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Utilizing New Temporal Complex Neutrosophic Aczel-Alsina Aggregation Operators for Multi-Criteria Decision Making: An Application to Tourist Destination Selection

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Abstract: The Temporal Complex Neutrosophic Set (TCNS) is a powerful extension of the Neutrosophic theory, and TCNS has added advantage to represents uncertain, indeterminate, inconsistent and time related factors. In the literature TCNS has been used for multi-criteria decision-making problems, especially in the time-aware systems. In order to tackle effectively decision-making problems in the TCNS environment effectively, it is crucial to make suite aggregation operators that enable decision makers to process preferences in a nuanced manner. In this paper, we introduce novel TCNS aggregation operators based on Aczel and Alsina t-norm and t-conorm using Aczel-Alsina operations, and we propose a multi-criteria decision-making model that utilizes those operators for TCNS-based multi criteria decision making problems (MCDMs). Theorytically, the soundness and completeness of new aggregation operators are proved, and to demonstrate the practicality and effectiveness of our model a real world case study for tourist destination selection in Vietnam is completed along with analysis. The result proved the application potential of the proposal to solve the time-related MCDMs in practical, such as creating a MCDM model for tourist destination selection.

Keywords: Aczel-Alsina operators; Complex Neutrosophic Set; Temporal Complex Neutrosophic Set; MCDM;

1. Introduction

Multi-criteria decision-making (MCDM), a sophisticated and skillful method to assisting human decision-making (DM) in the face of the challenges posed by complex real-world decisions, has applications in a wide range of industries including tourist destination selection, sustainable cement industry, and supply chain management. Recently, the explosion and complexity of information worldwide used in MCDMs have made it increasingly difficult to make the right decision. Among MCDMs challenges, tackling the uncertainty, ambiguity and time-aware context are attracted more attention from researchers and practitioners. The traditional MCDM strategies become outdated and weak to solve uncertainty and ambiguity of collected information. Therefore, new approaches for MCDMs are required, and appling extension of fuzzy set [1] is among the top.

Fuzzy set theory was introduced by Zadeh [2] in 1965, along with its broader applications, effectively addresses uncertainty and vagueness in data. However, these frameworks lack the capacity to express the partial ambiguity inherent in indeterminate and inconsistent data, particularly its temporal dynamics. Subsequently, Ali and Smarandache's [3] introduction of complex neutrosophic sets (CNSs) provides a new perspective on decision-making within the unit circle in complex space. This strategy is suitable for handling uncertainty, indeterminacy, inconsistency, and temporal cycles. In the field of neutrosophy, there is a widely recognized consensus that t-norm and t-conorm, such as algebraic, Einstein, Hamacher, Acz-Als's t-norm and t-conorm, play crucial roles as integral components of complex neutrosophic sets or neutrosophic systems in decision support.

In the field of complex neutrosophy set, numerous scholars have developed operations in this field that draw on various t-norms and t-conorms to aid decision-making in real-world circumstances. These operations include a wide range of topics, such as operators [4-7, 22-25, 28-29, 30], distance and similarity measures [8-10, 26, 30], and information entropy [5, 11-13]. Aggregation operators are often used in decision-making issues as crutical tools for summarizing information in the complex neutrosophic environment, assisting in decision support. In the complex neutrosophic set environment, several research has acknowledged their importance and presented mean operators. The detail is presented in next section.

Futhermore, the idea of multi-valued complex neutrosophic linguistic sets (MVCNLSs) was put forth by Mahmood et al. [16], leading to the creation of MVCNLSs that represent information of truth, abstinence, falsity, and uncertain. The presentation utilizes complex numbers whose real and imaginary components are constrained to a unit interval. Moreover, this paper provides a flexible framework for presenting Bonferroni mean operators and elaborates on important Dombi laws. Ali et al. [17] explored the notion of complex neutrosophic uncertain linguistic set (CNUL) and formulated valuable hypotheses using Heromian mean (HM) operators. CNUL data represents complex numbers consisting of uncertainty linguistic sets, truth grades, abstinence grades, and falsely grades. Furthermore, they endeavor to present well-known theories that are concerned with this idea. The first group of HM-CNUL aggregation operators based on HM presents such as weighted arithmetic, geometric. All these operators share some common objective: bringing together multiple alternatives into an element that shall be presented as one single thing.

Through a survey of various studies, it becomes evident that aggregation operators play a significant role in MCDM problems within the complex neutrosophic environment. Furthermore, the temporal complex neutrosophic environment proves to be an effective framework for representing the temporal aspects of decision-making problems. However, it is noticeable that there have been few studies that express interest in or have only partially considered the use of average operators for temporal neutrosophic sets employing Aczel and Alsina t-norms or t-conorms.

Take all into account, this paper introduces new aggregation operators for TCNS utilizing Aczel and Alsina t-norm and t-conorm. The following are the specific contributions of this paper:

1. The development of new TCNS aggregation operators based on Aczel and Alsina t-norm and t-conorm.

2. The proposal of a MCDM model grounded in the newly introduced TCNS aggregation operators.

3. The demonstration of the feasibility and soundness of addressing DM problems through the application of the proposed model. This is illustrated with a real-world case study involving the selection of a tourist destination in Vietnam, accompanied by comparative analysis.

The remain of work is presented as follows. The Section 2 presents related works, and required fundamentals are covered in Section 3, that includes the definition of the TCNS and its operators. The new Aczel-Alsina t-norm and t-cornom operations for TCNS is introduced in Section 4. Section 5 provides a comprehensive MCDM model for TCNS environment based on new proposed operators, and that is pertaining to a real-world application presented in Section 6. The application of designed

MCDM for tourist destination selection illustrates the pratical potential of new aggregation operations for TCNS. In Section 7 the summary and discusion are presented.

