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A robust framework for medical diagnostics based on interval-valued Q-neutrosophic soft sets with aggregation operators

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Abstract: The best way to deal with complicated life scenarios that accompany the decision-making process is to update previous concepts constantly. Therefore, researchers must constantly discover powerful mathematical tools that suit the accompanying circumstances. In this regard, we combine both soft set, neutrosophic set, and interval setting under Q-two-dimensional universal information to introduce a new hybrid innovative model called interval valued-Q-neutrosophic soft sets. The core goal of this model is to keep the features of previous models like soft sets, neutrosophic sets, and Q-Fuzzy sets in dealing with the lack of uncertainty and neutrality associated with real-life issues. This new approach allows decision-makers to employ interval-valued form with Q-two-dimensional universal information, which provides them with more stability and feasibility in describing uncertain information more completely and accurately. Under the our propose model, we discuss effectively set-theory operations such as subset, union, intersection, complement, AND operation, and OR operation for interval valued-Q-neutrosophic soft sets, as well as some special operations like the necessity and possibility operations of an interval valued-Q-neutrosophic soft sets. In addition, we presented many properties supported by numerical examples that explain how they work. Finally, this new model has been successfully tested in dealing with one of the medical diagnostic problems based on hypothetical data for a respiratory disease. Building an algorithm based on the aggregation operator for interval valued-Q-neutrosophic soft set data solved this issue (i.e., selecting the optimal alternative).

Keywords: fuzzy set; neutrosophic set; soft set; Q- neutrosophic set, Q- neutrosophic soft set

1. Introduction

In our daily lives, numerous complicated issues contain diverse uncertainties and vagueness in human thinking. The decision-making process associated with human thinking is affected by these issues, which can have a significant impact on the effectiveness of the decision-making process, leading to suboptimal or even incorrect decisions. To address these provocations, Zadeh [1] first initiated a mathematical instrument called fuzzy set (FS) as a mathematical structure consisting of one function called the membership function (MF) or truth- MF that works on universal discourse U as a domain and close intervals [0, 1] as a codomain. But from a logical standpoint, it indicates that for every degree of judgment with a degree of truthfulness, there is another degree called the degree of falsehood or the degree of diss truthfulness. Accordingly, Atanassov [2] introduced another concept called intuitionistic fuzzy sets (IFS) by adding a second function called the nonmembership function (NMF), or falsehood-MF. It works in parallel with the truth-MF of correctness through the manifestations of falsehood-MF. Both FS and IFS show better accuracy levels in dealing with different issues in real-life applications. Later, researchers realized that the membership and non-membership values of an FS and IFS are insufficient for dealing with ambiguous indefinite, and inconsistent information in a real-world situation. Based on this need, Smarandache [3] developed another mathematical idea called the neutrosophic set (NS) as a generalization of FS and IFS. This concept is related to three functions MF, NMF and indeterminacy-membership function (IMF), each of which starts from U and rests in the closed interval [0,1]. This idea attracted the curiosity of many scholars around the world and pushed them to applied in many areas, including decision-making, machine learning, pattern recognition, medical diagnosis, market prediction, and image processing. From a scientific point of view, the degree of truth, falsity, and indeterminacy that exist in all the models mentioned above are organized into one single value. Still, sometimes in real situations, these memberships are uncertain, and it is hard for an expert

to express their certainty with a single value. To clarify this issue, consider this example: when you ask someone about the expected temperature for tomorrow, it is challenging to organize this degree immediately with a single value, but when he or she puts this expected degree in the form of an interval value, this person will find it easy to guess the desired degree. As a result, many researchers have reorganized the above models into interval form to make them more flexible and adaptable for addressing real-life problems that include uncertain, unpredictable, and incomplete information. For instance, the notion of an interval neutrosophic set (INS) has been proposed by Wang et al. [4] as an extension of an interval fuzzy set (IVFS) [5] and interval intuitionistic fuzzy set (IVIFS) [6] and they also give the set-theoretic operators of INS. The INS can independently represent the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree, all of them in interval form. So, many investigators have studied it in depth and used it in many areas, such as making decisions, recognizing patterns, data mining, predicting the market, machine learning, and image processing. Molodtsov, on the other hand, pointed out that none of the above-mentioned models have good parameterization of the alternatives. This makes it hard to describe the alternatives to a problem because these parameters cannot be specified well enough. To address these difficulties, Molodtsov [7] came up with a soft set (SS) as a powerful parameter tool to deal with these problems.

This concept (SS), along with the concepts above (FS, IFS, NS), created a storm of important research work, for instance: Cagman et al. [8] introduced the fuzzy soft set (FSS) concept and provided its operations and properties. Following them, Maji [9] introduced neutrosophic soft set and its operations and properties. Deli [10] generalises the notions of SS and NS to interval-NSs under interval form. Saber et al. [11] started the research on the topological-NS information of soft sets by introducing a new approach called single-valued neutrosophic soft topological space. In complex spaces, a lot of research has been introduced [12-20].

1.1. Research gap: the fuzzy set environment and its extension lack the ability to handle two-dimensional information that is available in universal discourse U. For example, if we consider that U contains three patients, u_1, u_2, and u_3, who are suspected of being infected with a disease, it is difficult to describe their condition through a single object (one dimension). This motivates Adam and Hassan [21] to propose new strategies when they build a new model of Q-fuzzy sets (Q-FSs) to serve uncertainty and two-dimensionality simultaneously. After that, Broumi [22] extended to a Q-intuitionistic fuzzy soft set by combining IFSs and SSs by adding a two-dimensional non-membership function. These models are an extension of FSs and IFSs, so it is not feasible to deal with uncertain information that is saturated with positions of neutrality and ambiguity. To address this aspect, recently Abu Qamar and Hassan [23] established the notion of Q-neutrosophic soft sets (Q-NSSs) as a generalisation of NSSs and Q-FSs by upgrading the membership functions of NSSs to two dimensions. This approach has good capabilities compared to the works mentioned in this literature, but the outputs of this model are single values. As we mentioned previously, these values constitute an obstacle for the decision-maker and do not give him sufficient freedom to build numerical data that describes the information of the trouble to be clear up.

Moreover, in interactions process with the concepts described above and as a powerful tool, many researchers have used a technical known as Aggregation Operators (AO) to deal with various fields. This powerful tool allows us to summarize the data and data exploration emerging from the analysis of the problem using the above concepts, condense it, and extract the values with a clear meaning, thus facilitating the task of the user (decision maker) in the process of making clearly informed decisions. Xu [24] developed a new algorithm to solve the DM problem using AO for IFS environments. Chen and Ye [25] extend the Dombi Weighted AO (DWAO) for single-valued neutrosophic numbers (SVNNs) using the operations of both the Dombi T-norm and T-conorm and employ it in solving some real-life applications. Liu and Tang [26] generalised AO in interval-valued neutrosophic seting, and they showed their application to solve decision-making. Zulqarnain et al. [27] proposed the generalised aggregate operators on soft computing in a neutrosophic setting. Al-Sharqi et al. used this tool with many concepts within the fuzzy environment, such as fuzzy hypersoft [28], q-rung orthopair fuzzy neutrosophic valued [29], neutrosophic soft matrix [30], and bipolar neutrosophic hypersoft setting [31], and they employed all these concepts with AO in solving different real-life applications [32-35].

1.2. Novelity and Contributions: This manuscript aimed to suggest techniques a new idea called IV-Q-NSSs, which stands for interval-valued Q-neutrosophic soft sets. These are a more developed form of Q-NSSs, and each membership function is unique to Q-NSSs given in interval form. This format gives the user more freedom and efficiency when dealing with everyday scenarios, especially those saturated with neutral, two-dimensional uncertainty information.

The main contributions shown in this work that were made to achieve these objectives are:

i. A new technique (IV-Q-NSSs) is proposed to contain the effects of uncertainty information in two-dimensional.
ii. To demonstrate the theoretical side of this model, we presented the basic operations, supported by an illustrative numerical example. In addition to presenting the basic properties and theories of IV-Q-NSSs.
iii. On the applied side, these techniques have been added to solve one of the decision-making problems in the medical field by proposing a multi-step algorithm that works on IV-Q-NSS data.

1.3. The following diagram presents the stand down of the paper:

Figure 1: a representation of results.

2. Preliminaries

In this part, we recollect some critical consepts related to our proposed approach like FS, Q-FS, SS, and NS.

Definition 2.1. [1] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an FS $\mathcal{F}$ on $\mathfrak{U}$ is defined by following form:

$$\mathcal{F} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}: \mathfrak{U} \to [0,1]$ such that $\hat{P}^t \in [0,1]$ and called truth membership function (TMF).

Definition 2.2. [21] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set) and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \dots, \mathfrak{q}_n\}$ be nonempty set. Then an Q-FS $\mathcal{F}_{\mathfrak{Q}}$ on the order pair $(\mathfrak{U}, \mathfrak{Q})$is defined by following form:

$$\mathcal{F}_{\mathfrak{Q}} = \{(u, \mathfrak{q}), \hat{P}^t(\hat{u}, \breve{\mathfrak{q}}) | (\hat{u}, \breve{\mathfrak{q}}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}_{\mathfrak{Q}}: \mathfrak{U} \times \mathfrak{Q} \to [0,1]$ such that $\hat{P}^t_{\mathfrak{Q}} \in [0,1]$ and called Q-truth membership function (TMF).

