Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices

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Abstract – This study explores a new type of matrix called a range-symmetric Fermatean neutrosophic fuzzy matrix (FNFM), inspired by the concept of range-Hermitian matrices. We demonstrate that all FNFMs inherently possess a specific property we term "Pythagorean neutrosophic fuzzy," (PNFM) but the reverse is not always true. Furthermore, we delve into graphical representations of FNFMs with specific symmetry properties (kernel-symmetric (KS), column symmetric, and range-symmetric (RS)) and show that these properties hold for all isomorphic graphs. The study goes on to establish equivalent characterizations for range-symmetric FNFMs and identify conditions for KS FNFMs. We introduce a novel concept: k-KS and RS FNFMs. Examples illustrate that KS FNFMs inherently possess k-KS, but not necessarily the other way around. This research contributes to a deeper understanding of symmetric FNFM and their potential applications, highlighting their importance in mathematical and computational fields.

Keywords: PNFM, FNFM, NFM

1. Introduction

This paper delves into Generalized Symmetric FNFM, a recent development in representing uncertainty. We begin with the fundamental concept of FS, introduced by Zadeh [1], which use membership degrees to handle vagueness. Recognizing limitations in assigning non-membership values, Atanassov introduced intuitionistic fuzzy sets [2]. Smarandache further extended this framework with neutrosophic sets (NSs) to encompass indeterminacy [3]. Building on these ideas, Wang et al. [4, 5] proposed single-valued and interval-valued neutrosophic sets, expanding their applicability.

The concept of symmetric fuzzy matrices with properties based on range and kernel was explored by Kim and Roush [6]. They showed that range symmetry implies kernel symmetry, but not vice versa. Meenakshi [7] introduced FM with a fixed product, leading to further research on k-real and k-Hermitian matrices [8]. Baskett and Katz [9] and Schwerdtfeger [10] also contributed significantly to the field.

Recent studies by Meenakshi and colleagues [11, 12, 13, 14] explored various aspects of symmetric fuzzy matrices, including k-kernel symmetric and k-range symmetric properties. Sumathi and Arockiarani [15] proposed new operations on FNSM, while Meenakshi and Krishnamoorthy [16] introduced k-EP matrices. Jaya Shree [17] studied secondary κ-RSFM. Anandhkumar et al. [18] characterized Generalized Symmetric NFMs, and Broumi et al. [19] discussed Fermatean neutrosophic matrices. Silambarasan [20] further explored Fermatean fuzzy matrices.

Neutrosophic theory, introduced by Smarandache [3], embraces indeterminacy, acknowledging that truth, falsity, and indeterminacy can coexist. Neutrosophic sets address this by using membership degrees for truth, falsity, and indeterminacy. The study of matrices has evolved significantly to accommodate uncertainty, leading to the development of GSFNFM. This paper aims to explore the development of GSFNFM, discuss their theoretical foundations, mathematical properties, and potential applications. Anandhkumar [21] et.al, have studied Interval Valued Secondary k-Range Symmetric NFMs,

GSFNFM represent a novel approach to modeling uncertainty by combining Fermatean algebra, neutrosophic theory, and fuzzy logic. Fermatean algebra extends classical algebra to include three logical states: true, false, and indeterminate. This framework allows for a structured representation of uncertainty. We will explore the theoretical underpinnings of GSFNFM, their mathematical properties, and potential applications in various domains. Anandhkumar [22] et.al, have studied Secondary K- CSNFM.

1.1 Research Gap

In our research, we aim to introduce two innovative categories of NFM the RS-FNFM and the KS-FNFM. These matrices draw inspiration from EP-matrices within the complex domain and offer fresh perspectives on representing uncertainty and indeterminacy.

Our study establishes that while every FNFM qualifies as a PNFM, the reverse relationship does not always hold. Additionally, we depict visual representations of KS, CS, and RS adjacency FNFM to demonstrate their versatility in various scenarios, particularly in depicting relationships within isomorphic graphs.

Furthermore, we introduce RS-FNFM and derive environments for KS-FNFM, shedding light on their characteristics and interrelations. This exploration aids in comprehending the basic structures and constraints foremost these matrices.

Moreover, we present the idea of k-KS and RS-FNFM, providing illustrations that illustrate their relations. Exactly, we demonstrate that KS implies k-KS, thereby deepening our understanding of the interactions between different forms of SNFM.

Our research expands the comprehension of symmetric NFM and their practical applications, mostly in mathematical and computational sciences. By presenting novel matrix types and investigating their properties, we pay to forward both theoretic frameworks and applied uses of NFM. This underscores the worth of our findings in proceeding the understanding and utilization of SFM.

Prior research has laid a foundation for understanding symmetric fuzzy matrices, including explorations of k-kernel symmetric and k-range symmetric properties [11, 12, 13, 14]. However, the specific application of these symmetry concepts to Fermatean neutrosophic fuzzy matrices (FNFMs) remains underexplored. This gap in knowledge motivates our current investigation.