2. Related works

Zadeh [1] introduced fuzzy set theory in 1965, it successfully handles ambiguity and uncertainty in data, and it has larger uses theoritically and practically as well. These frameworks, however, are unable to capture the partial ambiguity that exists in inconsistent and ambiguous data, especially with regard to its temporal dynamics. Subsequently, the introduction of complex neutrosophic sets (CNSs) by Ali et al. [3] offers a fresh viewpoint on making decisions inside the unit circle in complex space. This approach works well for managing temporal cycles, uncertainty, indeterminacy, and consistency.

It is commonly acknowledged in the field of neutrosophy that t-norm and t-conorm, including algebraic, Einstein, Hamacher, and Acz-Als t-norm and t-conorm, are essential elements of complex neutrosophic sets or neutrosophic systems in decision support. There are many scholars that developed operations draw on various t-norm and t-conorm to aid MCDMs for reality applications. These operations include a wide range of topics, such as operators [4-7, 22-25, 28-30], distance and similarity measures [8-10, 26, 30], and information entropy [5, 11-13]. Aggregation operators are essential tools for combining data in the complex neutrosophic environment and supporting in MCDM situations. Several studies have given mean operators and acknowledged their significance in the complex neutrosophic set environment. Notably, Ajay et al. [12] introduced and investigated the extension of sine trigonometry operational laws to neutrosophic set. They explored the workings and functioning of these extended laws. Furthermore, the research focused on the core of the work, which involved expanding these ST-Ols to complex neutrosophic sets. Mathematical properties were rigorously proven based on the operations. The study delved into basic operations and distance measurements for CNSs, employing the operational laws. Xu et al. [14] presented the single-valued complex neutrosophic set (SVCN) as a valuable method for managing datasets characterized by unpredictability and periodicity. This study presents a SVCN-EDAS for green supplier selection. The paper begins with a concise introduction to the notion of single-valued complex neutrosophic set and their associated operational rules. Subsequently, in order to combine and synthesize the overall information from SVCN, the paper introduces the aggregation operators. These operators are rooted in SVCNs and utilize Einstein product and sum for aggregation. Rahman et al. [15] explore the emerging field of hypersoft sets, aimed at addressing the limitations of previous soft-set-like frameworks by considering and accommodating multi-argument approximate procedures. The authors introduced fresh ideas within the hypersoft set framework, specifically complex intuitionistic fuzzy sets and complex neutrosophic sets, in the context of interval-valued complex neutrosophic set. Two new structures, namely the interval-valued complex intuitionistic hypersoft set and the interval-valued complex neutrosophic hypersoft set, have been created using theoretical, axiomatic, graphical, and computational approaches.

The fundamental operational rules for the single-valued neutrosophic hesitant fuzzy set (SV-NHFS) under Einstein's t-norm and t-conorm were introduced by Kamran et al. [28]. Moreover, SV-NHFS perform key functions for Einstein operators, such as the scalar multiplication, product, and sum. Then, they suggested a list of names for aggregation operators in order to solve the issues in the SV-NHF context, they finally talk about a MCDM method based on the proposed aggregation operators. The authors of the following study [29] talked about fuzzy rough set, probabilistic hesitant fuzzy set, and neutrosophic set as mathematical answers for smart housing communities. In this study, a hybrid structure of these sets, the single valued neutrosophic probabilistic hesitant fuzzy rough set, is presented in a novel generalised sense. This approach works well for handling ambiguity and incomplete data, especially when taking into account several conflicting factors. The suggested decision support system performs better than present tools in terms of choice flexibility, according to the study's comparison of it with other methods. Future research on waste management, public safety diagnostics, housing societies, and hybridization is anticipated to benefit from this study.

The idea of multi-valued complex neutrosophic linguistic sets (MVCNLSs) was put forth by Mahmood et al. [16], leading to the creation of MVCNLSs that represent information of truth, abstinence, falsity, and uncertain. And the way they are presented is with complex numbers having real and imaginary components that are restricted in a unit interval. Moreover, this paper provides a flexible framework for presenting Bonferroni mean operators and elaborates on important Dombi laws. Ali et al. [17] introducted the notion of complex neutrosophic uncertain linguistic (CNUL) and their useful hypotheses based on Heromian mean (HM) operators. CNUL data represents complex numbers consisting of uncertainty linguistic sets, truth grades, abstinence grades, and falsely grades. Furthermore, they endeavor to present well-known theories that are concerned with this idea. The first group of HM-CNUL aggregation operators based on HM presents such as arithmetic, geometric and weighted arithmetic, weighted geometric. All these operators share some common objective: bringing together multiple alternatives into an element that shall be presented as one single thing.

In solving real-world situations and decision-making (DM) scenarios, researchers have shown interest in augmenting the conceptual framework's flexibility through multiple studies, including:

Dat et al. [18] have shown that the most effective means of modeling for decisions problems under uncertainties is through Interval-valued Complex Neutrosophic Set. In the context of DM, linguistic variables have long served as valuable instruments for addressing the issue of sharp neutrosophic membership degrees. This study endeavors to introduce innovative notions, specifically the single-valued and interval-valued linguistic complex neutrosophic set. These new concepts offer greater applicability and adaptability to practical application compared to their predecessors. Al-Quran et al. [19] emphasize that many real-world complicated circumstances frequently entail data in two dimensions characterized by uncertainty, incompleteness, and indeterminacy, which encompass characteristics and the periodicity of problem parameters. Recognizing the requirement for frameworks capable of accommodating such data, neutrosophic sets (NSs) were introduced and subsequently extended to complex neutrosophic sets. The authors presented the concept of Q-complex neutrosophic sets by expanding the variety of membership functions within Q-neutrosophic sets. Broumi et al. [11] note that in the past decade, numerous researchers have directed their focus toward quantifying npredictability and ambiguity in datasets by leveraging the attributes of neutrosophic sets. This section, in particular, is dedicated to the measurement of unpredictability and its variability employing the characteristics of complex neutrosophic sets. Lan et al. [20] proposed Temporal Complex Neutrosophic Sets (TCNSs) to address temporal uncertainty, inconsistency, and uncertainty in data, which is often represented by Complex Neutrosophic Sets. Then, the authors proposed TCNS operations and features. Next, the MCDM method driven by TCNSs is introduced, which extends the ANP method to identify group multi-criteria weights and TOPSIS for ranking options obtained over different time intervals. Furthermore, a case study of selecting a tourism location in Vietnam is given within the TCNSs to prove the viability and potentiality of the suggested approach. A comparison with current methods illustrates the proposed model's availability, consistency and effectiveness. Gamal et al. [29] introduced a framework of MCDM on type-2 neutrosophic numbers. The authors proved that this extension can tackle well the uncertainty resulted from human judgment in decision-making scenarios. Two real-life case studies were discussed in this study to evaluate the proposed method.