Definition 2.3. [3] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an NS N on $\mathfrak{U}$ is defined by following form:

$$\mathrm{N} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where N is a mapping defined as $\mathrm{N}: \mathfrak{U} \to [0,1]$ such that $\hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) \in [0,1]$ and named truth membership function (TMF), neutrality membership function (NMF), and falsity membership function (FMF) with stander condition $0 \le \hat{P}^t(\mathfrak{u}_j) + \hat{P}^i(\mathfrak{u}_j) + \hat{P}^f(\mathfrak{u}_j) \le 1$.

Definition 2.4. [24] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an Q-NS N on $(\mathfrak{U} \times \mathfrak{Q})$is defined by following form:

$$\mathrm{N}_{\mathfrak{Q}} = \{\mathfrak{u}_j, \hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{i}}_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) | (u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where $\mathrm{N}_{\mathfrak{Q}}$ is a mapping defined as $\mathrm{N}_{\mathfrak{Q}}: \mathfrak{U} \times \mathfrak{Q} \to [0,1]$ such that $\hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{i}}_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) \in [0,1]$ and called truth membership function (TMF), neutrality membership function (NMF), and falsity membership function (FMF) with stander condition $0 \le \hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}) + \hat{P}^{\mathfrak{i}}_{\mathfrak{Q}}(u, \mathfrak{q}) + \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) \le 1$.

Definition 2.5. [10] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an IVNS N on $\mathfrak{U}$ is defined by following form:

$$\mathrm{N} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\hat{P}^t(\mathfrak{u}_j) = [\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)]$, $\hat{P}^i(\mathfrak{u}_j) = [\hat{P}^{i,l}(\mathfrak{u}_j), \hat{P}^{i,u}(\mathfrak{u}_j)]$ and $\hat{P}^f(\mathfrak{u}_j) = [\hat{P}^{f,l}(\mathfrak{u}_j), \hat{P}^{f,u}(\mathfrak{u}_j)]$ Such that the domen of these terms is $\mathfrak{U}$ and the co-domen is $[0,1]$ and $\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)$ are lower and upper of TMF, $\hat{P}^{i,l}(\mathfrak{u}_j), \hat{P}^{i,u}(\mathfrak{u}_j)$ are lower and upper of IMF and $\hat{P}^{f,l}(\mathfrak{u}_j), \hat{P}^{f,u}(\mathfrak{u}_j)$ are lower and upper of FMF, with two stander conditions $0 \le \hat{P}^{t,l}(\mathfrak{u}_j) + \hat{P}^{i,l}(\mathfrak{u}_j) + \hat{P}^{f,l}(\mathfrak{u}_j) \le 1$ and $0 \le \hat{P}^{t.u}(\mathfrak{u}_j) + \hat{P}^{i,u}(\mathfrak{u}_j) + \hat{P}^{f,u}(\mathfrak{u}_j) \le 1$.

Definition 2.6. [10] Assume that $N_1 = \{\mathfrak{u}_j, \hat{P}^t_1(\mathfrak{u}_j), \hat{P}^i_1(\mathfrak{u}_j), \hat{P}^f_1(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$, $N_2 = \{\mathfrak{u}_j, \hat{P}^t_2(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$ be two INS on initial points space(non-empty universal set) $\mathfrak{U}$ where $\hat{P}^t_1(\mathfrak{u}_j) = [\hat{P}^{t,l}_1(\mathfrak{u}_j), \hat{P}^{t,u}_1(\mathfrak{u}_j)]$, $\hat{P}^i_1(\mathfrak{u}_j) = [\hat{P}^{i,l}_1(\mathfrak{u}_j), \hat{P}^{i,u}_1(\mathfrak{u}_j)]$ and $\hat{P}^f_1(\mathfrak{u}_j) = [\hat{P}^{f,l}_1(\mathfrak{u}_j), \hat{P}^{f,u}_1(\mathfrak{u}_j)]$ and $\hat{P}^t_2(\mathfrak{u}_j) = [\hat{P}^{t,l}_2(\mathfrak{u}_j), \hat{P}^{t,u}_2(\mathfrak{u}_j)]$, $\hat{P}^i_2(\mathfrak{u}_j) = [\hat{P}^{i,l}_2(\mathfrak{u}_j), \hat{P}^{i,u}_2(\mathfrak{u}_j)]$ and $\hat{P}^f_2(\mathfrak{u}_j) = [\hat{P}^{f,l}_2(\mathfrak{u}_j), \hat{P}^{f,u}_2(\mathfrak{u}_j)]$ Then,

- **i. Complement** $N_1^C = \{\mathfrak{u}_j, \hat{P}^f_1(\mathfrak{u}_j), 1 - \hat{P}^i_1(\mathfrak{u}_j), \hat{P}^t_1(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$
- **ii. Union**: $N_1 \cup N_2 = \{\mathfrak{u}_j, max[\hat{P}^t_1(\mathfrak{u}_j), \hat{P}^t_2(\mathfrak{u}_j)], \min[\hat{P}^i_1(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j)], \min[\hat{P}^f_1(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.
- **iii. Intersection**: $N_1 \cap N_2 = \{\mathfrak{u}_j, min[\hat{P}^t_1(\mathfrak{u}_j), \hat{P}^t_2(\mathfrak{u}_j)], \max[\hat{P}^i_1(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j)], \max[\hat{P}^f_1(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.
- **iv. Subset** $N_1 \subseteq N_2$ if $\hat{P}^t_1(\mathfrak{u}_j) \le \hat{P}^t_2(\mathfrak{u}_j)$, $\hat{P}^i_1(\mathfrak{u}_j) \ge \hat{P}^i_2(\mathfrak{u}_j)$, $\hat{P}^f_1(\mathfrak{u}_j) \ge \hat{P}^f_2(\mathfrak{u}_j)$.

Definition 2.7. [7] A pair $(\mathcal{F}, \bar{\mathrm{A}} \subseteq \mathcal{E})$ is named SSs over a non-empty universe of discourse $\mathfrak{U}$ if $\mathcal{F}$: $\bar{\mathrm{A}} \subseteq \mathcal{E} \longrightarrow P(\mathfrak{U})$, such that the term $P(\mathfrak{U})$ indicate the power set of $\mathfrak{U}$.

3. The Mathematical Structure of Interval Valued-Q-neutrosophic Soft Sets (IV-Q-NSSs)

This section proposes the general framework definition of our concept IV-Q-NSS with fundamental operations like empty IV-Q-NSS, absolute IV-Q-NSS, subset IV-Q-NSS, and equality between two IV-Q-NSS. Also, to clarify our model more, we will give some numerical examples.

Definition 3.1. Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set), $\mathfrak{Q} \neq \emptyset, ie$ $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_1, \mathfrak{q}_1, \dots, \mathfrak{q}_n\}$ and $\mathcal{E} = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of attribute (parameters set). Let $\bar{\mathrm{A}} \subseteq \mathcal{E}$ be sub set of attribute set , then a duet $(\hat{P}_{\mathfrak{Q}}, \bar{\mathrm{A}})$ is called a interval-valued $\mathfrak{Q}$-neutrosophic soft set over the initial points space (non-empty universal set) $\mathfrak{U}$, where $\hat{P}_{\mathfrak{Q}}$ given as following mapping

$$\hat{P}_{\mathfrak{Q}}: \bar{\mathrm{A}} \to \mathfrak{Q} - IVNS(\mathfrak{U})$$

Then ,the $IV - \mathfrak{Q} - NSS\mathfrak{U})$ can be characterized by the following get form

$$\left(\hat{P}_{\mathfrak{Q}}, \bar{\mathrm{A}}\right) = \hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}}} = \{e \in \bar{\mathrm{A}} \ , < \hat{P}^{t}_{\mathfrak{Q}}(u, \mathfrak{q})(e), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(u, \mathfrak{q})(e), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(u, \mathfrak{q})(e) > |(u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where

$$\hat{P}^{t}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) = [\hat{P}^{t,l}_{\mathfrak{Q}}(u, \mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e})]$$
$$\hat{P}^{i}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) = [\hat{P}^{i,l}_{\mathfrak{Q}}(u, \mathfrak{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e})]$$
$$\hat{P}^{f}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) = [\hat{P}^{f,l}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}), \hat{P}^{f,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e})]$$

Such that , the terms here $\hat{P}^{t,l}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}), \hat{P}^{t,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}), \hat{P}^{i,l}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}), \hat{P}^{i,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e})$ and $\hat{P}^{f,l}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}), \hat{P}^{f,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e})$ refer to true interval membership, indeterminacy interval membership, and falsehood interval membership of objects $(u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}$, with two stander conditions $0 \leq \hat{P}^{t,l}_{\mathfrak{Q}}(u, \mathfrak{q})(e) + \hat{P}^{i,l}_{\mathfrak{Q}}(u, \mathfrak{q})(e) + \hat{P}^{i,l}_{\mathfrak{Q}}(u, \mathfrak{q})(e) \leq 1$ and $0 \leq \hat{P}^{t,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) + \hat{P}^{i,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) + \hat{P}^{i,u}_{\mathfrak{Q}}(u, \mathfrak{q})(\mathrm{e}) \leq 1$.

Now, to shed more light on the above definition, we present below the following numerical example, which describes the mechanism of action of our approach presented in this work.

