An Integrated CODAS Method and Novel Surface-based Weighted Distance Measures under Neutrosophic Environment

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Abstract: Information theory provides suitable tools for solving practical problems, particularly multi-criteria decision-making (MCDM) problems under neutrosophic environments. Generally, a wide range of MCDM solution methods are constructed based on distance measures category. The defined distance measures for continuous neutrosophic numbers, especially its trapezoidal type, is very limited rather than discrete type. The main goals of this study is to come up with a way to add two new types of weighted distance measures based on meaningful surfaces: Euclidean and Hamming. To define these measures, all three components of the neutrosophic trapezoidal fuzzy number (truth, indeterminacy, and falsity membership functions) have been used simultaneously. The proof of some theorems and properties for the weighted distance measures demonstrates their validity. The CODAS algorithm is known as one of the distance-based methods for solving MCDM problems. The following represents the CODAS algorithm based on two novel distance measures. In addition, an explanatory example from the research literature is given to check the performance of the proposed hybrid algorithm. The results of this study indicate that the algorithm based on the proposed measures obtains a reasonable and appropriate ranking order between the options. Furthermore, the sensitivity parameter analysis and comparative analysis show the flexibility and accuracy of the suggested measures in the combined algorithm. The acceptable efficiency of proposed distance measures formed on the surfaces can shed light on research related to distance measures in the methodology and implicated aspects.

Keywords: Neutrosophic Set; Neutrosophic Trapezoidal Fuzzy Number; Distance Measure; Multi-Criteria Decision Making

1. Introduction

Information measure as an efficient tool for extracting the final result in the competitive environment and complex conditions of today's organizations, it is inevitable to encounter multi-criteria decision-making problems under uncertainty. Fuzzy sets (Zadeh (1965) [1]) and their innovative extensions have introduced acceptable covers to match the expression of data with the human mind. Among them, Neutrosophic Sets (NS) (Smarandache, (1999) [2]), which are considered as a multi-dimensional generalization of fuzzy sets to adequately describe the uncertainty involving the truth, indeterminacy, and falsity of decision makers' attitudes, have attracted the attention of many researchers. In general, many methods have been introduced to solve multi-criteria decision-making problems in conditions caused by uncertainty. Essential operators such as aggregating operators [3-7], preference relations [8], distance measures [9,10], similarity measures [11-17], correlation coefficient [18, 19], etc. [20-22] have a practical effect in solving MCDM problems.

Recently, Chen and Pan (2021) [23] presented a complete classification of MCDM problem-solving methods based on the overall structure of the solution techniques. This category includes numerical, distance-based, pairwise comparison, outranking methods, and so on, which can be considered for solution approaches. Most MCDM solution methods are in the group of distance-based methods such as COmplex Proportional Assessment (COPRAS) [24], Data Envelopment Analysis (DEA) [25, 26], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [27-31], and Compromise Programming (CP) [32], VlseKriterijumska Optimizacija I KOmpromisno Resenje (VIKOR) [33-35].

The CODAS method (Combinative Distance-based Assessment), which is known as one of the new distance-based methods to solve the MCDM problem, is proposed by Keshavarz Ghorabaee et al. (2016) [36] for the first time. In this sense, Euclidean and Taxicab distances are applied to calculate the assessment results of alternatives. In this idea, degrees are stated by a threshold parameter (Keshavarz Ghorabaee et al., 2016) [36]. Since its introduction, this method has had significant expansions from a theoretical perspective, combined with other existing methods and practical models.

From the theoretical point of view, we can mention the extension of the method from deterministic data to the fuzzy CODAS method proposed by Keshavarz Ghorabaee et al. [37] to cover uncertainty in MCDM which is employed for a selection problem (market segment evaluation) under fuzzy data. Bolturk (2018) [38] developed the CODAS method into Pythagorean fuzzy sets (PFSs) to handle more flexible data, and then the proposed method is used to explain the supplier selection problems. Yeni and Özçelik (2019) [39], after using interval-valued Atanassov intuitionistic fuzzy weighted aggregation, investigated extending the traditional fuzzy CODAS for interval-valued Atanassov intuitionistic fuzzy data in personnel selection problems. Later, the novel extension of fuzzy CODAS under the Interval-Valued Intuitionistic Trapezoidal Fuzzy Set (IVITrFS) was presented by Seker (2020) [40]. Subsequently, Wang et al. (2020) [41] established the CODAS method under the 2-tuple linguistic neutrosophic information, and expressed the computing steps for multiple attribute group decision-making (MAGDM). The idea of the CODAS method was extended by Deveci et al. (2022) [42] to support the evaluation of activities in mining sites under q-rung orthopter fuzzy sets (q-ROFSs). Menekse et al. (2022) [43] talk about an interval-valued spherical fuzzy CODAS process that can help clear up problems that aren't very clear.

From the combined point of view, more and more detailed research has been done so far. For instance, two alternatives to MCDM methods, named fuzzy AHP (Fuzzy Analytical Hierarchy Process) and CODAS, are integrated by Panchal et al. (2017) [44] for the evaluation process of a selection problem in a factory. Seker and Aydin (2020) [45] combined Interval-Valued Intuitionistic Fuzzy Analytical Hierarchy Process (IVIF-AHP) and IVIF-CODAS to depict an integrated MCDM framework, then they obtained the ranking of the alternatives in public transportation service quality. A multi-criteria group decision-making (MAGDM) framework process based on a combination of the full consistency method (FUCOM) and CODAS method has appeared in Biswas (2021) [46] for the first time in the literature. An integrated SWARA (Stepwise Weight Assessment Ratio Analysis) and CODAS methods are stated for the e-scooter charging station location selection problem in Pythagorean fuzzy information by Ayyildiz (2022) [47]. Recently, Mohamed and El-Saber (2023) [48] constructed the multi-stage intelligent decision-making model (MsIDMM) based on the CODAS method with interval-valued neutrosophic sets to evaluate the renewable energy sources.

Jafarzadeh et al. (2023) [49] combined the SWARA and the CODAS algorithm is applied to evaluate the clean energy barriers under a spherical fuzzy environment as a decision-making process. Garg et al. (2023) [50] constructed a theme of the (CODAS) method and the Dombi sine weighted arithmetic aggregation operator with complex intuitionistic fuzzy data for multi-criteria group decision-making problems. Sahmutoglu et al. (2023) [51] presented an integrated AHP-CODAS under Interval-Valued neutrosophic for risk assessment methodology in the district of Turkey, which is repeatedly exposed to floods. Dorfeshan et al. (2023) [52] presented the MABACODAS method, which includes MABAC and CODAS processes for MCDM under interval type-2 fuzzy information.

From a practical point of view, the use of CODAS and its combinations can be mentioned, such as Location selection problem [53, 54], Technological system evaluation problem [55], Material selection problem [56], Personnel selection problem [39], Service quality evaluation problem [45, 57], Cloud computing technology selection problem [58], Flexible Manufacturing System (FMS) selection [59], and so on [49, 60-62].

Distance measures (DMs) are substantial research topics for describing the distinctions and differences between various kinds of objects. The application of distance measures does not only include the procedure of decision-making problems based on distance measures, it can play a broad role in clustering algorithms, pattern recognition problems, medical diagnosis and image processing under uncertainty [63-67]. It is clear that acquaintance with distance measures that examine the nature of neutrosophic trapezoidal numbers from different perspectives can play an essential role in researchers' knowledge of MCDM problems and improvement of solution methods. Therefore, our primary focus in this research is introducing two new weighted measures based on surface distance under neutrosophic trapezoidal fuzzy information. These measures are used for presentation and productivity in the CODAS algorithm. In the proposed distances, the influences of all neutrosophic trapezoidal fuzzy numbers of the components are investigated. In addition, the choice of weights based on the decision maker's preferences determines the overall impact of each component on the final result. The meaningful structure of the proposed measures, along with their logical properties and characteristics, guarantees their proper performance in combination with other algorithms.

The rest of the study is constructed as follows: in Sect. 2, the required conceptions and operations of neutrosophic sets and numbers are explained. In Sect. 3, the conceptual structure of the main idea is given, along with two suggested distance measures for neutrosophic numbers. Then, in Sect. 4, theorems and properties are proven to ensure consistent and reasonable formulations of proposed distance measures. While in Sect. 5. The CODAS algorithm based on two novel distance measures for MCDM problems presented based on two new distance measures for MCDM problems. In Sect. 6, an illustrative example is solved in comparison to other existing methods. Furthermore, a sensitive analysis of parameters for the suggested hybrid approach is given to examine the effectiveness and robustness of the results. Lastly, in Sect. 6, some conclusions and future studies are stated.

2. Preliminaries

In this section, we consider a brief required definition of neutrosophic sets and neutrosophic trapezoidal fuzzy numbers (NTraFNs), along with some essential operators which are related to the subsequent sections of our study.

Definition 1 [2]. Assume that U is a universe of discourse, then a neutrosophic set $\widetilde{N}$ in U is defined by the following representation [2]:

$$\widetilde{N} = \{\langle u, \zeta_{\widetilde{N}}(u), \eta_{\widetilde{N}}(u), \theta_{\widetilde{N}}(u)\rangle |\ 0 \le \zeta_{\widetilde{N}}(u), \eta_{\widetilde{N}}(u), \theta_{\widetilde{N}}(u) \le 1, u \in U\}, \quad (1)$$

Where $\zeta_{\widetilde{N}}: U \to [0,1]$ is truth-membership function, $\eta_{\widetilde{N}}: U \to [0,1]$ is falsity-membership function, and $\eta_{\widetilde{N}}: U \to [0,1]$ is an indeterminacy-membership function. Furthermore $0 \le \zeta_{\widetilde{N}}(u) + \eta_{\widetilde{N}}(u) + \theta_{\widetilde{N}}(u) \le 3$.