Through a survey of various studies, it becomes evident that aggregation operators play a crucial role in MCDM problems within the complex neutrosophic environment. Furthermore, the temporal complex neutrosophic environment proves to be an effective framework for representing the temporal aspects of decision-making problems. However, it is noticeable that there have been few studies that express interest in or have only partially considered the use of average operators for temporal neutrosophic sets employing Aczel and Alsina t-norms or t-conorms.

It is clear from a review of numerous studies that aggregation operators are important when it comes to MCDM issues in the intricate neutrosophic environment. And to built aggregation operators on fuzzy theory, t-norms or t-conorms operators are crucial points. Moreover, it turns out that the temporal complex neutrosophic environment is a useful framework for expressing the temporal dimensions of decision-making issues. However, it is apparent that the implementation of average operators for temporal neutrosophic sets utilizing t-conorms or t-norms proposed by Alsina and Aczel has not received much attention in research, or has received only limited attention. Therefore, this study aims to introduce novel TCNS aggregation operators according to Aczel and Alsina t-norm and t-conorm using Aczel-Alsina operations, and we propose a MCDM model that utilizes those operators for TCNS-based multi criteria decision making problems.

3. Preliminaries

Definition 1 [20]: Assume there is a universal U, time periods $\tilde{\tau}=\{\tau_1,\tau_2,...,\tau_p\}$. A Temporal complex neutrosophic set (TCNS) A in U is defined as

$$A(\delta,\tilde{\tau})=\{\delta,\langle CVT(\delta,\tilde{\tau}),CVI(\delta,\tilde{\tau}),CVF(\delta,\tilde{\tau})\rangle | \delta\in U\} \tag{1}$$

Where $CVT(\delta,\tilde{\tau})=p(\delta,\tau_s)e^{j\mu(\delta,\tau_s)}$; $CVI(\delta,\tilde{\tau})=q(\delta,\tau_s)e^{j\nu(\delta,\tau_s)}$; $CVF(\delta,\tilde{\tau})=r(\delta,\tau_s)e^{j\eta(\delta,\tau_s)}$; $\tau_s\in\tilde{\tau}$;

In 1982, Aczel and Alsina [21] introducted Aczel-Alsina t-norm and t-conorm, representing a class of flexible aggregation operators adjustable through a parameter.

Definition 2: A mapping $\left(N_\alpha^\theta\right)_{\sigma\in[0,\infty]}$ is a Acz-Als t-norm if

$$\left(TN_\alpha^\theta\right)(a,b)=\begin{cases} TN_D(a,b) & if \quad \theta=0 \\ \min(a,b) & if \quad \theta=\infty \\ \exp\left(-\left((-\ln a)^\theta+(-\ln b)^\theta\right)^{\frac{1}{\theta}}\right) & otherwise \end{cases} \tag{2}$$

Where $TN_D(a,b)$ is t-norm define in equation (3) and $a,b\in[0,1]$, σ is positive constant

$$\left(TN_D\right)(a,b)=\begin{cases} a & if \quad b=1 \\ b & if \quad a=1 \\ 0 & otherwise \end{cases} \tag{3}$$

Definition 3: A mapping $\left(TCN_\alpha^\sigma\right)_{\sigma\in[0,\infty]}$ is a Acz-Als s-norm if

$$\left(TCN_\alpha^\theta\right)(a,b)=\begin{cases} TCN_D(a,b) & if \quad \theta=0 \\ \max(a,b) & if \quad \theta=\infty \\ \exp\left(-\left((-\ln(1-a))^\theta+(-\ln(1-b))^\theta\right)^{\frac{1}{\theta}}\right) & otherwise \end{cases} \tag{4}$$

Where $TCN_D(a,b)$ is s-norm, define in equation (5) and $a,b\in[0,1]$, σ is positive constant

$$\left(TCN_D\right)(a,b)=\begin{cases} a & when \quad b=0 \\ b & when \quad a=0 \\ 0 & otherwise \end{cases} \tag{5}$$

the t-norm $\left(TN_\alpha^\theta\right)_{\theta\in[0,\infty]},\forall\theta\in[0,\infty]$ and s-norm $\left(TCN_\alpha^\theta\right)_{\theta\in[0,\infty]},\forall\theta\in[0,\infty]$ are dual to each other

4. Aczel-Alsina-TCNS Aggregation Operators

In this section, we present Acz-Alz operations in relation to TCNS, taking in to account the Acz-Alz t-norm and t-conorm