Example 3.2. Assume that we are interested in analyzing the attractiveness of three houses that one person is thinking of buying one of them. Now, let us analyze this attractiveness according to our model (IV-Q-NSS), therefore we assume that the three houses present as following universal set $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3\}$ and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ be a set constituting two cities under consideration and $\mathcal{E} = \{e_1, e_2, e_3\}$ be a collection of

$$\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}}} =$$
$$\left\{\left(\mathrm{e}_1, \frac{\langle [0.2,0.8], [0.1,0.7], [0.4,0.8] \rangle}{(\mathfrak{u}_1, \mathfrak{q}_1)}, \frac{\langle [0.1,0.4], [0.5,0.8], [0.7,0.8] \rangle}{(\mathfrak{u}_1, \mathfrak{q}_2)}, \right.\right.$$
$$\frac{\langle [0.3,0.6], [0.2,0.7], [0.5,0.8] \rangle}{(\mathfrak{u}_2, \mathfrak{q}_1)}, \frac{\langle [0.4,0.6], [0.2,0.9], [0.5,0.7] \rangle}{(\mathfrak{u}_2, \mathfrak{q}_2)}$$
$$\left.\frac{\langle [0.1,0.5], [0.3,0.7], [0.2,0.8] \rangle}{(\mathfrak{u}_3, \mathfrak{q}_1)}, \frac{\langle [0.4,0.8], [0.4,0.6], [0.2,0.8] \rangle}{(\mathfrak{u}_3, \mathfrak{q}_2)}\right)$$

$\left(e_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(u_1,q_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(u_1,q_2)}\right.$

$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(u_2,q_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(u_2,q_2)}$

$\left.\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(u_3,q_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_3,q_2)}\right)$

$\left(e_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(u_1,q_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(u_1,q_2)}\right.$

$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(u_2,q_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(u_2,q_2)}$

$\left.\left.\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(u_3,q_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(u_3,q_2)}\right)\right\}$

Definition 3.3 Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A} \ , < \hat{P}^t_{\mathfrak{Q}}(u,q)(e), \hat{P}^i_{\mathfrak{Q}}(u,q)(e), \hat{P}^f_{\mathfrak{Q}}(u,q)(e) > |(u,q) \in \mathfrak{U} \times \mathfrak{Q}\}$ be a $IV-Q-NSS$ on initial point space (universal set). Then $\hat{P}_{Q_{\bar{A}}}$ knowing as $IV-Q-NS-nullset\ and\ refer\ as \hat{P}_{\emptyset_{(0)}} if \quad \hat{P}_{\emptyset_{(0)}}(u,q) = \{([0,0],[1,1],[1,1])\}$.

Example 2.4. The term $\hat{P}_{\emptyset_{(0)}(u_3,q_2)}(e_1) = \left(e_1, \frac{([0,0],[1,1],[1,1])}{(u_3,q_2)}\right)$ is consider $IV-Q-NS-nullset$ on $\mathfrak{U}$.

Definition 3.5 Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A} \ , < \hat{P}^t_{\mathfrak{Q}}(u,q)(e), \hat{P}^i_{\mathfrak{Q}}(u,q)(e), \hat{P}^f_{\mathfrak{Q}}(u,q)(e) > |(u,q) \in \mathfrak{U} \times \mathfrak{Q}\}$ be a $IV-Q-NSS$ on initial point space (universal set). Then $\hat{P}_{Q_{\bar{A}}}$ knowing as $IV-Q-NS-$ absolute $set\ and\ refer\ as\ \hat{P}_{\emptyset_{(1)}} if \quad \hat{P}_{\emptyset_{(1)}}(u,q) = \{([1,1],[0,0],[0,0])\}$.

Example 3.6. The term $\hat{P}_{\emptyset_{(1)}(u_3,q_2)(e_1)} = \left(e_1, \frac{([1,1],[0,0],[0,0])}{(u_3,q_2)}\right)$ is consider $IV-Q-NS-$ absolute set on $\mathfrak{U}$.

Definition 3.7 Let $\hat{P}_{Q_{\bar{A}}} and \hat{P}_{Q_{\bar{B}}}$ be two $IV-Q-NSSs$ on non empty universal set (initial points space) $\mathfrak{U}$ with $\mathfrak{Q}$ .Then we say that $\hat{P}_{Q_{\bar{A}}}$ is $IV-Q-NSS\ subset\ of\ \hat{P}_{Q_{\bar{B}}}$ and refer to this relation as $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$ if fulifed the following conditions
For $A \subseteq B$ and $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$ for all $e \in A\ and\ B\ ,(u,q) \in \mathcal{U} \times Q$
Then
$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,q) \leq \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,q), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,q) \leq \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,q),$
$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(u,q) \geq \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}}}(u,q), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(u,q) \geq \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,q),$

$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,q) \geq \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}}}(u,q), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(u,q) \geq \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}}}(u,q).$

Example 3.8. Assume that the two terms in example 3.2, where $B = \{e_1\}$,such that

$\hat{P}_{Q_{\bar{A}}(u_1,q_2)}(e_1) = \left(e_1, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(u_1,q_2)}\right)$

$\hat{P}_{Q_{\bar{B}}(u_1,q_2)}(e_1) = \left(e_1, \frac{\langle[0.2,0.5],[0.3,0.4],[0.2,0.8]\rangle}{(u_1,q_2)}\right)$

Then, it's clear $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$.

Definition 3.9. Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A} \ , < \hat{P}^t_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^i_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^f_{\mathfrak{Q}}(\hat{u},\breve{q})(e) > |(\hat{u},\breve{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

$\in \mathrm{IV}-\mathrm{Q}-\mathrm{NSS}(\mathfrak{U})$.Then, its complement given as $\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} or\ \ c\hat{P}_{\mathfrak{Q}_{\bar{A}}}$ and its defined as following:.

$$\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{\mathrm{A}}\ \ , < P^{t\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), P^{i\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), P^{f\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) > |(\hat{u},\breve{\mathrm{q}}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Or $c\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{\mathrm{A}}\ \ , < c\hat{P}^{t}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), c\hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), c\hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) > |(\hat{u},\breve{\mathrm{q}}) \in \mathfrak{U} \times \mathfrak{Q}\}$

Where

$$P^{t\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[P^{t,l\ c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), P^{t,u\,c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right],$$

$$P^{i\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[P^{i,l\ c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), P^{i,u\,c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[1-\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), 1-\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right],$$

$$P^{f\ c}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[P^{f,l\,c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), P^{f,u\,c}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{\mathrm{q}})(e)\right],$$

Or

$$c\hat{P}^{t}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[c\hat{P}^{t,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), c\hat{P}^{t,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[\hat{P}^{f,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), \hat{P}^{f,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right],$$

$$c\hat{P}^{i}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[c\hat{P}^{i,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), c\hat{P}^{i,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[1-\hat{P}^{i,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), 1-\hat{P}^{f,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right],$$

$$c\hat{P}^{f}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e) = \left[c\hat{P}^{f,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), c\hat{P}^{f,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right] = \left[\hat{P}^{t,l}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e), \hat{P}^{t,u}_{\mathfrak{Q}}(\hat{u},\breve{\mathrm{q}})(e)\right].$$

Based on a above definition we sat that this $\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} or\ \ c\hat{P}_{\mathfrak{Q}_{\bar{A}}}$ is the complement of $\mathrm{IV}-\mathrm{Q}-\mathrm{NSS}(\mathfrak{U})$.

Example3.10. Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2\}$ be initial point (universal set) , $\mathfrak{Q} = \{q_1, q_2\}$ and $\bar{A} = \{e_1, e_2\}$. Then

$$\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \left\{\left(\mathrm{e}_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(\mathfrak{u}_1,\mathrm{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(\mathfrak{u}_1,\mathrm{q}_2)}, \frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(\mathfrak{u}_2,\mathrm{q}_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(\mathfrak{u}_2,\mathrm{q}_2)}\right) \left(\mathrm{e}_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(\mathfrak{u}_1,\mathrm{q}_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(\mathfrak{u}_1,\mathrm{q}_2)}, \frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(\mathfrak{u}_2,\mathrm{q}_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(\mathfrak{u}_2,\mathrm{q}_2)}\right)\right\}$$

Then the complement operation defining $\hat{P}_{\mathfrak{Q}_{\bar{A}}}\ \ as\ \ P^c_{\mathfrak{Q}_{\bar{A}}}\ \ or\ \ cP_{\mathfrak{Q}_{\bar{A}}}$ basedon definition as following

$$\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} = c\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \left\{\left(\mathrm{e}_1, \frac{\langle[0.4,0.8],[0.3,0.9],[0.2,0.8]\rangle}{(\mathfrak{u}_1,\mathrm{q}_1)}, \frac{\langle[0.7,0.8],[0.2,0.5],[0.1,0.4]\rangle}{(\mathfrak{u}_1,\mathrm{q}_2)}, \frac{\langle[0.2,0.8],[0.3,0.7],[0.1,0.5]\rangle}{(\mathfrak{u}_2,\mathrm{q}_1)}, \frac{\langle[0.2,0.8],[0.4,0.6],[0.4,0.8]\rangle}{(\mathfrak{u}_2,\mathrm{q}_2)}\right) \left(\mathrm{e}_2, \frac{\langle[0.3,0.4],[0.3,0.5],[0.1,0.8]\rangle}{(\mathfrak{u}_1,\mathrm{q}_1)}, \frac{\langle[0.2,0.6],[0.3,0.6],[0.1,0.8]\rangle}{(\mathfrak{u}_1,\mathrm{q}_2)}, \frac{\langle[0.3,0.7],[0.5,0.8],[0.1,0.4]\rangle}{(\mathfrak{u}_2,\mathrm{q}_1)}, \frac{\langle[0.5,0.7],[0.5,0.6],[0.1,0.6]\rangle}{(\mathfrak{u}_2,\mathrm{q}_2)}\right)\right\}$$