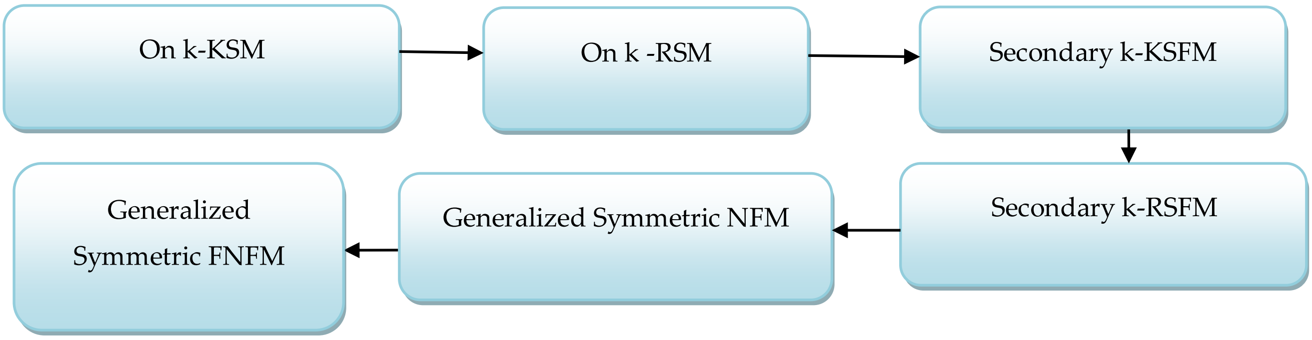
Building on the work of Anandhkumar et al. [18] who presented range and KS to NFM, we extend these principles to FNFMs. While their work represents a significant advancement, a critical research gap persists regarding generalized symmetric properties in FNFMs. In particular, no prior research has investigated how range and KS principles apply to FNFMs, nor have the properties of such matrices been well-characterized.

Our study addresses this gap by:

We propose two new types of FNFMs: RS-FNFM) and KS-FNFM. These matrices draw inspiration from EP-matrices in the complex domain and offer a fresh perspective for representing uncertainty and indeterminacy, especially when neutrosophic logic is relevant. We go beyond simply introducing new matrix types. Our research explores the connections between these novel FNFMs and existing concepts. We will establish that all FNFMs possess a property we call "Pythagorean NFM," but the converse is not always true. This distinction provides valuable insights into the characteristics of these matrices. To enhance understanding and highlight their applicability, we will explore graphical representations of KS, CS, and RS adjacency FNFMs. This visualization is particularly useful in representing relationships within isomorphic graphs. We will establish equivalent characterizations for RS-FNFM and identify conditions for KS-FNFM. These findings will provide a deeper understanding of the fundamental constructions and relationships leading these matrices. We will present the concept of k-KS and RS-FNFMs and illustrate their connections through examples. We will show that a kernel-symmetric FNFM inherently possesses k-KS, though the converse does not always hold. This analysis sheds light on the interplay between different types of symmetric NFM.

By addressing these research objectives, our work aims to significantly advance the understanding of symmetric NFM and their applications, particularly in mathematical and computational domains.

We introduce novel matrix types, explore their properties and relationships with existing concepts, and showcase their potential for representing uncertainty in various contexts.



1.2 Novelty

This study introduces several key elements that contribute to its originality and significance:

We propose two new types of matrices, RS-FNFM and KS-FNFM. Inspired by EP-matrices, these matrices offer a fresh perspective for representing uncertainty and indeterminacy, particularly when dealing with neutrosophic logic. Our research goes beyond simply introducing new matrices. We extend the well-established principles of symmetric matrices to the domain of FNFMs. This expansion leads to the development of equivalent characterizations for RS-FNFM and the identification of conditions for KS-FNFM. These findings provide deeper insights into the properties and underlying structures governing these novel matrices. We delve into the connections between different types of symmetric NFMs. For instance, we demonstrate that a kernel-symmetric FNFM inherently possesses k-KS, although the converse is not always true. This analysis sheds light on the intricate interplay between various symmetry properties within the framework of NFMs. To enhance understanding and showcase the applicability of these matrices, we explore graphical representations of KS, CS, and RS adjacency FNFMs.

This visualization is particularly valuable in representing relationships within isomorphic graphs. Our research identifies and addresses a critical gap in the existing literature. While prior studies have explored various types of symmetric fuzzy matrices, the specific application of these concepts to FNFM has remained largely unexamined. This work fills this void by introducing novel matrix types, exploring their properties, and highlighting their potential applications. By introducing these novel matrices and exploring their properties, we contribute to a more comprehensive understanding of symmetric NFM and their potential applications in mathematical and computational domains. This work advances the theoretical foundations of NFM and paves the way for their broader practical use. These elements collectively demonstrate the originality and significance of our research, offering valuable insights and expanding the existing framework for representing uncertainty in various fields.

1.3 Notations:

For FNFM of $P=[P_T,P_I,P_F]\in\left(FNFM\right)_n$

$[P_T,P_I,P_F]^T$: Transpose of $[P_T,P_I,P_F]$,

R($[P_T,P_I,P_F]$) : Row space of $[P_T,P_I,P_F]$,

C($[P_T,P_I,P_F]$) : Column space of $[P_T,P_I,P_F]$

N($[P_T,P_I,P_F]$) : Null Space of $[P_T,P_I,P_F]$,

$[P_T,P_I,P_F]^+$: Moore-Penrose inverse of $[P_T,P_I,P_F]$,

GSFNFM: Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices.

1.4. PRELIMINARIES: The permutation matrix K is satisfied using the subsequent

(P1) K = K^T , K^2 = I for all $P=[P_T,P_I,P_F]\in\left(FNFM\right)_n$

(P2) N($[P_T,P_I,P_F]$) = N($[P_T,P_I,P_F]$K) = N(K$[P_T,P_I,P_F]$)

(P3) $([P_T,P_I,P_F]K)^+=K[P_T,P_I,P_F]^+$ and $([P_T,P_I,P_F])^+=[P_T,P_I,P_F]^+$ K exists, if $[P_T,P_I,P_F]^+$ exists.

(P4) $[P_T,P_I,P_F]^T$ is a Generalized inverse of $[P_T,P_I,P_F]$ iff $[P_T,P_I,P_F]^+$ occur.

2. DEFINITIONS AND THEOREMS

Definition:2.1 (NFM): A NS P on the set X is well-defined as $P=\left\{<x,T,\mathrm{I},\mathrm{F}>,\mathrm{x}\in\mathrm{X}\right\}$, where

$T,I,F:X\rightarrow]^-0,1^+[$ and $0\le T+I+F\le 3$.

