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# Neutrosophic Labeling Graph

**M. Gomathi[1] and V. Keerthika[2]**

[1]Department of Science and Humanities, Sri Krishna College of Engineering and Technology, Coimbatore, Tamilnadu, India. gomathimathaiyan@gmail.com, gomathim@skcet.ac.in

[2]Department of Science and Humanities, Sri Krishna College of Engineering and Technology, Coimbatore, Tamilnadu, India. krt.keerthika@gmail.com, keerthikav@skcet.ac.in.

**Abstract:** In this paper, some new connectivity concepts in neutrosophic labeling graphs are portrayed. Definition of neutrosophic strong arc, neutrosophic partial cut node, Neutrosophic Bridge and block are introduced with examples. Also, neutrosophic labeling tree and partial intuitionistic fuzzy labeling tree is explored with interesting properties.

**Keywords:** neutrosophic graphs, neutrosophic labeling graphs, neutrosophic labeling tree, partial neutrosophic labeling tree.

## 1. Introduction

Fuzzy is a concept characterized by three basic criteria namely imprecision, uncertainty, and degrees of truthfulness of values. These criteria has been introduced by Zadeh in 1965 to give the detailed description for linguistic variables, representing size, age and temperature etc., used for system input and output. Once we collect the set of categories of the linguistic variables, it defines a fuzzy set along with the membership function developed for each member in that set. The membership function always takes values in the interval [0, 1] and this range is referred to as the membership grade or degree of membership. Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov (1986). Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function ($\mu_A(x)$) and a non-membership function ($\nu_A(x)$) with their sum being less than or equal to one ($\mu_A(x) + \nu_A(x) \leq 1$). This relaxes the enforced duality $\nu_A(x) = 1- \mu_A(x)$ from fuzzy set theory. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately.

Neutrosophic set is simply an extension of intuitionistic fuzzy set and fuzzy set. This concept came into existence when Floretic Smarandache, the professor of mathematics from university of New Mexico, proposed a paper in 1998 [26, 27]. He characterized the Neutrosophic set by using 3 values namely a truth-membership degree, an indeterminacy-membership degree and a falsity membership degree, whose sum lies between 0 and 3. This concept has been successfully applied to many fields such as medical diagnosis problem, decision making problem, etc. The graphical representation of fuzzy set was developed by Rosenfeld in1973. This induces several graphical concepts based of fuzzy-graph logics. Ansari in 2013 extended the fuzzy logic to neutrosophic logic and also developed neutrosophication of fuzzy models. In 2016, Rajab Ali Borzooei defined some basic concepts in fuzzy labeling graph and in 2017, Akram and shahzadi introduced the neutrosophic graph. Recently many applications of neutrosophic sets were developed by Abdel Basset [1-6] and Broumi [14-22].

In this paper, we extend the fuzzy- graph logics by introducing the neutrosophic labeling graphs which has a scope in the entire real world field which involves decision making problems. The new criteria that define neutrosophic labeling tree were introduced.

## 2. Preliminaries

**Definition 2.1:** A neutrosophic graph is of the form $G* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$

and $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \cdots, v_n\}$ such that $T_1: V \rightarrow [0, 1]$, $I_1: V \rightarrow [0, 1]$ and $F_1 : V \rightarrow [0, 1]$ denote the degree of truth-membership function, indeterminacy-membership function and falsity-membership function of the vertex $v_i \in V$ respectively, and $0 \leq T_1 (v) + I_1 (v) + F_1 (v) \leq 3$ $\forall$ $v_i \in V$ (i=1, 2, 3….n).

(ii) $T_2 : V \times V \rightarrow [0, 1]$, $I_2 : V \times V \rightarrow [0, 1]$ and $F_2 : V \times V \rightarrow [0, 1]$, where $T_2(v_i, v_j)$ , $I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function, indeterminacy membership function and falsity-membership function of the edge $(v_i, v_j)$ respectively such that for every $(v_i, v_j)$,

$T_2 (v_i, v_j) \leq \min \{T_1(v_i), T_1(v_j)\}$,

$I_2 (v_i, v_j) \leq \min \{I_1(v_i), I_1(v_j)\}$,

$F_2 (v_i, v_j) \leq \max \{F_1 (v_i), F_1(v_j)\}$, and $0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$ .

**Example 2.2:** Let $G^* = (V, \sigma, \mu)$ be an neutrosophic graph, where $\sigma = (T_1(v), I_1(v), F_1(v))$,

$\mu = (T_2(v_i, v_j) , I_2(v_i, v_j), F_2(v_i, v_j))$. Let the vertex set be $V= \{v_1, v_2, v_3, v_4, v_5\}$ and $\sigma$ $(v_1) = (0.5,0.3,0.4)$, $\sigma$ $(v_2) = (0.2,0.2,0.6)$, $\sigma$ $(v_3) = (0.6,0.45,0.3)$, $\sigma$ $(v_4) = (0.4,0.8,0.35)$, $\sigma$ $(v_5) = (0.4,0.6,0.5)$, $\mu$ $( v_1, v_2) = (0.1,0.2,0.5)$, $\mu$ $(v_2, v_3) = (0.15,0.1,0.5)$, $\mu$ $(v_3, v_4) = (0.3,0.35,0.3)$, $\mu$ $(v_4, v_5) = (0.35,0.5,0.45)$ $\mu$ $(v_5, v_1) = (0.4,0.2,0.4)$, $\mu$ $(v_5, v_2) = (0.15,0.15,0.4)$, $\mu$ $(v_1, v_4) = (0.3,0.25,0.3)$, $\mu$ $( v_4, v_2) = (0.05,0.1,0.4)$.