Definition 4: Suppose θ is positive constant and three TCNS corresponding, $\Gamma(\delta,\tilde{\tau})=\{\delta,\langle CVT_\Gamma(\delta,\tilde{\tau}),CVI_\Gamma(\delta,\tilde{\tau}),CVF_\Gamma(\delta,\tilde{\tau})\rangle\}$; $\Gamma_1(\delta,\tilde{\tau})=\left\{\delta,\left\langle CVT_{\Gamma_1}(\delta,\tilde{\tau}),CVI_{\Gamma_1}(\delta,\tilde{\tau}),CVF_{\Gamma_1}(\delta,\tilde{\tau})\right\rangle\right\}$; $\Gamma_2(\delta,\tilde{\tau})=\left\{\delta,\left\langle CVT_{\Gamma_2}(\delta,\tilde{\tau}),CVI_{\Gamma_2}(\delta,\tilde{\tau}),CVF_{\Gamma_2}(\delta,\tilde{\tau})\right\rangle\right\}$. Then Aczel-Alsina-TCNS operations are presented as follows:

i. ***Addition***

$$\Gamma_1\oplus\Gamma_2=\left\{\delta,\left\langle CVT_{\Gamma_{12}}(\delta,\tilde{\tau}),CVI_{\Gamma_{12}}(\delta,\tilde{\tau}),CVF_{\Gamma_{12}}(\delta,\tilde{\tau})\,|\,\delta\in U\right\rangle\right\} \tag{6}$$

Where

$$CVT_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

$$CVI_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

$$CVF_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

ii. Multiplication

$$\Gamma_1\otimes\Gamma_2=\left\{\delta,\left\langle CVT_{\Gamma_{12}}(\delta,\tilde{\tau}),CVI_{\Gamma_{12}}(\delta,\tilde{\tau}),CVF_{\Gamma_{12}}(\delta,\tilde{\tau})\right\rangle \mid \delta\in U\right\} \tag{7}$$

Where $CVT_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$

$$CVI_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

$$CVF_{\Gamma_{12}}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\left(-\ln\left(1-r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

iii. Scalar Multiplication

$$\zeta.\Gamma=\left\{\delta,\left\langle CVT_{\Gamma}(\delta,\tilde{\tau}),CVI_{\Gamma}(\delta,\tilde{\tau}),CVF_{\Gamma}(\delta,\tilde{\tau})\right\rangle \mid \delta\in U\right\} \tag{8}$$

Where $CVT_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-p_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-\mu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$

$$CVI_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(q_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(\nu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

$$CVF_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(r_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(\eta_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

iv. Power

$$\Gamma^{\zeta}=\left\{\delta,\left\langle CVT_{\Gamma}(\delta,\tilde{\tau}),CVI_{\Gamma}(\delta,\tilde{\tau}),CVF_{\Gamma}(\delta,\tilde{\tau})\right\rangle \mid \delta\in U\right\} \tag{9}$$

Where $CVT_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(p_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(\mu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$

$$CVI_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-q_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-\nu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

$$CVF_{\Gamma}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-r_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\zeta\left(-\ln\left(1-\eta_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)};$$

Theorem 1: The properties for any two TCNS are as follows:

(1) $\Gamma_1 \oplus \Gamma_2 = \Gamma_2 \oplus \Gamma_1$

(2) $\Gamma_1 \otimes \Gamma_2 = \Gamma_2 \otimes \Gamma_1$

(3) $\lambda(\Gamma_1 \oplus \Gamma_2) = \lambda\Gamma_1 \oplus \lambda\Gamma_2$

(4) $(\lambda_1 + \lambda_2)\Gamma = \lambda_1\Gamma_1 \oplus \lambda_2\Gamma$

(5) $(\Gamma_1 \oplus \Gamma_2)^{\lambda} = {\Gamma_1}^{\lambda} \oplus {\Gamma_2}^{\lambda}$

(6) $\Gamma^{\lambda_1} \otimes \Gamma^{\lambda_2} = \Gamma^{\lambda_1+\lambda_2}$

Proof.

(1)
$$\Gamma_1 \oplus \Gamma_2 = \left\langle \begin{array}{l} \left(1-\exp\left(-\left(\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(1-\exp\left(-\left(\left(-\ln\left(1-p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array} \right\rangle$$

$$= \Gamma_2 \oplus \Gamma_1$$

(2)
$$\Gamma_1 \otimes \Gamma_2 = \left\langle \begin{array}{l} \left(1-\exp\left(-\left(\left(-\ln\left(p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\left(-\ln\left(1-r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array} \right\rangle$$

$$
= \left\langle \begin{array}{l}
\left(1-\exp\left(-\left(\left(-\ln\left(p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\left(-\ln\left(1-r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}
\end{array} \right\rangle
$$

$$= \Gamma_2 \otimes \Gamma_1$$

(3)

$$
\lambda(\Gamma_1 \oplus \Gamma_2) = \lambda \left\langle \begin{array}{l}
\left(1-\exp\left(-\left(\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\left(-\ln\left(\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}
\end{array} \right\rangle
$$

$$
= \left\langle \begin{array}{l}
\left(1-\exp\left(-\left(\lambda\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(1-p_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(1-\mu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\lambda\left(-\ln\left(1-q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(1-q_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda\left(-\ln\left(1-\nu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(1-\nu_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\lambda\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(r_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}+\lambda\left(-\ln\left(\eta_{\Gamma_2}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}
\end{array} \right\rangle
$$

$$= \lambda\Gamma_1 \oplus \lambda\Gamma_2$$

(4) $(\lambda_1+\lambda_2)\Gamma = \lambda_1\Gamma \oplus \lambda_2\Gamma$

$$
\lambda_1\Gamma \oplus \lambda_2\Gamma = \left\langle \begin{array}{l}
\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(1-p_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(1-\mu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(q_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(\nu_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\
\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(r_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_1\left(-\ln\left(\eta_{\Gamma}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}
\end{array} \right\rangle
$$

$$\oplus\left\langle\begin{array}{l}\left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(1-p_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(1-\mu_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(q_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(v_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(r_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\lambda_2\left(-\ln\left(\eta_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)}\end{array}\right\rangle$$

$$=\left\langle\begin{array}{l}\left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(1-p_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(1-\mu_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(q_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(v_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(r_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left((\lambda_1+\lambda_2)\left(-\ln\left(\eta_\Gamma(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)}\end{array}\right\rangle$$

$$=(\lambda_1+\lambda_2)\Gamma$$

(5) $(\Gamma_1\oplus\Gamma_2)^\lambda=\Gamma_1{}^\lambda\oplus\Gamma_2{}^\lambda$