Proposition 3.11 If $\hat{P}_{\mathfrak{Q}_{\bar{A}}} \in \mathrm{IV}-\mathrm{Q}-\mathrm{NSS}(\mathcal{X})$. Then $c(c\hat{P}_{\mathfrak{Q}_{\bar{A}}}) = \left(\hat{P}^{c}_{\mathfrak{Q}_{\bar{A}}}\right) or \left(\hat{P}^{c}_{\mathfrak{Q}_{\bar{A}}}\right)^{c} = \hat{P}_{\mathfrak{Q}_{\bar{A}}}$

Proof: From above definition, we have

$\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}\ , < \hat{P}^{t}_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^{i}_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^{f}_{\mathfrak{Q}}(\hat{u},\breve{q})(e) > |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\}$

Then,

$$\hat{P}^{c}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}\ , < {P^{t}_{\mathfrak{Q}}}^{c}(\hat{u},\breve{q})(e), {P^{i}_{\mathfrak{Q}}}^{c}(\hat{u},\breve{q})(e), {P^{f}_{\mathfrak{Q}}}^{c}(\hat{u},\breve{q})(e) > |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\}$$

$$= \Big\{e \in \bar{A}\ , \Big\langle\Big[{P^{t,l}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e), {P^{t,u}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e)\Big], \Big[{P^{i,l}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e), {P^{i,u}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e)\Big],$$

$$\Big[{P^{f,l}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e), {P^{f,u}_{\mathfrak{Q}_{\bar{A}}}}^{c}(\hat{u},\breve{q})(e)\Big]\Big\rangle : |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\Big\}$$

$$\left(\hat{P}^{c}_{\mathfrak{Q}_{\bar{A}}}\right)^{c} = \Big\{e \in \bar{A}\ , \Big\langle\Big[\Big(\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big], \Big[\Big(1-\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(1-\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big],$$

$$\Big[\Big(\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big]\Big\rangle : |(u,q) \in \mathfrak{U}\times\mathfrak{Q}\Big\}$$

$$= \Big\{e \in \bar{A}\ , \Big\langle\Big[\Big(\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big], \Big[\Big(1-\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(1-\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big],$$

$$\Big[\Big(\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}, \Big(\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)^{c}\Big]\Big\rangle : |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\Big\}$$

$$= \Big\{e \in \bar{A}\ , \Big\langle\Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big], \Big[\Big(1-\Big(1-\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)\Big), \Big(1-\Big(1-\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big)\Big)\Big],$$

$$\Big[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big]\Big\rangle : |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\Big\}$$

$$= \Big\{e \in \bar{A}\ , \Big\langle\Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big], \Big[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big],$$

$$\Big[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(\hat{u},\breve{q})(e)\Big]\Big\rangle : |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\Big\}$$

$$= \hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}\ , < \hat{P}^{t}_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^{i}_{\mathfrak{Q}}(\hat{u},\breve{q})(e), \hat{P}^{f}_{\mathfrak{Q}}(\hat{u},\breve{q})(e) > |(\hat{u},\breve{q}) \in \mathfrak{U}\times\mathfrak{Q}\}$$

Hence we get $c(c\hat{P}_{\mathfrak{Q}_{\bar{A}}}) = \hat{P}_{\mathfrak{Q}_{\bar{A}}}$.

Definition 3.12 The union of two $IV-QNSS$ $\hat{P}_{\mathfrak{Q}_{\bar{C}}}$ and written as $\hat{P}_{\mathfrak{Q}_{\bar{A}}} \cup \hat{P}_{\mathfrak{Q}_{\bar{B}}} = \hat{P}_{\mathfrak{Q}_{\bar{C}}}$, where $\bar{C} = \bar{A} \cup \bar{B}$ and for all $c \in \bar{C}$, $(u,q) \in \mathfrak{U}\times\mathfrak{Q}$,the three $IV-Q-NSS$ member ships function given as follows :.

$$\hat{P}^{t}_{\mathfrak{Q}_{\bar{C}}}(u,q)\begin{cases} P^{t}_{\mathfrak{Q}_{\bar{A}}}(u,q) = \Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,q)(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,q)(e)\Big] & if\ \ c \in \bar{A}-\bar{B} \\ P^{t}_{\mathfrak{Q}_{\bar{B}}}(u,q) = \Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,q)(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,q)(e)\Big] & if\ \ c \in \bar{B}-\bar{A} \\ P^{t}_{\mathfrak{Q}_{\bar{\bar{C}}}}(u,q) = \max\Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,q)(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,q)(e)\Big] & if\ \ c \in \bar{A}\cap\bar{B} \end{cases}$$

$$\hat{P}^{i}_{\mathfrak{Q}_{\bar{C}}}(u,q)\begin{cases} P^{f}_{\mathfrak{Q}_{\bar{A}}}(u,q) = \Big[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(u,q)(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(u,q)(e)\Big] & if\ \ c \in \bar{A}-\bar{B} \\ P^{f}_{\mathfrak{Q}_{\bar{B}}}(u,q) = \Big[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}}}(u,q)(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,q)(e)\Big] & if\ \ c \in \bar{B}-\bar{A} \\ P^{f}_{\mathfrak{Q}_{\bar{\bar{C}}}}(u,q) = \min\Big[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,q)(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,q)(e)\Big] & if\ \ c \in \bar{A}\cap\bar{B} \end{cases}$$

$$\hat{P}^{f}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})\begin{cases} P^{f}_{\mathfrak{Q}_{\bar{A}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}-\bar{B} \\ P^{f}_{\mathfrak{Q}_{\bar{B}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e),\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{B}-\bar{A} \\ P^{f}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathrm{u},\mathfrak{q}) = \min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}\cap\bar{B} \end{cases}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right].$$

Here. The max represents the largest value of $IV-QNSS$ and min represents the smallest value of $IV-QNSS$.

Definition 3.13 The intersection of two $IV-QNSS$ $\hat{P}_{\mathfrak{Q}_{\bar{C}}}$ and written as $\hat{P}_{\mathfrak{Q}_{\bar{A}}}\cup\hat{P}_{\mathfrak{Q}_{\bar{B}}}=\hat{P}_{\mathfrak{Q}_{\bar{C}}}$, where $\bar{C}=\bar{A}\cap\bar{B}$ and for all $c\in\bar{C}$,$(\mathrm{u},\mathfrak{q})\in\ \mathfrak{U}\times\mathfrak{Q}$,the three $IV-Q-NSS$ member ships function given as follows :.

$$\hat{P}^{t}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})\begin{cases} P^{t}_{\mathfrak{Q}_{\bar{A}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}-\bar{B} \\ P^{t}_{\mathfrak{Q}_{\bar{B}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e),\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{B}-\bar{A} \\ P^{t}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathrm{u},\mathfrak{q}) = \min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}\cap\bar{B} \end{cases}$$

$$\hat{P}^{i}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})\begin{cases} P^{f}_{\mathfrak{Q}_{\bar{A}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}-\bar{B} \\ P^{f}_{\mathfrak{Q}_{\bar{B}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e),\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{B}-\bar{A} \\ P^{f}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathrm{u},\mathfrak{q}) = \max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}\cap\bar{B} \end{cases}$$

$$\hat{P}^{f}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})\begin{cases} P^{f}_{\mathfrak{Q}_{\bar{A}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}-\bar{B} \\ P^{f}_{\mathfrak{Q}_{\bar{B}}}(\mathrm{u},\mathfrak{q}) = \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e),\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{B}-\bar{A} \\ P^{f}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathrm{u},\mathfrak{q}) = \mathrm{mix}\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ \ c\in\bar{A}\cap\bar{B} \end{cases}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e),\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

$$\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right].$$

Here. The max represents the largest value of $IV - QNSS$ and min represents the smallest value of $IV - QNSS$.

Example 3.14. Let $\mathcal{X} = \{u_1, u_2\}$ be non-empty initial universal set, $\mathfrak{L} = \{e_1, e_2\} and\ Q = \{\mathfrak{q}_1\}. then\ , if\ \ \bar{A} = \{e_1\} \subseteq \mathfrak{L}\ ,$
$\bar{B} = \{e_1, e_2\} \subseteq \mathfrak{L}\ , then\ the\ two\ IV - Q - NSSs\ \left(\hat{P}_Q, \bar{A}\right), \left(\hat{P}_Q, \bar{B}\right)$
Will be analyze as following

$$\left(\hat{P}_Q, \bar{A}\right) =$$
$$\left\{\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_2,\mathfrak{q}_1)}\right)\right\}$$
$$\left(\hat{P}_Q, \bar{B}\right) =$$
$$\left\{\left(e_1, \frac{\langle[0.3,0.5],[0.2,0.4],[0.6,0.7]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.6,0.9],[0.5,0.8],[0.4,0.6]\rangle}{(u_2,\mathfrak{q}_1)}\right)\right.$$
$$\left.\left(e_2 \frac{\langle[0.6,1],[0.7,0.9],[0.6,0.8]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.3,0.5],[0.8,0.8],[0.6,0.7]\rangle}{(u_2,\mathfrak{q}_1)}\right)\right\}$$
Then, $\left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right) \cup \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) =$
$$\left\{\left(e_1, \frac{\langle[0.3,0.8],[0.1,0.7],[0.4,0.7]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.6,0.9],[0.4,0.5],[0.4,0.6]\rangle}{(u_1,\mathfrak{q}_2)}\right)\right.$$
$$\left.\left(e_2, \frac{\langle[0.6,1],[0.7,0.9],[0.6,0.8]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.3,0.5],[0.8,0.8],[0.6,0.7]\rangle}{(u_2,\mathfrak{q}_1)}\right)\right\}$$
$$\left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right) \cap \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) =$$
$$\left\{\left(e_1, \frac{\langle[0.2,0.5],[0.2,0.7],[0.6,0.8]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],[0.5,0.8],[0.5,0.7]\rangle}{(u_1,\mathfrak{q}_2)}\right)\right.$$
$$\left.\left(e_2, \frac{\langle[0.6,1],[0.7,0.9],[0.6,0.8]\rangle}{(u_1,\mathfrak{q}_1)}, \frac{\langle[0.3,0.5],[0.8,0.8],[0.6,0.7]\rangle}{(u_2,\mathfrak{q}_1)}\right)\right\}$$