Definition 2 [11]. Assume $\tilde{n} = \langle \zeta_{\tilde{n}}(u), \eta_{\tilde{n}}(u), \theta_{\tilde{n}}(u)\rangle$ is a neutrosophic fuzzy number in the set of real numbers. Then, its truth membership function is

$$\zeta_{\tilde{n}}(u) = \begin{cases} \zeta_{\tilde{n}}^{l}(u), & \alpha_1 \le u \le \alpha_2 \\ 1, & \alpha_2 \le u \le \alpha_3 \\ \zeta_{\tilde{n}}^{r}(u), & \alpha_3 \le u \le \alpha_4 \\ 0, & o.w \end{cases} \quad (2)$$

Its falsity membership function is

$$\eta_{\tilde{n}}(u) = \begin{cases} \eta_{\tilde{n}}^{l}(u), & \beta_1 \le u \le \beta_2 \\ 1, & \beta_2 \le u \le \beta_3 \\ \eta_{\tilde{n}}^{r}(u), & \beta_3 \le u \le \beta_4 \\ 0, & o.w \end{cases} \quad (3)$$

And its indeterminacy membership function is

$$\theta_{\tilde{n}}(u) = \begin{cases} \theta_{\tilde{n}}^{l}(u), & \gamma_1 \le u \le \gamma_2 \\ 1, & \gamma_2 \le u \le \gamma_3 \\ \theta_{\tilde{n}}^{r}(u), & \gamma_3 \le u \le \gamma_4 \\ 0, & o.w \end{cases} \quad (4)$$

Where $0 \le \zeta_{\tilde{n}}(u), \eta_{\tilde{n}}(u), \theta_{\tilde{n}}(u) \le 1$ and $0 \le \zeta_{\tilde{n}}(u) + \eta_{\tilde{n}}(u) + \theta_{\tilde{n}}(u) \le 3$.

Definition 3 [2]. Let two neutrosophic fuzzy numbers be $\tilde{n}_1 = \langle \zeta_{\tilde{n}_1}(u), \eta_{\tilde{n}_1}(u), \theta_{\tilde{n}_1}(u)\rangle$ and $\tilde{n}_2 = \langle \zeta_{\tilde{n}_2}(u), \eta_{\tilde{n}_2}(u), \theta_{\tilde{n}_2}(u)\rangle$. Then,

$$\tilde{n}_1 \subseteq \tilde{n}_2 \Leftrightarrow \quad \zeta_{\tilde{n}_1}(u) \le \zeta_{\tilde{n}_2}(u), \quad \eta_{\tilde{n}_1}(u) \ge \eta_{\tilde{n}_2}(u), \theta_{\tilde{n}_1}(u) \ge \theta_{\tilde{n}_2}(u) \quad , for \ \forall \ u \in \mathrm{U}$$

Definition 4 [11]. Assume U be a universe of discourse, $\tilde{n} = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4), (\beta_1, \beta_2, \beta_3, \beta_4), (\gamma_1, \gamma_2, \gamma_3, \gamma_4)\rangle$ is a neutrosophic trapezoidal fuzzy number in U that its truth-membership function is defined as

$$\zeta_{\tilde{n}}(u) = \begin{cases} \frac{(u-\alpha_1)}{\alpha_2-\alpha_1}, & \alpha_1 \le u \le \alpha_2 \\ 1, & \alpha_2 \le u \le \alpha_3 \\ \frac{(\alpha_4-u)}{\alpha_4-\alpha_3},, & \alpha_3 \le u \le \alpha_4 \\ 0, & o.w \end{cases} \tag{5}$$

Its falsity-membership function is defined as

$$\eta_{\tilde{n}}(u) = \begin{cases} \frac{(\beta_2-u)}{\beta_2-\beta_1}, & \beta_1 \le u \le \beta_2 \\ 1, & \beta_2 \le u \le \beta_3 \\ \frac{(u-\beta_3)}{\beta_4-\beta_3}, & \beta_3 \le u \le \beta_4 \\ 0, & o.w \end{cases} \tag{6}$$

and its indeterminacy-membership function is defined as

$$\theta_{\tilde{n}}(u) = \begin{cases} \frac{(\gamma_2-u)}{\gamma_2-\gamma_1}, & \gamma_1 \le u \le \gamma_2 \\ 1, & \gamma_2 \le u \le \gamma_3 \\ \frac{(u-\gamma_3)}{\gamma_4-\gamma_3}, & \gamma_3 \le u \le \gamma_4 \\ 0, & o.w \end{cases} \tag{7}$$

Where $0 \le \zeta_{\tilde{n}}(u), \eta_{\tilde{n}}(u), \theta_{\tilde{n}}(u) \le 1$ and $0 \le \zeta_{\tilde{n}}(u) + \eta_{\tilde{n}}(u) + \theta_{\tilde{n}}(u) \le 3$.

Figure 1 depicts the general representation of a neutrosophic trapezoidal fuzzy number.

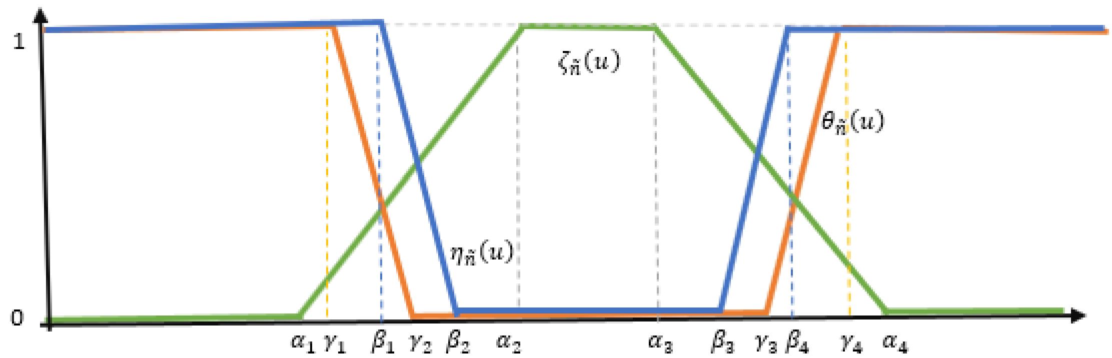

Figure 1. Trapezoidal neutrosophic fuzzy number.

Definition 5 [11]. Assume λ is a positive actual number, and consider two neutrosophic trapezoidal fuzzy numbers

$\tilde{n}_1 = \langle(\alpha_{1\tilde{n}_1}, \alpha_{2\tilde{n}_1}, \alpha_{3\tilde{n}_1}, \alpha_{4\tilde{n}_1}), (\beta_{1\tilde{n}_1}, \beta_{2\tilde{n}_1}, \beta_{3\tilde{n}_1}, \beta_{4\tilde{n}_1}), (\gamma_{1\tilde{n}_1}, \gamma_{2\tilde{n}_1}, \gamma_{3\tilde{n}_1}, \gamma_{4\tilde{n}_1})\rangle$

$\tilde{n}_2 = \langle(\alpha_{1\tilde{n}_2}, \alpha_{2\tilde{n}_2}, \alpha_{3\tilde{n}_2}, \alpha_{4\tilde{n}_2}), (\beta_{1\tilde{n}_2}, \beta_{2\tilde{n}_2}, \beta_{3\tilde{n}_2}, \beta_{4\tilde{n}_2}), (\gamma_{1\tilde{n}_2}, \gamma_{2\tilde{n}_2}, \gamma_{3\tilde{n}_2}, \gamma_{4\tilde{n}_2})\rangle$

Then, the following operations are valid.

1) $\tilde{n}_1 \oplus \tilde{n}_2 = \langle(\alpha_{1\tilde{n}_1} + \alpha_{1\tilde{n}_2} - \alpha_{1\tilde{n}_1}\alpha_{1\tilde{n}_2}, \alpha_{2\tilde{n}_1} + \alpha_{2\tilde{n}_2} - \alpha_{2\tilde{n}_1}\alpha_{2\tilde{n}_2}, \alpha_{3\tilde{n}_1} + \alpha_{3\tilde{n}_2} - \alpha_{3\tilde{n}_1}\alpha_{3\tilde{n}_2}, \alpha_{4\tilde{n}_1} + \alpha_{4\tilde{n}_2} - \alpha_{4\tilde{n}_1}\alpha_{4\tilde{n}_2}), (\beta_{1\tilde{n}_1}\beta_{1\tilde{n}_2}, \beta_{2\tilde{n}_1}\beta_{2\tilde{n}_2}, \beta_{3\tilde{n}_1}\beta_{3\tilde{n}_2}, \beta_{4\tilde{n}_1}\beta_{4\tilde{n}_2}), (\gamma_{1\tilde{n}_1}\gamma_{1\tilde{n}_2}, \gamma_{2\tilde{n}_1}\gamma_{2\tilde{n}_2}, \gamma_{3\tilde{n}_1}\gamma_{3\tilde{n}_2}, \gamma_{4\tilde{n}_1}\gamma_{4\tilde{n}_2})\ \rangle$,

2) $\tilde{n}_1 \otimes \tilde{n}_2 = \langle(\alpha_{1\tilde{n}_1}\alpha_{1\tilde{n}_2}, \alpha_{2\tilde{n}_1}\alpha_{2\tilde{n}_2}, \alpha_{3\tilde{n}_1}\alpha_{3\tilde{n}_2}, \alpha_{4\tilde{n}_1}\alpha_{4\tilde{n}_2}), (\beta_{1\tilde{n}_1} + \beta_{1\tilde{n}_2} - \beta_{1\tilde{n}_1}\beta_{1\tilde{n}_2}, \beta_{2\tilde{n}_1} + \beta_{2\tilde{n}_2} - \beta_{2\tilde{n}_1}\beta_{2\tilde{n}_2}, \beta_{3\tilde{n}_1} + \beta_{3\tilde{n}_2} - \beta_{3\tilde{n}_1}\beta_{3\tilde{n}_2}, \beta_{4\tilde{n}_1} + \beta_{4\tilde{n}_2} - \beta_{4\tilde{n}_1}\beta_{4\tilde{n}_2}), (\gamma_{1\tilde{n}_1} + \gamma_{1\tilde{n}_2} - \gamma_{1\tilde{n}_1}\gamma_{1\tilde{n}_2}, \gamma_{2\tilde{n}_1} + \gamma_{2\tilde{n}_2} - \gamma_{2\tilde{n}_1}\gamma_{2\tilde{n}_2}, \gamma_{3\tilde{n}_1} + \gamma_{3\tilde{n}_2} - \gamma_{3\tilde{n}_1}\gamma_{3\tilde{n}_2}, \gamma_{4\tilde{n}_1} + \gamma_{4\tilde{n}_2} - \gamma_{4\tilde{n}_1}\gamma_{4\tilde{n}_2})\ \rangle$,

3) $\lambda\tilde{n}_1 = \langle \left(\left(1-(1-\alpha_{1\tilde{n}_1})^{\lambda}\right), \left(1-(1-\alpha_{2\tilde{n}_1})^{\lambda}\right), \left(1-(1-\alpha_{3\tilde{n}_1})^{\lambda}\right), \left(1-(1-\alpha_{4\tilde{n}_1})^{\lambda}\right)\right), (\beta_{1\tilde{n}_1}^{\lambda}, \beta_{2\tilde{n}_1}^{\lambda}, \beta_{3\tilde{n}_1}^{\lambda}, \beta_{4\tilde{n}_1}^{\lambda}), (\gamma_{1\tilde{n}_1}^{\lambda}, \gamma_{2\tilde{n}_1}^{\lambda}, \gamma_{3\tilde{n}_1}^{\lambda}, \gamma_{4\tilde{n}_1}^{\lambda}) \rangle$,