Example2.1: Consider a NFM $P=\begin{bmatrix}(0.7,0.8,0.5) & (0.2,0.4,0.6) & (0.3,0.7,0.4)\\(0.4,0.5,0.6) & (0.3,0.2,0.1) & (0.3,0.2,0.1)\\(0.1,0.2,0.3) & (0.7,0.2,0.1) & (0.2,0.2,0.2)\end{bmatrix}$

Definition 2.2 PNFM: PNFM P with m × n matrix is given by $P=[X_{ij},<T,I,F>]_{mxn}$ where $T,I,F\in[0,1]$ are referred to as the degrees of the truth, the indeterminacy, and the falsity of in P, which preserving the form $0\le T^2+I^2+F^2\le 2$ where $0\le T^2+F^2\le 2$ and $0\le I^2\le 1$.

Example2.2: Consider a PNFM $P = \begin{bmatrix} (1,1,0) & (0.5,0.3,0.4) & (0.3,0.4,0.1) \\ (0.6,1,0.2) & (0.4,0.1,0.6) & (1,1,0) \\ (0.5,0.5,1) & (1,1,0) & (0.4,0.4,0.5) \end{bmatrix}$

Definition 2.3 FNFM: FNFM with dimensions m × n is given by $P = [X, <T, I, F>]_{mxn}$ where $T, I, F \in [0,1]$ are referred to as the degrees of the truth, the indeterminacy, and the falsity of in P, which preserving the state $0 \le T^3 + I^3 + F^3 \le 2$ where $0 \le T^3 + F^3 \le 1$ and $0 \le I^3 \le 1$.

Example2.3: Consider a NFM $P = \begin{bmatrix} <0.7,0.7,0.8> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.3,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}$ is not PNFM but it P is a FNFM.

$(0.7)^2 + (0.7)^2 + (0.8)^2 = 1.62 \le 2$, $(0.7)^2 + (0.8)^2 = 1.13 > 1$ not PNFM

$(0.7)^3 + (0.7)^3 + (0.8)^3 = 1.198 \le 2$, $(0.7)^3 + (0.8)^3 = 0.855 < 1$ is FNFM

Therefore every FNFM is PNFM but converse need not be true.

Definition: 2.4 Let $P = [P_T, P_I, P_F] \in (FNFM)_n$ be a FNFM, if R $[[P_T, P_I, P_F]]$ = R $[[P_T, P_I, P_F]^T]$ and R $[[P_T, P_I, P_F]]$ = R $[[P_T, P_I, P_F]^T]$ then $P = [P_T, P_I, P_F]$ is called as RS.

Example: 2.4 Consider a FNFM

$$P = [P_T, P_I, P_F] = \begin{bmatrix} <0.4,0,1> & <1,0,1> & <0.2,0.3,0.4> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.2,0.3,0.4> & <1,0,1> & <1,0,1> \end{bmatrix}$$

Here, R $[\big[(0.2,0.4,0.4)\ (0.1,0.2,0.3)\ (0.1,0.2,0.3)\big] \notin R([P_T, P_I, P_F]^T)]$ = R $[[P_T, P_I, P_F]^T]$

The following matrix does not meet the RS-FNFM condition.

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.3,0,0.2> & <0.1,0.2,0.3> & <0.5,0.6,0.4> \\ <0.1,0.2,0.3> & <0.1,0.2,0.3> & <0.1,0.2,0.3> \\ <0.2,0.4,0.4> & <0.1,0.2,0.3> & <0.1,0.2,0.3> \end{bmatrix},$$

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.3,0,0.2> & <0.1,0.2,0.3> & <0.2,0.4,0.4> \\ <0.1,0.2,0.3> & <0.1,0.2,0.3> & <0.1,0.2,0.3> \\ <0.5,0.6,0.4> & <0.1,0.2,0.3> & <0.1,0.2,0.3> \end{bmatrix},$$

$\big[(0.3,0,0.2)\ (0.1,0.2,0.3)\ (0.5,0.6,0.4)\big] \in R([P_T, P_I, P_F])$,

$\left[(0.3,0,0.2)\ (0.1,0.2,0.3)\ (0.5,0.6,0.4)\right] \notin R([P_T,P_I,P_F]^T)$

$\left[(0.1,0.2,0.3)\ (0.1,0.2,0.3)\ (0.1,0.2,0.3)\right] \in R([P_T,P_I,P_F])$,

$\left[(0.1,0.2,0.3)\ (0.1,0.2,0.3)\ (0.1,0.2,0.3)\right] \in R([P_T,P_I,P_F]^T)$

$\left[(0.2,0.4,0.4)\ (0.1,0.2,0.3)\ (0.1,0.2,0.3)\right] \in R([P_T,P_I,P_F])$,

$\left[(0.2,0.4,0.4)\ (0.1,0.2,0.3)\ (0.1,0.2,0.3)\right] \notin R([P_T,P_I,P_F]^T)$

$R([P_T,P_I,P_F]) \notin R([P_T,P_I,P_F]^T)$

Note:2.1 For FNFM P with det $[P_T,P_I,P_F]$ > <0,1,1> has non- null rows columns, hereafter N($[P_T,P_I,P_F]$) = <0,1,1> = N($[P_T,P_I,P_F]^T$). Additionally, a SM $[P_T,P_I,P_F]$ = $[P_T,P_I,P_F]^T$ that is N($[P_T,P_I,P_F]$)= N($[P_T,P_I,P_F]^T$).

Definition : 2.5 Let $P=[P_T,P_I,P_F]\in\left(FNFM\right)_n$ if N($[P_T,P_I,P_F]$) = N($[P_T,P_I,P_F]^T$) and P is said to be KS- FNFM where N($[P_T,P_I,P_F]$) = {y/y$[P_T,P_I,P_F]$ = (0,1,1) and y∈ $F_{1\times n}$}.

