_1	1.06252	0.142724324
S_2	0.132168332	0.052504021
S_3	0.113622049	0.070743093

S_4	0.138092728	0.059237447

Table 16. Distance to PIS and NIS according to SWGM operator

Alternatives	$D\left(S_i, S^*\right)$	$D\left(S_i, S^-\right)$
S_1	0.028542681	0.136333827
S_2	0.117780274	0.067011546
S_3	0.119173652	0.07645776
S_4	0.145694319	0.053394324

We calculate the maximum and minimum distances to the NS-NIS and NS-PIS, respectively, from Tables 15 and 16. The closeness ratios are computed using Equation (28), and they are shown in Tables 17 and 18.

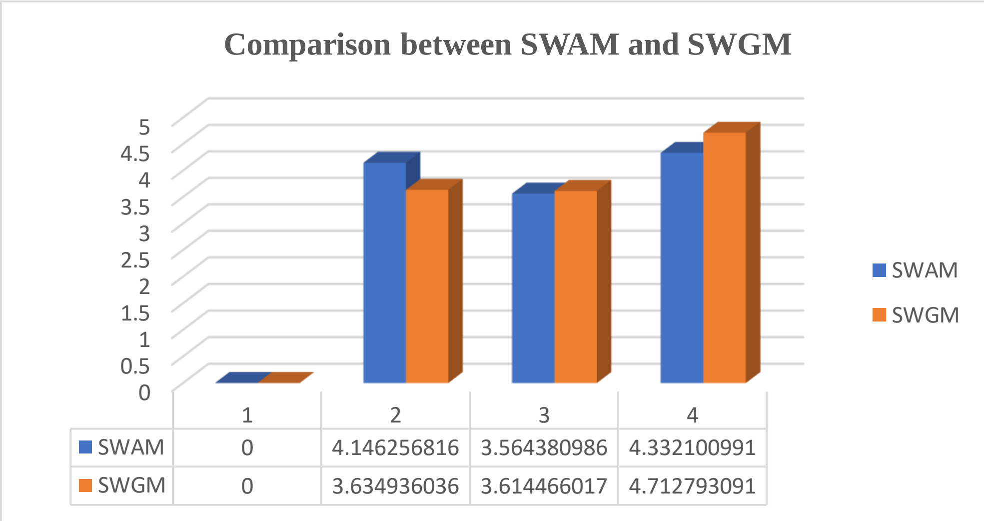
Table 17. Every alternative's closeness ratio according to the SWAM operator

Alternatives	Closeness Ratio	Rank
S_1	0	1
S_2	12438.77	3
S_3	10693.143	2
S_4	12996.303	4

Table 18. Closeness ratio of each alternative according to SWGM operator

Alternatives	Closeness Ratio	Rank
S_1	0	1
S_2	3.634936	3
S_3	3.614466	2
S_4	4.7127931	4

According to the SWAM operator, the closeness ratio for each alternative show that the best option is S_1, and over all ranking is $S_1 > S_3 > S_2 > S_4$. The closest alternative, according to the proximity ratios based on the SWGM operator, is S_1, and overall ranking is $S_1 > S_3 > S_2 > S_4$.The aggregation operators determine how the ranks differ. However, in both strategies, the best and worst options are the same.



6. Conclusions

This study introduces two novel accuracy functions, neutrosophic SWGM and SWAM, which represent significant advancements over conventional aggregation operators by integrating neutrosophic spherical sets. Through the development and application of an algorithm for the CODAS technique, we have effectively addressed the supplier selection problem. Our approach prioritizes alternatives based on distance measurements, utilizing the neutrosophic spherical CODAS approach to compute closeness ratios between criteria. Significantly, our comparison between SWAM and SWGM operators demonstrates comparable rankings and their efficacy in assessing alternatives. This research contributes to the advancement of decision-making methodologies, particularly in complex scenarios where traditional methods may fall short.

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