NSS
University of New Mexico

Neutrosophic Semilattices and Their Properties

M.Reehana Parveen[1] and P.Sekar[2]

[1] Department of Mathematics, Justice Baheer Ahmed Sayeed College, Chennai, India. E-mail: reehana2@gmail.com
[2] Principal, Kandasamy Naidu College, Chennai, India.

Abstract: In this paper authors study neutrosophic semi-lattices and their properties. These neutrosophic semi-lattices are built using either $\cup$ or $\cap$ operation only. This application of these concepts is also discussed in this paper.

Keywords: Neutrosophic semi-lattices, pure neutrosophic semi-lattice.

1 Introduction

In this paper the new notion of neutrosophic semilattices is introduced for the first time. However the study of neutrosophic lattices started in 2004 [5,6]. But those neutrosophic lattices are of a special type as they were mainly defined to cater to the needs of applications in fuzzy models. This paper has three sections. Section one is introductory in nature. In section two different types of neutrosophic semilattices are defined and their properties developed.

Section three gives the probable applications of these concepts in data mining, sorting etc. Finally we give the conclusions based on this work.

2 Neutrosophic semilattices of different types and their properties

In this section neutrosophic semilattices of various types are defined and described. This study is new and innovative and certainly can provide lots of applications to various fields where semilattices are applied.

Example 2.1: Let S(P) = {1, a, aI, I, 0} be the neutrosophic set, {S(P), $\cap$} is a neutrosophic semilattice whose diagram is as follows.

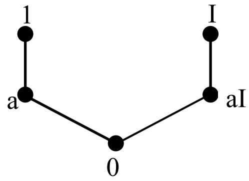


Figure 2.1

Example 2.2. Let {S(P), $\cap$} be the neutrosophic semilattice given by the following example.

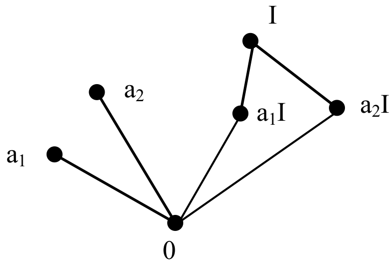


Figure 2.2

Example 2.3. Let {S(P), $\cap$} be the neutrosophic semilattice given by the following example.

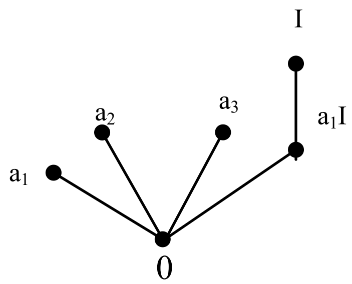


Figure 2.3

In view of this the following neutrosophic semilattice is defined.

Definition 2.1: *Let {S(P)} $\cap$} be the partially ordered neutrosophic set with 0. {S(P), $\cap$} is defined as the neutrosophic semilattice if*

$$\min\{x, y\} = x \cap y \in S(P)$$

for all $x, y \in S(P)$.

The examples given above are neutrosophic semilattices of finite order.

Similarly one can define {S(P), ∪} the neutrosophic semilattice.

Now examples of neutrosophic semilattice {S(P), ∪} are as follows.

Example 2.4: Let {S(P), ∪} be the neutrosophic semilattice which has the following figure.

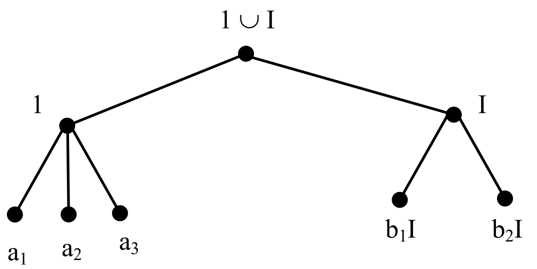


Figure 2.4

Example 2.5: Let {S(P), ∪} be the neutrosophic semilattice which has the following figure.

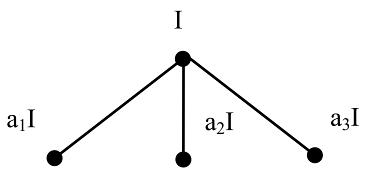


Figure 2.5

Example 2.6. Let {S(P), ∪} be the neutrosophic semilattice given by the following figure.