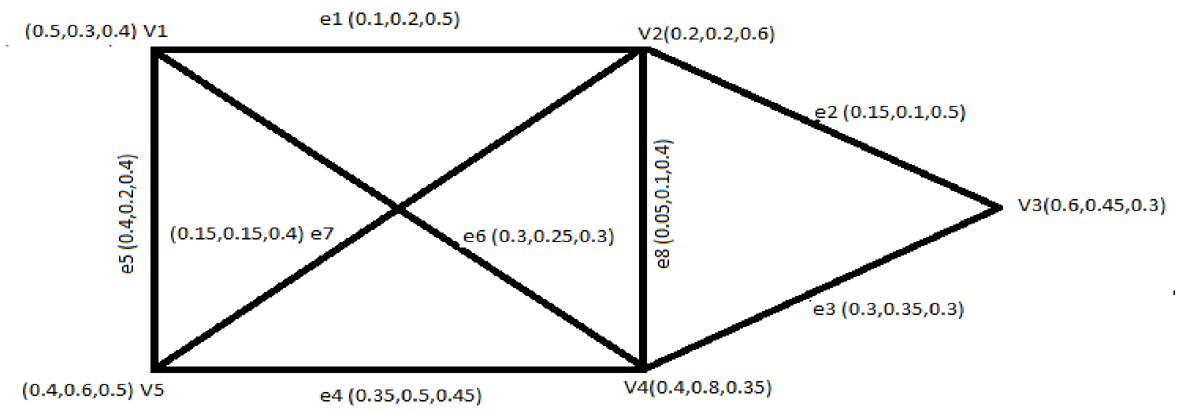


Fig 1: **NEUTROSOPHIC GRAPH**

## 3. Neutrosophic labeling graph

In this section we introduce neutrosophic labeling graph, neutrosophic labeling subgraph, connectedness in neutrosophic labeling graph, neutrosophic partial cut node and neutrosophic partial bridge and investigated some of the properties with suitable examples.

**Definition 3.1:** A neutrosophic graph $G^* = (V, \sigma, \mu)$ is said to be an neutrosophic labeling graph if $T_1 : V \to [0, 1]$, $I_1 : V \to [0, 1]$ $F_1 : V \to [0, 1]$ and $T_2 : V \times V \to [0, 1]$, $I_2 : V \times V \to [0, 1]$, $F_2 : V \times V \to [0, 1]$ is bijective such that truth-membership function, indeterminacy-membership function and falsity-membership of the vertices and edges are distinct and for every edges $(v_i, v_j)$,

$T_2(v_i, v_j) \le \min\{T_1(v_i), T_1(v_j)\}$,

$I_2(v_i, v_j) \le \min\{I_1(v_i), I_1(v_j)\}$,

$F_2(v_i, v_j) \le \max\{F_1(v_i), F_1(v_j)\}$, and

$0 \le T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \le 3$

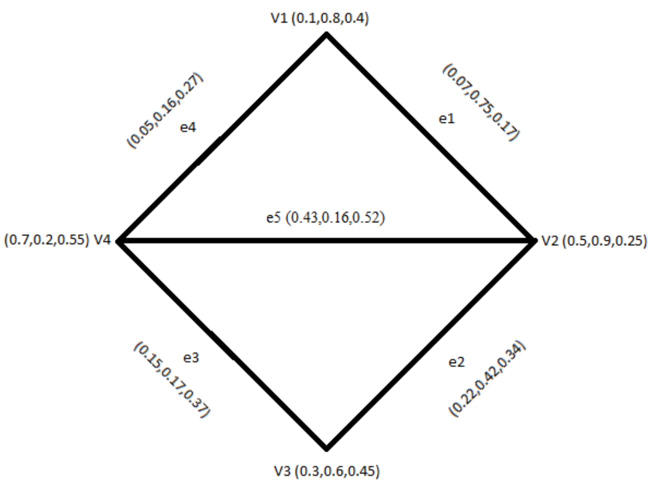


Fig.2 NEUTROSOPHIC LABELING GRAPH

**Example 3.2:** In the above figure 2, all the vertices and edges have distinct values for membership, indeterminacy and falsity. Therefore $\sigma$ , I and $\mu$ are one to one and onto functions.