4) $\tilde{n}_1{}^{\lambda} = \langle (\alpha_{1\tilde{n}_1}^{\lambda}, \alpha_{2\tilde{n}_1}^{\lambda}, \alpha_{3\tilde{n}_1}^{\lambda}, \alpha_{4\tilde{n}_1}^{\lambda}), \left(\left(1-(1-\beta_{1\tilde{n}_1})^{\lambda}\right), \left(1-(1-\beta_{2\tilde{n}_1})^{\lambda}\right), \left(1-(1-\beta_{3\tilde{n}_1})^{\lambda}\right), \left(1-(1-\beta_{4\tilde{n}_1})^{\lambda}\right)\right), \left(\left(1-(1-\gamma_{1\tilde{n}_1})^{\lambda}\right), \left(1-(1-\gamma_{2\tilde{n}_1})^{\lambda}\right), \left(1-(1-\gamma_{3\tilde{n}_1})^{\lambda}\right), \left(1-(1-\gamma_{4\tilde{n}_1})^{\lambda}\right)\right) \rangle$

Example 1: Assume $\lambda = 2$, and consider two neutrosophic trapezoidal fuzzy numbers

$\tilde{n}_1 = \langle (0.10, 0.15, 0.20, 0.25), (0.05, 0.10, 0.30, 0.35), (0.05, 0.20, 0.30, 0.45) \rangle$

$\tilde{n}_2 = \langle (0.15, 0.20, 0.30, 0.35), (0.20, 0.30, 0.40, 0.50), (0.35, 0.40, 0.50, 0.55) \rangle$

Then, according to Definition 5, we have

1) $\tilde{n}_1 \oplus \tilde{n}_2 = \langle (0.23, 0.32, 0.44, 0.51), (0.01, 0.03, 0.12, 0.18), (0.02, 0.08, 0.15, 0.25) \rangle$,
2) $\tilde{n}_1 \otimes \tilde{n}_2 = \langle (0.02, 0.03, 0.06, 0.09), (0.24, 0.37, 0.58, 0.67), (0.38, 0.52, 0.65, 0.75) \rangle$,
3) $2\tilde{n}_1 = \langle (0.19, 0.28, 0.36, 0.44), (0.002, 0.01, 0.09, 0.12), (0.002, 0.04, 0.09, 0.20) \rangle$,
4) $\tilde{n}_1{}^2 = \langle (0.01, 0.02, 0.04, 0.06), (0.10, 0.19, 0.51, 0.58), (0.10, 0.36, 0.51, 0.70) \rangle$.

3. Suggested weighted distance measures for neutrosophic trapezoidal fuzzy numbers

The distance measure concept is one of the most important theoretical and practical tools in information theorem that can be applied to evaluate the difference and distance of objects.

Here, we propose the conceptual scheme to model the distance measure between two neutrosophic trapezoidal fuzzy numbers (see Figure 2).

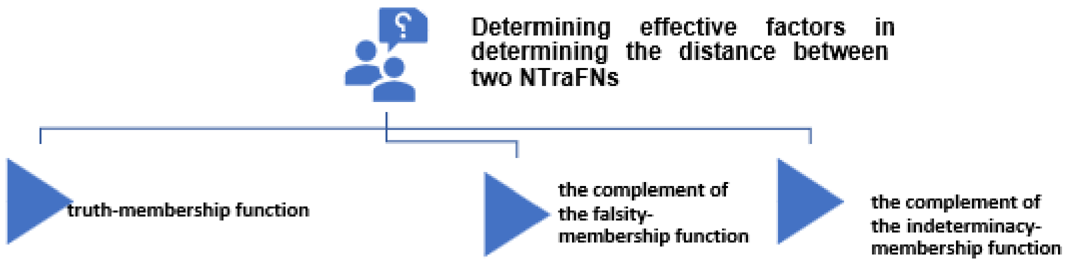

Figure 2. The main factors to determine the distance measure between NTraFNs.

Suppose $\tilde{n}_i = \langle (\alpha_{1\tilde{n}_i}, \alpha_{2\tilde{n}_i}, \alpha_{3\tilde{n}_i}, \alpha_{4\tilde{n}_i}), (\beta_{1\tilde{n}_i}, \beta_{2\tilde{n}_i}, \beta_{3\tilde{n}_i}, \beta_{4\tilde{n}_i}), (\gamma_{1\tilde{n}_i}, \gamma_{2\tilde{n}_i}, \gamma_{3\tilde{n}_i}, \gamma_{4\tilde{n}_i}) \rangle$ and $\tilde{n}_j = \left\langle \left(\alpha_{1\tilde{n}_j}, \alpha_{2\tilde{n}_j}, \alpha_{3\tilde{n}_j}, \alpha_{4\tilde{n}_j}\right), \left(\beta_{1\tilde{n}_j}, \beta_{2\tilde{n}_j}, \beta_{3\tilde{n}_j}, \beta_{4\tilde{n}_j}\right), \left(\gamma_{1\tilde{n}_j}, \gamma_{2\tilde{n}_j}, \gamma_{3\tilde{n}_j}, \gamma_{4\tilde{n}_j}\right) \right\rangle$ are two neutrosophic trapezoidal fuzzy numbers.

Step 1. Obtain the left and right line formulas corresponding to the truth-membership function;$[f^{l_{\tilde{n}_i}}(z), f^{r_{\tilde{n}_i}}(z)]$, the complement of the falsity-membership function; $[g^{l_{\tilde{n}_i}}(z), g^{r_{\tilde{n}_i}}(z)]$ and the complement of the indeterminacy-membership function; $[h^{l_{\tilde{n}_i}}(z), h^{r_{\tilde{n}_i}}(z)]$ for each trapezoidal neutrosophic fuzzy numbers with respect to the vertical lines z =0 and $z = 1$ respectively. In this sense, we can write:

$$[f^{l_{\tilde{n}_i}}(z), f^{r_{\tilde{n}_i}}(z)] = [(\alpha_{1\tilde{n}_i}) + z(\alpha_{2\tilde{n}_i} - \alpha_{1\tilde{n}_i}), (1-\alpha_{4\tilde{n}_i}) + z(\alpha_{4\tilde{n}_i} - \alpha_{3\tilde{n}_i})] \quad (8)$$

$$\left[g^{l_{\tilde{n}_i}}(z), g^{r_{\tilde{n}_i}}(z)\right] = \left[(\beta_{1\tilde{n}_i}) + \mathrm{z}\left(\beta_{2\tilde{n}_i} - \beta_{1\tilde{n}_i}\right), \left(1 - \beta_{4\tilde{n}_i}\right) + \mathrm{z}(\beta_{4\tilde{n}_i} - \beta_{3\tilde{n}_i})\right] \quad (9)$$

$$\left[h^{l_{\tilde{n}_i}}(z), h^{r_{\tilde{n}_i}}(z)\right] = \left[(\gamma_{1\tilde{n}_i}) + \mathrm{z}\left(\gamma_{2\tilde{n}_i} - \gamma_{1\tilde{n}_i}\right), \left(1 - \gamma_{4\tilde{n}_i}\right) + \mathrm{z}(\gamma_{4\tilde{n}_i} - \gamma_{3\tilde{n}_i})\right] \quad (10)$$

Step 2.a. Calculate the area of the left half of each interval function respective to the vertical axis. So, we can express

$$Q^{l_\zeta}(\tilde{n}_i) = \int_0^1 \left(\left(\alpha_{1\tilde{n}_i}\right) + z\left(\alpha_{2\tilde{n}_i} - \alpha_{1\tilde{n}_i}\right)\right) dz \quad = \left(\alpha_{1\tilde{n}_i}\right) + \frac{\left(\alpha_{2\tilde{n}_i} - \alpha_{1\tilde{n}_i}\right)}{2} \quad (11)$$

$$Q^{l_\eta}(\tilde{n}_i) = \int_0^1 \left(\left(\beta_{1\tilde{n}_i}\right) + z\left(\beta_{2\tilde{n}_i} - \beta_{1\tilde{n}_i}\right)\right) dz \quad = \left(\beta_{1\tilde{n}_i}\right) + \frac{\left(\beta_{2\tilde{n}_i} - \beta_{1\tilde{n}_i}\right)}{2} \quad (12)$$

$$Q^{r_\theta}(\tilde{n}_i) = \int_0^1 \left((\gamma_{1\tilde{n}_i}) + z(\gamma_{2\tilde{n}_i} - \gamma_{1\tilde{n}_i})\right) dz = \left(\gamma_{1\tilde{n}_i}\right) + \frac{(\gamma_{2\tilde{n}_i} - \gamma_{1\tilde{n}_i})}{2} \quad (13)$$

b. Calculate the area of the right half of each interval function respective to the vertical axis. So, we can express

$$Q^{r_\zeta}(\tilde{n}_i) = \int_0^1 \left((1 - \alpha_{4\tilde{n}_i}) + z(\alpha_{4\tilde{n}_i} - \alpha_{3\tilde{n}_i})\right) dz \quad = \left(1 - \alpha_{4\tilde{n}_i}\right) + \frac{\left(\alpha_{4\tilde{n}_i} - \alpha_{3\tilde{n}_i}\right)}{2} \quad (14)$$

$$Q^{r_\eta}(\tilde{n}_i) = \int_0^1 \left((1 - \beta_{4\tilde{n}_i}) + z(\beta_{4\tilde{n}_i} - \beta_{3\tilde{n}_i})\right) dz \quad = \left(1 - \beta_{4\tilde{n}_i}\right) + \frac{\left(\beta_{4\tilde{n}_i} - \beta_{3\tilde{n}_i}\right)}{2} \quad (15)$$

$$Q^{r_\theta}(\tilde{n}_i) = \int_0^1 \left((1 - \gamma_{4\tilde{n}_i}) + z(\gamma_{4\tilde{n}_i} - \gamma_{3\tilde{n}_i})\right) dz \quad = \left(1 - \gamma_{4\tilde{n}_i}\right) + \frac{(\gamma_{4\tilde{n}_i} - \gamma_{3\tilde{n}_i})}{2} \quad (16)$$