Proposition 3. 15. Let $\left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right), \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) and \left(\hat{P}_{\mathfrak{Q}}, C\right)$ be three $IV - Q - NSSs\ \ on\ \ non-universal\ \ set\ \ U$. The, the following points are satisfied:

1. $\left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right) \cup \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) = \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) \cup \left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right)$
2. $\left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right) \cap \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) = \left(\hat{P}_{\mathfrak{Q}}, \hat{B}\right) \cap \left(\hat{P}_{\mathfrak{Q}}, \hat{A}\right)$

3. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup \left((\hat{P}_{\mathfrak{Q}},\hat{B}) \cup (\hat{P}_{\mathfrak{Q}},C)\right) = \left((\hat{P}_{\mathfrak{Q}},\hat{A}),(\hat{P}_{\mathfrak{Q}},\hat{B})\right) \cup (\hat{P}_{\mathfrak{Q}},C)$

4. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup \left((\hat{P}_{\mathfrak{Q}},\hat{B}) \cup (\hat{P}_{\mathfrak{Q}},C)\right) = \left((\hat{P}_{\mathfrak{Q}},\hat{A}),(\hat{P}_{\mathfrak{Q}},\hat{B})\right) \cup (\hat{P}_{\mathfrak{Q}},C)$

5. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup \phi = \phi \cup (\hat{P}_{\mathfrak{Q}},\hat{A}) = (\hat{P}_{\mathfrak{Q}},\hat{A})$
6. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cap \phi = \phi \cap (\hat{P}_{\mathfrak{Q}},\hat{A}) = \phi$
7. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup U = U \cup (\hat{P}_{\mathfrak{Q}},\hat{A}) = U$
8. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cap U = U \cap (\hat{P}_{\mathfrak{Q}},\hat{A}) = (\hat{P}_{\mathfrak{Q}},\hat{A})$

Proof (1). Now we will show that $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B})$=$(\hat{P}_{\mathfrak{Q}},\hat{B}) \cup (\hat{P}_{\mathfrak{Q}},\hat{A})$based on **Definition 3.12** Also, in this case we will consider the case $c \in \bar{A} \cap \bar{B}$ and other case are trivial .

Now, take the left side $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B}), then$

$$(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B}) = \{< c \ \ (\max\{\hat{P}^{t}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{t}_{\mathfrak{Q}_{\bar{B}}}(u,t),$$

$$\min\left\{\hat{P}^{i}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{i}_{\mathfrak{Q}_{\bar{B}}}(u,q)\right\}, \min\left\{\hat{P}^{f}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{f}_{\mathfrak{Q}_{\bar{B}}}(u,q)\right\} : (u,q) \in U \times Q >$$

$$=\left\{< c(\max\left[\max P^{t,i}_{Q_{\bar{A}}}(u,q), P^{t,l}_{Q_{\bar{B}}}(u,q)\right], \left[\max \ \ [P^{t,u}_{Q_{\bar{A}}}(u,q), P^{t,u}_{Q_{\bar{B}}}(u,q)\right]\right\}$$

$$, \min\left\{ \ \ \min\left[P^{i,l}_{Q_{\bar{A}}}(u,q), P^{i,l}_{Q_{\bar{B}}}(u,q)\right], \min\left[P^{i,u}_{Q_{\bar{A}}}(u,q), P^{i,u}_{Q_{\bar{B}}}(u,q)\right]\right\},$$

$$\min\left\{\min\left[P^{f,u}_{Q_{\bar{A}}}(u,q), P^{f,u}_{Q_{\bar{B}}}(u,q)\right], \min\left[P^{f,u}_{Q_{\bar{A}}}(u,q), P^{f,u}_{Q_{\bar{B}}}(u,q)\right] > (u,q) \in U \times Q\right\}$$

$$= \{< c \ \ (\max\{\hat{P}^{t}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{t}_{\mathfrak{Q}_{\bar{B}}}(u,t),$$

$$\min\left\{\hat{P}^{i}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{i}_{\mathfrak{Q}_{\bar{B}}}(u,q)\right\}, \min\left\{\hat{P}^{f}_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^{f}_{\mathfrak{Q}_{\bar{B}}}(u,q)\right\} : (u,q) \in U \times Q >\}$$

$$= (\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B})$$

Definition 3.16. Assume that $(\hat{P}_{\mathfrak{Q}},\hat{A})$ and $(\hat{P}_{\mathfrak{Q}},\hat{B})$ are two $IV-Q-NSSs$ on initial point space (non-empty universal set) U ,then $(\hat{P}_{\mathfrak{Q}},\hat{A})$ $AND(\hat{P}_{\mathfrak{Q}},\hat{B})$ is an $IV-Q-NSSs$ and denoted by $(\hat{P}_{\mathfrak{Q}},\hat{A}) \wedge (\hat{P}_{\mathfrak{Q}},\hat{B})$ and it defined by the following formalh $(\hat{P}_{\mathfrak{Q}},\hat{A})$ $AND(\hat{P}_{\mathfrak{Q}},\hat{B}) = (\hat{P}_{\mathfrak{Q}},\bar{A} \times \hat{B}), where$

$$\hat{P}_{\mathfrak{Q}}(\bar{a},\bar{b})^{(u,q)} = \hat{P}_{\mathfrak{Q}_{(\bar{a})}}{}^{(u,q)} \cap \hat{P}_{\mathfrak{Q}_{(\bar{b})}}{}^{(u,q)}$$

For all $(\bar{a},\bar{b}) \in \bar{A} \times \bar{B}$, where $\cap\, is$ tha $intersection$ operation of two $IV-Q-NSSs$ on initial points pace (non-empty universal set)

Now , based on the intersection definition the three $IV-Q-NSSs$ membership function defined as following

$$P^{t}_{Q_{(\bar{a},\bar{b})}}(u,q) = \min\{P^{t}_{Q_{(\bar{a})}}(u,q), P^{t}_{Q_{(\bar{b})}}(u,q)\} = \min\left\{\min\left[P^{t,l}_{Q_{(\bar{a})}}(u,q), P^{t,l}_{Q_{(\bar{b})}}(u,q)\right], \min\left[P^{t,u}_{Q_{(\bar{a})}}(u,q), P^{t,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

$$P^{i}_{Q_{(\bar{a},\bar{b})}}(u,q) = \mathrm{mix}\{P^{i}_{Q_{(\bar{a})}}(u,q), P^{i}_{Q_{(\bar{b})}}(u,q)\} = \mathrm{mix}\left\{\mathrm{mix}\left[P^{i,l}_{Q_{(\bar{a})}}(u,q), P^{i,l}_{Q_{(\bar{b})}}(u,q)\right], \mathrm{mix}\left[P^{i,u}_{Q_{(\bar{a})}}(u,q), P^{i,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

$$P^{f}_{Q_{(\bar{a},\bar{b})}}(u,q) = \mathrm{mix}\{P^{f}_{Q_{(\bar{a})}}(u,q), P^{f}_{Q_{(\bar{b})}}(u,q)\} = \mathrm{mix}\left\{\mathrm{mix}\left[P^{f,l}_{Q_{(\bar{a})}}(u,q), P^{f,l}_{Q_{(\bar{b})}}(u,q)\right], \mathrm{mix}\left[P^{f,u}_{Q_{(\bar{a})}}(u,q), P^{f,u}_{Q_{(\bar{b})}}(u,q)\right]\right\}.$$

Definition 3.17. Assume that $(\hat{P}_{\mathfrak{Q}},\hat{A})$ and $(\hat{P}_{\mathfrak{Q}},\hat{B})$ are two $IV-Q-NSSs$ on initial point space (non-empty universal set) U ,then $(\hat{P}_{\mathfrak{Q}},\hat{A})$ $OR(\hat{P}_{\mathfrak{Q}},\hat{B})$ is an $IV-Q-NSSs$ and denoted by $(\hat{P}_{\mathfrak{Q}},\hat{A})\vee(\hat{P}_{\mathfrak{Q}},\hat{B})$ and it defined by the following formalh $(\hat{P}_{\mathfrak{Q}},\hat{A})OR(\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\bar{A}\times\hat{B}), where$

$$\hat{P}_{\mathfrak{Q}}(\bar{a},\bar{b})^{(u,q)}=\hat{P}_{\mathfrak{Q}_{(\bar{a})}}{}^{(u,q)}\cup\hat{P}_{\mathfrak{Q}_{(\bar{b})}}{}^{(u,q)}$$

For all $(\bar{a},\bar{b})\in\bar{A}\times\bar{B}$, where $\cup$ is tha $union$operation of two $IV-Q-NSSs$ on initial points pace (non-empty universal set) U.