It is obvious as (3)

(6) $\Gamma^{\lambda_1}\otimes\Gamma^{\lambda_2}=\Gamma^{\lambda_1+\lambda_2}$

It is obvious as (4)

Definition 5: The Temporal Commplex Neutrosophic Weighted Averaging Operator (TCN-A) has a computational form:

$$TCN-A(\Gamma_1,\Gamma_2,\Gamma_3,...,\Gamma_n)=\oplus_{k=1}^n(w_k\Gamma_k)=\left\langle\overline{CVT}(\delta,\tilde{\tau}),\overline{CVI}(\delta,\tilde{\tau}),\overline{CVF}(\delta,\tilde{\tau})\right\rangle \tag{10}$$

Where $\overline{CVT}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^n w_k\left(-\ln\left(1-p_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^n w_k\left(-\ln\left(1-\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)}$

$$\overline{CVI}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^n w_k\left(-\ln\left(q_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^n w_k\left(-\ln\left(v_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)}$$

$$\overline{CVF}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^n w_i\left(-\ln\left(r_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^n w_i\left(-\ln\left(\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^\theta\right)^{\frac{1}{\theta}}\right)\right)}$$

Theorem 2: Let a set of TCNS; $\Gamma=\Gamma_1,\Gamma_2,...,\Gamma_n$, then the result obtained by utilizing TCN-A operator is still a TCNS.

Proof.

Using Mathematical induction. Equation (10) holds because it simplifies the outcome to the trivial one, which is obviously TCNS when $n=1$,

$$TCN-A\left(\Gamma_1\right)=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(w_1\left(-\ln\left(1-p_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_1\left(-\ln\left(1-\mu_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_1\left(-\ln\left(q_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_1\left(-\ln\left(v_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_1\left(-\ln\left(r_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_1\left(-\ln\left(\eta_{\Gamma_1}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

Assume that equation (10) is true for $n=m$,

$$TCN-A\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_m\right)=$$

$$\left\langle \begin{array}{l} \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(1-p_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(1-\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(q_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(v_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(r_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

When $n=m+1$

$$TCN-A\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_n\right)=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(1-p_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(1-\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(q_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(v_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(r_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m}w_k\left(-\ln\left(\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

$$\oplus\left\langle \begin{array}{l} \left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-p_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-\mu_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(q_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(v_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(r_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(\eta_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-p_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(q_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(v_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(r_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle \square$$

Definition 6: The Temporal Commplex Neutrosophic Weighted Geometric Operator (TCN-G) has a computational form:

$$TCN-G\left(\Gamma_1,\Gamma_2,...,\Gamma_n\right)=\otimes_{k=1}^{n}\left(w_k\Gamma_k\right)=\left\langle \overline{CVT}(\delta,\tilde{\tau}),\overline{CVI}(\delta,\tilde{\tau}),\overline{CVF}(\delta,\tilde{\tau})\right\rangle \tag{11}$$

Where $\overline{CVT}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(p_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}$

$$\overline{CVI}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-q_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-v_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}$$

$$\overline{CVF}(\delta,\tilde{\tau})=\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-r_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n} w_k\left(-\ln\left(1-\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\rho}\right)^{\frac{1}{\theta}}\right)\right)}$$

Theorem 3: Let a set of TCNS; $\Gamma=\Gamma_1,\Gamma_2...,\Gamma_n$, then the result obtained by utilizing TCN-G is still a TCNS.

Proof.

Using Mathematical induction. Equation (11) holds because it simplifies the outcome to the trivial one, which is obviously TCNS when $n=1$,

$$TCN-A\left(\Gamma_1\right)=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(w_1\left(-\ln\left(p_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_1\left(-\ln\left(\mu_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_1\left(-\ln\left(1-q_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_1\left(-\ln\left(1-\nu_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(w_1\left(-\ln\left(1-r_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_i\left(-\ln\left(1-\eta_{\Gamma_1}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

Assume that equation (11) is true for $n=m$,

$$TCN-A\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_m\right)=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(p_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(\mu_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-q_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-\nu_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-r_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-\eta_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

When $n=m+1$

$$TCN-A\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_n\right)=\left\langle \begin{array}{l} \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(p_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(\mu_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-q_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-\nu_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}, \\ \left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-r_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{m} w_k\left(-\ln\left(1-\eta_{\Gamma_k}\left(\delta,\tau_s\right)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)} \end{array}\right\rangle$$

$$\otimes\left\langle\begin{array}{l}\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(p_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(\mu_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-q_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-\nu_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-r_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(w_{m+1}\left(-\ln\left(1-\eta_{\Gamma_{m+1}}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}\end{array}\right\rangle$$

$$=\left\langle\begin{array}{l}\left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(p_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(\mu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(1-q_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(1-\nu_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)},\\ \left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(1-r_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{k=1}^{n}w_k\left(-\ln\left(1-\eta_{\Gamma_k}(\delta,\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}\end{array}\right\rangle \quad \square$$

5. MCDM based on Aggregation Operators of TCNS

In this section, the paper presents an approach to MCDM using the proposed Aczel-Alsina TCNS operators. The MCDM problem on TCNS is defined as follow.

Assume $\tilde{D}=\{D_1,D_2,...,D_{n_D}\}$, $\tilde{C}=\{C_1,C_2,...,C_{n_C}\}$ and $\tilde{A}=\{A_1,A_2,...,A_{n_A}\}$ are sets of decision makers, criteria and alternatives. The evaluation characteristic of an alternatives A_{i_a} ; $i_a=1,2,...,n_A$ on an criteria C_{i_c} ; $i_c=1,2,...,n_C$ in time period τ_s ; $s=1,2,...,n_\tau$ by decision maker D_{i_d} ; $i_d=1,2,...,n_D$, is determined by matrix $X^{i_d}(\tilde{\tau})=\left(\theta^{i_d}_{i_ai_c}(\tilde{\tau})\right)_{n_A\times n_C}$. where $\theta^{i_d}_{i_ai_c}(\tau_l)$ taken language value of CNS on a time period τ_l .