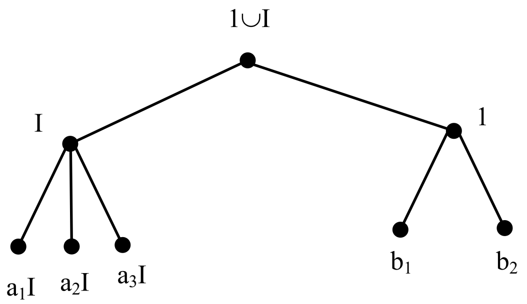


Figure 2.6

The above neutrosophic semilattice is not pure for it contains 1, b_1 and b_2 as elements of S(P).

Thus the notion of neutrosophic pure semilattice is one in which all elements of S(P) are only neutrosophic elements.

Example 2.7. Let {S(P), ∪} be the pure vertex neutrosophic semilattice whose figure is given in the following.

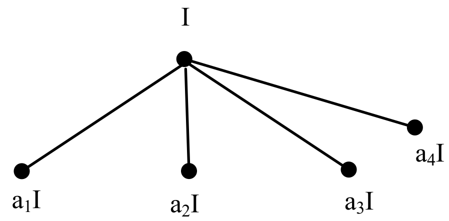


Figure 2.7

Next the notion of edge neutrosophic semilattice under {S, ∪} and {S, ∩} are defined and described in the following.

Definition 2.2: *{S, ∪} or {S, ∩} is defined to be the edge neutrosophic semilattice if all elements in S are real and is a partial ordered set. There are some edges which are neutrosophic are indeterminate.*

Examples of them are given in the following.

Example 2.8. Let {S, ∪} be the edge neutrosophic semilattice which is given by the following figure.

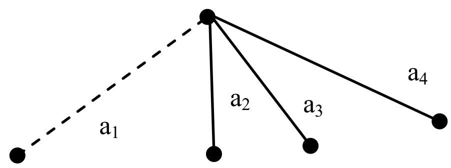


Figure 2.8

Example 2.9. The following figure gives the edge neutrosophic semilattice {S, ∩}.

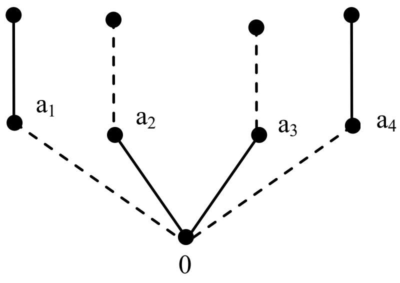


Figure 2.9

Example 2.10. Let {S, ∩} be the edge neutrosophic semilattice given by the following figure.

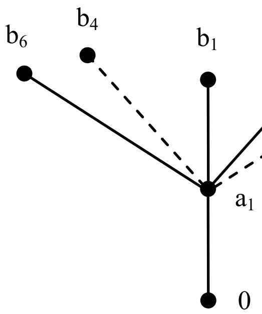


Figure 2.10

Next the notion of pure neutrosophic semilattice is defined in the following.

Definition 2.3: *Let {S, ∪} (or {S, ∩}) be the partial ordered set all of its vertes elements are neutrosophic and if every edge is also neutrosophic or an indeterminacy then {S, ∪} (or {S, ∩}) is defined as the pure neutrosophic semilattice.*

Examples of pure neutrosophic semilattices is given below.

Example 2.11: Let {S(P), ∪} be the pure neutrosophic semilattice the figure of which is as follows;

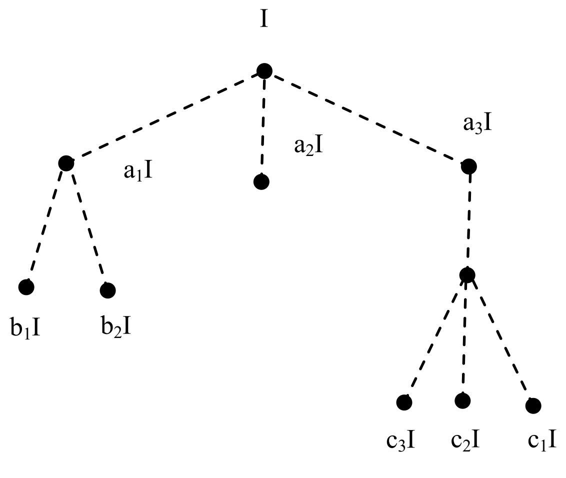


Figure 2.11

The above semilattice is a pure neutrosophic semilattice whose cardinality is 10.