**Definition 3.3:** Neutrosophic labeling graph R= (V, $\alpha$, β) where $\alpha = (\alpha_1(c), \alpha_2(c), \alpha_3(c))$ and $\beta = (\beta_1(c,d), \beta_2(c,d), \beta_3(c,d))$ is called an neutrosophic labeling subgraph of $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1(c), I_1(c), F_1(c))$ and $\mu = (T_2(c,d), I_2(c,d), F_2(c,d))$, if $\alpha_1(c) \le T_1(c)$, $\alpha_2(c) \le I_1(c)$, $\alpha_3(c) \ge F_1(c)$ for all $c \in V$ and $\beta_1(c,d) \le T_2(c,d)$, $\beta_2(c,d) \le I_2(c,d)$, $\beta_3(c,d) \le F_2(c,d)$ for all edges (c,d).

**Theorem 3.4:** If R=(V, $\alpha$, β) is an neutrosophic labeling subgraph of $G* = (V, \sigma, \mu)$, then

$\beta_1^\infty(c,d) \le T_2^\infty(c,d)$, $\beta_2^\infty(c,d) \le I_2^\infty(c,d)$, $\beta_3^\infty(c,d) \ge F_2^\infty(c,d)$, for all $c,d \in V$.

**Proof:** Let $G* = (V, \sigma, \mu)$ be any neutrosophic labeling graph and R = (v, $\alpha$, β) be its subgraph. Let (c,d) be any path in G* then its strength be $((T_2^\infty(c,d), I_2^\infty(c,d), F_2^\infty(c,d))$. Since R in a subgraph of G* ,then $\alpha_1(c) \le T_1(c)$, $\beta_1(c,d) \le T_2(c,d)$, $\alpha_2(c) \le I_1(c)$, $\beta_2(c,d) \le I_2(c,d)$, $\alpha_3(c) \ge F_1(c)$ and $\beta_3(c,d) \ge F_2(c,d)$, which implies that $\beta_1^\infty(c,d) \le T_2^\infty(c,d)$, $\beta_2^\infty(c,d) \le I_2^\infty(c,d)$, $\beta_3^\infty(c,d) \ge F_2^\infty(c,d)$, for all $c,d \in V$.

**Theorem 3.5:** The union of any two neutrosophic labeling graph $G^{*}=(\mathrm{V}^{1},\sigma_1,\mu_1)$ and $G^{**}=(\mathrm{V}^{11},\sigma_2,\mu_2)$ where $\sigma_1=(T_1(c),I_1(c),F_1(c))$ , $\mu_1=(T_2(c,d),I_2(c,d),F_2(c,d))$ , $\sigma_2=(T_3(c),I_3(c),F_3(c))$, $\mu_2=(T_4(c,d),I_4(c,d),F_4(c,d))$, is also an neutrosophic labeling graph, if the Truth membership, Indeterminacy, Falsity membership values of the edges between $G^{*}$ and $G^{**}$ are distinct.

**Proof:** Let $G^{*}=(\mathrm{V}^{1},\sigma_1,\mu_1)$ and $G^{**}=(\mathrm{V}^{11},\sigma_2,\mu_2)$ be any two neutrosophic labeling graph such that, the Truth membership, Indeterminacy, Falsity membership values of the edges between $G^{*}$ and $G^{**}$ are distinct and $G=(\mathrm{V},\sigma,\mu)$ , where $\sigma=(\sigma_M,\sigma_I,\sigma_N)$ and $\mu=(\mu_M,\mu_I,\mu_N)$, be the union of two neutrosophic labeling graph $G^{*}$ and $G^{**}$.

**To prove:** G is a Neutrosophic labeling graph.

Now,

For Truth membership values $\sigma_M(c)=\begin{cases} T_1(c), & if\ \mathrm{c}\in \mathrm{V}^1-\mathrm{V}^{11} \\ T_3(c), & if\ \mathrm{c}\in \mathrm{V}^{11}\text{ - }\mathrm{V}^1 \\ T_1(c)\vee T_3(c), & if\ \mathrm{c}\in \mathrm{V}^1\bigcap \mathrm{V}^{11} \end{cases}$

For Indeterminacy values $\sigma_I(c)=\begin{cases} I_1(c), & if\ \mathrm{c}\in \mathrm{V}^1-\mathrm{V}^{11} \\ I_3(c), & if\ \mathrm{c}\in \mathrm{V}^{11}\text{ - }\mathrm{V}^1 \\ I_1(c)\vee I_3(c), & if\ \mathrm{c}\in \mathrm{V}^1\bigcap \mathrm{V}^{11} \end{cases}$

For Falsity membership values $\sigma_F(u)=\begin{cases} F_1(u), & if\ \mathrm{u}\in \mathrm{V}^1-\mathrm{V}^{11} \\ F_3(u), & if\ \mathrm{u}\in \mathrm{V}^{11}\text{ - }\mathrm{V}^1 \\ F_1(u)\wedge F_3(u), & if\ \mathrm{u}\in \mathrm{V}^1\bigcap \mathrm{V}^{11} \end{cases}$