Let $\theta^{i_d}_{i_ai_c}(\tau_s)=\left\langle p^{i_d}_{i_ai_c}(\tau_s)e^{j\mu^{i_d}_{i_ai_c}(\tau_l)},q^{i_d}_{i_ai_c}(\tau_s)e^{j\nu^{i_d}_{i_ai_c}(\tau_l)},r^{i_d}_{i_ai_c}(\tau_s)e^{j\eta^{i_d}_{i_ai_c}(\tau_s)}\right\rangle$

A model for MCDM on TCNS based on TOPSIS method and the proposed Aczel-Alsina TCNS operators includes 8 consequent steps. This MCDM model can be clearly described as follow.

Step 1. Normalize the evaluation matrix $X^{i_d}(\tau_s)$ to $\tilde{X}^{i_d}(\tau_s)$,

For benefit-type attributes C_{i_c}, $\psi^{i_d}_{i_ai_c}(\tau_s)=\left\langle p^{i_d}_{i_ai_c}(\tau_s)e^{j\mu^{i_d}_{i_ai_c}(\tau_s)},q^{i_d}_{i_ai_c}(\tau_s)e^{j\nu^{i_d}_{i_ai_c}(\tau_s)},r^{i_d}_{i_ai_c}(\tau_s)e^{j\eta^{i_d}_{i_ai_c}(\tau_s)}\right\rangle$

For cost-type attributes C_{i_c},

$\psi^{i_d}_{i_ai_c}(\tau_s)=\left\langle r^{i_d}_{i_ai_c}(\tau_s)e^{j\left(2\pi-\eta^{i_d}_{i_ai_c}(\tau_s)\right)},\left(1-q^{i_d}_{i_ai_c}(\tau_s)\right)e^{j\left(2\pi-\nu^{i_d}_{i_ai_c}(\tau_s)\right)},p^{i_d}_{i_ai_c}(\tau_s)e^{j\left(2\pi-\mu^{i_d}_{i_ai_c}(\tau_s)\right)}\right\rangle$

Step 2. According $\tilde{X}^{i_d}(\tau_l)$ matrix and the weight vector $w^d=\{w^d_1,w^d_2,...,w^d_{n_D}\}$ of decision makers, the aggregated decision making matrix $\overline{X}(\tau_l)$ of decision makers denoted as (utilizing it within the working space).

$$\overline{X}(\tau_l)=\left(\theta_{i_a i_c}(\tau_s)\right)_{m\times n}=\left\langle p_{i_a i_c}(\tau_s)e^{j\mu_{i_a i_c}(\tau_s)}, q_{i_a i_c}(\tau_s)e^{j\nu_{i_a i_c}(\tau_s)}, r_{i_a i_c}(\tau_s)e^{j\eta_{i_a i_c}(\tau_s)}\right\rangle \quad (12)$$

Step 3. Determines the average solution $AvS=\left(AvS_j\right)_{1\times n_C}$ with $AvS_j=\frac{1}{n_A}\bigoplus_{i_a=1}^{n_A}\psi_{i_a i_c}(\tau_l)$.

Based on Definition 4, 5, we can get

$$AvS_j(\tilde{\tau})=\frac{1}{n_A}\bigoplus_{i_a=1}^{n_A}\theta_{i_a i_c}(\tau_s)=\left\langle \overline{T}_{AvS_j}, \overline{I}_{AvS_j}, \overline{F}_{AvS_j}\right\rangle \quad (13)$$

$$\overline{T}_{AvS_j}=\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(1-p_{i_a}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(1-\mu_{i_a}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

$$\overline{I}_{AvS_j}=\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(q_{i_a}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(v_{i_a}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

$$\overline{F}_{AvS_j}=\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(r_{i_a}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right).e^{j2\pi\left(1-\exp\left(-\left(\sum_{i_a=1}^{n_A}\frac{1}{n_A}\left(-\ln\left(\eta_{\Gamma_i}(\tau_s)\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)\right)}$$

Step 4. Compute $PDA=\left(P_{i_a i_c}\right)_{n_A\times n_C}$ and $NDA=\left(N_{i_a i_c}\right)_{n_A\times n_C}$ are positive/negative distance from average

$$PDA_{i_A i_C}=\frac{\max\left\{0, s\left(\theta_{i_A i_C}(\tilde{\tau})\right)-s\left(AvS_j(\tilde{\tau})\right)\right\}}{s\left(AvS_j(\tilde{\tau})\right)};\ NDA_{i_A i_C}=\frac{\max\left\{0, s\left(AvS_j(\tilde{\tau})\right)-s\left(\theta_{i_A i_C}(\tilde{\tau})\right)\right\}}{s\left(AvS_j(\tilde{\tau})\right)}; \quad (14)$$

Where $s\left(\theta_{i_A i_C}(\tilde{\tau})\right)$ and $s\left(AvS_j(\tilde{\tau})\right)$ are score functions,

Step 5. Compute weighted sums *SP* and *SN* from *PDA* and *NDA*.

$$SP_{i_A}=\sum_{i_C=1}^{n_C}w_{i_C}PDA_{i_A i_C};\ SN_{i_A}=\sum_{i_C=1}^{n_C}w_{i_C}NDA_{i_A i_C}; \quad (15)$$

Where w_{i_C} is the weight of the creteria i_C

Step 6. Normalize SP_{i_A}, SN_{i_A} to obtain NSP_{i_A}, NSN_{i_A}

$$NSP_{i_A}=\frac{SP_{i_A}}{\max_{i_A}\left\{SP_{i_A}\right\}};\ NSN_{i_A}=\frac{SN_{i_A}}{\max_{i_A}\left\{SN_{i_A}\right\}}; \quad (16)$$