Step 3.a. The suggested surface-based weighted hamming distance measure is introduced as

$$\mathrm{D}^{HS}\left(\tilde{n}_i, \tilde{n}_j\right) = \left\{ \omega^{l_\zeta}\left(\left|Q^{l_\zeta}(\tilde{n}_i) - Q^{l_\zeta}(\tilde{n}_j)\right| + \omega^{r_\zeta}\left|Q^{r_\zeta}(\tilde{n}_i) - Q^{r_\zeta}(\tilde{n}_j)\right|\right) + \omega^{l_\eta}\left(\left|Q^{l_\eta}(\tilde{n}_i) - Q^{l_\eta}(\tilde{n}_j)\right| + \omega^{r_\eta}\left|Q^{r_\eta}(\tilde{n}_i) - Q^{r_\eta}(\tilde{n}_j)\right|\right) + \omega^{l_\theta}\left(\left|Q^{l_\theta}(\tilde{n}_i) - Q^{l_\theta}(\tilde{n}_j)\right| + \omega^{r_\theta}\left|Q^{r_\theta}(\tilde{n}_i) - Q^{r_\theta}(\tilde{n}_j)\right|\right)\right\} \quad (17)$$

Where $\omega^{l_\zeta}, \omega^{r_\zeta}, \omega^{l_\eta}, \omega^{r_\eta}, \omega^{l_\theta}, \omega^{r_\theta} \in [0,1]$ and satisfies $\omega^{l_\zeta} + \omega^{r_\zeta} + \omega^{l_\eta} + \omega^{r_\eta} + \omega^{l_\theta} + \omega^{r_\theta} = 1$.

If $\omega^{l_\zeta} = \omega^{r_\zeta} = \omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta}$, then

$$\mathrm{D}^{HS}\left(\tilde{n}_i, \tilde{n}_j\right) = \frac{1}{6}\left\{\left(\left|Q^{l_\zeta}(\tilde{n}_i) - Q^{l_\zeta}(\tilde{n}_j)\right| + \left|Q^{r_\zeta}(\tilde{n}_i) - Q^{r_\zeta}(\tilde{n}_j)\right|\right) + \left(\left|Q^{l_\eta}(\tilde{n}_i) - Q^{l_\eta}(\tilde{n}_j)\right| + \left|Q^{r_\eta}(\tilde{n}_i) - Q^{r_\eta}(\tilde{n}_j)\right|\right) + \left(\left|Q^{l_\theta}(\tilde{n}_i) - Q^{l_\theta}(\tilde{n}_j)\right| + \left|Q^{r_\theta}(\tilde{n}_i) - Q^{r_\theta}(\tilde{n}_j)\right|\right)\right\} \quad (18)$$

b. Similarly, the suggested surface-based weighted Euclidean distance measure introduced as

$$\mathrm{D}^{ES}\left(\tilde{n}_i, \tilde{n}_j\right) = \left\{ \omega^{l_\zeta}\left(Q^{l_\zeta}(\tilde{n}_i) - Q^{l_\zeta}(\tilde{n}_j)\right)^2 + \omega^{r_\zeta}\left(Q^{r_\zeta}(\tilde{n}_i) - Q^{r_\zeta}(\tilde{n}_j)\right)^2 + \omega^{l_\eta}\left(Q^{l_\eta}(\tilde{n}_i) - Q^{l_\eta}(\tilde{n}_j)\right)^2 + \omega^{r_\eta}\left(Q^{r_\eta}(\tilde{n}_i) - Q^{r_\eta}(\tilde{n}_j)\right)^2 + \omega^{l_\theta}\left(Q^{l_\theta}(\tilde{n}_i) - Q^{l_\theta}(\tilde{n}_j)\right)^2 + \omega^{r_\theta}\left(Q^{r_\theta}(\tilde{n}_i) - Q^{r_\theta}(\tilde{n}_j)\right)^2 \right\}^{1/2} \quad (19)$$

Where $\omega^{l\zeta}, \omega^{r\zeta}, \omega^{l\eta}, \omega^{r\eta}, \omega^{l\theta}, \omega^{r\theta} \in [0,1]$ and satisfies $\omega^{l\zeta} + \omega^{r\zeta} + \omega^{l\eta} + \omega^{r\eta} + \omega^{l\theta} + \omega^{r\theta} = 1$.
If $\omega^{l\zeta} = \omega^{r\zeta} = \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta}$, then

$$\mathrm{D}^{ES}(\tilde{n}_i, \tilde{n}_j) = \frac{1}{\sqrt{6}}\Big\{ \left(Q^{l\zeta}(\tilde{n}_i) - Q^{l\zeta}(\tilde{n}_j)\right)^2 + \left(Q^{r\zeta}(\tilde{n}_i) - Q^{r\zeta}(\tilde{n}_j)\right)^2 + \left(Q^{l\eta}(\tilde{n}_i) - Q^{l\eta}(\tilde{n}_j)\right)^2 + \left(Q^{r\eta}(\tilde{n}_i) - Q^{r\eta}(\tilde{n}_j)\right)^2 + \left(Q^{l\theta}(\tilde{n}_i) - Q^{l\theta}(\tilde{n}_j)\right)^2 + \left(Q^{r\theta}(\tilde{n}_i) - Q^{r\theta}(\tilde{n}_j)\right)^2 \Big\}^{1/2} \tag{20}$$

Example 2: Suppose $\tilde{n}_1 = \langle(0.10,0.15,0.20,0.25),(0.05,0.10,0.30,0.35),(0.05,0.20,0.30,0.45)\rangle$ and $\tilde{n}_2 = \langle(0.15,0.20,0.30,0.35),(0.20,0.30,0.40,0.50),(0.35,0.40,0.50,0.55)\rangle$ are two neutrosophic trapezoidal fuzzy numbers. Then, according to Eqs 19 and 20, we have

$$\mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_2) = \frac{1}{6}\{(0.05 + 0.10) + (0.10 + 0.125) + (0.275 + 0.15)\} = 0.13$$

$$\mathrm{D}^{ES}(\tilde{n}_1, \tilde{n}_2) = \frac{1}{\sqrt{6}}\{ 0.0025 + 0.01 + 0.01 + 0.016 + 0.076 + 0.0225 \}^{1/2} = 0.15$$

4. Theorems and properties

In this part, we focus on noteworthy features of suggested surface-based weighted hamming and Euclidean distance measures.

Theorem 1: let $\tilde{n}_1$, $\tilde{n}_2$, and $\tilde{n}_3$ are three neutrosophic trapezoidal fuzzy numbers on U. We represent the distance measure between the two numbers $\tilde{n}_1$ and $\tilde{n}_2$ is denoted as $D(\tilde{n}_1, \tilde{n}_2)$. Demonstrate that equation (18) satisfies the following distance measure principles.

A 1) $0 \le D(\tilde{n}_1, \tilde{n}_2) \le 1$
A 2) $D(\tilde{n}_1, \tilde{n}_2) = 0 \Leftrightarrow \tilde{n}_1 \sim \tilde{n}_2$
A 3) $D(\tilde{n}_1, \tilde{n}_2) = D(\tilde{n}_2, \tilde{n}_1)$
A 4) If $\tilde{n}_1 \subseteq \tilde{n}_2 \subseteq \tilde{n}_3 \Rightarrow D(\tilde{n}_1, \tilde{n}_2) \le D(\tilde{n}_1, \tilde{n}_3)$ and $D(\tilde{n}_1, \tilde{n}_2) \le D(\tilde{n}_1, \tilde{n}_3)$
Proof:
1) In every term of (18) the result of each absolute value is positive and smaller than 1, the structure of the relation and the fact that the sum of the weights is one, thematic principle 1 is valid.
Therefore:

$$0 \le \mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_2) \le 1$$

2) If $\mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_2) = 0$, then:

$$Q^{l\zeta}(\tilde{n}_1) = Q^{l\zeta}(\tilde{n}_2), \quad Q^{r\zeta}(\tilde{n}_1) = Q^{r\zeta}(\tilde{n}_2), \quad Q^{l\eta}(\tilde{n}_1) = Q^{l\eta}(\tilde{n}_2), \quad Q^{r\eta}(\tilde{n}_1) = Q^{r\eta}(\tilde{n}_2)$$
$$Q^{l\theta}(\tilde{n}_1) = Q^{l\theta}(\tilde{n}_2), \quad Q^{r\theta}(\tilde{n}_1) = Q^{r\theta}(\tilde{n}_2)$$

Therefore

$$\tilde{n}_1 \sim \tilde{n}_2$$

The converse of principle 2 is also can be proven in a similar way.
3) Since each of the expressions is in absolute value. Therefore:

$$\mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_2) = \mathrm{D}^{HS}(\tilde{n}_2, \tilde{n}_1)$$

4) If $\tilde{n}_1 \subseteq \tilde{n}_2 \subseteq \tilde{n}_3$, then:

$$\left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_2)\right| \le \left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_3)\right|, \qquad \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_2)\right| \le \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_3)\right|$$
$$\left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_2)\right| \le \left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_3)\right|, \qquad \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_2)\right| \le \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_3)\right|$$
$$\left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_2)\right| \le \left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_3)\right|, \qquad \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_2)\right| \le \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_3)\right|$$

Hence

$$\begin{aligned} \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_2) = \frac{1}{6}\{ & \left(\left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_2)\right| + \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_2)\right|\right) \\ & + \left(\left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_2)\right| + \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_2)\right|\right) \\ & + \left(\left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_2)\right| + \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_2)\right|\right)\} \end{aligned}$$

$$\le \frac{1}{6}\{\left(\left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_3)\right| + \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_3)\right|\right) + \left(\left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_3)\right| + \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_3)\right|\right) +$$

$$\left(\left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_3)\right| + \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_3)\right|\right)\} = \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3)$$

As a result

$$\mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_2) \le \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3)$$

It can be shown in a similar way

$$\mathrm{D}^{HS}(\tilde{n}_2,\tilde{n}_3) \le \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3)$$

Therefore, relation (20) satisfies all measure properties.