Similarly,

For Truth membership values $\mu_M(c,d)=\begin{cases} T_2(c,d), & if\ (c,d)\in \mathrm{E}^1-\mathrm{E}^{11} \\ T_4(c,d), & if\ (c,d)\in \mathrm{E}^{11}\text{ - }\mathrm{E}^1 \\ T_2(c,d)\vee T_4(c,d), & if\ (c,d)\in \mathrm{E}^1\bigcap \mathrm{E}^{11} \end{cases}$

For Indeterminacy values $\mu_I(c,d)=\begin{cases} I_2(c,d), & if\ (c,d)\in \mathrm{E}^1-\mathrm{E}^{11} \\ I_4(c,d), & if\ (c,d)\in \mathrm{E}^{11}\text{ - }\mathrm{E}^1 \\ I_2(c,d)\vee I_4(c,d), & if\ (c,d)\in \mathrm{E}^1\bigcap \mathrm{E}^{11} \end{cases}$

For Falsity membership values $\mu_F(c,d)=\begin{cases} F_2(c,d), & if\ (c,d)\in \mathrm{E}^1-\mathrm{E}^{11} \\ F_4(c,d), & if\ (c,d)\in \mathrm{E}^{11}\text{ - }\mathrm{E}^1 \\ F_2(c,d)\wedge F_4(c,d), & if\ (c,d)\in \mathrm{E}^1\bigcap \mathrm{E}^{11} \end{cases}$

Thus the Truth membership, Indeterminacy and Falsity membership values of the vertices and edges are distinct. Hence, $G=(\mathrm{V},\sigma,\mu)$ is a Neutrosophic labeling graph.

**Definition 3.6:** Let $G^{*}=(V,\sigma,\mu)$ be an neutrosophic labeling graph. The strength of the path P of n edges $e_i$ for i = 1,2,……,n is denoted by S(P) = ($S_1$(P), $S_2$(P), $S_3$(P)) and denoted by $S_1(P)=\min_{1\le i\le n} T_2(e_i)$, $S_2(R)=\min_{1\le i\le n} I_2(e_i)$ and $S_3(R)=\max_{1\le i\le n} F_2(e_i)$.

**Definition 3.7:** Let G = (V, $\sigma$, $\mu$) be a neutrosophic labeling graph. Then for a pair of vertices c,d ∈ V, the strength of connectedness, denoted by $CONN_G(c,d) = (CONN_{1G}(c,d), CONN_{2G}(c,d), CONN_{3G}(c,d))$ and is defined as

$CONN_{1G}(c,d) = \max\{S_1(P)\}$, $CONN_{2G}(c,d) = \max\{S_1(P)\}$ and $CONN_{3G}(c,d) = \min\{S_2(P)\}$, where P is a path connecting the vertices c,d in G. If c and d are isolated vertices of G, then $CONN_G(c,d)$ = (0, 0).

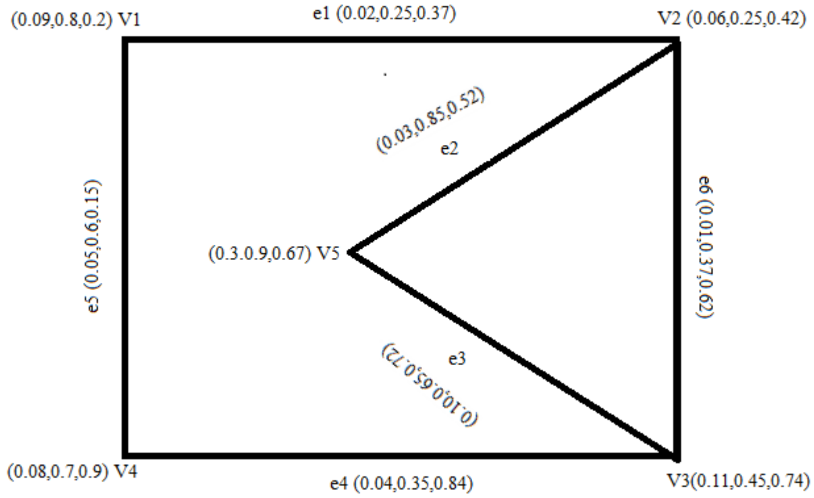


Fig. 3 **CONNECTEDNESS IN NEUTROSOPHIC LABELING GRAPH**

**Example 3.8:** Figure 3 is an example of neutrosophic labeling graph G having $CONN_G(v_1, v2)$ = (0.02, 0.75, 0.37), $CONN_G(v_1, v_3)$ = (0.04, 0.6, 0.62), $CONN_G(v_1, v_5)$ = (0.04, 0.65, 0.52) and so on.

**Proposition 3.9:** Let G be an neutrosophic labeling graph and R is an neutrosophic labeling subgraph of G. Then for every pair of vertices c,d ∈ V, we have $CONN_{1R}(c,d) \leq CONN_{1G}(c,d)$, $CONN_{2R}(c,d) \leq CONN_{2G}(c,d)$ and $CONN_{3R}(c,d) \geq CONN_{3G}(c,d)$.