Example: 2.5 Consider a FNFM

$$[A_T,A_I,A_F]=\begin{bmatrix} <0.4,0.4,0.6> & <0.6,0.7,0.7> & <0.5,0.6,0.7> \\ <0.6,0.8,0.7> & <0.7,0.9,0.2> & <0.3,0.7,0.2> \\ <0.7,0.6,0.7> & <0.5,0.6,0.6> & <0.5,0.5,0.6> \end{bmatrix}$$

$N([P_T,P_I,P_F]) = N([P_T,P_I,P_F]^T) = (0,1,1).$

Definition 2.6. Symmetric FNFM. If $P=[P_T,P_I,P_F]\in\left(FNFM\right)_n$ is called SFNFM if $p_{ij} = p_{ji}$.

Example: 2.6 Consider a FNFM

$$[P_T,P_I,P_F]=\begin{bmatrix} <0.5,0,1> & <1,0,1> & <0.5,0.6,0.7> \\ <1,0,1> & <1,0,1> & <1,0,1> \\ <0.5,0.6,0.7> & <1,0,1> & <1,0,1> \end{bmatrix}$$

Here, $[P_T,P_I,P_F]$ = $[P_T,P_I,P_F]^T$

Definition 2.7. Permutation NFM

A NFPM is a square matrix where every row and every column contain exactly one <1,1,0> entry, with all other entries being <0,0,1>.

Example: 2.7 Consider a NFPM,

$$K=\begin{bmatrix}(0,0,1) & (0,1,1) & (0,0,1)\\ (0,0,1) & (0,0,1) & (1,1,0)\\ (1,1,0) & (1,0,0) & (0,0,1)\end{bmatrix}$$

3**. Graphical Representation of Range symmetric, CS and KS Adjacency NFM.**

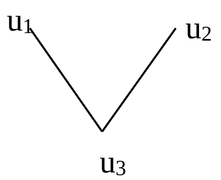
Definition 3.1. Adjacency FNFM

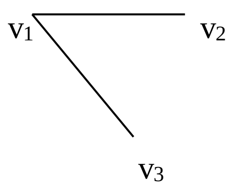
An adjacency FNFM is a square matrix used to represent a finite graph. The elements of this matrix indicate whether pairs of vertices in the graph are connected. For a finite simple graph, this matrix can be defined as a binary matrix, often referred to as a (1,1,0) and (0,0,1) matrix, where all diagonal elements are consistently set to (0,0,1). Let G(V, E) represent a simple graph with n vertices. The adjacency matrix P = [Pij] is a SM defined P = [Pij] = (1,1,0) vi is adjacent to vj and (0,0,1) otherwise denoted by P(G).

Example: 3.1 Consider a FNFM

$$A_G=\begin{bmatrix}(0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (0,0,1) & (1,1,0)\\ (1,1,0) & (1,1,0) & (0,0,1)\end{bmatrix}, A_H=\begin{bmatrix}(0,0,1) & (1,1,0) & (1,1,0)\\ (1,1,0) & (0,0,1) & (0,0,1)\\ (1,1,0) & (0,0,1) & (0,0,1)\end{bmatrix}$$

Equivalent adjacency graph.





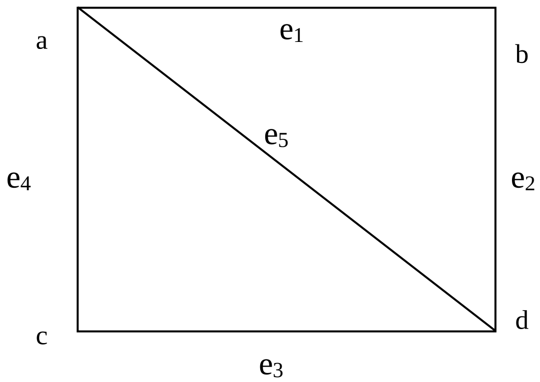
Definition 3.2. Incidence FNFM

The incidence NFM I = [m_{ij}] is a $n \times m$ matrix defined by I = [mij] = (1,1,0), vi is incident to vj and (0,0,1) otherwise denoted by P(G).

Example:3.2 Consider a FNFM and its equivalent graph.

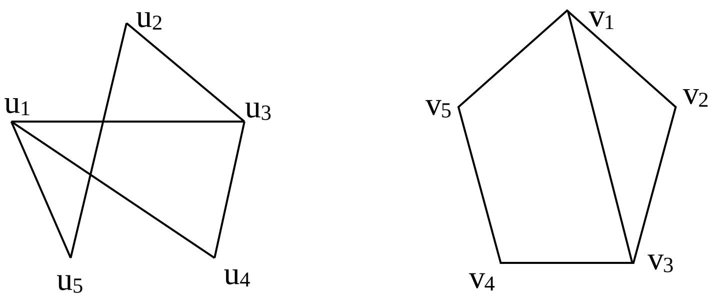
$$I=\begin{matrix}a\\ b\\ c\\ d\end{matrix}\begin{bmatrix}(1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0)\\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1)\\ (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1)\\ (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0)\end{bmatrix}$$