Theorem 2. let $\tilde{n}_1 \subseteq \tilde{n}_2 \subseteq \tilde{n}_3$ then, demonstrate

$$\mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3) \le \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_2) + \mathrm{D}^{HS}(\tilde{n}_2,\tilde{n}_3) \tag{21}$$

Proof: Starting from the left side of (18), we can write:

$$\begin{aligned} \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3) = \frac{1}{6}\{ & \left(\left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_2) + Q^{l\zeta}(\tilde{n}_2) - Q^{l\zeta}(\tilde{n}_3)\right| + \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_2) + Q^{r\zeta}(\tilde{n}_2) - Q^{r\zeta}(\tilde{n}_3)\right|\right) \\ & + \left(\left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_2) + Q^{l_\eta}(\tilde{n}_2) - Q^{l_\eta}(\tilde{n}_3)\right|\right. \\ & \left. + \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_2) + Q^{r_\eta}(\tilde{n}_2) - Q^{r_\eta}(\tilde{n}_3)\right|\right) \\ & + \left(\left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_2) + Q^{l\theta}(\tilde{n}_2) - Q^{l\theta}(\tilde{n}_3)\right|\right. \\ & \left. + \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_2) + Q^{r\theta}(\tilde{n}_2) - Q^{r\theta}(\tilde{n}_3)\right|\right)\} \end{aligned}$$

According to the Triangular inequality property of absolute value, we can say

$$\begin{aligned} \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3) \le \frac{1}{6}\{ & \left(\left|Q^{l\zeta}(\tilde{n}_1) - Q^{l\zeta}(\tilde{n}_2)\right| + \left|Q^{r\zeta}(\tilde{n}_1) - Q^{r\zeta}(\tilde{n}_2)\right|\right) \\ & + \left(\left|Q^{l_\eta}(\tilde{n}_1) - Q^{l_\eta}(\tilde{n}_2)\right| + \left|Q^{r_\eta}(\tilde{n}_1) - Q^{r_\eta}(\tilde{n}_2)\right|\right) \\ & + \left(\left|Q^{l\theta}(\tilde{n}_1) - Q^{l\theta}(\tilde{n}_2)\right| + \left|Q^{r\theta}(\tilde{n}_1) - Q^{r\theta}(\tilde{n}_2)\right|\right)\} \end{aligned}$$

$$\begin{aligned} + \frac{1}{6}\{ & \left(\left|Q^{l\zeta}(\tilde{n}_2) - Q^{l\zeta}(\tilde{n}_3)\right| + \left|Q^{r\zeta}(\tilde{n}_2) - Q^{r\zeta}(\tilde{n}_3)\right|\right) + \left(\left|Q^{l_\eta}(\tilde{n}_2) - Q^{l_\eta}(\tilde{n}_3)\right| + \left|Q^{r_\eta}(\tilde{n}_2) - Q^{r_\eta}(\tilde{n}_3)\right|\right) \\ & + \left(\left|Q^{l\theta}(\tilde{n}_2) - Q^{l\theta}(\tilde{n}_3)\right| + \left|Q^{r\theta}(\tilde{n}_2) - Q^{r\theta}(\tilde{n}_3)\right|\right)\} \\ & = \mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_3) + \mathrm{D}^{HS}(\tilde{n}_2,\tilde{n}_3). \end{aligned}$$

Now, let us demonstrate some meaningful properties of the proposed measures. For this aim, consider the following NTraFNs:

$$\tilde{n}_1 = \langle (a,a,a,a),(a,a,a,a),(0,0,0,0)\rangle, \tilde{n}_2 = \langle (b,b,b,b),(0,0,0,0),(0,0,0,0)\rangle,$$
$$\tilde{n}_3 = \langle (0,0,0,0),(0,0,0,0),(0,0,0,0)\rangle, \tilde{n}_4 = \langle (1,1,1,1),(0,0,0,0),(0,0,0,0)\rangle.$$

Property 1: If $\omega^{l\zeta} = \omega^{r\zeta} = 0.5, \omega^{l_\eta} = \omega^{r_\eta} = \omega^{l\theta} = \omega^{r\theta} = 0, then$

$$\mathrm{D}^{HS}(\tilde{n}_1,\tilde{n}_2) = \mathrm{D}^{ES}(\tilde{n}_1,\tilde{n}_2) = |a-b|.$$

Property 2: If $\omega^{l\zeta} = \omega^{r\zeta} = 0.5, \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 0, then$

$$\mathrm{D}^{HS}(\tilde{n}_3, \tilde{n}_4) = \mathrm{D}^{ES}(\tilde{n}_3, \tilde{n}_4) = 1.$$

Property 3: If $\omega^{l\zeta} = \omega^{r\zeta} = 0.5, \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 0, then$

$$\mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_3) = \mathrm{D}^{ES}(\tilde{n}_1, \tilde{n}_3) = a.$$

Property 4: If $\omega^{l\zeta} = \omega^{r\zeta} = 0.5, \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 0, then$

$$\mathrm{D}^{HS}(\tilde{n}_1, \tilde{n}_4) = \mathrm{D}^{ES}(\tilde{n}_1, \tilde{n}_4) = 1 - a.$$

Example 3: Find D^{HS} and D^{ES} between the following numbers

$$\tilde{n}_1 = \langle(0.5,0.5,0.5,0.5), (0,0,0,0), (0,0,0,0)\rangle, \tilde{n}_2 = \langle(0.7,0.7,0.7,0.7), (0,0,0,0), (0,0,0,0)\rangle,$$
$$\tilde{n}_3 = \langle(0,0,0,0), (0,0,0,0), (0,0,0,0)\rangle, \tilde{n}_4 = \langle(1,1,1,1), (0,0,0,0), (0,0,0,0)\rangle.$$

Regard to three following cases for weights as

Case1: $\omega^{l\zeta} = \omega^{r\zeta} = 0.5, \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 0$

Case2: $\omega^{l\zeta} = \omega^{r\zeta} = \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 1/6$

Case3: $\omega^{l\zeta} = \omega^{r\zeta} = 0.3, \omega^{l\eta} = \omega^{r\eta} = \omega^{l\theta} = \omega^{r\theta} = 0.1$

Since the mentioned numbers in example 1 are all deterministic, we expect the proposed distance measures to verify an acceptable performance with changes in the weighting coefficients. We consider three modes for weight variation according to what was mentioned earlier. The weighted Euclidean and Hamming distance between the numbers $\tilde{n}_1, \tilde{n}_2, \tilde{n}_3$, and $\tilde{n}_4$ under states 1, 2, and 3 are given in tables 1 to 6.

Case1	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.2000	0.5000	0.5000
$\tilde{n}_2$	0.2000	0	0.7000	0.3000
$\tilde{n}_3$	0.5000	0.7000	0	1.0000
$\tilde{n}_4$	0.5000	0.3000	1.0000	0

Table 1. Hamming distance measure for case 1 of Ex 1.

Case1	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.2000	0.5000	0.5000
$\tilde{n}_2$	0.2000	0	0.7000	0.3000
$\tilde{n}_3$	0.5000	0.7000	0	1.0000
$\tilde{n}_4$	0.5000	0.3000	1.0000	0

Table 2. Euclidean distance measure for case 1 of Ex 1.

Case2	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.0667	0.1667	0.1667
$\tilde{n}_2$	0.0667	0	0.2333	0.1000
$\tilde{n}_3$	0.1667	0.2333	0	0.3333
$\tilde{n}_4$	0.1667	0.3333	0.3333	0

Table 3. Hamming distance measure for case 2 of Ex 1.

Case2	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.1155	0.2877	0.2877
$\tilde{n}_2$	0.1155	0	0.4041	0.1732
$\tilde{n}_3$	0.2877	0.4041	0	0.5774
$\tilde{n}_4$	0.2877	0.1732	0.5774	0

Table 4. Euclidean distance measure for case 2 of Ex 1.

Case3	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.1200	0.3000	0.3000
$\tilde{n}_2$	0.1200	0	0.4200	0.1800
$\tilde{n}_3$	0.3000	0.4200	0	0.6000

Case3	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$
$\tilde{n}_1$	0	0.1549	0.3873	0.3873
$\tilde{n}_2$	0.1549	0	0.5422	0.2324
$\tilde{n}_3$	0.3873	0.5422	0	0.7746

$\tilde{n}_4$	0.3000	0.1800	0.6000	0

Table 5. Hamming distance measure for case 3 of Ex 1.

$\tilde{n}_4$	0.3873	0.2324	0.7746	0

Table 6. Euclidean distance measure for case 3 of Ex 1.

As expected, the results of Tables 1 and 2 in case 1 reflect the logical and uniform performance of the proposed distance measures. In addition, by increasing the weighting coefficients related to the indeterminacy and falsity-membership functions for cases 2 and 3 (Tables 3, 4, 5, and 6), smaller values for Euclidean and Hamming distances are obtained compared to case 1.

5. NTraFNs-CODAS method based on novel weighted distance measures

Keshavarz Ghorabaee et al. (2016) [36] introduced the CODAS method as one of the distance-based methods in 2016 to solve the multi-criteria decision-making problem. In this attitude, the Euclidean and Hamming distances of each option are used to determine the most desirable option from the negative ideal. Based on our current studies and knowledge from the research literature, no extension of this method has been done on neutrosophic trapezoidal fuzzy data for selection problems. Therefore, in this part, we want to present the CODAS algorithm under neutrosophic trapezoidal fuzzy information and using two surface-based weighted distance measures. Figure 3 shows the general structure of the NTraFNs-CODAS algorithm.

More precisely, the steps of the CODAS method are expressed as follows:

Step 1: Record the alternative sets $O = \{o_1, o_2, \dots, o_s\}$, attribute sets $E = \{e_1, e_2, \dots, e_t\}$ and relevant weight sets $\Xi = \{\xi_1, \xi_2, \dots, \xi_t\}$. Then construct the NTraFNs decision matrix, which is denoted as $\widetilde{U} = [\tilde{u}_{ij}]_{s*t}$ such that each array is given by

$$\tilde{u}_{ij} = \left\langle \left(\alpha_{1\tilde{u}_{ij}}, \alpha_{2\tilde{u}_{ij}}, \alpha_{3\tilde{u}_{ij}}, \alpha_{4\tilde{u}_{ij}}\right), \left(\beta_{1\tilde{u}_{ij}}, \beta_{2\tilde{u}_{ij}}, \beta_{3\tilde{u}_{ij}}, \beta_{4\tilde{u}_{ij}}\right), \left(\gamma_{1\tilde{u}_{ij}}, \gamma_{2\tilde{u}_{ij}}, \gamma_{3\tilde{u}_{ij}}, \gamma_{4\tilde{u}_{ij}}\right) \right\rangle \tag{22}$$

Where $i \epsilon \{1,2, \dots, s\}$ and $i \epsilon \{1,2, \dots, t\}$.

