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Solution of Multi-Criteria Assignment Problem using Neutrosophic Set Theory

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Abstract: Assignment Problem (AP) is a very well-known and also useful decision making problem in real life situations. It becomes more effective when different criteria are added. To solve Multi-Criteria Assignment Problem (MCAP), the different criteria have been considered as neutrosophic elements because Neutrosophic Set Theory (NST) is a generalization of the classical sets, conventional fuzzy sets, Intuitionistic Fuzzy Sets (IFS) and Interval Valued Fuzzy Sets (IVFS). In this paper two different methods have been proposed for solving MCAP. In the first method, we have calculated evaluation matrix, score function matrix, accuracy matrix and ranking matrix of the MCAP. The rows represent the alternatives and columns represent the projects of the MCAP. From the ranking matrix, the ranking order of the alternatives and the projects are determined separately. From the above two matrices, composite matrix is formed and it is solved by Hungarian Method to get the optimal assignment.

In the second one, Cosine formula for Vector Similarity Measure [1] on neutrosophic set is used to calculate the degree of similarity between each alternative and the ideal alternative. From the similarity matrix, the ranking order of the alternatives and the projects are determined in the same way as above. Finally the problem is solved by Hungarian Method to obtain the optimal solution.

Keywords: Assignment, Neutrosophic Set, Similarity Measures.

1. Introduction:-

NST is a powerful formal framework which generalizes the concepts of classical set, fuzzy set, IFS, IVFS etc. In the year 1965 Zadeh [2] first introduced the concept of fuzzy set which is a very effective tool to measure uncertainty in real life situation. After two decades, Turksen [3] proposed the concept of IVFS. Atanassov [4] introduced IFS which not only describes the degree of membership, but also the degree of non-membership function. Wang et. al [5] proposed a different concept of imprecise data which gives indeterminate information. F. Smarandache introduced the degree of indeterminacy/neutrality [6] as independent concept in 1995 (published in 1998) and defined the neutrosophic set. He coined the words 'neutrosophy' and 'neutrosophic'. In 2013, he refined the neutrosophic set to 'n' components: t1, t2,…… ; i1, i1,….; f1, f2,….. .

Different authors have solved Multi-Criteria Decision Making (MCDM) problems in different ways. But in neutrosophy, MCAP has not been solved earlier. In real life situation, truth value and falsity (membership and non-membership function) are not sufficient; indeterminacy is also a very important part for decision making problem. NST is a different and more practical concept of fuzzy set where degree of truth value, falsity and indeterminacy are all considered and so it is more relevant to solve MCDM problems.

Several mathematicians have worked on the concept of similarity measures of fuzzy sets. Xu. Z. S [7] used similarity measures of IFS and their applications to multiple attribute DM problems. Li et. al [8] also worked on IFS using similarity measures. Zhizhen Liang, Pengfei Shi (2003) [9] also worked on similarity measures on IFS. Smeg-Hyuk Cha [10] worked on distance similarity measures between probability density functions. Santini S. et. al [11] developed a similarity measure which is based on fuzzy logic. The model is dubbed Fuzzy Feature Contrast (FFC) and they used it to model similarity assessment from fuzzy judgment of properties. Wen-Liang Hung et. al [12] worked on similarity measures between two IFSs. Said Broumi, F. Samarandache [13] calculated the degree of similarity between neutrosophic sets.

In this paper we have developed two methods to solve MCAP. One is based on score function and another one is on vector similarity measure for neutrosophic set. The methods have been demonstrated by a numerical example. The paper is organized as follows- In section 2 preliminaries have been given; section 3 describes the MCAP method and its solution procedures along with the two algorithms. Section 4 illustrates the numerical example and finally section 5 concludes the paper.

2. Preliminaries:-

2.1 Neutrosophic Set:-

Let U be the space of points (or objects) with generic element 'x'. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non-standard subsets of $]^-0, 1^+[$, i.e $T_A: x \rightarrow]^-0, 1^+[$

$F_A: x \rightarrow]^-0, 1^+[$

$I_A: x \rightarrow]^-0, 1^+[$

A neutrosophic set A upon U as an object is defined as -

$$\frac{x}{T_A(x), I_A(x), F_A(x)} = \left\{ \frac{x}{T_A, I_A, F_A} : x \in U \right\}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of [0, 1].

2.2 Algebraic Operations with Neutrosophic Set:-

For two neutrosophic sets A and B where and

a> Complement of A
$A' = \{ \frac{x}{T,I,F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \}$

b> Intersection of A and B
$A \cap B = \{ \frac{x}{T,I,F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \}$

c> Union of A and B
$A \cup B = \{ \frac{x}{T,I,F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \}$

d> Cartesian Product of A and B

$$A \times B = \{ (\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B}) \mid \frac{x}{T_A, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \}$$

e> A is a subset of B

$$A \subseteq B \ \forall \ \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leq T_B \text{ and } F_A \geq F_B$$

f> Difference of A and B
$A \backslash B = \{ \frac{x}{T,I,F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \}$

NST can be used in assignment problem (AP) and Generalized Assignment Problem (GAP).