Corresponding graph



3.1 Relation between isomorphism , non-isomorphism and KS, RS and CS- FNFM

Graph: I

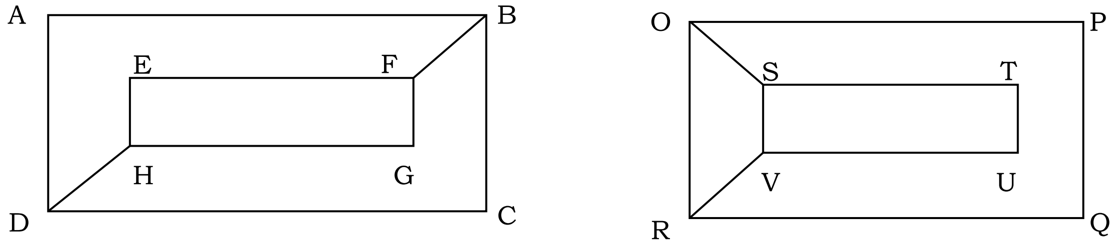


$$A_U = \begin{bmatrix} & u_1 & u_2 & u_3 & u_4 & u_5 \\ u_1 & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) \\ u_2 & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) \\ u_3 & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \\ u_4 & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ u_5 & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

$$A_V = \begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 \\ v_1 & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) \\ v_2 & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) \\ v_3 & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) \\ v_4 & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) \\ v_5 & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

Graph: I Two graphs have the equal number of vertices, the equal number of edges, the equal degree sequence, and the FNFM are equal. Consequently, the given graphs are isomorphic and also KS, CS, RS adjacency FNFM.

Graph: II



$$A_U = \begin{bmatrix} & O & R & P & Q & S & V & T & U \\ O & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ R & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) \\ P & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ Q & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ S & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \\ V & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ T & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ U & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \end{bmatrix}$$

$$A_V = \begin{bmatrix} & A & B & D & C & E & F & H & G \\ A & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ B & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) \\ D & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) \\ C & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ E & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \\ F & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ H & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ G & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) \end{bmatrix}$$

Two graphs have the equal number of vertices, the equal number of edges, the degree sequence are not equal.

Consequently, the graphs II, G and H are not isomorphic.

4.Theorems and Results

Theorem:4.1 For a FNFM $P=[P_T,P_I,P_F]$, $Q=[Q_T,Q_I,Q_F]$ and K be a NFPM if

$$N([P_T,P_I,P_F])=N([Q_T,Q_I,Q_F]) \Leftrightarrow N(K[P_T,P_I,P_F]\,K^T)=N(K[Q_T,Q_I,Q_F]K^T).$$

Proof: Let $z \in N(K[P_T,P_I,P_F]K^T)$

$$\Rightarrow z(K[P_T,P_I,P_F]K^T)=(0,1,1)$$

$\Rightarrow xK^T = (0,1,1),$

everywhere $x = zK([P_T, P_I, P_F])$

$\Rightarrow x \in N(K^T)$

$\det K = \det K^T > (0,1,1)$

Consequently,

$N(K^T) = (0,1,1)$

Hereafter

$x = (0,0,1)$

$\Rightarrow zK([P_T, P_I, P_F]) = (0,1,1)$

$\Rightarrow zK \in N([P_T, P_I, P_F]) = N([Q_T, Q_I, Q_F])$

$\Rightarrow zK([Q_T, Q_I, Q_F])K^T = (0,0,1)$

$\Rightarrow z \in N(K([Q_T, Q_I, Q_F])K^T)$

$N(K[\mathrm{P}_\mu, \mathrm{P}_\lambda, \mathrm{P}_\nu]_L K^T) \subseteq N(K[Q_T, Q_I, Q_F]K^T)$

Also, $N(K[Q_T, Q_I, Q_F]K^T) \subseteq N(K[P_T, P_I, P_F]K^T)$

Consequently,

$N([P_T, P_I, P_F]) = N([Q_T, Q_I, Q_F]) \Leftrightarrow N(K[P_T, P_I, P_F]K^T) = N(K[Q_T, Q_I, Q_F]K^T)$

Conversely, if $N(K[P_T, P_I, P_F]K^T) = N(K[Q_T, Q_I, Q_F]K^T)$,

N $([P_T, P_I, P_F])$ = N (K^T (K$[P_T, P_I, P_F]$ K^T) K)

= N (K^T (K$([Q_T, Q_I, Q_F])$ K^T) K)

$N([P_T, P_I, P_F]) = N([Q_T, Q_I, Q_F])$

Example: 4.1 Consider a FNFM

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$$

$$K = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$[Q_T, Q_I, Q_F] = \begin{bmatrix} <0.5,0.3,0.7> & <0.4,0.3,0.6> & <0.3,0.4,0.4> \\ <0.5,0.7,0.5> & <0.4,0.8,0.2> & <0.2,0.5,0.2> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$$

$$K[P_T, P_I, P_F]K^T = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$\begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix} \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$$= \begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$$

Let $w \in N(K[P_T, P_I, P_F]K^T)$

$$w = [(0,0,1) \quad (0,0,1) \quad (0,0,1)]$$

By definition 2.7

$$\Rightarrow w(K[P_T, P_I, P_F]K^T)$$

$$= [(0,0,1) \quad (0,0,1) \quad (0,0,1)] \begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix} = (0,0,1)$$

$$\Rightarrow xK^T = (0,1,1) \text{ where } x = zK([P_T, P_I, P_F])$$

Hence

$$x = (0,1,1)$$

$$K[P_T,P_I,P_F]=\begin{bmatrix} <0.3,0.3,0.4> & <0.5,0.3,0.6> & <0.2,0.3,0.4> \\ <0.4,0.7,0.6> & <0.4,0.8,0.1> & <0.2,0.5,0.1> \\ <0.2,0.4,0.4> & <0.4,0.4,0.5> & <0.4,0.5,0.6> \end{bmatrix}$$

$$\Rightarrow zK([P_T,P_I,P_F])=(0,1,1)$$

$$\Rightarrow zK\in N([P_T,P_I,P_F])=N([Q_T,Q_I,Q_F])$$

$$\Rightarrow zK([Q_T,Q_I,Q_F])K^T=(0,0,1)$$

$$\Rightarrow z\in N(K([Q_T,Q_I,Q_F])K^T)$$

$$N(K[\mathrm{P}_\mu,\mathrm{P}_\lambda,\mathrm{P}_v]_L K^T)\subseteq N(K[Q_T,Q_I,Q_F]K^T)$$