Step 2: Obtain the normalized and then the weighted normalized NTraFNs decision matrix, which can denote as $\widetilde{U}^n = [\tilde{u}_{ij}^n]_{s*t}$ and $\widetilde{U}^{wn} = [\tilde{u}_{ij}^{wn}]_{s*t}$ where

$$\tilde{u}_{ij}^{wn} = \tilde{u}_{ij}^n * \xi_j \tag{23}$$

$$\tilde{u}_{ij}^{wn} = \tilde{u}_{ij} = \left\langle \left(\alpha_{1\tilde{u}_{ij}}^{wn}, \alpha_{2\tilde{u}_{ij}}^{wn}, \alpha_{3\tilde{u}_{ij}}^{wn}, \alpha_{4\tilde{u}_{ij}}^{wn}\right), \left(\beta_{1\tilde{u}_{ij}}^{wn}, \beta_{2\tilde{u}_{ij}}^{wn}, \beta_{3\tilde{u}_{ij}}^{wn}, \alpha_{4\tilde{u}_{ij}}^{wn}\right), \left(\gamma_{1\tilde{u}_{ij}}^{wn}, \gamma_{2\tilde{u}_{ij}}^{wn}, \gamma_{3\tilde{u}_{ij}}^{wn}, \gamma_{4\tilde{u}_{ij}}^{wn}\right) \right\rangle \tag{24}$$

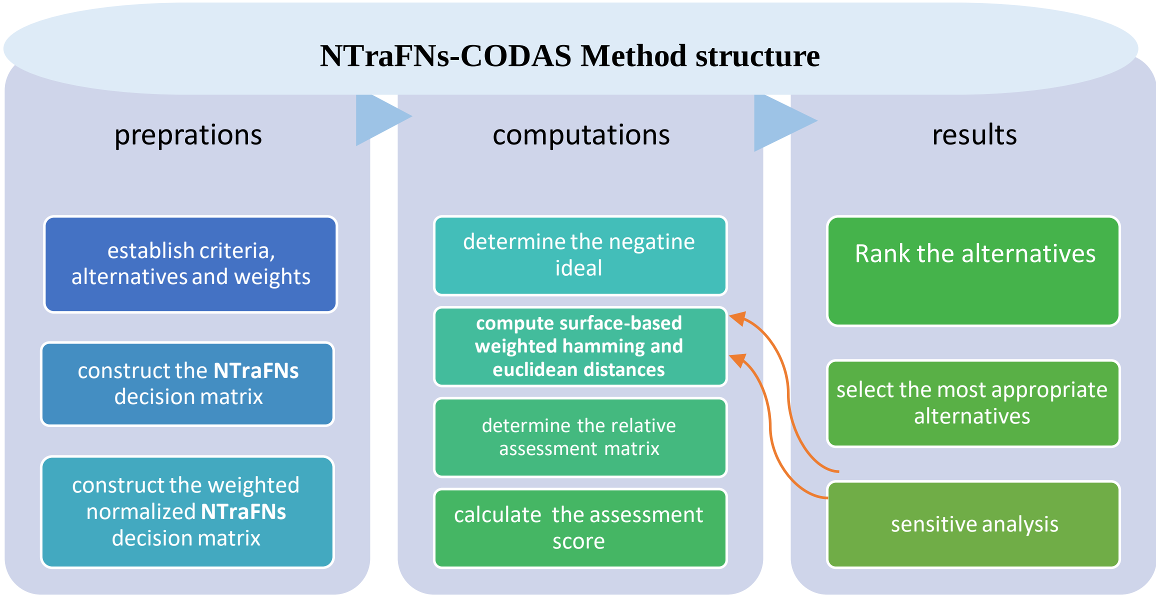

Figure 3. NTraFNs-CODAS Method structure.

Step 3: Recognize the negative ideal solution. This matrix defines by $\widetilde{NI} = [\widetilde{ni}_{ij}]_{1*t}$ such that:

$$\widetilde{ni}_{ij} = \min_i \tilde{u}_{ij}^{wn} \tag{25}$$

In such a way as

$$\min_i \tilde{u}_{ij}^{wn} = \left\langle \begin{array}{c} \left(\min_i(\alpha^{wn}_{1\tilde{u}_{ij}}), \min_i(\alpha^{wn}_{2\tilde{u}_{ij}}), \min_i(\alpha^{wn}_{3\tilde{u}_{ij}}), \min_i(\alpha^{wn}_{4\tilde{u}_{ij}})\right), \\ \left(\max_i(\beta^{wn}_{1\tilde{u}_{ij}}), \max_i(\beta^{wn}_{2\tilde{u}_{ij}}), \max_i(\beta^{wn}_{3\tilde{u}_{ij}}), \max_i(\alpha^{wn}_{4\tilde{u}_{ij}})\right), \\ \left(\max_i(\gamma^{wn}_{1\tilde{u}_{ij}}), \max_i(\gamma^{wn}_{2\tilde{u}_{ij}}), \max_i(\gamma^{wn}_{3\tilde{u}_{ij}}), \max_i(\gamma^{wn}_{4\tilde{u}_{ij}})\right) \end{array} \right\rangle \tag{26}$$

Step 4: Calculate the surface-based weighted Hamming and Euclidean distances between alternatives and $\widetilde{NI}$ according to Eqs (17, 19). Then D_i^{HS} and D_i^{ES} are computed as aggregated distances in Eqs (27, 28)

$$D_i^{HS} = \sum_{j=1}^{t} \mathrm{D}^{HS}\left(\tilde{u}_{ij}^{wn}, \widetilde{ni}_{ij}\right) \tag{27}$$

$$D_i^{ES} = \sum_{j=1}^{t} \mathrm{D}^{ES}\left(\tilde{u}_{ij}^{wn}, \widetilde{ni}_{ij}\right) \tag{28}$$

Step 5: Organize the relative assessment matrix as follows

$$RA = [p_{ik}]_{s*s} \tag{29}$$

Where each array of RA is obtained by applying Eqs 30 and 31

$$p_{ik} = \left\{(D_i^{ES} - D_k^{ES}) + \left(\gamma(D_i^{ES} - D_k^{ES}) * (D_i^{HS} - D_k^{HS})\right)\right\}, \tag{30}$$

$$\Phi(u) = \begin{cases} 1 & |u| \geq \varphi, \\ 0 & |u| \leq \varphi, \end{cases} \tag{31}$$

as a threshold value chosen by the decision-maker's opinion for the function $\Phi(u)$. In this paper, $\varphi = 0.02$ is assumed for computations.

Step 6: The assessment value for each alternative is determined as the following equation

$$AS_i = \sum_{k=1}^{s} p_{ik}. \tag{32}$$

Step 7: The highest Assessment value of step 6 indicates the most desirable choice.

5 -1 Illustrative example

In order to show the efficiency of the proposed hybrid method, we adopted the illustrative example of the material selection problem discussed by Jana and Karaaslan [68]. The customer desires to buy a tablet from the list of primarily selected five alternatives $O = \{o_1, o_2, o_3, o_4, o_5\}$. The following four attributes are considered by the customer (Figure 4):

(1) Options (e1);
(2) Hardware (e2);
(3) Affordable price (e3); and
(4) Customer support (e4).

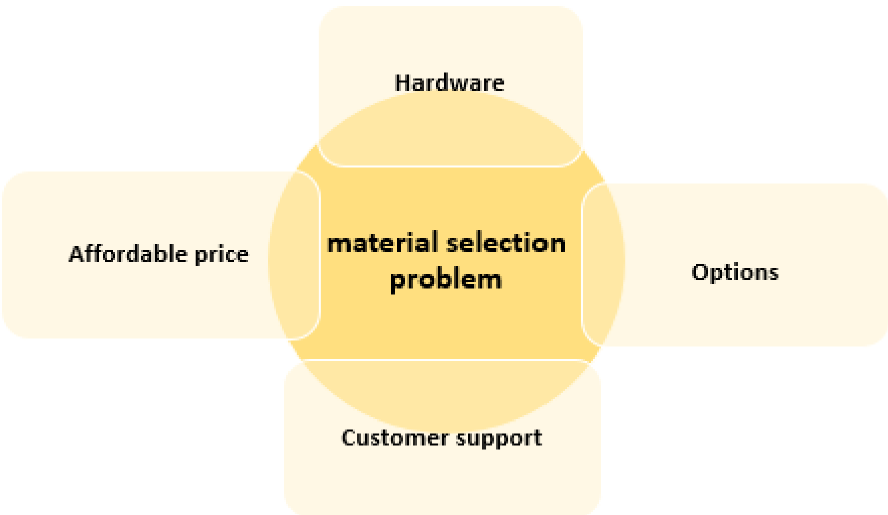

Figure 4. Criteria of material selection problem.

Assume that the weight vectors are provided by experts for the four attributes under the TrNFNs as follows:

$$\xi_1 = \langle (0.3,0.5,0.8,0.9), (0.1,0.3,0.6,0.7), (0.2,0.3,0.6,0.6) \rangle$$
$$\xi_2 = \langle (0.5,0.6,0.7,0.9), (0.3,0.5,0.6,0.8), (0.2,0.4,0.7,0.8) \rangle$$
$$\xi_3 = \langle (0.6,0.7,0.8,0.9), (0.0,0.1,0.2,0.3), (0.1,0.1,0.2,0.3) \rangle$$
$$\xi_4 = \langle (0.4,0.6,0.7,0.7), (0.2,0.3,0.4,0.5), (0.2,0.3,0.6,0.6) \rangle$$

Table 7 provides information (neutrosophic trapezoidal fuzzy numbers) on the experts' opinions of the five alternatives on the relevant criteria for the decision-making process.