Example 2.12. {S(P), ∩} be the pure neutrosophic semilattice whose Hasse diagram is as follows.

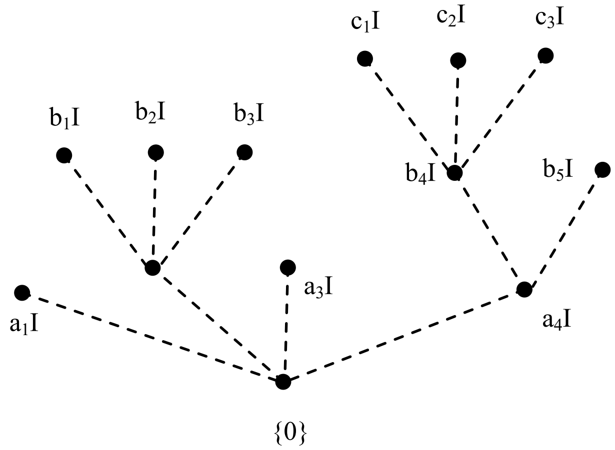


Figure 2.12

This is again a pure neutrosophic semilattice of order 13.

Now having seen examples of pure vertex neutrosophic semilattices, edge neutrosophic semilattices and pure neutrosphic semilattices, examples of neutrosophic subsemilattices are provided.

Example 2.13. Let {S(P), ∪} be the vertex neutrosophic semilattice whose Hasse diagram is as follows.

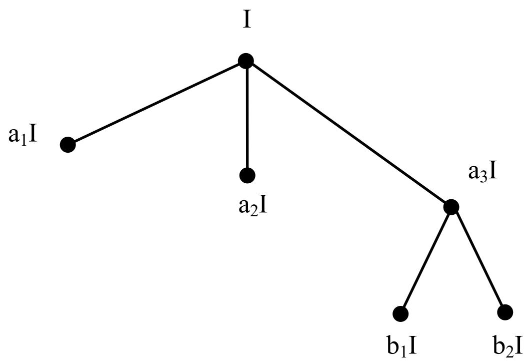
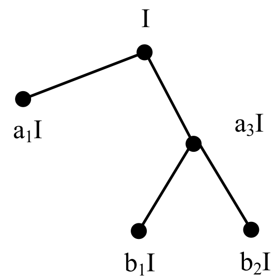
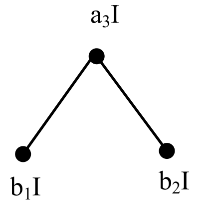


Figure 2.13

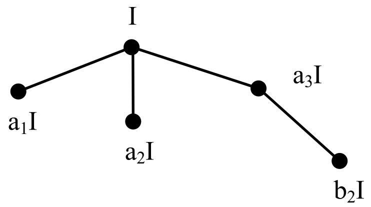
Clearly the following figures gives the vertex neutrosophic subsemilattice.



(a)



(b)



(c)

Figure 2.14 (a), (b) (c)

Next examples of edge neutrosophic semilattice and their subsemilattices are obtained.

Example 2.14. Let {S(P), ∩} be the edge neutrosophic semilattice whose figure is given below.

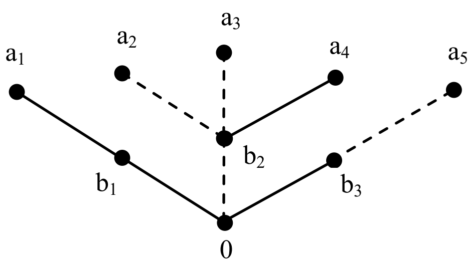
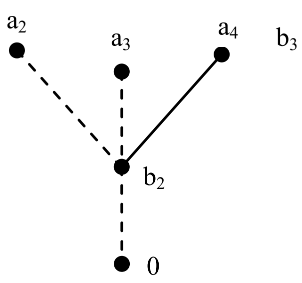


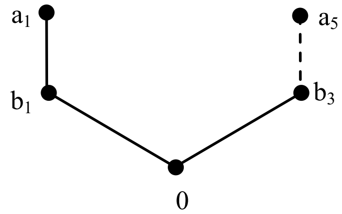
Figure 2.15

The subsemilattices of {S(P), ∩} need not in general be edge neutrosophic subsemilattices. They can be usual subsemilattices as well as edge neutrosophic subsemilattices.