2.3 Cosine formula for vector similarity measure:-

Cosine formula for vector similarity measure is

$WS_c(A_i, A^*) =$

$$\frac{\sum_{j=1}^{n} w_j [a_{ij} a_j^* + b_{ij} b_j^* + c_{ij} c_j^*]}{\sqrt{\sum (a_{ij}^2 + b_{ij}^2 + c_{ij}^2)}\sqrt{\sum (a_j^{*2} + b_j^{*2} + c_j^{*2})}}$$

…………………………..[1]

Where A_i is the alternative, A^* is the ideal alternative, w_j represents the weight of the alternatives s.t. $\sum_{j=1}^{n} w_j = 1$.

The criteria are divided into two types – one is cost criterion and the other is benefit criterion (profit, efficiency, quality etc). For these two types ideal alternatives have been defined as –

a> Ideal alternative for cost criterion, A^* is $\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> =$
$$\left\langle \left[\min_i (a_{ij}), \max_i (b_{ij}), \max_i (c_{ij}) \right] \right\rangle$$
…………………….[2]

b> Ideal alternative for benefit criterion, A^* is $\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> =$
$$\left\langle \left[\max_i (a_{ij}), \min_i (b_{ij}), \min_i (c_{ij}) \right] \right\rangle$$
……………………..[3]

3. MCAP using NST:-

In this section we have formulated the MCAP using NST. The AP has been solved by different mathematicians in various ways [14], [15]. Here we have proposed two methods – i> In the first method, to compute the best final result, the evaluation of the alternatives with regard to each criteria are must. So from the decision matrix, evaluation matrix has been calculated. Then score function of each alternative has been computed. To find the degree of accuracy ($H(A_i)$) of neutrosophic elements, accuracy matrix has been evaluated. The larger value of $H(A_i)$, the more is the degree of accuracy of an alternative A_i. To evaluate all the above matrices weights must be considered because the larger the value of $W(E(A_i))$, the more is the suitability to which the alternative A_i satisfies the decision maker's requirement. Using the above, ranking matrix has been computed. Then the alternatives (Teams) are ranked with respect to the criteria (Projects) row-wise and the opposite is done column-wise. From the above two matrices, composite matrix has been formed. Finally assignment is done using Hungarian method. ii> By using the cosine formula for vector similarity measure on neutrosophic elements. Similarity matrix is computed and the Hungarian method, as mentioned earlier, is again used to get the optimal assignment.

3.1 Solution procedure for MCAP:-

Method 1:

To solve MCAP we have considered the elements of the criteria as neutrosophic elements (T, I, F), where T is the truth membership degree, I is indeterminacy and F represents falsity degree. From the input data, evaluation matrix E(A), score function matrix S(A) and accuracy matrix H(A) of the alternatives are determined. Algorithm 1 is applied to find the ranking matrix R(A) using the above three matrices and weights of the criteria.

Algorithm 1:

Step 1: Construct the matrix of neutrosophic MCAP.

Step 2: Determine the evaluation matrix $E(A) = (E(A_{ij}))_{mxn}$ of the alternatives as $E(A) = \left[T_{A_i}^l, T_{A_i}^u\right]$ where

$$[T_{A_i}^l, T_{A_i}^u] = \begin{bmatrix} \min\left(\left(\frac{T_{A_{ij}} + I_{A_{ij}}}{2}\right), \left(\frac{1 - F_{A_{ij}} + I_{A_{ij}}}{2}\right)\right), \\ \max\left(\left(\frac{T_{A_{ij}} + I_{A_{ij}}}{2}\right), \left(\frac{1 - F_{A_{ij}} + I_{A_{ij}}}{2}\right)\right) \end{bmatrix}$$ ……[4]

Step 3: Compute the score function matrix $S(A) = (S(A_{ij}))_{mxn}$ of an alternative using the formula

$S(A) = 2\left[T_{A_i}^U - T_{A_i}^L\right]$ where $0 \leq S(A_i) \leq 1$……[5]

Step 4: Compute Accuracy matrix H(A) = $(H(A_{ij}))_{mxn}$ to evaluate degree of accuracy of the neutrosophic elements as –

$$H(A) = 0.5\left[T_{A_i}{}^{U} + T_{A_i}{}^{L}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots[6]$$

Step 5: Using E(A), H(A), S(A) and w_j, ranking matrix R(A) = $(R(A_{ij}))_{mxn}$ of the alternatives is determined by the formula

$$R(A)=\sum_{j=1}^{n}\left[(S(A_{ij}))^2 - \frac{1-H(A_{ij})}{2}\right] w_j \quad \ldots\ldots\ldots[7]$$

Step 6: Form R_1 matrix by considering the rank of the teams and R_2 matrix for the projects.

