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The indefinite integrals of refined neutrosophic trigonometric functions

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Abstract: in this paper, we presented the indefinite integrals of refined neutrosophic trigonometric functions, where we discussed the integrals by distinguishing several cases of these types of refined neutrosophic integrals. Also, we introduced the refined neutrosophic trigonometric identities, which makes it easier for us to find these types of integrals. In addition, to supporting this with appropriate examples.

Keywords: refined; neutrosophic; indefinite integral; indeterminacy; neutrosophic trigonometric.

1. Introduction and Preliminaries

As an alternative to the existing logics, Smarandache proposed the neutrosophic Logic to characterize a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction. Smarandache provided the following form of refined neutrosophic numbers: $(a, b_1I_1, b_2I_2, \dots, b_nI_n)$, where: $a, b_1, b_2, \dots, b_n \in R \text{ or } C$ [1]

Agboola introduced the concept of refined neutrosophic algebraic structures [2]. Also, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. The refined indefinite neutrosophic integral was studied and included the following set of rules and theorem [14]:

Theorem:

If $\int f(x, I_1, I_2)dx = \varphi(x, I_1, I_2)$, then:

$$\int f\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)\,dx$$

$$= \left(\frac{1}{a} + \left[\frac{-ab}{a(a+c)(a+b+c)}\right] I_1 - \left[\frac{c}{a(a+c)}\right] I_2\right) \varphi\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big) + C$$

where a, b, c, r, s, t are real numbers and $a \neq 0$, $a \neq -c$ and $a \neq -b - c$
C is an indeterminate real constant (i.e. constant of the form $a_0 + a_1I_1 + a_2I_2$,and a_0, a_1, a_2 are real numbers, while $I_1, I_2 =$ indeterminacy).

In addition, there are many papers presenting studies on refined neutrosophic numbers [4-5-6-7-8].
Smarandache discussed neutrosophic indefinite integral (Refined Indeterminacy) [11]
Let $g: \mathbb{R} \to \mathbb{R} \cup \{I_1\} \cup \{I_2\} \cup \{I_3\}$, where I_1, I_2, and I_3 are types of sub indeterminacies,

$$g(x) = 7x - 2I_1 + x^2I_2 + 4x^3I_3$$

then:

$$F(x) = \int [7x - 2I_1 + x^2I_2 + 4x^3I_3]dx$$

$$= \frac{7x^2}{2} - 2xI_1 + \frac{x^3}{3}I_2 + x^4I_3 + a_0 + a_1I_1 + a_2I_2 + a_3I_3$$

where a_0, a_1, a_2 and a_3 are real constants.

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [9-10]. Several studies were also presented in the field of neutrosophic logic in statistics and others [12-13].

2. Main Discussion

We consider C is an indeterminate real constant (i.e. constant of the form $a + bI_1 + cI_2$, where a, b, c are real numbers, while $I_1, I_2 =$ indeterminacy).

Integrating products of neutrosophic trigonometric function:

I. $\int \sin^m\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big) \cos^n\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)\ dx$, where m and n are positive integers.

The two cases below can be distinguished in order to find this integral:

- Case n *is* odd:

  - Split of $cos\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$
  - Apply $cos^2\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big) = 1 - sin^2\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$
  - We substitute $u = sin\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$

- Case m *is* odd:

  - Split of $sin\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$
  - Apply $sin^2\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big) = 1 - cos^2\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$
  - We substitute $u = cos\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$

Example 1

Find:

$$\int sin^2\big((5 + I_1 - 3I_2)x + 7I_1\big) cos^3\big((5 + I_1 - 3I_2)x + 7I_1\big)\ dx$$