Table 7. NTraFNs decision matrix.

	e_1	e_2	e_3	e_4
o_1	(0/1,0/2,0/3,0/3), ⟨(0/0,0/3,0/4,0/4),⟩ (0/2,0/5,0/6,0/7)	(0/4,0/5,0/6,0/6), ⟨(0/1,0/1,0/4,0/6),⟩ (0/2,0/5,0/6,0/7)	(0/2,0/2,0/3,0/4), ⟨(0/5,0/6,0/6,0/8),⟩ (0/0,0/2,0/2,0/5)	(0/5,0/5,0/6,0/6), ⟨(0/2,0/7,0/7,0/7),⟩ (0/2,0/3,0/3,0/3)
o_2	(0/2,0/2,0/4,0/4), ⟨(0/3,0/3,0/5,0/6),⟩ (0/1,0/2,0/2,0/5)	(0/3,0/5,0/6,0/7), ⟨(0/2,0/2,0/3,0/4),⟩ (0/4,0/5,0/8,0/9)	(0/4,0/5,0/5,0/7), ⟨(0/3,0/3,0/4,0/6),⟩ (0/2,0/3,0/4,0/5)	(0/1,0/1,0/2,0/8), ⟨(0/6,0/6,0/7,0/8),⟩ (0/0,0/1,0/2,0/4)
o_3	(0/5,0/7,0/8,0/9), ⟨(0/2,0/4,0/5,0/8),⟩ (0/3,0/3,0/5,0/5)	(0/1,0/2,0/2,0/3), ⟨(0/2,0/5,0/6,0/6),⟩ (0/1,0/2,0/3,0/4)	(0/3,0/3,0/4,0/5), ⟨(0/1,0/4,0/4,0/6),⟩ (0/2,0/2,0/3,0/7)	(0/0,0/2,0/3,0/9), ⟨(0/1,0/7,0/7,0/8),⟩ (0/6,0/7,0/7,0/8)
o_4	(0/0,0/2,0/3,0/7), ⟨(0/4,0/5,0/6,0/8),⟩ (0/4,0/5,0/5,0/9)	(0/5,0/5,0/7,0/8), ⟨(0/4,0/5,0/6,0/6),⟩ (0/5,0/6,0/7,0/8)	(0/5,0/6,0/6,0/9), ⟨(0/3,0/5,0/5,0/6),⟩ (0/1,0/5,0/5,0/6)	(0/5,0/7,0/8,0/9), ⟨(0/5,0/6,0/6,0/6),⟩ (0/2,0/3,0/3,0/3)
o_5	(0/2,0/4,0/4,0/5), ⟨(0/3,0/6,0/6,0/9),⟩ (0/0,0/2,0/3,0/5)	(0/1,0/5,0/7,0/9), ⟨(0/2,0/3,0/3,0/6),⟩ (0/6,0/7,0/7,0/9)	(0/4,0/4,0/7,0/7), ⟨(0/1,0/4,0/4,0/7),⟩ (0/2,0/4,0/4,0/6)	(0/0,0/1,0/2,0/3), ⟨(0/2,0/2,0/4,0/5),⟩ (0/1,0/1,0/3,0/4)

From step 2, the weighted normalized NTraFNs decision matrix is shown in Table 8.

Table 8. Weighted normalized NTraFNs decision matrix.

	e_1	e_2	e_3	e_4
o_1	(0/03,0/1,0/24,0/27) ⟨ (0/1,0/51,0/76,0/82) (0/36,0/65,0/84,0/88	(0/2,0/3,0/42,0/54), ⟨(0/37,0/55,0/76,0/92 (0/44,0/64,0/82,0/9)	(0/12,0/14,0/24,0/36 ⟨ (0/5,0/64,0/68,0/86) (0/1,0/28,0/36,0/65)	(0/2,0/3,0/42,0/42), ⟨(0/36,0/79,0/82,0/85 (0/28,0/44,0/51,0/58
o_2	(0/06,0/1,0/32,0/36), ⟨(0/37,0/51,0/8,0/88), (0/28,0/44,0/68,0/8)	(0/15,0/3,0/42,0/63), ⟨(0/44,0/6,0/72,0/88), (0/52,0/7,0/94,0/98)	(0/24,0/35,0/4,0/63) ⟨ (0/3,0/37,0/52,0/72) (0/28,0/37,0/52,0/65	(0/04,0/06,0/14,0/56 ⟨ (0/68,0/72,0/82,0/9) (0/1,0/28,0/44,0/64)
o_3	(0/15,0/35,0/64,0/81 ⟨ (0/28,0/58,0/8,0/94) (0/44,0/51,0/8,0/8)	(0/05,0/12,0/14,0/27 ⟨(0/44,0/75,0/84,0/92 (0/28,0/52,0/79,0/88	(0/18,0/21,0/32,0/45 ⟨ (0/1,0/46,0/52,0/72) (0/28,0/28,0/44,0/79	(0/0,0/12,0/21,0/63) ⟨ (0/28,0/79,0/82,0/9) (0/64,0/76,0/79,0/88
o_4	(0/0,0/1,0/24,0/63), ⟨(0/46,0/65,0/84,0/94 (0/52,0/65,0/8,0/96)	(0/25,0/3,0/49,0/72) ⟨(0/58,0/75,0/84,0/92 (0/6,0/76,0/91,0/96)	(0/3,0/42,0/48,0/81), ⟨ (0/3,0/55,0/6,0/72), (0/19,0/55,0/6,0/72)	(0/2,0/42,0/56,0/63) ⟨ (0/6,0/72,0/76,0/8), (0/28,0/44,0/51,0/58
o_5	(0/06,0/2,0/32,0/45) ⟨(0/37,0/72,0/84,0/97 (0/2,0/44,0/72,0/8)	(0/05,0/3,0/49,0/81) ⟨(0/44,0/65,0/72,0/92 (0/68,0/82,0/91,0/98	(0/24,0/28,0/56,0/63 ⟨ (0/1,0/46,0/52,0/79) (0/28,0/46,0/52,0/72	(0/0,0/06,0/14,0/21) ⟨(0/36,0/44,0/64,0/75 (0/19,0/28,0/51,0/64

Due to Eqs. 25 and 26 of step 3, the $\widetilde{NI}$ matrix obtained as:

$$\widetilde{NI}=\begin{bmatrix}\langle(0.00,0.10,0.24,0.27),(0.46,0.72,0.84,0.97),(0.52,0.65,0.84,0.96)\rangle\\ \langle(0.05,0.12,0.14,0.27),(0.58,0.75,0.84,0.92),(0.68,0.82,0.94,0.98)\rangle\\ \langle(0.12,0.14,0.24,0.36),(0.50,0.64,0.68,0.86),(0.28,0.55,0.60,0.79)\rangle\\ \langle(0.00,06,0.14,0.21),(0.68,0.70,0.82,0.90),(0.64,0.76,0.79,0.88)\rangle\end{bmatrix}'$$

Then, the calculations of D_i^{HS} and D_i^{ES} with regard to Eqs. 17, 19, 27, and 28 are summarized in Table 9.

Table 9. Surface-based weighted Hamming and Euclidean distances of alternatives.

Alts	o_1	o_2	o_3	o_4	o_5
D_i^{HS}	0.5180	0.5585	0.5430	0.7470	0.5980
D_i^{ES}	0.6448	0.6829	0.7504	0.8942	0.7950

Now, following steps 5 and 6, once the arrays of the relative assessment matrix have been found, Eq. 32 is used to find the assessment value of each option.

Table 10. Ranking alternatives based on RA matrix.

	Relative assessment matrix						
Alts	o_1	o_2	o_3	o_4	o_5	AS	Rank
o_1	0	-0.0382	-0.1056	-0.2483	-0.1500	-0.5421	5
o_2	0.0382	0	-0.0675	-0.2105	0.1120	-0.3517	4
o_3	0.1057	0.0675	0	-0.1432	-0.0445	-0.0145	3
o_4	0.2506	0.2121	0.1444	0	0.0995	0.7065	1
o_5	0.1505	0.1122	0.0446	-0.0989	0	0.2084	2

Finally, the highest assessment value of step 6 shows the most desirable material. The ranking order of all candidates is available in table 10.

5 -2 Sensitive analysis

As mentioned in the previous part, the evaluation scores and the ranking order corresponding to each alternative in the multi-criteria decision-making problem were obtained using the NTraFNs-CODAS method for $\varphi=0.02$ (in step 5) and the specified weights ($\omega^{l\zeta}=\omega^{r\zeta}=0.3, \omega^{l\eta}=\omega^{r\eta}=\omega^{l\theta}=\omega^{r\theta}=0.1$) as parameters of the problem (Table 10). In this part, we want to investigate the effect of the sensitive analysis of φ and weights on the evaluation values and ranking of options for the material selection problem in the environment with neutrosophic trapezoidal fuzzy data.

A. Change of the parameter φ

The results of evaluating and ranking the options for different values are obtained in Table 11. As can be seen, although these changes have had a slight effect on the evaluation values, they have not had any effect on the final ranking of the options.

Table 11. Ranking alternatives based on sensitive analysis of φ values.

	AS related to sensitive analysis of φ values					
Alts	0.01	0.02	0.03	0.04	0.05	Rank
o_1	-0.5428	-0.5421	-0.5414	-0.5406	-0.5399	5
o_2	-0.3522	-0.3517	-0.3513	-0.3506	-0.3504	4
o_3	-0.0149	-0.0145	-0.0142	-0.0139	-0.0135	3
o_4	0.7051	0.7065	0.7079	0.7093	0.7108	1
o_5	0.2080	0.2084	0.2087	0.2091	0.2094	2

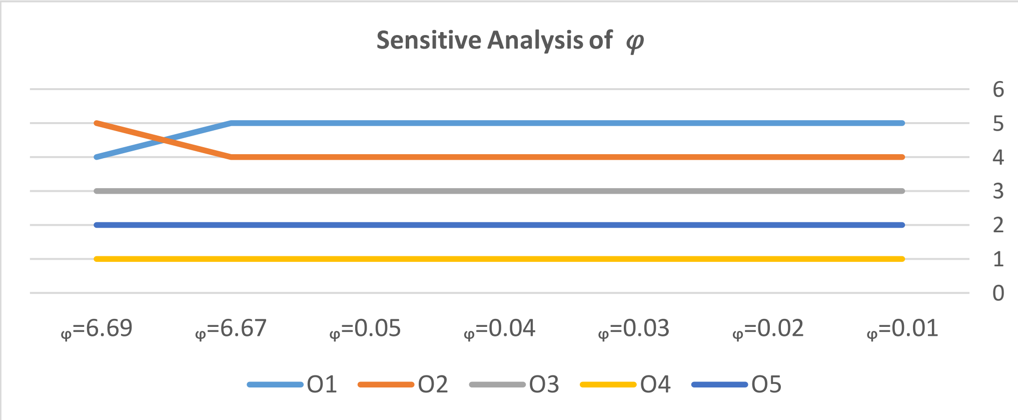

Figure 5. Ranking results based on sensitive analysis of φ.

Figure 5, clearly emphasizes the sameness of ranking results for sensitive analysis of φ.

Now, we desire to discuss on the admissible increase of φ, which does not any effect on the ranking result.

By increasing the value of φ to 6.67, the results of the evaluation options are obtained as follows:

$$AS_1 = -0.0544,\ AS_2 = -0.0541, AS_3 = 0.2074, AS_4 = 1.6446, AS_5 = 0.4324$$

And for $\varphi = 6.69$, we have

$$AS_1 = -0.0530, AS_2 = -0.0533, AS_3 = 0.2081, AS_4 = 1.6474, AS_5 = 0.4330$$

So, based on the analyses above, the stability and efficiency of the proposed NTraFNs-CODAS algorithm to changing of φ on $[0.01, 6.67]$ are observed.