Step 7: Form the composite matrix by taking the product of R_1 and R_2.

Step 8: Solve the composite matrix by Hungarian method to get the optimal assignment.

Step 9: End.

Method 2:

This method is based on the concept of similarity measures. Here the cosine formula, previously mentioned, has been used. The ideal alternatives for the two types of criteria (cost and benefit) have been defined in the equations [2] and [3]. The cosine formula for similarity measures which is defined in equation [1] has been used to find the degree of similarity and the ranking matrix has been evaluated. The alternative which has the maximum value of the degree of similarity is more similar to the ideal alternative A^* and can be considered as the best choice.

Algorithm 2:

Step 1: Categorize the criteria in two ways – cost criterion and benefit criterion.

Step 2: Determine the ideal alternative for both of the types of criteria defined as in equation [2] and [3].

Step 3: Consider the weights of the criteria w_j and use cosine formula (equation [1]) for vector similarity measures on NS to find the similarity matrix.

Step 4: Follow steps 6 to 8 of Method 1 Algorithm 1.

Step 5: End.

Numerical Example:-

Let us consider an AP consisting of three projects and four teams with three criteria. The three criteria are – cost, profit and efficiency of the team which are considered as neutrosophic elements and the data are as follows.

Table – 1

Input Data Table

Teams	Projects								
	I			II			III		
	c_1	c_2	c_3	c_1	c_2	c_3	c_1	c_2	c_3
A	[(0.75,0.39,0.1)	(0.6,0.5,0.25)	(0.8,0.4,0.2)]	[(0.3,0.2,0.5)	(0.8,0.6,0.1)	(0.2,0.3,0.5)]	[(0.1,0.2,0.4)	(0.2,0.55,0.6)	(0.3,0.5,0.7)]
B	[(0.8,0.6,0.15)	(0.68,0.46,0.2)	(0.45,0.1,0.05)]	[(0.1,0.3,0.4)	(0.5,0.6,0.8)	(0.4,0.5,0.6)]	[(0.2,0.3,0.5)	(0.4,0.5,0.7)	(0.3,0.2,1)]
C	[(0.4,0.8,0.45)	(0.75,0.9,0.05)	(1,0.5,1)]	[(0.25,0.2,0.4)	(0.3,0.5,0.6)	(0.4,0.7,0.8)]	[(0.6,0.5,0.1)	(0.3,0.5,0.6)	(0.6,0.7,0.8)]
D	[(0.4,0.6,0.3)	(0.5,0.4,0.8)	(0.5,0.6,0.9)]	[(0.15,0.3,0.5)	(0.4,0.5,0.6)	(0.7,0.8,1)]	[(0.3,0.4,0.5)	(0.2,0.3,0.4)	(0.6,0.7,1)]

The weights of the criteria are $w_1 = 0.35$, $w_2 = 0.40$ and $w_3 = 0.25$ such that $\sum_{j=1}^{3} w_j = 1$.

The problem is to find the optimal assignment.

Solution:

Method 1:

First we calculate the evaluation matrix $E(A_i)$ of the alternatives by applying formula [4] –

Table 2:

Evaluation Matrix ($E(A_i)$)

Projects \ Teams	I	II	III
A	(0.57,0.645),(0.55,0.625),(0.6,0.6)	(0.25,0.35),(0.7,0.75),(0.25,0.4)	(0.15,0.4),(0.375,0.475),(0.4,0.4)
B	(0.7,0.725),(0.57,0.63),(0.275,0.525)	(0.2,0.45),(0.4,0.55),(0.45,0.45)	(0.25,0.4),(0.4,0.45),(0.1,0.25)
C	(0.6,0.675),(0.825,0.925),(0.25,0.75)	(0.225,0.4),(0.4,0.45),(0.45,0.55)	(0.55,0.7),(0.4,0.45),(0.45,0.65)
D	(0.5,0.65),(0.3,0.45),(0.35,0.55)	(0.225,0.4),(0.45,0.45),(0.4,0.75)	(0.35,0.45),(0.25,0.45),(0.35,0.65)

The score function matrix is calculated by the formula [5].

Table 3:

Score Function Matrix ($S(A_i)$)

Projects \ Teams	I	II	III
A	(0.15,0.15,0)	(0.2,0.1,0.3)	(0.5,0.2,0)
B	(0.05,0.12,0.5)	(0.5,0.3,0)	(0.3,0.1,0.3)
C	(0.15,0.2,1)	(0.35,1,0.2)	(0.3,0.1,0.4)
D	(0.3,0.3,0.4)	(0.35,0,0.7)	(0.2,0.4,0.6)

Accuracy matrix $H(A_i)$ has been calculated by formula [6].