Now , based on the $union$ definition the three $IV-Q-NSSs$ membership function defined as following

$$P^{t}_{Q_{(\bar{a},\bar{b})}}(u,q)=\text{mix}\{P^{t}_{Q_{(\bar{a})}}(u,q),P^{t}_{Q_{(\bar{b})}}(u,q)\}=\text{mix}\left\{\text{mix}\left[P^{t,l}_{Q_{(\bar{a})}}(u,q),P^{t,l}_{Q_{(\bar{b})}}(u,q)\right],\text{mix}\left[P^{t,u}_{Q_{(\bar{a})}}(u,q),P^{t,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

$$P^{i}_{Q_{(\bar{a},\bar{b})}}(u,q)=\min\{P^{i}_{Q_{(\bar{a})}}(u,q),P^{i}_{Q_{(\bar{b})}}(u,q)\}=\min\left\{\min\left[P^{i,l}_{Q_{(\bar{a})}}(u,q),P^{i,l}_{Q_{(\bar{b})}}(u,q)\right],\min\left[P^{i,u}_{Q_{(\bar{a})}}(u,q),P^{i,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

$$P^{f}_{Q_{(\bar{a},\bar{b})}}(u,q)=\min\{P^{f}_{Q_{(\bar{a})}}(u,q),P^{f}_{Q_{(\bar{b})}}(u,q)\}=\min\left\{\min\left[P^{f,l}_{Q_{(\bar{a})}}(u,q),P^{f,l}_{Q_{(\bar{b})}}(u,q)\right],\min\left[P^{f,u}_{Q_{(\bar{a})}}(u,q),P^{f,u}_{Q_{(\bar{b})}}(u,q)\right]\right\}.$$

Example 3.18. Let $\mathcal{X}=\{u_1,u_2\}$ be non-empty initial universal set, $\mathfrak{L}=\{e_1,e_2,e_3\}and$ $Q=\{q_1\}.then$ $,if$ $\bar{A}=\{e_1\}\subseteq\mathfrak{L}$,

$\bar{B}=\{e_2,e_3\}\subseteq\mathfrak{L}$ $,then$ the two $IV-Q-NSSs$ $(\hat{P}_Q,\bar{A}),(\hat{P}_Q,\bar{B})$

Will be analyze as following

$$(\hat{P}_Q,\bar{A})=\left\{\left(e_1,\frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(u_1,q_1)},\frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_2,q_1)}\right)\right\}$$

$$(\hat{P}_Q,\bar{B})=\left\{\left(e_2,\frac{\langle[0.3,0.5],[0.2,0.4],[0.6,0.7]\rangle}{(u_1,q_1)},\frac{\langle[0.6,0.9],[0.5,0.8],[0.4,0.6]\rangle}{(u_2,q_1)}\right)\right.$$
$$\left.\left(e_3\frac{\langle[0.6,1],[0.7,0.9],[0.6,0.8]\rangle}{(u_1,q_1)},\frac{\langle[0.3,0.5],[0.8,0.8],[0.6,0.7]\rangle}{(u_2,q_1)}\right)\right\}$$

Then,

$$(\hat{P}_{\mathfrak{Q}},\hat{A})\ AND(\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\bar{A}\times\hat{B})=$$
$$\left\{\left((e_1,e_2),\frac{\langle[0.2,0.5],[0.2,0.7],[0.6,0.8]\rangle}{(u_1,q_1)},\frac{\langle[0.1,0.3],[0.5,0.8],[0.7,0.8]\rangle}{(u_1,q_2)}\right)\right.$$
$$\left.\left((e_1,e_3),\frac{\langle[0.3,0.5],[0.7,0.9],[0.6,0.8]\rangle}{(u_2,q_1)},\frac{\langle[0.3,0.5],[0.8,0.8],[0.6,0.7]\rangle}{(u_2,q_2)}\right)\right\}$$

And $(\hat{P}_{\mathfrak{Q}},\hat{A})\ OR(\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\bar{A}\times\hat{B})=$

$$\left\{\left((e_1,e_2),\frac{\langle[0.3,0.8],[0.1,0.4],[0.4,0.7]\rangle}{(u_1,q_1)},\frac{\langle[0.6,0.9],[0.4,0.5],[0.4,0.6]\rangle}{(u_1,q_2)}\right)\right.$$

$$\left((e_1, e_3), \frac{\langle[0.6,1],[0.1,0.7],[0.4,0.8]\rangle}{(u_2, q_1)}, \frac{\langle[0.3,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_2, q_2)}\right)\}$$

Proposition 3.19 Assume that $(\hat{P}_Q, \bar{A}), (\hat{P}_Q, \bar{B}) and\ (\hat{P}_Q, \bar{C})$ be three $IV - Q - NSSs$ no non-empty initial universal set U. Then following point (properties) will be satisfied:

1. $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C})$

2. $(\hat{P}_Q, \bar{A}) \vee (\hat{P}_Q, \bar{B}) \vee (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \vee (\hat{P}_Q, \bar{B})\right) \vee (\hat{P}_Q, \bar{C})$

Proof.1. Assume that $\bar{a} \in \bar{A}, \bar{b} \in \bar{B}$ and the thired one $\bar{c} \in \bar{C}$ and $(\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = (\hat{P}_Q, \bar{B} \times \bar{C}), such\ \ that$ $\hat{P}_Q(\bar{b}, \bar{c}) = \hat{P}_Q(\bar{b}) \cap \hat{P}_Q(\bar{c})$

Now , we have $(\hat{P}_Q, \bar{A}) \wedge \left((\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C})\right) = (\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B} \times \bar{C}) = (\hat{P}_Q, \bar{A} \times \bar{B} \times \bar{C}),$

Such that

$(\hat{P}_Q, \bar{a} \times \bar{b} \times \bar{c}) = \hat{P}_Q(a) \cap \hat{P}_Q(b, c) = \hat{P}_Q(a) \cap \hat{P}_Q(b) \cap \hat{P}_Q(c)$

Also we have $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) = (\hat{P}_Q, \bar{A} \times \bar{B})$ such that

$$\hat{P}_Q(\bar{a}, \bar{b}) = \hat{P}_Q(\bar{a}) \cap \hat{P}_Q(\bar{b})$$

Therefor $\left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C}) = (\hat{P}_Q, \bar{A} \times \bar{B}) \wedge (\hat{P}_Q, \bar{C})$

$= (\hat{P}_Q, \bar{A} \times \bar{B} \times \bar{C})\ \ where\ \ (\hat{P}_Q, \bar{a} \times \bar{b} \times \bar{c}) = \hat{P}_Q(\bar{a}, \bar{b}) \cap \hat{P}_Q(\bar{c}) = \hat{P}_Q(\bar{a}) \cap \hat{P}_Q(\bar{b}) \cap \hat{P}_Q(\bar{c}).$

Hence $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C})$

Proof 2. Same proof (1)

Definition 3.20. (Necessity operation (NO)). The NO define on $\mathrm{IV} - \mathrm{Q} - \mathrm{NSS}$ $(\hat{P}_Q, \bar{A})$ on non-empty initial universal set U and denoted ass following, for all $\bar{a} \in \bar{A}$

$$\widehat{\boxdot}\,(\hat{P}_Q, \bar{A}) = \{< \bar{a}[(u, q), \hat{P}^t_{Q(\bar{a})}(u, q), \hat{P}^i_{Q(\bar{a})}(u, q), 1 - \hat{P}^t_{Q(\bar{a})}(u, q)] : (a, q) \in U \times Q >\}$$
$$= \{< \bar{a}\ \ [(u, q)[\hat{P}^{t,l}_{Q(\bar{a})}(u, q), \hat{P}^{t,u}_{Q(\bar{a})}(u, q)], [\hat{P}^{i,l}_{Q(\bar{a})}(u, q), \hat{P}^{i,u}_{Q(\bar{a})}(u, q)], [1 - \hat{P}^{t,u}_{Q(\bar{a})}(u, q), 1 - \hat{P}^{t,l}_{Q(\bar{a})}(u, q)]; (u, q) \in U \times Q\}$$

Proposition 3.21 Assume that $(\hat{P}_Q, \bar{A})$ and $(\hat{P}_Q, \bar{B})$ be two $IV - Q - NSSs\ \ on\ \ U$. Then

1. $\widehat{\boxdot}\left((\hat{P}_Q, \bar{A}) \cup \widehat{\boxdot}\,(\hat{P}_Q, \bar{B})\right) = \widehat{\boxdot}\,(\hat{P}_Q, \bar{A}) \cup \widehat{\boxdot}\,(\hat{P}_Q, \bar{A})$

2. $\widehat{\boxdot}\left((\hat{P}_Q, \bar{A}) \cap \widehat{\boxdot}\,(\hat{P}_Q, \bar{B})\right) = \widehat{\boxdot}\,(\hat{P}_Q, \bar{A}) \cap \widehat{\boxdot}\,(\hat{P}_Q, \bar{A})$

3. $\widehat{\boxdot}\left(\widehat{\boxdot}\,(\hat{P}_Q, \bar{A})\right) = (\hat{P}_Q, \bar{A})$

Proof. The proof of these facts is directly based on the definitions above.

