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Developing Neutrosophic Cubic Spherical Fuzzy Sets along with their Exponential Aggregation Operators for Decision-Making Problems

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Abstract:

In this article, we have devised the notion of neutrosophic cubic spherical fuzzy sets (NCSFSs) for the first time and discussed their basic binary operations along with important properties. This proposition has been framed superimposing the existing notions of spherical neutrosophic set (SNS) as well as the interval-valued neutrosophic spherical fuzzy set (IVNSFS). This proposition computationally helps in handling incompatible situations where each element has been addressed by truth, indeterminacy, and false membership values. Next, some important exponential operational laws have been relationally established for NCSFSs, highlighting their significant properties. Furthermore, to address decision-making challenges in the NCSFS environment, we have developed exponential weighted aggregate operators based on the proposed & defined operational laws with important results. Finally, an algorithm for solving a decision-making problem has been presented by using the proposed exponential weighted aggregate operators, where the best site for waste material has been identified with the help of a numerical example by keeping the environmental factors into account. In the numerical example, we created a decision matrix based on experts' opinions and then applied the suggested NCSFWEA operators to a waste disposal site selection issue that has five alternatives and four attributes. At last, by utilizing the scoring function, the alternatives have been ranked.

Keywords: Spherical fuzzy set; Neutrosophic spherical fuzzy set; Aggregation operators; Multi-criteria decision making (MCDM); Solid waste disposal.

1. Introduction

As our society faces an ever-increasing volume and variety of solid wastes, addressing the obstacles of responsible management and sustainable disposal has become critical to shaping a cleaner, environmentally conscious future. Solid waste includes everyday objects like packaging for goods, green waste, textiles apparel, containers, debris from food, daily papers, equipment, color, and fuel cells [1]. If these materials are not removed, they may seriously harm the environment and the populace, particularly if they end up in the sewage system, water supply, or areas used for gathering and storing trash. Proper disposal is essential for a clean and sustainable environment, benefiting both community well-being and the earth's preservation. The process of choosing a waste material disposal technology is complicated and involves a multitude of qualitative and quantitative factors.

The primary cause is that managing the disposal of solid waste is not only a societal issue but also a multi-criteria decision problem that takes into account financial, technical, political, sociocultural, and environmental factors [2]. The population is rapidly growing, which is having a significant impact on the environment and threatening the globe's viability. Among the most visible effects of population growth is urban development. The growth of cities is affecting the natural balance of the environment. Every urban center produces a massive amount of solid matter [3]. According to Barzehkar et al. [4] and Kamdar et al. [5], landfills are an important component of the garbage management chain, which includes garbage bargaining, decomposition, recycling, reusing and disposal facilities. The GM (1, 1) model for pythagorean fuzzy numbers (PFNs) was presented by Li et al. [6], who also used their suggested model for solid waste management (SWM) site selection. Ren et al. [7] identified a novel multi-attribute decision- making (MADM) procedure for determining an appropriate location for SWM by applying the power Muirhead mean operator to a q-rung ortho-pair probabilistic hesitant fuzzy set. The REGIME method for Pythagorean fuzzy set (PFS) was first presented by Basar et al. [8] and employed to the selection of waste disposal sites. In order to have an overview of the generalizations/extensions of fuzzy sets, we may refer to the following Figure 1.