Also, $N(K[Q_T,Q_I,Q_F]K^T)\subseteq N(K[P_T,P_I,P_F]K^T)$

Consequently,

$$N([P_T,P_I,P_F])=N([Q_T,Q_I,Q_F])\Leftrightarrow N(K[P_T,P_I,P_F]K^T)=N(K[Q_T,Q_I,Q_F]K^T)$$

Conversely, if $N(K[P_T,P_I,P_F]K^T)=N(K[Q_T,Q_I,Q_F]K^T)$,

N $([P_T,P_I,P_F])$ = N (K^T (K$[P_T,P_I,P_F]$ K^T) K)

= N (K^T (K$([Q_T,Q_I,Q_F])$ K^T) K)

$$N([P_T,P_I,P_F])=N([Q_T,Q_I,Q_F])$$

Theorem:4.2 For a FNFM $P=[P_T,P_I,P_F]\in(FNFM)_n$ and K be a NFPM if

$N([P_T,P_I,P_F])=N([P_T,P_I,P_F]^T)\Leftrightarrow$N(K$[P_T,P_I,P_F]$ K^T) = N(K $[P_T,P_I,P_F]^T$ K^T) .

Proof: The proof is like to theorem 4.1

Theorem: 4.3 For $P=[P_T,P_I,P_F]\in(FNFM)_n$ is KS- FNFM, then N($[P_T,P_I,P_F]$ $[P_T,P_I,P_F]^T$) =

N($[P_T,P_I,P_F]$) = N($[P_T,P_I,P_F]^T[P_T,P_I,P_F]$).

Example: 4.2 Consider a FNFM

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.1, 0.3, 0.5> & <0.4, 0.3, 0.6> & <0.2, 0.3, 0.4> \\ <0.4, 0.7, 0.6> & <0.4, 0.7, 0.1> & <0.2, 0.5, 0.1> \\ <0.2, 0.3, 0.5> & <0.4, 0.3, 0.5> & <0.4, 0.2, 0.1> \end{bmatrix}$$

Theorem:4.4 For a FNFM $P = [P_T, P_I, P_F], Q = [Q_T, Q_I, Q_F] \in \left(FNFM\right)_n$ and K NFPM,

R ($[P_T, P_I, P_F]$) = R ($[Q_T, Q_I, Q_F]$) $\Leftrightarrow$ R (K$[P_T, P_I, P_F]$K^T) = R (K$[Q_T, Q_I, Q_F]$K^T) .

Proof: Let R ($[P_T, P_I, P_F]$) = R ($[Q_T, Q_I, Q_F]$)

Then, R ($[P_T, P_I, P_F]$ K^T) = R ($[P_\mu, P_\lambda, P_v]_L$) K^T

= R ($[P_T, P_I, P_F]$) K^T

= R ($[P_T, P_I, P_F]$ K^T)

Let $z \in \left\{R(K[P_T, P_I, P_F]K^T)\right\}$

$z = w(K[P_T, P_I, P_F]K^T)$ for some $w \in V^n$

$z = r[P_T, P_I, P_F]K^T, \; r = wK$

$z \in R\left([P_T, P_I, P_F]K^T\right) = R\left([Q_T, Q_I, Q_F]\left(K^T\right)\right)$

$z = u[Q_T, Q_I, Q_F]K^T$ for some $u \in V^n$

$z = \left(uK^T\right)K[Q_T, Q_I, Q_F]K^T$

$z = vK[Q_T, Q_I, Q_F]K^T$ for some $v \in V^n$

$z \in R(K[Q_T, Q_I, Q_F]K^T)$

Therefore, $R(K[P_T, P_I, P_F]K^T) \subseteq R(K[Q_T, Q_I, Q_F]K^T)$

Similarly, $R(K[Q_T, Q_I, Q_F]K^T) \subseteq R(K[P_T, P_I, P_F]K^T)$

Therefore, R(K$[P_T, P_I, P_F]$K^T) = R(K$[Q_T, Q_I, Q_F]$K^T)

Conversely, Let R(K$[P_T, P_I, P_F]$K^T) = R(K$[Q_T, Q_I, Q_F]$K^T)

$$R([P_T, P_I, P_F]) = R[K^T (K[P_T, P_I, P_F]K^T)K]$$

$$= R[K^T (K[Q_T, Q_I, Q_F]K^T)K]$$

$$= R([Q_T, Q_I, Q_F])$$

$$R([P_T, P_I, P_F]) = R([Q_T, Q_I, Q_F])$$

Example: 4.3 Consider a FNFM

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.5, 0.6, 0.4> & <0.3, 0.1, 0.2> & <0.2, 0.4, 0.5> \\ <0.3, 0.1, 0.2> & <0.4, 0.5, 0.1> & <0.2, 0.5, 0.5> \\ <0.2, 0.4, 0.5> & <0.2, 0.5, 0.5> & <0.4, 0.2, 0.1> \end{bmatrix}$$

$$[Q_T, Q_I, Q_F] = \begin{bmatrix} <0.2, 0.4, 0.5> & <0.2, 0.5, 0.5> & <0.4, 0.2, 0.1> \\ <0.3, 0.1, 0.2> & <0.4, 0.5, 0.1> & <0.2, 0.5, 0.5> \\ <0.5, 0.6, 0.4> & <0.3, 0.1, 0.2> & <0.2, 0.4, 0.5> \end{bmatrix}$$

$$K = \begin{bmatrix} (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \\ (1,1,0) & (0,0,1) & (0,0,1) \end{bmatrix}$$

Theorem:4.5 For $P = [P_T, P_I, P_F] \in (FNFM)_n$ be the FNFM and K be a NFPM, R($[P_T, P_I, P_F]$)

= R($[P_T, P_I, P_F]^T$) ⇔ R(K$[P_T, P_I, P_F]$K^T) = R(K $[P_T, P_I, P_F]^T$ K^T) .