Example 3.22 Reconsider the term in example 3.2, then

$$\widehat{\boxdot}\, \hat{P}_{Q(u_1,q_2)}(e_1) = \widehat{\boxdot}\left(e_1, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(u_1,q_2)}\right) = \left(e_1, \frac{\langle[0.1,0.4],[0.5,0.8],[0.6,0.9]\rangle}{(u_1,q_2)}\right)$$

Definition 3.23 (Possibility operation (PO)). The PO on an $\mathrm{IV-Q-NSS}(\hat{P}_Q, \bar{A})$ on non empty universal set U is indicated by $\widehat{\triangle}\,(\hat{P}_Q, \bar{A})$ and given by , for all $\bar{a} \in \bar{A}$

$$\widehat{\triangle}\,(\hat{P}_Q, \bar{A}) = \{< \bar{a}[(u,q), 1 - \hat{P}^{f}_{Q(\bar{a})}(u,q), \hat{P}^{i}_{Q(\bar{a})}(u,q), \hat{P}^{f}_{Q(\bar{a})}(u,q)] : (a,q) \in U \times Q >\}$$

$$= \{< \bar{a}\ \ [(u,q)[1 - \hat{P}^{t,u}_{Q(\bar{a})}(u,q), 1 - \hat{P}^{f,l}_{Q(\bar{a})}(u,q)], [\hat{P}^{i,l}_{Q(\bar{a})}(u,q), \hat{P}^{i,u}_{Q(\bar{a})}(u,q)], [\hat{P}^{t,u}_{Q(\bar{a})}(u,q), \hat{P}^{t,l}_{Q(\bar{a})}(u,q)]; (u,q) \in U \times Q\}$$

Example 3.24 Reconsider the term in example 3.2, then

$$\widehat{\boxdot}\, \hat{P}_{Q(u_1,q_2)}(e_1) = \widehat{\boxdot}\left(e_1, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(u_1,q_2)}\right) = \left(e_1, \frac{\langle[0.2,0.3],[0.5,0.8],[0.7,0.8]\rangle}{(u_1,q_2)}\right).$$

Proposition 3.25 Assume that $(\hat{P}_Q, \bar{A})$ and $(\hat{P}_Q, \bar{B})$ be two $IV-Q-NSSs\ \ on\ \ U.$
Then:

1. $\widehat{\triangle}\left((\hat{P}_Q, \bar{A}) \cup\ \widehat{\triangle}\,(\hat{P}_Q, \bar{B})\right) =\ \widehat{\triangle}\,(\hat{P}_Q, \bar{A}) \cup \widehat{\triangle}\,(\hat{P}_Q, \bar{A}).$

2. $\widehat{\triangle}\left((\hat{P}_Q, \bar{A}) \cap\ \widehat{\triangle}\,(\hat{P}_Q, \bar{B})\right) = \widehat{\triangle}\,(\hat{P}_Q, \bar{A})\ \ \cap \widehat{\triangle}\,(\hat{P}_Q, \bar{B}).$

3. $\widehat{\triangle}\left(\widehat{\triangle}\,(\hat{P}_Q, \bar{A})\right) = (\hat{P}_Q, \bar{A}).$

Proof. The proof of these facts is directly based on the definitions above**.**

Proposition 3.26 Let $(\hat{P}_Q, \bar{A})$ be an $\mathrm{IV-Q-NSS}$ over U, then we have the following point (proportion)
1. $\widehat{\triangle}\widehat{\boxdot}\,(\hat{P}_Q, \bar{A}) = \widehat{\boxdot}\,(\hat{P}_Q, \bar{A})$
2. $\widehat{\boxdot}\widehat{\triangle}\,(\hat{P}_Q, \bar{A}) = \widehat{\triangle}\,(\hat{P}_Q, \bar{A})$

Proof. The proof of these facts is directly based on the definitions above**.**

4. An Application of IV-Q-NSs in Medical Field Under Uncertainty

In this section, we will show the apparatus for appealing to our put-forward model in dealing with daily life situations. By narrating an issue in the medical field and showing the mechanism for representing its data proposed by our proposed model. After that, we will work on creating an algorithm consisting of a number of sequential steps that analyze the algebraic structure of our proposed model and the data it represents. Now we will provide some definitions that will be useful to us in building the above algorithm.

Definition 4.1 Let $(\hat{P}_Q, \bar{A})$ be IV-QNSS on non-empty initial universal set U. Then, an IV-QNS aggregation operator of $(\hat{P}_Q, \bar{A})$ and denoted by $\breve{\Pi}^{agg}_{Q}$ is defined by

$$\breve{\Pi}^{agg}_{Q} = \left\{< \bar{a}[(u,q), \hat{P}^{t,agg}_{Q(\bar{a})}(u,q), \hat{P}^{i,agg}_{Q(\bar{a})}(u,q), \hat{P}^{f,agg}_{Q(\bar{a})}(u,q)] : (u,q) \in U \times Q >\right\}$$

Where $\hat{P}^{t,agg}_{Q}, \hat{P}^{i,agg}_{Q}, \hat{P}^{f,agg}_{Q} : U \times Q \rightarrow [0,1]$, such that

$\hat{P}_Q^{t,l,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q((u,q))}^{t,l}, \hat{P}_Q^{t,u,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{t,u},$

$\hat{P}_Q^{i,l,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{i,l}, \hat{P}_Q^{i,u,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{i,u},$

$\hat{P}_Q^{f,l,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{f,l}, \hat{P}_Q^{f,u,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{f,u},$

Remark: The value of $(\hat{P}_Q, \bar{A})$can be reduce to IV-QFS using the following definition .

Definition 4.2. The IV-QNS can be reduced to Interval-Valued-Q-fuzzy set (IV-QFS)

$$(\hat{P}_Q, \bar{A}) = \{< \bar{a}, [(u,q), \hat{P}_{Q(\bar{a})}^t(u,q)] : (u,q) \in U \times Q >\}$$

Where $\hat{P}_Q^t : U \times Q \to [0,1]$ such that

$\hat{P}_Q^{t,l} = \frac{1}{3}[\hat{P}_Q^{t,l} + \hat{P}_Q^{i.l} + \hat{P}_Q^{f.l}],\ \hat{P}_Q^{t,u} = \frac{1}{3}[\hat{P}_Q^{t,u} + \hat{P}_Q^{i.u} + \hat{P}_Q^{f.u}]$

Now, using the above definitions, we lever up the following algorithm for a decision medical field method:

<u>Algorithm</u>

Step 1. Put up an IV-Q-NSSs on U.

Step 2. Calculate IV-Q-NS aggregation operator

Step 3. Calculate the reduced value of the IV-Q-NS aggregation operator to IV-QFS aggregation operator.

Step 4. Convert IV-QFS aggregation operator $(\hat{P}_Q^{t,l}, \hat{P}_Q^{t,u})$ to SV-QFS aggregation operator $(\hat{P}_Q^t)$,i.e. $\hat{P}_Q^t = \frac{\hat{P}_Q^{t,l}, +_Q^{t,u}}{2}$.

Step 5. The optimal decision is the element available in M, such that $M = max_{(u,q)\in U\times Q}\{\hat{P}_Q^t\}$.

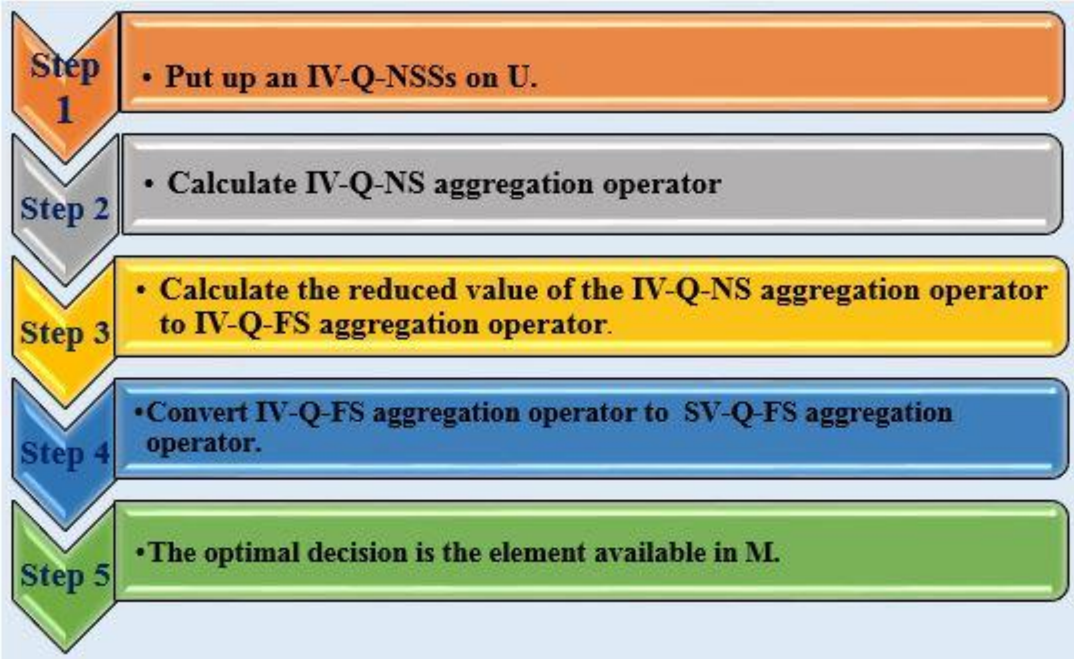

Figure 2: a representation of algorithm in an abbreviated way.

Now, we provide a case study related to the medical field for IV-Q-NSS strategic decision-making method.

On a cold winter day, many patients visited the office of a respiratory doctor to diagnose their health condition (COVID-positive or not) based on the symptoms they were experiencing. To help the doctor organize and analyze patient data based on our proposed model, we asked him to select a value between 0 and 1 that describes the severity of symptoms and their association with the disease (Covid), where the closer the ratio is to 1, the more serious the symptoms are (impact of symptoms). Therefore:
Suppose that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \mathfrak{u}_4\}$ be a patient set contains four patients, $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ where $\mathfrak{q}_1$ =infected and $\mathfrak{q}_2$ =uninfected,while $\bar{A} \subseteq \mathcal{E} = \{\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4\}$ be a set of symptoms contains four symptoms such that $\bar{a}_1$ =Headache, $\bar{a}_2$ =Sore throat, $\bar{a}_3$ =Muscle pain, $\bar{a}_4$ =Chest pain.