Step 7. Compute the score values S_{i_A} ; $i_{A=1,2,3,\dots,m}$ for all alternatives.

$$S_{i_A}=\frac{1}{2}\left(NSP_{i_A}+NSN_{i_A}\right); \quad (17)$$

Step 8. Ranking the alternatives according to the values of S_{i_A}

6. Application

6.1. The case study

In this study, we adopt an illustrative case based on Lan et al.'s work (2023) [20] to exemplify the practicality and efficiency of our proposed MCDM. The scenario we examine involves a company's decision process in selecting a tourist destination for the next summer within Vietnam. After initial pre-evaluation, the company selected set of candidates that includes five potential locations in example for evaluation. Any destination is judged according to a set of features that present the acttractiveness of a destination to traveller. These features are considered as criteria for decision maker in MCDM. To enhance the precision of the assessment, we identify the top twenty crucial criteria, that were proved to be best to Vietnamese tourist, which are as follows in Figure 1.

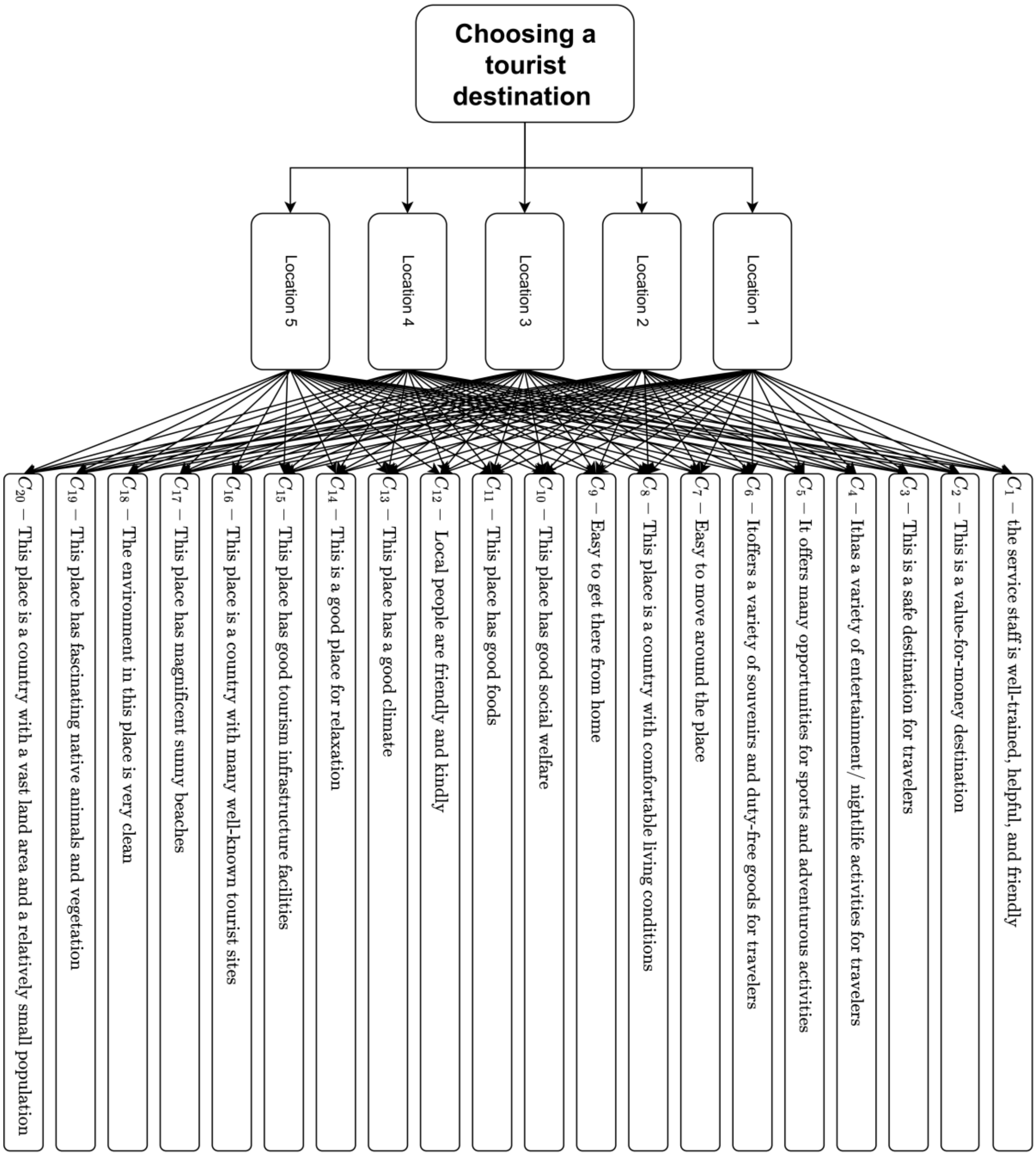

Figure 1. Tourist Destination Choice Problem

6.2. Address MCDM

The input data of this study is similar to the work presented in [20], and the new MCDM on TCNS has some remarkable steps as follow.

(1) We consider 20 criteria as benefit criteria within the experimental scope of the article

(2) Compute the value of the average solution using equation (12) with $\theta = 5$, the results of AvS shows in Table 1.

Table 1. The results of AvS

Criteria	AvS	Criteria	AvS	Criteria	AvS	Criteria	AvS
C_1	0.85544	C_6	0.86562	C_{11}	0.86694	C_{16}	0.87568
C_2	0.86356	C_7	0.85077	C_{12}	0.85654	C_{17}	0.87558
C_3	0.87216	C_8	0.86154	C_{13}	0.86779	C_{18}	0.86487
C_4	0.85742	C_9	0.84721	C_{14}	0.84973	C_{19}	0.86283
C_5	0.84606	C_{10}	0.85732	C_{15}	0.85001	C_{20}	0.86655

(3) Compute the positive/negative distance from average using equation (14). Table 2 shows the results of PDA and NDA for 5 locations.