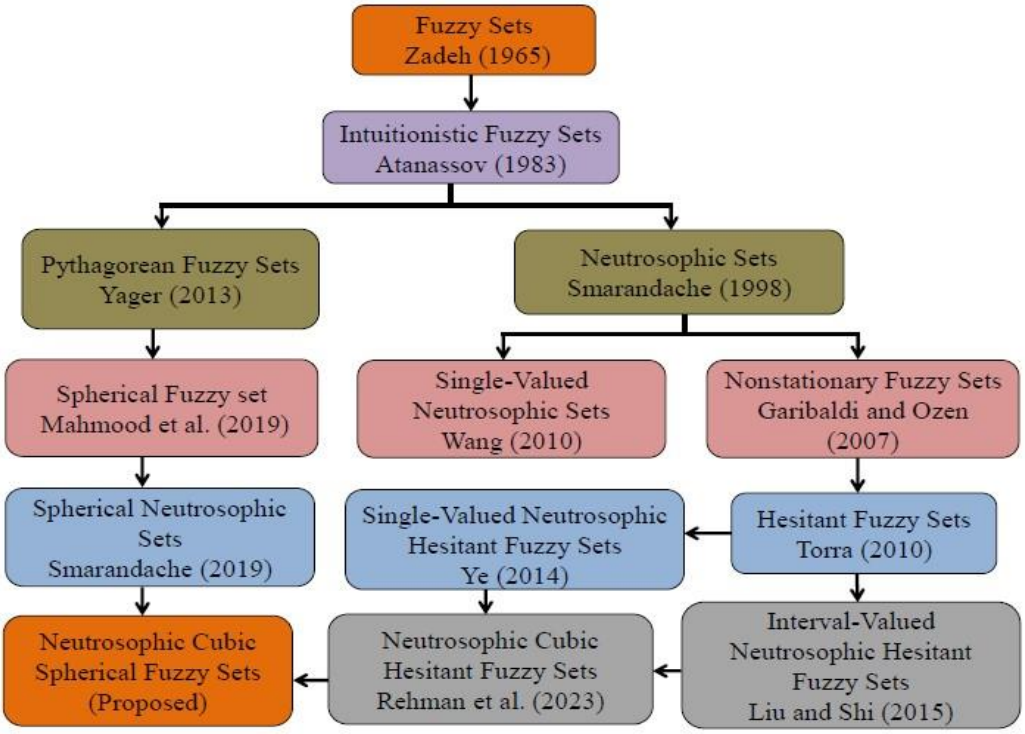

Fig 1: Overview on Generalized Extensions of Fuzzy Sets.

The most effective method for selecting a suitable alternative from all of the available options is the MCDM problem. Maximum evaluations were conducted in situations where goals and constraints are frequently ambiguous or unclear. In 1965, Zadeh [9] introduced a new way of dealing with vagueness and imperfectness called fuzzy set (FS) theory as a modified form of conventional theory. This theory is based on the concept of membership degree, which falls between zero and one. After that, he extended this theory to an interval-valued fuzzy set (IVFS). Since there are some limitations in this theory, many researchers have expanded it, which is discussed here. Attanasov [10] suggested intuitionistic fuzzy set (IFS) theory, a new form of the idea of fuzzy sets that includes both types of degrees: membership degrees and non-membership degrees. IFSs have been effectively implemented in numerous areas of science, including decision-making processes. Moreover, Smarandache [11] presented the first theory on the neutrosophic set (NS) in 1998, which one is a broadening of intuitionistic fuzzy sets and also fuzzy sets theory. This set presents the following kinds of membership functions, such as truth, indeterminacy, and falsity membership functions, which range from 0 to 1. Then in 1999, Molodtsov [12] introduced a new concept known as the soft set (SS) which is a generalization of fuzzy theory. One particular kind of the NS is the single-valued neutrosophic set (SVNS) created via Smarandache et al. [13] in 2010. Values of the membership function for a neutrosophic set can exceed 1. The truth membership value of a given scenario could be bigger than one, if one element of the neutrosophic set gets valued greater. A single-valued neutrosophic set has membership values ranging from 0 to 1, as well as the sum of membership values ranging from 0 to 3. Another fuzzy set extension that can be used as a practical tool is the hesitant fuzzy set (HFS) [14], which permits the membership degree to be a component of the fuzzy set and considers multiple membership grade values ranging from 0 to 1. These sets have been used to solve MCDM in uncertain environments. Many researchers [15, 16] investigated algebraic concepts and developed a potential theory in cubic sets. In 2012, Jun [17] utilized the IVFS and the FS to form the cubic set. The intuitionistic fuzzy set and the interval intuitionistic fuzzy sets are generalized into the cubic set. Cubic sets have become essential tools for dealing with ambiguous data. Then, neutrosophic soft set theory was developed by Maji [18] as a new notion with the help of SS and also provided basic operations and important properties. After that, Yager [19] extended this concept to Pythagorean fuzzy set (PFS) with the condition that the total squares of the membership and non-membership degree is equal to or lesser than 1. Further, there are many extensions of this theory, including picture fuzzy set (PFS) [20], generalized ortho-pair fuzzy set (OPFS) [21], spherical fuzzy set (SFS) [22], etc. Next, Ye [23] introduced the single-valued neutrosophic hesitant fuzzy set (SVNHFS), combining SVNS and HFS to handle challenges involving multi-attribute decision-making (MADM). Moreover, based on an interval neutrosophic hesitant set, Liu and Shi [24] proposed hybrid weighted average aggregation operators as an application of MADM challenges. Exponential operations along with aggregation operators for interval neutrosophic set (INS) are given by Ye [25], along with their application in decision-making obstacles. In 2016, Mehmood et al. [26] recommended a theory of cubic hesitant fuzzy sets. Smarandache et al. [27] expanded the cubic set into the neutrosophic cubic fuzzy set, which includes both the NS and the INS. After that for single-valued neutrosophic numbers, Lu and Ye [28] suggested the exponential laws. To convert the single-valued function into a multiple-valued one, Smarandache [29] introduced a hybridized version of the soft set called hypersoft sets (HSS). Next, in the environment of probabilistic single–valued neutrosophic hesitant fuzzy (PSVNHF) Shao et al. [30] developed the probabilistic interval neutrosophic hesitant fuzzy weighted averaging (PINHFWA) and geometric (PINHFWG) operators for dealing with decision–making challenges.