B. Changes of the weights in proposed measures

The weights in Eqs 17 and 19 are the other parameters of the proposed NTraFNs-CODAS method, which can show the flexibility of the results. In the NTraFNs-CODAS method, the results are calculated according to the weights in the weighted Hamming and Euclidean distances. It is clear that by changing the weight coefficients related to the proposed measures Eqs (17, 19), the effectiveness of each term will be different in calculating the final assessment scores. As a result, various categories of solution are available for the decision maker. In Table 12, six cases are considered for weight variation; hence the values of evaluation and ranking of the options are obtained based on each case.

Table 12. Ranking alternatives based on sensitive analysis of different weights.

cases	alternatives				
	o_1	o_2	o_3	o_4	o_5
$Case\ \ 1: \omega^{l_\zeta} = \omega^{r_\zeta} = 0,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0.25$	$0/3690$	$0/0609$	$0/3452$	$-1/2390$	$0/4766$
Ranking result	2	4	3	5	1
$Case\ \ 2: \omega^{l_\zeta} = \omega^{r_\zeta} = 0.1,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0.2$	$-0/0392$	$-0/1507$	$0/2252$	$-0/4172$	$0/3841$
Ranking result	3	4	2	5	1
$Case\ \ 3: \omega^{l_\zeta} = \omega^{r_\zeta} = 0.2,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0.15$	$-0/3104$	$-0/2646$	$0/1107$	$0/1667$	$0/2987$
Ranking result	5	4	3	2	1
$Case\ \ 4: \omega^{l_\zeta} = \omega^{r_\zeta} = 0.3,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0.1$	$-0/5421$	$-0/3517$	$-0/0145$	$0/7065$	$0/2084$
Ranking result	5	4	3	1	2
$Case\ \ 5: \omega^{l_\zeta} = \omega^{r_\zeta} = 0.4,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0.05$	$-0/7646$	$-0/4169$	$-0/1599$	$1/2569$	$0/1030$
Ranking result	5	4	3	1	2
$Case\ \ 6: \omega^{l_\zeta} = \omega^{r_\zeta} = 0.5,$ $\omega^{l_\eta} = \omega^{r_\eta} = \omega^{l_\theta} = \omega^{r_\theta} = 0$	$-1/0968$	$-0/3169$	$-0/3942$	$2/0162$	$-0/1670$
Ranking result	5	3	4	1	2

In addition, Figure 6 depicts the changes in the rank of the options concerning the variant in different modes (weights). In the suggested modes, the lowest fluctuation in the ranks has been observed for o_2 and o_5.

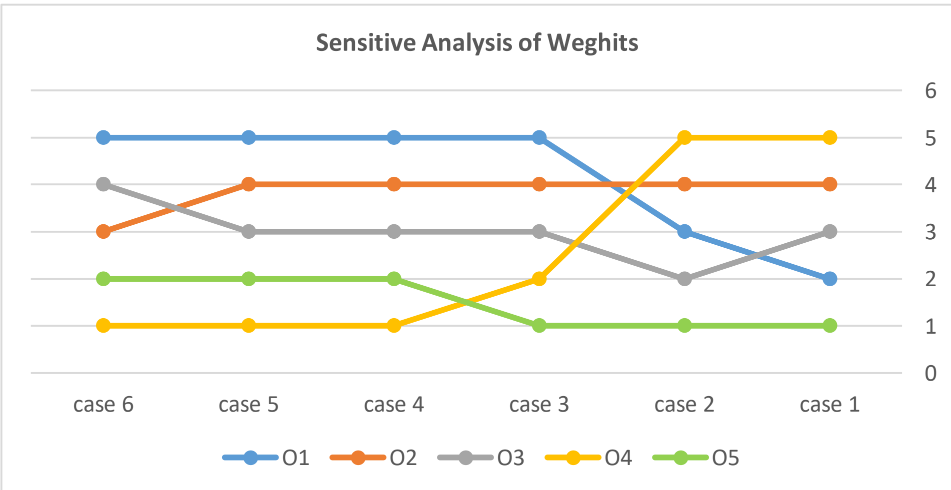

Figure 6. Ranking results based on different cases of weights.

5 -3 Comparative discussions

The performance of the proposed NTraFNs-CODAS Method is compared with some of the existing approaches (Biswas et al. [69], Pramanik et al. [70], Jana and Karaasslan [68], Suresh [71]) in this section. Researchers usually have investigated similarity measures and ranking methods to evaluate the alternatives in multi-criteria decision-making with NTraFNs. For example, Biswas et al. [69] extended the concepts of the Cosine similarity measure and weighted Cosine similarity measure according to an expected interval (EI) and expected value (EV) definitions with NtraFNs. Also, they find the desirable candidate for the MCDM problem based on this similarity measure. Later, Pramanik et al. [70] developed the TOPSIS method for MADM, where the weight information of attributes is incompletely known or completely unknown, under trapezoidal neutrosophic information for the first time. In another research, Jana and Karaasslan [68] introduced Dice and Jaccard similarity measures and weighted Dice and Jaccard similarity measures between NTraFNs for solving the MCDM method. Recently, Suresh [71] proposed a ranking strategy for MCDM under neutrosophic trapezoidal fuzzy numbers according to the Euclidean Distance measure and the centroid concept.

Differing from these studies, the proposed NTraFNs-CODAS Method is established based on two novel distance measures for the material selection problems. The results of applying these methods are summarized to the Table 13 and Figure 7.

Table 13. Ranking alternatives based on different methods.

Approaches		alternatives				
		o_1	o_2	o_3	o_4	o_5
Biswas et al.[69]	SC	$0/846$	$0/852$	$0/823$	$0/884$	$0/844$
	Ranking result	3	2	5	1	4
	SWC	$0/837$	$0/863$	$0/828$	$0/896$	$0/857$
	Ranking result	4	2	5	1	3
Pramanik et al. [70]	RCW	$0/349$	$0/433$	$0/407$	$0/784$	$0/417$
	Ranking result	5	2	4	1	3
Jana and Karaasslan[68]	SWD	$0/876$	$0/904$	$0/881$	$0/927$	$0/873$
	Ranking result	3	2	4	1	5
	SWJ	$0/728$	$0/762$	$0/719$	$0/803$	$0/722$
	Ranking result	3	2	5	1	4
Suresh [71]	R	$0/228$	$0/199$	$0/258$	$0/191$	$0/230$
	Ranking result	3	2	5	1	4
Proposed method	NTraFNs-CODAS	$-0/542$	$-0/352$	$-0/015$	$0/707$	$0/208$
	Ranking result	5	4	3	1	2

As can be seen, although the methods do not have the same performance in ranking all the options, they all choose option 2 as the best option. According to Figure 7, it can be said that the proposed method has the most similarity in the ranking results with the method presented by Pramanik et al. [70].

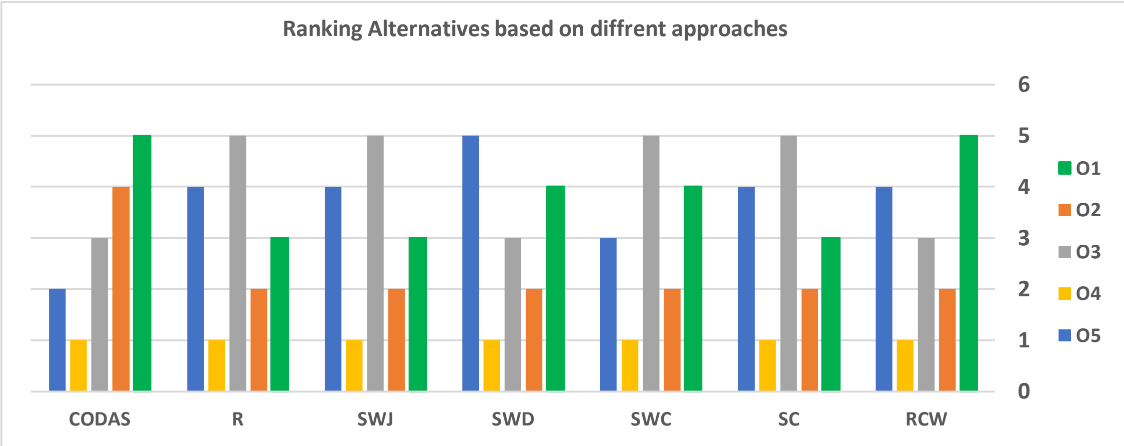

Figure 7. Ranking alternatives based on different methods.

6. Conclusions

The distance measures that can investigate the discriminationn between two neutrosophic trapezoidal fuzzy numbers do not gain indispensable comprehensiveness in the literature from the various perspectives. There is much less research in this field compared to the discrete neutersophic numbers. However, due to the greater flexibility in continuous neutrosophic numbers, examining issues under this type of numbers can be more preferred by decision-makers. Therefore, in this research, two surface-based distances were presented. The proposed weighted distance measures not only establish the basic principles of the measure but also apply to some logical properties of the measure, as shown in Example 1. Therefore, it can be properly used in distance-based decision-making algorithms. In the following, the CODAS algorithm was considered under neutrosophic trapezoidal fuzzy data for the first time in this manuscript. The effectiveness of the proposed NTraFNs-CODAS algorithm was shown for solving MCDM. According to Table 10, the ranking of the options is as follows: $o_4 \succ o_5 \succ o_3 \succ o_2 \succ o_1$. The sensitivity analysis of the threshold parameter (φ) showed that the ranking of alternatives remains constant until the value of φ is selected from the $[0.01,6.65]$. However, it cannot be expected that the ranking of the options will remain constant with the changes in the weighting coefficients (cases 1 to 6). Table 12 and the Figure 4 interpret the results of the impact of weight changes in the ranking of options. In addition, a comparative analysis of the NTraFNs-CODAS method with some existing methods demonstrates that the performance of our method is most similar to Pramanik et al. [70]. As suggestions for future research, the following can be considered:

1-Explore more features and properties for surface-based distance measures.

2- The conceptual structure of the method should be developed to other fuzzy extensions from a theoretical and practical point of view.

3- The suggested distance measures should be used in other decision-making methods based on distance, and its results should be compared with the method.

4- Weighted distance measures based on the area of surfaces can be used in other fields related to optimization, such as clustering, classification, medical diagnosis, and location problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Received: Mar 6, 2024. Accepted: May 30, 2024