**Definition 3.10:** If $S_1(P) = CONN_{1G}(c,d)$ $S_2(P) = CONN_{2G}(c,d)$ and $S_3(P) = CONN_{3G}(c,d)$, where P is a path connecting the vertices c,d in the neutrosophic labeling graph G then P is called the strongest path connecting c, d in G.

**Definition 3.11:** Let G be an neutrosophic labeling graph. A node z is called a neutrosophic partial cut node ( Neu p-cut node) of G if there exists a pair of nodes c,d ∈ G such that c $\neq$ d $\neq$ z and $CONN_{1(G-z)}(c,d) < CONN_{1G}(c,d)$, $CONN_{2(G-z)}(c,d) < CONN_{2G}(c,d)$ and $CONN_{3(G-z)}(c,d) > CONN_{3G}(c,d)$

A neutrosophic partial block (Neu p-block) is a neutrosophic labeling graph which is connected and does not contain any Neu p-cut nodes in it.

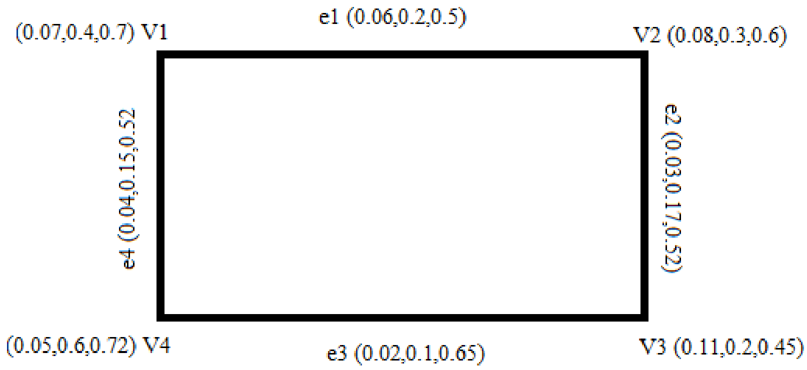


Fig.4 **NEUTROSOPHIC LABELING GRAPH**

**Example 3.12** : Let G be an neutrosophic labeling graph, which is shown in above Figure 4.
Node $v_1$ is a neutrosophic partial cut node, since
$CONN_{1(G-v_1)}(v_2, v_4) = 0.02 < 0.04 = CONN_{1G}(v_2, v_4)$,
$CONN_{2(G-v_1)}(v_2, v_4) = 0.1 < 0.15 = CONN_{2G}(v_2, v_4)$ and
$CONN_{3(G-v_1)}(v_2, v_4) = 0.65 > 0.55 = CONN_{3G}(v_2, v_4)$.
Similarly, Node $v_2$ is a neutrosophic partial cut node, since,
$CONN_{1(G-v_2)}(v_1, v_3) = 0.02 < 0.03 = CONN_{1G}(v_1, v_3)$,
$CONN_{2(G-v_2)}(v_1, v_3) = 0.1 < 0.17 = CONN_{2G}(v_1, v_3)$ and
$CONN_{1(G-v_2)}(v_1, v_3) = 0.65 > 0.52 = CONN_{3G}(v_1, v_3)$.

**Definition 3.13:** Let G be an neutrosophic labeling graph. An arc e = (c,d) is called neutrosophic partial bridge (Neu p- bridge) if $CONN_{1(G-e)}(c,d) < CONN_{1G(c,d)}$, $CONN_{1(G-e)}(c,d) < CONN_{1G(c,d)}$ and $CONN_{3(G-e)}(c,d) > CONN_{3G(c,d)}$.
A neutrosophic p-bridge is said to be a neutrosophic partial bond (Neu p-bond) if
$CONN_{1(G-e)}(x, y) < CONN_{1G(x, y)}$, $CONN_{2(G-e)}(x, y) < CONN_{2G(x, y)}$, $CONN_{3(G-e)}(x, y) > CONN_{3G(x, y)}$ with at least one of x or y different from both u and v and is said to be a neutrosophic partial cut bond (p-cut bond) if both x or y are different from u and v.

**Example 3.14 :** In the Figure 4, for all arcs except the arc ($v_4$, $v_3$) are neutrosophic partial bridge. In specific particular, arc ($v_2$, $v_3$) is a neutrosophic partial cut bond, since
$CONN_{1(G-(v2,v3))}(v_3, v_4) = 0.03 < 0.06 = CONN_{1G(v3, v4)}$ , $CONN_{2(G-(v2,v3))}(v_3, v_4) = 0.03 < 0.06 = CONN_{2G(v3, v4)}$ and $CONN_{3(G-(v2,v3))}(v_3, v_4) = 0.55 > 0.5 = CONN_{3G(v3, v4)}$.

## 4. Types of Arcs in a Neutrosophic Labeling Graph

In this section we discussed some types of neutrosophic $\alpha$ strong, δ strong, β strong arcs.