Next, to create the TOPSIS extension subject to fuzziness, Gundogdu and Kahraman [31] introduced new interval-valued spherical fuzzy set (IVSFS) with aggregate operators, which include IVSF weighted arithmetic as well as geometric mean operators. Further, Smarandache [32] also provided the spherical neutrosophic set (SNS), which integrates the spherical and Pythagorean fuzzy sets. Some new neutrosophic hesitant fuzzy set (NHFS) aggregation operators for MADM problems were developed by Liu and Luo [33]. Further, the stability evaluation with a neutrosophic cubic set for certain applications has been addressed by Al-Shumrani et al. [34]. In a neutrosophic cubic structures, Khan et al. [35] constructed exponential aggregation operators, which they then applied to MADM problems. By considering a spherical neutrosophic set (SNS), Molla and Giri [36] introduced the MCDM problem using the aggregation operator of SNS. Further, Dhumaras and Bajaj [37] proposed a new type of Hellinger divergence measure for symptomatic COVID-19 detection in the context of a single-valued neutrosophic hypersoft set. To promote sustainable soil practices in construction foundation projects, Ahmed et al. [38] introduced neutrosophic multi-criteria model as an application of life cycle assessment (LCA). Zenat et al. [39] presented a framework for sustainable supplier selection as a crucial part of supply chain management, encouraging ethical and environmentally friendly practices, using a neutrosophic multi-criteria decision-making methodology. Addressing vagueness and impreciseness in decision-making processes Dhumars and Bajaj [40] presented a bi-parametric discriminant measure for neutrosophic sets with application to symptomatic detection in medical diagnosis. After that, Kamran et al. [41] developed averaging and geometric aggregation operators coupled with confidence level within a probabilistic neutrosophic hesitant fuzzy rough environment to enhance the strategy for managing multi-criteria decision-making problems. Further, different types of aggregation operators proposed by Monika et al. [42] under the circumstances of T-spherical fuzzy hypersoft sets (TSFHSSs) also utilized these operators for choosing the best site in the context of renewable energy sources. After that, Muthukumaran [43] proposed n-hyperspherical neutrosophic matrices (n-HSNMs), expanding upon spherical neutrosophic matrices along with their algebraic properties and proof. To address environmental protection issues, Rehaman et al. [44] established exponential aggregate operators under neutrosophic cubic hesitant fuzzy sets. In 2024, Wang et al. [45] established a complex spherical fuzzy information-based selection framework for evaluating the food waste treatment system that includes mutual support and regularity decision information. Furthermore, Grag [46] proposed an exponential-logarithm-based SVNS that incorporates EL (exponential-logarithm) operations into the regular neutrosophic set. This integration aims to improve their effectiveness in solving decision-making problems. Next, for identifying the best alternative supplier type for strategic and non-strategic products categories, Onden et al. [47] provided micro mobility service using a type-2 neutrosophic number based decision-making approach.

From the literature, we observe that there are no theories or methods available that address cubic set theory in the context of a NCSFS, which is a hybrid of SNS and IVNSFS. This theory efficiently handles complex information in decision-making problems. Also, the decision-makers have more flexibility due to releasing the experts from the constraint, i.e., the sum of all uncertainty elements is less than 3. NCSFS and its aggregation operators are primarily intended to be used in real-life decision-making scenarios. This approach makes it possible to prepare ahead and minimize unfavorable effects in real-world situations like risk management, industrial operations, and artificial intelligence. Additionally, aggregation operators and exponential operational laws are in demand with urbanization growing rapidly and exponentially.

First, we have presented a new theory called the neutrosophic cubic spherical fuzzy set with some basic operations, this is a mixture of SNS and IVNSFS. Defined the accuracy function, the score function and the certainty function for NCSFS to compare the results. Next, we established exponential operational laws in NCSFS and presented important results. Moreover, we introduced a new aggregation operator known as the neutrosophic cubic spherical exponential weighted aggregation operator with the help of exponential laws and some theorems provided with proof. We then developed an algorithm based on proposed exponential aggregation operators and applied this in the application of decision-making challenges.