Table 4:

Accuracy Matrix ($H(A_i)$)

Projects \ Teams	I	II	III
A	(0.6075,0.5875,0.6)	(0.3,0.725,0.325)	(0.275,0.425,0.4)
B	(0.7125,0.6,0.4)	(0.325,0.475,0.45)	(0.325,0.425,0.175)
C	(0.6375,0.875,0.5)	(0.3125,0.475,0.5)	(0.625,0.425,0.55)
D	(0.575,0.375,0.45)	(0.3125,0.45,0.575)	(0.4,0.35,0.5)

Now we calculate the ranking matrix $R(A_i)$ using formula [7].

Table 5:

Ranking Matrix ($R(A_i)$)

Projects \ Teams	I	II	III
A	- 0.184	- 0.222	- 0.075
B	- 0.136	- 0.169	- 0.279
C	0.124	0.165	- 0.161
D	- 0.161	- 0.118	- 0.129

Table 6:

Ranking Indices (R_1) of the project w.r.t the team

Projects / Teams	I	II	III
A	2	3	1
B	1	2	3
C	2	1	3
D	3	1	2

Table 7:

Ranking Indices (R_2) of the team w.r.t the project

Projects / Teams	I	II	III
A	4	4	1
B	2	3	4
C	1	1	3
D	3	2	2

Table 8:

Composite matrix R_1R_2

Projects / Teams	I	II	III
A	8	12	1
B	2	6	12
C	2	1	9
D	9	2	4

To solve the above matrix, a dummy column has been added and the AP is solved by Hungarian method to get the optimal assignment.

Table 9:

Solution Matrix (a)

Projects / Teams	I	II	III	IV
A	6	11	[0]	0
B	[0]	5	11	0
C	0	[0]	8	0
D	7	1	3	[0]

Therefore optimal assignment is A→III, B→I, C→II, and D→IV.

Method 2:

Here the three criteria are –

c_1 → Cost, c_2→ Profit, c_3 → Efficiency

For cost criterion, ideal alternative A^* is –

$$\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> = \left\langle \left[\min_i (a_{ij}), \max_i (b_{ij}), \max_i (c_{ij}) \right] \right\rangle$$

For benefit criteria (profit and efficiency), ideal alternative A^* is –

$$\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> = \left\langle \left[\max_i (a_{ij}), \min_i (b_{ij}), \min_i (c_{ij}) \right] \right\rangle$$

Therefore cosine formula for similarity measure is –

$$WS_c(A_i, A^*) = \frac{\sum_{j=1}^{n} w_j [a_{ij} a_j^* + b_{ij} b_j^* + c_{ij} c_j^*]}{\sqrt{\sum (a_{ij}^2 + b_{ij}^2 + c_{ij}^2)} \sqrt{\sum (a_j^{*2} + b_j^{*2} + c_j^{*2})}}$$

where weights of the criteria c_1, c_2 and c_3 are w_1 = 0.35, w_2 = 0.40 and w_3 = 0.25 such that $\sum_{j=1}^{3} w_j = 1$.

Table 10:

Degree of similarity matrix ($WS_c(A_i, A^*)$)

Projects / Teams	I	II	III
A	0.289	0.323	0.441
B	0.303	0.275	0.293
C	0.280	0.251	0.264
D	0.244	0.263	0.274

Table 11:

Ranking Indices (R_3) of the project w.r.t the team

Projects / Teams	I	II	III
A	3	2	1
B	1	3	2
C	1	3	2
D	3	2	1

Table 12:

Ranking Indices (R_4) of the team w.r.t the project

Projects / Teams	I	II	III
A	2	1	1
B	1	2	2
C	3	4	4
D	4	3	3

Table 13:

Composite matrix R_3R_4

Projects / Teams	I	II	III
A	6	2	1
B	1	6	4
C	3	12	8
D	12	6	3

Solving composite matrix by Hungarian method-

Table 14:

Projects / Teams	I	II	III	IV
A	5	0	0	0
B	0	4	3	0
C	2	10	7	0
D	11	4	2	0

Table 15:

Solution Matrix (b)

Projects / Teams	I	II	III	IV
A	7	[0]	0	2
B	[0]	2	1	0
C	2	8	5	[0]
D	11	2	[0]	0

Therefore optimal assignment is A→II, B→I, C→IV, and D→III.

4. Conclusion:-

This paper proposes two different approaches to solve MCAP. Both of them are simple but very efficient and have not been used earlier. The numerical example demonstrates the application and effectiveness of the methods with the incomplete and indeterminate information which exist commonly in real life situations.

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