**Definition 4.1**: If all the arcs of cycle C in the neutorsophic labeling graph G are strong, then C is called the strong cycle in G.

**Definition 4.2:** An arc (n,m) of G is called neutrosophic $\alpha$ strong if $T_2(c,d) > CONN_{1(G-(n,m))}(n,m)$**,** $I_2(c,d) > CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) < CONN_{3(G-(n,m))}(n,m)$

**Definition 4.3:** An arc (n,m) of G is called neutrosophic δ strong if $T_2(c,d) < CONN_{1(G-(n,m))}(n,m)$, $I_2(c,d) < CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) > CONN_{3(G-(n,m))}(n,m)$

**Definition 4.4:** An arc (n,m) of G is called neutrosophic β strong if $T_2(c,d) = CONN_{1(G-(n,m))}(n,m)$, $I_2(c,d) = CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) = CONN_{3(G-(n,m))}(n,m)$

**Definition 4.5:** An n-m path P in G is called a strong n-m path if all the arcs of P are strong. In particular, if all the arcs of P are neutrosophic $\alpha$-strong, then P is called neutrosophic $\alpha$ strong path. Obviously, An arc (n,m) is strong if it is neutrosophic $\alpha$-strong, if (n,m) is strong arc, then n and m are said to be strong neighbors of each other.

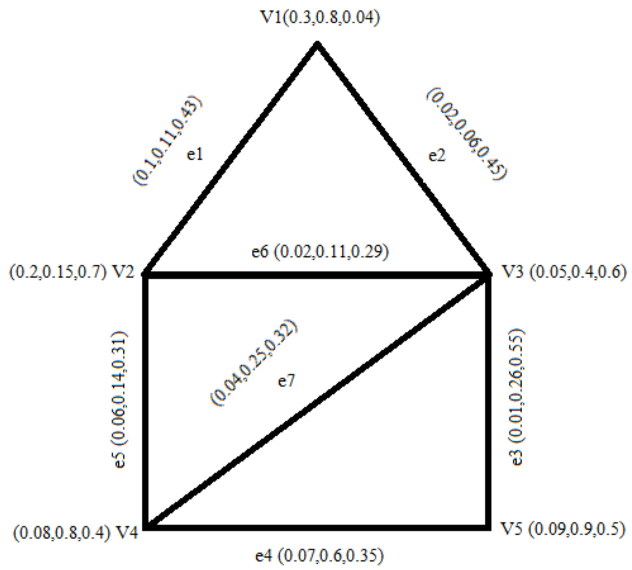


Fig.5 **NEUTROSOPHIC LABELING GRAPH**

**Example 4.6:** In the above figure 5, the arcs $(V_1, V_2)$, $(V_2, V_4)$, $(V_4, V_5)$ are neutrosophic $\alpha$ strong, the arc $(V_3, V_4)$ is neutrosophic δ strong, the arcs $(V_1, V_3)$ is neutrosophic β strong and $P = V_1V_2V_4V_5$ is a neutrosophic $\alpha$ strong path.

**Theorem 4.7.** Let G be a connected neutrosophic labeling graph and let r and s be any two nodes in G. Then there exists a strong path from c to do.

**Proof.**

Assume that $G = (V, \sigma, \mu)$ is a connected neutrosophic labeling graph. Let r and s be any two nodes of G. If the arc (r, s) is strong, then there is nothing to prove. Otherwise, either (r, s) is a δ arc or there exist a path of length more than one from r to s.

In the first case, we can find a path P (say) such that $S_1(P) > T_2(r,s)$, $S_2(P) > I_2(r,s)$ and $S_3(P) < F_2(r,s)$ In either case, the path from c to d of length more than one. If some arc on this path is not strong, replace it by a path having more strength. Hence P is a path from r to s, whose arcs are strong and thus P is a strong path from r to s.

**Theorem 4.8:** A connected neutrosophic labeling graph G is a neutrosophic partial block if and only if any two nodes x, y ∈ V such that (x y) is not neutrosophic $\alpha$ strong are joined by two internally disjoint strongest path.

**Proof:**

Suppose that G is a neutrosophic partial block. Let x, y ∈ V such that (x, y) is not neutrosophic $\alpha$ strong arc. Now, we shall prove that there exist two internally disjoint strongest x–y paths. If not, i.e

there exist exactly one internally disjoint strongest x-y path in G. Since (x, y) is not $\alpha$ strong, length of all strongest x - y path must be at least two. Also for all strongest x - y paths in G, there must be a common vertex. Let z be such node in G. Then $CONN_{1\,(G-z)}$ (x, y) > $CONN_{1G(x,\,y)}$, $CONN_{2\,(G-z)}$(x, y) > $CONN_{2G(x,\,y)}$ and $CONN_{3(G-z)}$(x, y) < $CONN_{3G(x,\,y)}$, which contradict the fact that G has no P-cut nodes. Hence there exist two internally disjoint strongest x - y paths.
Conversely, let any two nodes of G are joined by two internally disjoint strongest paths. Let w be a node in G. For any pair of nodes c,d $\in$ V such that u $\neq$ v $\neq$ w, there always exists a strongest path not containing w. So, we cannot be a neutrosophic p-cut node. Hence G is a neutrosophic partial block.