The remaining part of this paper is structured as follows. Some fundamental definitions and basic notions are presented in Section 2. We introduced a new notion/proposition coined as neutrosophic cubic spherical fuzzy sets (NCSFSs) in Section 3 and performed some fundamental operations on them. Then we presented some important exponential operational laws for NCSFSs along with some useful results in Section 4. Furthermore, we developed some exponential weighted aggregation operators for NCSFSs with important results in Section 5 for dealing with complex decision-making problems. Next, in Section 6, we presented an MCDM approach based on the proposed neutrosophic cubic spherical fuzzy weighted exponential aggregation operator (NCSFWEAO) and utilized this algorithm for the decision-making site selection issue of waste material disposal. Further, numerical simulations and computations for solid waste disposal site selection were conducted to better understand the proposed algorithms. Finally, the paper has been concluded in Section 7 outlining the scope for future work.

2. Basic Notions

This section has introduced few basic ideas which are helpful in the proposed work, such as such as spherical fuzzy set (SFS), interval-valued spherical fuzzy set (IVSFS), neutrosophic set (NS), spherical neutrosophic set (SNS) and neutrosophic cubic hesitant fuzzy set (NCHFS).

Definition 1. *[22] "Let $\mathfrak{R}$ represent the universal set. A spherical fuzzy set (SFS) ∇ is defined as:*

$$\nabla = \{< r, \mu_{\nabla}(r), \pi_{\nabla}(r), \vartheta_{\nabla}(r) > | r \in \mathfrak{R}\}$$

where $u_{\nabla}(r) : \mathfrak{R} \to [0,1]$ is the membership degree, $\pi_{\nabla}(r) : \to [0,1]$ is the indeterminacy degree and $\vartheta_{\nabla}(r) : \mathfrak{R} \to [0,1]$ is the non-membership degree and satisfies the condition s.t.

$$0 \leq (\mu_{\nabla}(r))^2 + (\pi_{\nabla}(r))^2 + (\vartheta_{\nabla}(r))^2 \leq 1."$$

Example: Suppose the universal set $\mathfrak{R} = \{r_1, r_2, r_3, r_4\}$ represents the 4 car company's brand.

Let the criteria be: $a_1 =$ average, $a_2 =$automatic and $a_3 =$ safety. Then, the spherical fuzzy set (∇), for determining the best car among the alternatives based on these criteria is defined as:

$$\nabla =< r_1\{average\ (0.7, 0.1, 0.2), automatic(0.2, 0.6, 0.5), saftey(0.8, 0.2, 0.3)\},$$
$$r_2\{average\ (0.5, 0.5, 0.6), automatic(0.7, 0.6, 0.3), saftey(0.7, 0.4, 0.2)\},$$
$$r_3\{average\ (0.9, 0.1, 0.1), automatic(0.1, 0.2, 0.8), saftey(0.8, 0.2, 0.1)\},$$
$$r_4\{average\ (0.8, 0.1, 0.2), automatic(0.9, 0.1, 0.2), saftey(0.9, 0.2, 0.2)\} >.$$

In this set each element is characterized by three parameters such as membership degree $\{\mu(r)\}$, indeterminancy degree $\{\pi(r)\}$ and non-membership degree $\{\nu(r)\}$ with condition s.t.

$$\mu^2(r) + \pi^2(r) + \nu^2(r) \leq 1.$$

Definition 2. [31] *"Let $\mathfrak{N}$ represent the universal set and I' is defined as an interval-valued spherical fuzzy set (IVSFS):*

$$I' = \{< r, \ ([\mu_{I'}(r)^L, \mu_{I'}(r)^U], \ [\pi_{I'}(r)^L, \pi_{I'}(r)^U], \ [\vartheta_{I'}(r)^L, \vartheta_{I'}(r)^U) | \ r \in \mathfrak{N} >]\}$$

where $1 \geq \ \mu_{I'}(r)^U \geq \mu_{I'}(r)^L \geq 0, \ 1 \geq \ \pi_{I'}(r)^U \geq \pi_{I'}(r)^L \geq 0, \ 1 \geq \vartheta_{I'}(r)^U \geq \vartheta_{I'}(r)^L \geq 0$ *and*

$$0 \leq (\mu_{I'}(r)^U)^2 + (\pi_{I'} \ (r)^U)^2 + (\vartheta_{I'}(r)^U)^2 \leq 1$$

For each $r \in \mathfrak{N}$, $\mu_{I'}(r)^U$, $\pi_{I'}(r)^U$ and $\vartheta_{I'}(r)^U$ are the upper membership degree, hesitancy degree and non-mebership degree of r to A respectively."