Proof: The proof is comparable to theorem 4.4

Example: 4.4 Consider a FNFM

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.2, 0.1, 0.3> & <0.3, 0.3, 0.2> & <0.4, 0.5, 0.6> \\ <0.3, 0.3, 0.2> & <0.4, 0.7, 0.1> & <0.4, 0.5, 0.2> \\ <0.4, 0.5, 0.6> & <0.4, 0.5, 0.2> & <0.3, 1, 1> \end{bmatrix}$$

Theorem:4.6 For a FNFM $P = [P_T, P_I, P_F], Q = [Q_T, Q_I, Q_F] \in (FNFM)_n$ and K NFPM

C($[P_T, P_I, P_F]$) = C($[Q_T, Q_I, Q_F]$) ⇔ C(K$[P_T, P_I, P_F]$K^T) = C(K $[Q_T, Q_I, Q_F]$ K^T).

Proof: The proof is like to theorem 4.4

Example: 4.5 Consider a FNFM

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.1,0.2,0.3> & <0.4,0.5,0.1> & <0.3,0.2,0.1> \\ <0.1,0.4,0.5> & <0.3,0.1,0.8> & <0.6,0.5,0.4> \\ <0.2,0.3,0.1> & <0.8,0.2,0.1> & <0.5,0.6,0.2> \end{bmatrix}$$

$$[Q_T, Q_I, Q_F] = \begin{bmatrix} <0.3,0.2,0.1> & <0.4,0.5,0.1> & <0.1,0.2,0.3> \\ <0.6,0.5,0.4> & <0.3,0.1,0.8> & <0.1,0.4,0.5> \\ <0.5,0.6,0.2> & <0.8,0.2,0.1> & <0.2,0.3,0.1> \end{bmatrix}$$

$$[P_T, P_I, P_F] = \begin{bmatrix} <0.1,0.2,0.3> & <0.4,0.5,0.1> & <0.3,0.2,0.1> \\ <0.1,0.4,0.5> & <0.3,0.1,0.8> & <0.6,0.5,0.4> \\ <0.2,0.3,0.1> & <0.8,0.2,0.1> & <0.5,0.6,0.2> \end{bmatrix}$$

$$[Q_T, Q_I, Q_F] = \begin{bmatrix} <0.3,0.2,0.1> & <0.4,0.5,0.1> & <0.1,0.2,0.3> \\ <0.6,0.5,0.4> & <0.3,0.1,0.8> & <0.1,0.4,0.5> \\ <0.5,0.6,0.2> & <0.8,0.2,0.1> & <0.2,0.3,0.1> \end{bmatrix}$$

5.k-KERNEL SYMMETRIC IVNFM

Definition: 5.1 Let $P = [P_T, P_I, P_F] \in (FNFM)_n$ is said to be k-KS- FNFM if N($[P_T, P_I, P_F]$) = N(K$[P_T, P_I, P_F]^T$K) .

Theorem: 5.1 The subsequent conditions are equivalent for $P = [P_T, P_I, P_F] \in (FNFM)_n$

(i) N($[P_T, P_I, P_F]$) = N(K$[P_T, P_I, P_F]^T$K),

(ii) N(K$[P_T, P_I, P_F]$) = N((K$[P_T, P_I, P_F]$)^T),

(iii) N($[P_T, P_I, P_F]$K) = N(($[P_T, P_I, P_F]$K)^T),

(iv) N($[P_T, P_I, P_F]^T$) = N(K$[P_T, P_I, P_F]$),

(v) N($[P_T, P_I, P_F]$) = N(($[P_T, P_I, P_F]$K)^T),

(vi) $[P_T, P_I, P_F]^+$ is k-KSIVNFM,

(vii) N($[P_T, P_I, P_F]$) = N($[P_T, P_I, P_F]^+$K),

(viii) K $[P_T, P_I, P_F]^+[P_T, P_I, P_F]$ = $[P_T, P_I, P_F][P_T, P_I, P_F]^+$K,

(ix) $[P_T, P_I, P_F]^{+}[P_T, P_I, P_F]\mathrm{K} = \mathrm{K}[P_T, P_I, P_F]\,[P_T, P_I, P_F]^{+}$

Proof: The proof is like to Ref [12]

6. Comparison Study:

Fuzzy Matrices	**Neutrosophic Fuzzy Matrices (NFM)**	**GSFNFM**
Representation of Uncertainty		
This research leverages fuzzy sets, a mathematical framework that utilizes membership degrees between 0 and 1. These degrees quantify the extent to which an element belongs to a particular set, enabling the representation of uncertainty with varying levels of inclusion.	In NFM, the authors extend the capabilities of traditional fuzzy matrices by incorporating neutrosophic logic. Neutrosophic sets introduce membership degrees for truth (T), indeterminacy (I), and falsity (F), allowing us to capture not only vagueness but also ambiguity and inconsistency within a single framework.	This study introduces GSFNFM, a novel extension of neutrosophic fuzzy matrices. GSFNFM incorporates symmetric properties and Fermatean neutrosophic elements, offering a more comprehensive and nuanced approach to representing complex forms of uncertainty encountered in real-world scenarios.
Symmetry Properties		
Prior research on fuzzy or neutrosophic matrices may not have explicitly addressed symmetry properties as a core concept.	While some studies might have introduced the idea of symmetry, they likely did not delve into it as deeply as the current work.	This research emphasizes the significance of various symmetry properties, such as kernel symmetry, range symmetry, and column symmetry. By extensively exploring these properties, the study enhances our understanding and broadens the potential applications of the matrices.
Computational Complexity		
Many fundamental matrix operations, such as addition, multiplication, and inversion, have	Introducing neutrosophic elements into the matrix framework can potentially increase the computational complexity compared to	Incorporating symmetric properties like kernel or range symmetry can further add to the computational complexity. Specialized algorithms may be