Table 2. The results of PDA and NDA

Criteria	L_1		L_2		L_3		L_4		L_5	
	PDA	*NDA*	*PDA*	*NDA*	*PDA*	*NDA*	*PDA*	*NDA*	*PDA*	*NDA*
C_1	0.03178	0.00000	0.01413	0.00000	0.03154	0.00000	0.03019	0.00000	0.03019	0.00000
C_2	0.03004	0.00000	0.01474	0.00000	0.02207	0.00000	0.02050	0.00000	0.00735	0.00000
C_3	0.01200	0.00000	0.00000	0.00349	0.00674	0.00000	0.00000	0.00007	0.01634	0.00000
C_4	0.00413	0.00000	0.00000	0.04281	0.02970	0.00000	0.02780	0.00000	0.02780	0.00000
C_5	0.02819	0.00000	0.02819	0.00000	0.04161	0.00000	0.00000	0.02652	0.04161	0.00000
C_6	0.01964	0.00000	0.00000	0.04852	0.01765	0.00000	0.01109	0.00000	0.01807	0.00000
C_7	0.03174	0.00000	0.00590	0.00000	0.03541	0.00000	0.02874	0.00000	0.02874	0.00000
C_8	0.02288	0.00000	0.00806	0.00000	0.02247	0.00000	0.02288	0.00000	0.02288	0.00000
C_9	0.05082	0.00000	0.05317	0.00000	0.03776	0.00000	0.03232	0.00000	0.02937	0.00000
C_{10}	0.03109	0.00000	0.01435	0.00000	0.02792	0.00000	0.03774	0.00000	0.03774	0.00000
C_{11}	0.01652	0.00000	0.00068	0.00000	0.02202	0.00000	0.00955	0.00000	0.01652	0.00000
C_{12}	0.02886	0.00000	0.02886	0.00000	0.02886	0.00000	0.02886	0.00000	0.02886	0.00000
C_{13}	0.01116	0.00000	0.00496	0.00000	0.01709	0.00000	0.00856	0.00000	0.01552	0.00000
C_{14}	0.04206	0.00000	0.04206	0.00000	0.03711	0.00000	0.03000	0.00000	0.03000	0.00000
C_{15}	0.03598	0.00000	0.02341	0.00000	0.03966	0.00000	0.02966	0.00000	0.02966	0.00000
C_{16}	0.00000	0.00659	0.00000	0.00052	0.01245	0.00000	0.00637	0.00000	0.00000	0.00659
C_{17}	0.01130	0.00000	0.00000	0.00040	0.00081	0.00000	0.00649	0.00000	0.00000	0.00647
C_{18}	0.02382	0.00000	0.01160	0.00000	0.01320	0.00000	0.01124	0.00000	0.01895	0.00000
C_{19}	0.02015	0.00000	0.00807	0.00000	0.01560	0.00000	0.02452	0.00000	0.02452	0.00000
C_{20}	0.01860	0.00000	0.00929	0.00000	0.02665	0.00000	0.01001	0.00000	0.01698	0.00000

(4) Calculate the values of SP_{i_A} and SN_{i_A} using equation (15). Table 3 shows results of SP and SN values for 5 locations.

Table 3. The results of SP and SN

Location	SP	SN
L_1	0.02354	0.00033
L_2	0.01337	0.00479
L_3	0.02432	0.00000
L_4	0.01883	0.00133
L_5	0.02205	0.00065

(5) Normalize SP_{i_A} and SN_{i_A} values to obtain NSP_{i_A} and NSN_{i_A} using equations (16). Using eqaution (17) to calculate score values and the results shows in Table 5.

Table 5. The results of NSP and NSN and S

Location	NSP	NSN	S	Ranking
L_1	0.96802	0.06886	0.51844	4
L_2	0.55002	1	0.77501	1
L_3	1	0	0.5	5

L_4	0.77425	0.27770	0.52598	2
L_5	0.90700	0.13648	0.52174	3

6.3. Comparison

Herein, to illustrate the impact of our novel operators, we compare our model's performance to Lan et al. [20]. Table 6 shows that the ranking of 5 locations are $L_2 \succ L_1 \succ L_3 \succ L_5 \succ L_4$ and L_2 is best location. Meanwhile, the result of the our model gives the raking $L_2 \succ L_4 \succ L_5 \succ L_1 \succ L_3$ and L_2 is best location. This result shows that the proposed model uses the Aczel-Alsina aggregation operators which works well in temporal complex neutrosophic environments.

Table 6. Compare table at ANP - TCNS - TOPSIS

Location	Score	Ranking
L_1	0.166891	2
L_2	0.175318	1
L_3	0.157247	3
L_4	0.145068	5
L_5	0.157223	4

7. Conclusions

Among other t-norm and t-conorm, the Aczel-Alsina t-norm and t-conorm have the advantage of being adjustable by changing a parameter. To take this advantage, this paper presents some novel aggregation operators of Temporal complex neutrosophic set (TCNS) based on Aczel-Alsina t-norm and t-conorm. A MCDM model utilizing the proposed TCNS aggregate operator is presented, this model is suitable for the domains in which time effect is matter and adjustable parameter of Aczel-Alsina t-norm and t-conorm can give benefit. Along with this theoretical model, to demonstration of the feasibility and soundness of addressing MCDM problems through the application of the proposed model, a real-world case study involving the selection of a tourist destination in Vietnam, accompanied by comparative analysis. The result of case study showed that proposed aggregation operators according to Aczel-Alsina t-norm and t-conorm for TCNS has pratical potential. However, in practical this result is limited in a case study and it can be evaluating in a larger scale. The future work of this study will focus on this aspect and will design a mechanism for tourist destination selection for wide range.

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