## 5. Neutrosophic Labeling Tree

In this section we define neutrosophic labeling tree as follows

**Definition 5.1:** A graph G* = (V, $\sigma, \mu$) where $\sigma$ (v)= ($T_1$(r), $I_1$(r), $F_1$(r)) and $\mu$ = ($T_2$(r,s) , $I_2$(r,s), $F_2$(r,s)) is said to be neutrosophic labeling tree, if it has neutrosophic labeling graph and an neutrosophic spanning subgraph M= (V, $\alpha$, β) where $\alpha$(r)= ($\alpha_1$(r), $\alpha_2$(r), $\alpha_3$(r)) and β= ($\beta_1$(r,s), $\beta_2$(r,s), $\beta_3$(r,s)) which is a tree, where for all arcs (r, s) not in $T_2$(r,s) < $\beta_1^\infty$ (r,s), $I_2$(r,s) < $\beta_2^\infty$ (r,s), $F_2$(r,s) > $\beta_3^\infty$ (r,s).

**Theorem 5.2:** If G* = (V, $\sigma, \mu$) is a neutrosophic labeling tree, then the arcs of neutrosophic spanning subgraph M= (V, $\alpha$, β) are neutrosophic bridges of G*.

**Proof:** Let G* = (V, $\sigma, \mu$) be a neutrosophic labeling tree and M= (V, $\alpha$, β) be its spanning subgraph. Let (r, s) be an arc in M. Then $\beta_1^\infty$ (r,s) < $T_2$(r,s) ≤ $T_2^\infty$ (c,d), $\beta_2^\infty$ (r,s) < $I_2$(r,s) ≤ $I_2^\infty$ (r,s), $\beta_3^\infty$ (r,s) > $F_2$(r,s) ≥ $F_2^\infty$ (r,s), which implies that the arc (r, s) is an neutrosophic bridge of G*. Since the arc (r, s) is an arbitrary, then the arcs of M are the neutrosophic bridges of G*.

**Theorem 5.3:** Every neutrosophic labeling graph is a neutrosophic labeling tree.
**Proof:** Let G* = (V, $\sigma, \mu$) be a neutrosophic labeling graph. Since is $\mu$ is bijective, each and every vertex of G* will have at least one arc as neutrosophic bridge. Therefore, the spanning subgraph M will exist, such that whose arcs are neutrosophic bridges. Hence, by above theorem 5.2, every neutrosophic labeling graph is an neutrosophic labeling tree.

## 6. Partial Neutrosophic Labeling Tree

Finally, we define partial neutrosophic labeling tree and discussed some of the properties.
**Definition 6.1:** A connected neutrosophiclabeling graph G* = (V, $\sigma, \mu$) is called a partial neutrosophic labeling tree if G* has a spanning subgraph M= (V, $\alpha$, β) which is a tree, where for all arc (r, s) of G* which are not in M, $CONN_{1G}$(r,s) > $T_2$(r,s), $CONN_{2G}$(r,s) > $I_2$(r,s) and $CONN_{3G}$(r,s) < $F_2$(r,s).
If all the components of disconnected graph G* satisfies above condition, then G* is called a partial forest.

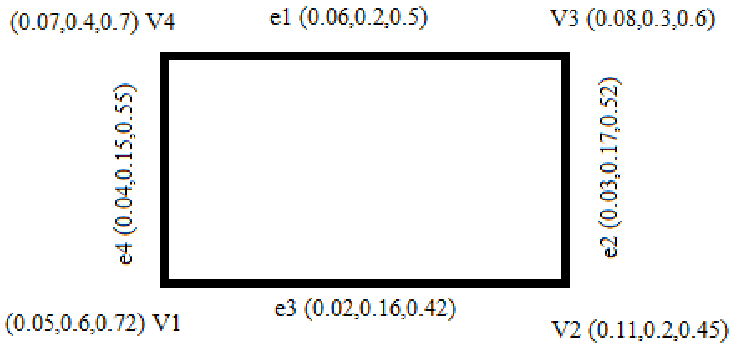


Fig.6 **PARTIAL NEUTROSOPHIC LABELING TREE**

**Example 6.2:** If we remove the arc ($v_1$, $v_2$) figure 6, we will get a spanning tree M. Also for the arc ($v_1$, $v_2$), $CONN_{1G}$ ($v_1$, $v_2$) = 0.03 > 0.02 = $T_1$ ($v_1$, $v_2$), CONN2G ($v_1$, $v_2$) = 0.16 > 0.15 = $I_1$ ($v_1$, $v_2$), and $CONN_{3G}$ ($v_1$, $v_2$) = 0.42 < 0.55 = $F_1$ ($v_1$, $v_2$). Thus figure 6 is an example of partial neutrosophic labeling tree.