well-established and efficient algorithms. These algorithms allow for fast and reliable computations involving traditional matrices.	traditional matrices. This is because neutrosophic elements involve additional membership degrees (truth, indeterminacy, falsity) compared to the single membership value used in classical matrices.	necessary to handle these properties efficiently when performing matrix operations involving GSFNFM. Developing such algorithms will be crucial for practical applications of GSFNFM.
Interpretability		
Traditional fuzzy sets offer a high degree of interpretability due to their use of single membership values between 0 and 1. These values directly correspond to the likelihood of an element belonging to a set, making the results easy to understand.	The introduction of neutrosophic elements, including indeterminacy and falsity memberships, can introduce some complexity into the interpretation process. Researchers and users need to be aware of the nuances of these additional degrees to avoid misinterpretations.	The complexity of symmetric properties, like kernel or range symmetry, can affect interpretability. While these properties offer valuable insights, understanding their impact on the overall meaning of the matrix might require advanced visualization techniques. These techniques can help to represent the data visually and enhance clarity, especially when dealing with intricate symmetric relationships.
Real-world Applications		
Traditional fuzzy set theory has proven valuable in numerous real-world applications, including decision-making, pattern recognition, and control systems. These applications leverage the ability of fuzzy sets to represent uncertainty with varying degrees of membership.	Neutrosophic sets extend the capabilities of fuzzy sets by incorporating indeterminacy and falsity memberships. This additional information makes them particularly well-suited for domains with inherent uncertainty and ambiguity, such as medical diagnosis and financial forecasting.	Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices (GSFNFM) represent a novel approach with the potential to address even more complex real-world scenarios. The combination of symmetric properties and Fermatean neutrosophic elements opens doors for applications in network analysis, image processing, and expert systems, where these features can play a significant role in modeling and reasoning with intricate forms

		of uncertainty.
Research Focus		
Much of the existing research in fuzzy logic and related fields has primarily focused on improving the efficiency of computational methods and expanding the applicability of these techniques to various real-world domains. This focus ensures that these tools can be used effectively in practical applications.	This research takes a broader perspective, shifting the focus towards enhancing the representational capabilities of these frameworks. By introducing concepts like neutrosophic elements, we aim to address complex uncertainty scenarios that may not be adequately captured by traditional methods.	Our study delves deeper by exploring the intricate interplay between symmetric properties and neutrosophic elements within the context of GSFNFM. This exploration opens doors for the development of novel algorithms specifically tailored for efficient manipulation of these matrices. Furthermore, by investigating these properties and algorithms, we aim to extend the applicability of GSFNFM to a wider range of diverse domains, enabling researchers and practitioners to leverage its strengths in tackling intricate problems.

Generalized Symmetric FFNFM emerge as a powerful new tool for representing and reasoning with uncertainty. This framework builds upon traditional fuzzy sets and neutrosophic fuzzy matrices by incorporating symmetric properties and FNFM elements. This innovation offers a more nuanced and comprehensive approach to capturing the complexities of uncertainty encountered in real-world scenarios.

The introduction of symmetric properties within GSFNFM unlocks new possibilities for analyzing relationships and structures. Furthermore, Fermatean neutrosophic elements provide a richer framework for representing situations involving not only vagueness but also indeterminacy and inconsistency.

While GSFNFM holds significant promise, future research is crucial to unlocking its full potential. In particular, further exploration is needed to validate its effectiveness through empirical studies and practical applications across diverse domains. Additionally, research efforts focused on developing efficient algorithms for GSFNFM operations will be essential for ensuring its widespread adoption.

In conclusion, GSFNFM represents a significant leap forward in the field of uncertainty modeling. By harnessing the strengths of prior frameworks and introducing novel features, GSFNFM paves the way for more robust and insightful decision-making processes in various fields grappling with intricate uncertainties.

7. CONCLUSION

Our investigation into Generalized Symmetric FNFM sheds light on a powerful tool for representing and reasoning with uncertain information. This novel framework holds significant promise for various fields, offering a more nuanced approach to decision-making under uncertainty.

Key Contributions:

We have delved into the theoretical foundations of GSFNFM, unveiling their core properties and relationships between different types, such as Range-Symmetric (RS-FNFM) and Kernel-Symmetric (KS-FNFM) matrices. Our exploration has introduced the concept of k-kernel-symmetric FNFMs (k-KS-FNFM) and illustrated their connections to KS-FNFM through examples. The graphical representation of adjacency and incidence FNFMs with specific symmetry properties provides valuable insights into their applicability, particularly in the context of isomorphic graphs.

Future works:

This research lays a solid foundation for further exploration of GSFNFM's potential. Future endeavors can extend our findings in several key directions: Investigating properties related to generalized inverses of k-Kernel Symmetric FNFMs can offer deeper understanding and potential applications. Devising efficient algorithms for performing matrix operations involving GSFNFM is crucial for practical applications. Integration with existing computational frameworks for uncertainty management holds significant promise. While theoretical advancements are important, further research should focus on validating the effectiveness of GSFNFM in real-world scenarios. Empirical studies demonstrating its practical utility across various domains are essential for broader adoption. The efficacy of GSFNFM relies on the quality and availability of data inputs. Future research should explore strategies for obtaining accurate and comprehensive data that can be effectively represented using GSFNFM, particularly in domains with inherent uncertainty and variability. By addressing these future directions, the potential of GSFNFM can be fully realized, leading to more robust and reliable decision-making processes in complex real-world problems. As we navigate the ever-present uncertainties in various fields, GSFNFM emerges as a powerful tool, paving the way for a more informed future.

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