Now, after the doctor has examined each patient and set a numerical value between 0 and 1 for each of the symptoms above, our proposed model can be built in a way that is consistent with the examining doctor's report.

$$\hat{P}_{\mathfrak{Q}_{\bar{A}}} =$$
$$\Bigg\{\Bigg(\bar{a}_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}$$
$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$$
$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}$$

$$\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(u_4,q_1)},\frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(u_4,q_2)}\Bigg)$$

$$\Bigg(e_2,\frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(u_1,q_1)},\frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(u_1,q_2)}$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(u_2,q_1)},\frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(u_2,q_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(u_3,q_1)},\frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(u_3,q_2)}$$

$$\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(u_4,q_1)},\frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_4,q_2)}\Bigg)$$

$$\Bigg(e_3,\frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(u_1,q_1)},\frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(u_1,q_2)}$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(u_2,q_1)},\frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(u_2,q_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(u_3,q_1)},\frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(u_3,q_2)}$$

$$\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(u_4,q_1)},\frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(u_4,q_2)}\Bigg)$$

$$\Bigg(e_4,\frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(u_1,q_1)},\frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(u_1,q_2)}$$

$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(u_2,q_1)},\frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(u_2,q_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(u_3,q_1)},\frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(u_3,q_2)}$$

$$\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(u_4,q_1)},\frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(u_4,q_2)}\Bigg)\Bigg\}$$

Step 2. The IV-Q-NS aggregation operator is given as

$$\widetilde{\Pi}_{ivQ-NS}^{agg} =$$
$$\{\big((u_1,q_1),\langle[0.275,0.825],[0.325,0.725],[0.327,0.550]\rangle\big),$$
$$\big((u_1,q_2),\langle[0.124,0.342],[0.451,0.537],[0.463,0.643]\rangle\big),$$
$$\big((u_2,q_1),\langle\langle[0.335,0.673],[0.326,0.673],[0.421,0.568]\rangle\rangle\big),$$
$$\big((u_2,q_2),\langle\langle[0.453,0.765],[0.321,0.547],[0.322,0.629]\rangle\rangle\big),$$
$$\big((u_3,q_1),\langle\langle[0.237,0.763],[0.327,0.743],[0.382,0.639]\rangle\rangle\big),$$
$$\big((u_3,q_2),\langle\langle[0.287,0.325],[0.210,0.482],[0.238,0.734]\rangle\rangle\big),$$
$$\big((u_4,q_1),\langle\langle[0.128,0.342],[0.438,0.983],[0.364,0.754]\rangle\rangle\big),$$
$$\big((u_4,q_2),\langle\langle[0.234,0.432],[0.543,0.578],[0.334,0.749]\rangle\rangle\big)\}$$

Step 3. Calculate the reduced value of the IV-Q-NS aggregation operator to IV-QFS aggregation operator.
$$\widetilde{\Pi}_{ivQ-FS}^{agg} =$$

$\{((\mathfrak{u}_1, \mathfrak{q}_1), \langle[0.309, 0.700]\rangle), \ ((\mathfrak{u}_1, \mathfrak{q}_2), \langle[0.346, 0.507]\rangle),$
$((\mathfrak{u}_2, \mathfrak{q}_1), \langle[0.346, 0.638]\rangle), ((\mathfrak{u}_2, \mathfrak{q}_2), \langle[0.365, 0.647]\rangle),$
$((\mathfrak{u}_3, \mathfrak{q}_1), \langle[0.285, 0.715]\rangle), ((\mathfrak{u}_3, \mathfrak{q}_2), \langle[0.245, 0.513]\rangle),$
$((\mathfrak{u}_4, \mathfrak{q}_1), \langle[0.310, 0.693]\rangle), ((\mathfrak{u}_4, \mathfrak{q}_2), \langle[0.370, 0.586]\rangle)\}$

Step 4. Convert IV-Q-FS aggregation operator $(\hat{P}_Q^{t,l}, \hat{P}_Q^{t,u})$ to SV-QFS aggregation operator $(\hat{P}_Q^{t})$.

$\breve{\Pi}_{ivQ-FS}^{agg} =$
$\{((\mathfrak{u}_1, \mathfrak{q}_1), \langle 0.515\rangle), \ ((\mathfrak{u}_1, \mathfrak{q}_2), \langle 0.426\rangle),$
$((\mathfrak{u}_2, \mathfrak{q}_1), \langle 0.492\rangle), ((\mathfrak{u}_2, \mathfrak{q}_2), \langle 0.506\rangle),$
$((\mathfrak{u}_3, \mathfrak{q}_1), \langle 0.500\rangle), ((\mathfrak{u}_3, \mathfrak{q}_2), \langle 0.379\rangle),$
$((\mathfrak{u}_4, \mathfrak{q}_1), \langle 0.501\rangle), ((\mathfrak{u}_4, \mathfrak{q}_2), \langle 0.478\rangle)\}$

Step 5. The optimal decision is the element available inM_i, such that

$M_1 = max_{(\mathfrak{u}_1, \mathfrak{q}_{1,2}) \in U \times Q}\{0.515, 0.426\} = 0.515.$

$M_2 = max_{(\mathfrak{u}_2, \mathfrak{q}_{1,2}) \in U \times Q}\{0.492, 0.506\} = 0.506.$

$M_3 = max_{(\mathfrak{u}_3, \mathfrak{q}_{1,2}) \in U \times Q}\{0.500, 0.379\} = 0.500.$

$M_4 = max_{(\mathfrak{u}_4, \mathfrak{q}_{1,2}) \in U \times Q}\{0.501, 0..478\} = 0.501.$

By looking at Table 1. below, which contains a comparison between the results obtained, it is clear that all patients $\mathbf{u_1}, \mathbf{u_3}, \ \mathbf{u_4}$ are infected except the patient $\mathbf{u_2}$**.**

Table 1: Comparison of the results obtained from the above algorithm

Patients	Degree of $(\mathfrak{u}, \mathfrak{q}_1)$	Degree of $(\mathfrak{u}, \mathfrak{q}_2)$	Comparison	Result
$\mathfrak{u}_1$	**0.515**	**0.426**	$\mathfrak{q}_1 > \mathfrak{q}_2$	**Yes**
$\mathfrak{u}_2$	**0.492**	**0.506**	$\mathfrak{q}_1 < \mathfrak{q}_2$	**NO**
$\mathfrak{u}_3$	**0.500**	**0.379**	$\mathfrak{q}_1 > \mathfrak{q}_2$	**Yes**
$\mathfrak{u}_4$	**0.501**	**0.478**	$\mathfrak{q}_1 > \mathfrak{q}_2$	**Yes**

5. Comparison with existing works

Now in this part, the proposed model is compared with some prevailing works like Adam and Hassan [21], Abu Qamar and Hassan [23], and Zhang et al. [36]. This comparison will focus on the structural structure of these methods compared to our method presented in this work, where the similarities and differences between these concepts were analyzed. Firstly, Abu Qamar and Hassan developed the notion of Q-NSSs as an extension of Adam and Hassan's notion, and this notion depicts decision-making data that has two diminutions in a single value, which causes some constraint for the decision maker when analysing data for the problem. Secondly, Zhang et al. defined INSs as a generalisation of FSs and IFSs and NSs to address real situations with a set of numbers in the real unit interval. This model has the ability to represent decision-making information that is characterised by uncertainty, indeterminacy, and inconsistency in one dimension (one universal set).

On the other hand, our model addresses all the complexities that appeared in the concepts referred to above, as its structural structure provides it with all the advantages that the currently prevailing methods lack. Moreover, Table 2 provides a further comparison between our proposed method and other prevailing methods based on some of the criteria fixed in the table.

Propose Methoed	**TM**	**IM**	**FM**	**SS**	**TD**	**IV**
FS	√	×	×	×	×	×
Q-FS	√	×	×	×	√	×
IFS	√	×	√	×	×	×
NS	√	√	√	×	√	×
Q-NSS	√	√	√	√	√	×
Our model IV-Q-NSS	√	√	√	√	√	√

Table 2: Comparison between our proposed method and other prevailing methods

In this table, each of (TM, IM, FM, SS, TD and IV) point out to True, Indeterminacy, Falsity, Matching with Soft set, Tow dimension, and Interval- Valued.

6. Conclusion

IV-Q-NSS is a useful tool for dealing with Q-two-dimensional universal information in interval form. It is made up of three NS membership degrees in interval form. Also, this tool was created to deal with the relationship between parameters in the SS environment when these parameters play a key role in the deep description of two-dimensional universal information. So, in this paper, we suggested an interval-valued Q-neutrosophic soft set (IV-Q-NSS) mean set theory. This theory includes special operations like the necessity and possibility operations of an IV-Q-NSS, as well as operations like the complement of an IV-Q-NSS, the union of two IV-Q-NSSs, the intersection of two IV-Q-NSSs, and the AND and OR operation of two IV-Q-NSSs. In addition, we presented many properties supported by numerical examples that explain how they work. Future, this new model has been successfully tested in dealing with one of the medical diagnostic problems based on hypothetical data for a respiratory disease when a new algorithm based on the aggregation operator for IV-Q-NSS data was built to solve this issue. Finally, directions will likely focus on improving some of the gaps in this work in the soft computing environment, as it is preferable to expand the work in this environment by integrating these tools with the hypersoft set (HSS) [37], where this environment will enable us to give a more accurate description of the parameters related to the SS environment. In addition to applying some mathematical tools, such as the similarity measure, the distance measure, or other measures on IV-Q-NSSs. In addition, this environment can be combined with other environments such as the algebraic environment [38-42] and the soft topological environment [43-46] and the use of other techniques such as techniques for measuring similarity and distance [47-49] between two objects.

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