Solution:

$$\int sin^2\big((5+I_1-3I_2)x+7I_1\big)\,cos^3\big((5+I_1-3I_2)x+7I_1\big)\ \ dx$$

$$=\int sin^2\big((5+I_1-3I_2)x+7I_1\big)\,cos^2\big((5+I_1-3I_2)x+7I_1\big)\ \ cos\big((5+I_1-3I_2)x+7I_1\big)\ \ dx$$

$$=\int sin^2\big((5+I_1-3I_2)x+7I_1\big)\left(1-sin^2\big((5+I_1-3I_2)x+7I_1\big)\right)\ \ cos\big((5+I_1-3I_2)x+7I_1\big)\ \ dx$$

$$=\int\left(sin^2\big((5+I_1-3I_2)x+7I_1\big)-sin^4\big((5+I_1-3I_2)x+7I_1\big)\right)\ \ cos\big((5+I_1-3I_2)x+7I_1\big)\ \ dx$$

$$u=sin\big((5+I_1-3I_2)x+7I_1\big)\quad\Longrightarrow\quad\frac{1}{5+I_1-3I_2}\ \ du=cos\big((5+I_1-3I_2)x+7I_1\big)\ \ dx$$

$$\Longrightarrow\quad\int sin^2\big((5+I_1-3I_2)x+7I_1\big)\,cos^3\big((5+I_1-3I_2)x+7I_1\big)\ \ dx=\frac{1}{5+I_1-3I_2}\int(u^2-u^4)\,du$$

$$=\left(\frac{1}{5}-\frac{1}{18}I_1+\frac{3}{10}I_2\right)\left(\frac{u^3}{3}-\frac{u^5}{5}\right)+C$$

$$=\left(\frac{1}{5}-\frac{1}{18}I_1+\frac{3}{10}I_2\right)\left(\frac{sin^3\big((5+I_1-3I_2)x+7I_1\big)}{3}-\frac{sin^5\big((5+I_1-3I_2)x+7I_1\big)}{5}\right)+C$$

II. $\int tan^m\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)\,sec^n\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)\ \ dx$, where m and n are positive integers.

The two cases below can be distinguished in order to find this integral:

- Case n is even:

- Split of $sec^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- Apply $sec^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)=1+tan^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- We substitute $u=tan\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$

- Case m is odd:

- Split of $sec\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)tan\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- Apply $tan^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)=sec^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)-1$
- We substitute $u=sec\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$

- Case m even and n odd:

- Apply $tan^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)=sec^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)-1$
- We substitute $u=sec\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$ or $u=tan\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$, depending on the case.

Example 2

Find:

$$\int tan^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ sec^4\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

Solution:

$$n=4\ \ \text{(Even)}$$

$$\int tan^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ sec^4\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

$$=\int tan^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ sec^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ \ sec^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

$$=\int \Big(tan^2\big((3+I_1+2I_2)x-4+I_2\big)+tan^4\big((3+I_1+2I_2)x-4+I_2\big)\Big)\ \ \ sec^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

$$u=tan\big((3+I_1+2I_2)x-4+I_2\big)\qquad\Rightarrow\qquad \frac{1}{3+I_1+2I_2}\ du=\ sec^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

$$\Rightarrow\int \Big(tan^2\big((3+I_1+2I_2)x-4+I_2\big)+tan^4\big((3+I_1+2I_2)x-4+I_2\big)\Big)\ \ \ sec^2\big((3+I_1+2I_2)x-4+I_2\big)\ \ dx$$

$$=\ \frac{1}{3+I_1+2I_2}\int (u^2+u^4)\,du$$

$$=\left(\frac{1}{3}-\frac{1}{30}I_1-\frac{2}{15}I_2\right)\left(\frac{u^3}{3}+\frac{u^5}{5}\right)+C$$

$$=\left(\frac{1}{3}-\frac{1}{30}I_1-\frac{2}{15}I_2\right)\left(\frac{tan^3\big((3+I_1+2I_2)x-4+I_2\big)}{3}+\frac{tan^5\big((3+I_1+2I_2)x-4+I_2\big)}{5}\right)+C$$

Example 3

Find:

$$\int tan^3\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ sec^3\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ dx$$