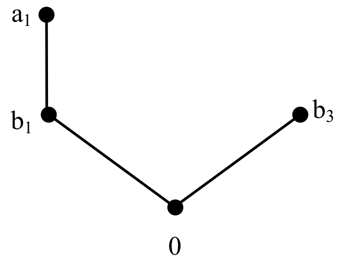
The figures associated with subsemilattices of {S(P), ∩} is as follows.



(a)



(b)



(c)

Figure 2.16 (a) (b) (c)

Fig 2.16(a) and (b) are edge neutrosophic subsemilattices whereas 2.16(c) is a usual subsemilattice.

Next the subsemilattices of a pure neutrosophic semilattices is described by the following example.

Example 2.15: Let *{S(P), ∩}* be the pure neutrosophic semilattice whose figure is given below.

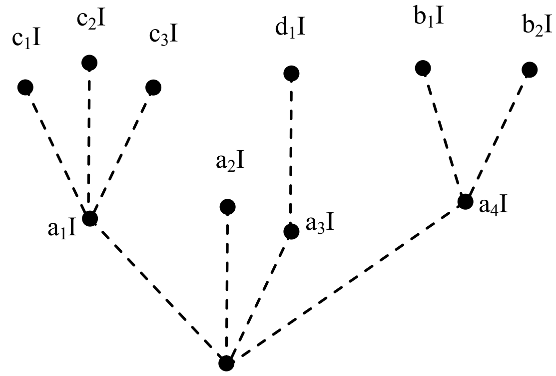


Figure 2.17

All subsemilattices of S(P) are pure neutrosophic subsemilattices only.

In view of this the following theorem is proved.

Theorem 2.1: *Let {S(P), ∪} (or {S(P), ∩} be the pure neutrosophic semilattice. Then every subsemilattice of {S(P), ∪} or {(S(P), ∩}) are also pure neutrosophic.*

Proof: Follows from the fact in a pure neutrosophic semilattice all vertices and edges are neutrosophic. Hence the claim.

Proposition 2.1: *Let {S(P), ∪} (or {S(P), ∩}) be a edge neutrosophic semilattice. Every subsemilattice need not be a edge neutrosophic subsemilattice.*

Proof: Follows from the fact a edge neutrosophic semilattice can have subsemilattices which are not in general neutrosophic edges subsemilattice.

Proposition 2.2: *Let {S(P), ∪} (or{S(P),∩}) be a neutrosophic semilattice. Every subsemilattice of a neutrosophic semilattice need not be a neutrosophic subsemilattice.*

Proof: Evident from the examples given.

3 Application of Neutrosophic Semilattices

In this section applications of neutrosophic semilattices is briefly given.

Infact all neutrosophic semilattices are neutrosophic trees. So these will find application in all places where neutrosophic trees find their applications.

So one can apply these neutrosophic semilattices when the research or the investigator feels that indeterminancy is present in that analysis.

Conclusion

For the first time the new notion of neutrosophic semilattices is introduced and their properties are discussed. These neutrosophic lattices are also neutrosophic tress and they find their application in data mining and sorting.

References

[1]. Birkhoff G, Lattice Theory, 2nd Edition, Amer, Math. Soc, 1948.
[2]. Gratzer G, General Lattice Theory, Birkhauser Ver-lageBasel and Stullgart, 1978.
[3]. Iqbal Unnisa and W.B. Vasantha, On a certain class of Modular lattices, J. of Madras Univ, Vol. 41, 166-169, 1978,
[4]. Iqbal Unnisa, W.B. Vasantha Kandasamy and Florentin Smarandache, Super modular Lattices, Educational Publisher, Ohio, 2012.
[5]. W.B.Vasantha Kandasamy and Florentin Smarandache, Fuzzy Relational Equations and Neutrosophic Relational Equations, Hexis, 2004
[6]. W.B.Vasantha Kandasamy and Florentin Smarandache, Neutrosophic lattice, Neutrosophic sets and Systems, Vol.2, 42-47, 2013

Received: October 30, 2015. Accepted: November 26, 2015