**Theorem 6.3:** Let $G^*$ = (V, $\sigma, \mu$) be a connected neutrosophic labeling graph. Then the necessary and sufficient condition for $G^*$ to be a neutrosophic partial tree is that , for any cycle C in $G^*$, there must exists an arc $\gamma$ = (r, s) such that $T_2(\gamma) < CONN_{1(G^*-\gamma)}(r, s)$, $I_2(\gamma) < CONN_{2(G^*-\gamma)}(r, s)$ and $F_2(\gamma) > CONN_{3(G^*-\gamma)}(r, s)$, where $G^*- \gamma$ is the subgraph of $G^*$ obtained by deleting the arc $\gamma$ from $G^*$.

**Proof:** Assume that $G^*$ is a connected neutrosophic labeling graph. If $G^*$ has no cycle, then $G^*$ itself behave as a partial tree.

If $G^*$ has a cycle C and let $\gamma$ = (r,s) be an arc of C with minimum weightage for truth membership, indeterminacy and maximum weightage for falsity membership in $G^*$. Now, remove the arc $\gamma$ = (r,s) from $G^*$ and continue this process until we get a tree M which is the subgraph of $G^*$.

The arcs deleted in each process were stronger than the one which removed preceding process. Since M is a tree and the arc $\gamma$ = (r, s) having minimum membership value, minimum indeterminacy and maximum falsity membership value from the arcs of a cycle in $G^*$ does not belongs to M, we can conclude that there exists a path from r to s whose membership value greater than $T_2$(r, s), indeterminacy value greater than $I_2$(r, s) and falsity membership value less than $F_2$(r, s), and that does not involve (r, s) or any arcs deleted prior to it. It contains only the arcs of M. Thus $G^*$ is a partial neutrosophic labeling tree.

Conversely, if $G^*$ is a partial neutrosophic labeling tree and P is cycle, then some arc $\gamma$ = (r, s) of P does not belong to M. Thus by definition we have $T_2(\gamma) < CONN_{1(G^*-\gamma)}(r, s)$, $I_2(\gamma) < CONN_{2(G^*-\gamma)}(r, s)$ and $F_2(\gamma) > CONN_{3(G^*-\gamma)}(r, s)$.

**Theorem 6.4:** Between any two nodes of $G^*$, If there exist at most one strongest path, then $G^*$ must be a partial forest.

**Proof:**

Assume that $G^*$ is not a partial forest. Then $G^*$ must contain a cycle C such that $T_2(r, s) \geq CONN_{1G}(r, s)$, $I_2(r, s) \geq CONN_{2G}(r, s)$ and $F_2(r, s) \leq CONN_{3G}(r, s)$ for all arcs $\gamma$ = (r, s) of the cycle C. Thus, $\gamma$ = (r, s) is the strongest path from r to s. If we choose (r, s) to be a weakest arc of C, it follows that the rest of the cycle C is also a strongest path from r to s, which is a contradiction. Hence, $G^*$ must be a partial forest.

**Theorem 6.5:** If $G^*$ is a not a tree but partial tree, then has $G^*$ at least one arc $\gamma = (r, s)$ for which $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$.

**Proof:**

Assume that $G^*$ is a partial tree, then by definition of partial tree, $G^*$ must contain a spanning tree M such that $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$, for all arcs $\gamma = (r, s)$ not in M. Thus has $G^*$ at least one arc $\gamma = (r, s)$ (since $G^*$ is not a tree), which satisfies the above condition.

**Theorem 6.6:** If M is the spanning tree of the partial tree $G^*$, then the arcs of M are the partial bridges of $G^*$.

**Proof:**

Let $\gamma = (r, s)$ be an arc in M. Since, M is a spanning tree, this arc $\gamma$ form a unique path between the nodes r and s in M.

If $G^*$ has no other paths between r and s, then clearly $\gamma = (r, s)$ is a bridge of $G^*$ and hence it is a partial bridge of $G^*$.

On the other hand, if P is a path connecting r and s in $G^*$, then P must contain an arc $\gamma = (r, s)$ which is not in M such that $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$. Then $\gamma = (r, s)$ is not a weakest arc of any cycle in $G^*$ and hence (r, s) is a partial bridge.

## 7. Conclusion

Connectivity concepts are the major key in neutrosophic graph problems. This paper presented new connectivity concepts in neutrosophic labeling graphs. Definition of neutrosophic strong arc, neutrosophic partial cut node, Neutrosophic Bridge and block based on connectivity concepts of intuitionistic fuzzy graph was introduced. The neutrosophic labeling tree and partial neutrosophic labeling tree concepts were established with interesting properties on them. We extended our research work to bipolar neutrosophic graph, covering problem on neutrosophic graphs, Chromatic number in neutrosophic graphs, Colouring of neutrosophic graphs.

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## Conflicts of Interest

The authors declare no conflict of interest.

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