Solution:

$$m=4\ \ \text{(Odd)}$$

$$\int tan^3\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ sec^3\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ dx$$

$$=\int tan^3\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ sec^2\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ sec\big((7-4I_1+I_2)x+I_1+I_2\big)\ \ tan\big((7-4I_1+I_2)x+I_1+I_2\big)\,dx$$

$$= \int \Big(sec^4\big((7-4I_1+I_2)x+I_1+I_2\big) - sec^2\big((7-4I_1+I_2)x+I_1+I_2\big)\Big)\ sec\big((7-4I_1+I_2)x+I_1+I_2\big)\ tan\big((7-4I_1+I_2)x+I_1+I_2\big)\, dx$$

$$u = sec\big((7-4I_1+I_2)x+I_1+I_2\big)$$

$$\Rightarrow \quad \frac{1}{7-4I_1+I_2}\ du = \ sec\big((7-4I_1+I_2)x+I_1+I_2\big)\ tan\big((7-4I_1+I_2)x+I_1+I_2\big)\ dx$$

$$\Rightarrow \int \Big(sec^4\big((7-4I_1+I_2)x+I_1+I_2\big) - sec^2\big((7-4I_1+I_2)x+I_1+I_2\big)\Big)\ sec\big((7-4I_1+I_2)x+I_1+I_2\big)\ tan\big((7-4I_1+I_2)x+I_1+I_2\big)\, dx$$

$$= \frac{1}{7-4I_1+I_2}\int (u^4-u^2)\, du$$

$$= \left(\frac{1}{7}+\frac{1}{8}I_1-\frac{1}{56}I_2\right)\left(\frac{u^5}{5}-\frac{u^3}{3}\right)+C$$

$$= \left(\frac{1}{7}+\frac{1}{8}I_1-\frac{1}{56}I_2\right)\left(\frac{sec^5\big((7-4I_1+I_2)x+I_1+I_2\big)}{5}-\frac{sec^3\big((7-4I_1+I_2)x+I_1+I_2\big)}{3}\right)+C$$

III. $\int cot^m\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)\, csc^n\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)\ dx,$ where m and n are positive integers.

To find this integral, we can distinguish the following cases:

- Case n *is* even:

- Split of $csc^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- Apply $csc^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big) = 1+cot^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- We substitute $u = cot\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$

- Case m *is* odd:

- Split of $csc\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)\ cot\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$
- Apply $cot^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big) = csc^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)-1$
- We substitute $u = csc\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)$

- Case m even and n odd:

  - Apply $cot^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big) = csc^2\big((a+bI_1+cI_2)x+r+sI_1+tI_2\big)-1$

- We substitution $u = csc\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$ or $u = cot\big((a + bI_1 + cI_2)x + r + sI_1 + tI_2\big)$, depending on the case.

Example 4

Find:

$$\int \sqrt{cot\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)}\ \ csc^4\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

Solution:

$$n = 4 \ \text{(Even)}$$

$$\int \sqrt{cot\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)}\ \ csc^4\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

$$= \int cot^{\frac{1}{2}}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ csc^2\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big) csc^2\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

$$= \int \Big(cot^{1/2}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big) + cot^{3/2}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\Big) csc^2\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

$$u = cot\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)$$

$$\Rightarrow \quad \frac{-1}{2 + 4I_1 + 6I_2}\ du = \ csc^2\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

$$\Rightarrow \int \left(cot^{\frac{1}{2}}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big) + cot^{\frac{3}{2}}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\right) csc^2\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)\ \ dx$$

$$= \ \frac{-1}{2 + 4I_1 + 6I_2}\int \big(u^{1/2} + u^{3/2}\big)\, du$$

$$= -\left(\frac{1}{2} - \frac{1}{24}I_1 - \frac{3}{8}I_2\right)\left(\frac{2\ u^{\frac{3}{2}}}{3} + \frac{2\ u^{\frac{5}{2}}}{5}\right) + C$$

$$= \left(-\frac{1}{2} + \frac{1}{24}I_1 + \frac{3}{8}I_2\right)\left(\frac{2\ \ cot^{3/2}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)}{3} + \frac{2\ \ cot^{5/2}\big((2 + 4I_1 + 6I_2)x - 7 + 5I_1 + 9I_2\big)}{5}\right) + C$$

The refined neutrosophic trigonometric identities:

1) $sin(a + bI_1 + cI_2)x\, cos(r + sI_1 + tI_2)\, x = \frac{1}{2}\ \ [sin(a + r + (b + s)I_1 + (c + t)I_2)x + sin(a - r + (b - s)I_1 + (c - t)I_2)\, x]$

2) $cos(a + bI_1 + cI_2)x\, sin(r + sI_1 + tI_2)\, x = \frac{1}{2}\ \ [sin(a + r + (b + s)I_1 + (c + t)I_2)x - sin(a - r + (b - s)I_1 + (c - t)I_2)\, x]$

3) $cos(a + bI_1 + cI_2)x\, cos(r + sI_1 + tI_2)\, x = \frac{1}{2}\ [cos(a + r + (b + s)I_1 + (c + t)I_2)x + cos(a + r + (b + s)I_1 + (c + t)I_2)x]$

4) $sin(a + bI_1 + cI_2)x\, sin(r + sI_1 + tI_2)\, x = \frac{-1}{2}\ [cos(a + r + (b + s)I_1 + (c + t)I_2)x - cos(a + r + (b + s)I_1 + (c + t)I_2)x]$

Example 5
Find:

1) $\int sin(1 - 5I_1 + 6I_2)x\, cos(2 + 4I_1 + I_2)\, x\ \ dx$

$$= \int \frac{1}{2}\ [sin(3 - I_1 + 7I_2)x + sin(-1 - 9I_1 + 5I_2)x]\ \ dx$$

$$= \frac{1}{2}\left[-\left(\frac{1}{3 - I_1 + 7I_2}\right) cos(3 - I_1 + 7I_2)x - \frac{1}{-1 - 9I_1 + 5I_2} cos(-1 - 9I_1 + 5I_2)x\right] + C$$

$$= -\left(\frac{1}{6} + \frac{1}{180}I_1 - \frac{7}{60}I_2\right) cos(3 - I_1 + 7I_2)x - \left(\frac{1}{2} - \frac{9}{40}I_1 + \frac{5}{8}I_2\right) cos(-1 - 9I_1 + 5I_2)x + C$$

2) $\int cos(2 - I_1 + I_2)x\, cos(3 + I_1 + 3I_2)\, x\ \ dx = \int \frac{1}{2}\ [cos(5 + 4I_2)x + cos(-1 - 2I_1 - 2I_2)\, x]\ \ dx$

$$= \frac{1}{2}\ \left[\frac{1}{5 + 4I_2} sin(5 + 4I_2)x + \frac{1}{-1 - 2I_1 - 2I_2} sin(-1 - 2I_1 - 2I_2)\, x\right] + C$$

$$= \left(\frac{1}{10} - \frac{2}{45}I_2\right) sin(5 + 4I_2)x + \left(-\frac{1}{2} + \frac{1}{15}I_1 + \frac{1}{3}I_2\right) sin(-1 - 2I_1 - 2I_2)x + C$$

3) $\int sin(5 + I_2)x\, sin(2 + I_1)\, x\ \ dx = \int \frac{-1}{2}\ \ [cos(7 + I_1 + I_2)x - cos(3 - I_1 + I_2)x]\ \ dx$

$$= -\frac{1}{2}\left[\left(\frac{1}{7 + I_1 + I_2}\right) sin(7 + I_1 + I_2)x - \left(\frac{1}{3 - I_1 + I_2}\mathrm{I}\right) sin(3 - I_1 + I_2)x\right] + C$$

$$= \left(-\frac{1}{14} + \frac{1}{144}I_1 + \frac{1}{112}I_2\right) sin(7 + I_1 + I_2)x + \left(\frac{1}{6} + \frac{1}{24}I_1 - \frac{1}{24}I_2\right) sin(3 - I_1 + I_2)x + C$$

3. Conclusions

This paper is consider one of the important papers that presented integrals of refined neutrosophic functions, using several cases of integrals of refined neutrosophic trigonometric functions. Also, in order to make it easier to compute the integrals of refined neutrosophic functions, we developed the refined neutrosophic trigonometric identities, and precise results were obtained.. Furthermore, the paper is thought to be significant for further research.

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