Example: Now based on the above example the interval-valued spherical fuzzy set is defined as;

$$I' = < r_1\{average \ ([0.7, 0.5], [0.3, 0.1], [0.4, 0.1]), automatic([0.3, 0.2], [0.7, \ 0.5], [0.5, 0.3]),$$
$$saftey([0.8, 0.5], [0.2, 0.1], [0.3, 0.1])\},$$
$$r_2\{average \ ([0.5, 0.2], [0.6, 0.3], [0.5, 0.4]), automatic([0.7, 0.5], [0.6, 0.3], [0.4, 0.2]),$$
$$saftey([0.6, 0.4], [0.4, 0.3], [0.2, 0.1])\}$$
$$r_3\{average \ ([0.8, 0.7], [0.2, 0.1], [0.2, 0.1]), automatic([0.1 \ \ 0.1], [0.2, 0.1], [0.8, 0.6]),$$
$$saftey([0.9, 0.7], [0.2, 0.1], [0.2, 0.1])\}$$
$$r_4\{average \ ([0.8, 0.7], [0.1, 0.1], [0.2, 0.1]), automatic([0.9, 0.7], [0.2, 0.2], [0.2, 0.1]),$$
$$saftey([0.8, 0.7], [0.2, 0.1], [0.2, 0.1])\} >.$$

Definition 3. *[11] "Suppose $\mathfrak{N}$ represent the universal set. Then a neutrosopic set (NS) is expressed as:*

$$\mathcal{N} = \{< r, \mu_{\mathcal{N}}(r), \pi_{\mathcal{N}}(r), \vartheta_{\mathcal{N}}(r) > | r \in \mathfrak{N}, u_{\mathcal{N}}(r), \ \pi_{\mathcal{N}}(r), \ \vartheta_{\mathcal{N}}(r) \in \ [\ 0^-, 1^+ \ [\}$$

where $u_{\mathcal{N}}(r)$, $\pi_{\mathcal{N}}(r)$ and $\vartheta_{\mathcal{N}}(r)$ are real standard element mapping from $\mathfrak{N}$ to $[0,1]$ called truth membership degree, indeterminacy membership degree and falsity membership degree respectively, with condition s.t.

$$0^- \leq \mu_{\mathcal{N}}(r) + \pi_{\mathcal{N}}(r) + \vartheta_{\mathcal{N}}(r) \leq 3^+."$$

Example: Suppose there is a need to evaluate the customer satisfaction with a newly launched smartphone model. Consider the set of customers $< X, Y, Z >$. Customer Feedback is collected and analyzed according to three criteria: Satisfaction, Indeterminancy, Dissatisfaction. Then the neutrosophic set is defined as:

$$\mathcal{N} = \{< X(0.8, 0.3, 0.2) >, < \ Y(0.5, 0.7, 0.7) >, < Z(0.5, 0.7, 0.5) >\};$$

where, all values of degree of truth, indeterminancy and falsity lies in [0, 1], and the sum of each components is less than or equal to 3. This example shows how neutrosophic sets provide a more detailed and nuanced approach to controlling uncertainty and variability in data, making them useful in complex decision-making scenarios due to their 3-D dimensional space.

Definition 4. *[32] "Let $\mathfrak{N}$ represent the universal set. The spherical neutrosophic set (SNS) A is defined as:*

$$A = \{< r, \mu_A(r), \pi_A(r), \vartheta_A(r) > | r \in \mathfrak{N}\}$$

where $u_A(r)$ mapping from $\mathfrak{R}$ to $[0,1]$ called truth membership degree, $\pi_A(r)$ mapping from $\mathfrak{R}$ to $[0,1]$ called indeterminant membership degree and $\vartheta_A(r)$ mapping from $\mathfrak{R}$ to $[0,1]$ called falsity membership degree and satisfies the following condition s.t.

$$0 \le \left(\mu_A(r)\right)^2 + \left(\pi_A(r)\right)^2 + \left(\vartheta_A(r)\right)^2 \le 3.\text{''}$$

Example: Based on the above example defined for neutosophic set, the spherical neutrosophic set is defined as:

$$A = \{< X(0.88, 0.78, 0.88) >, < Y(0.55, 0.29, 0.77) >, < Z(0.99, 0.59, 0.69) >\};$$

with the condition that sum of the square of each component is less than or equal to 3, i.e.

For customer X: $0.88^2 + 0.78^2 + 0.88^2 = 2.1572$.

For customer Y: $0.55^2 + 0.29^2 + 0.77^2 = 0.9795$.

For customer Z: $0.99^2 + 0.59^2 + 0.69^2 = 1.8043$.

Here, it can be noticed that the set meets the standard criteria for a spherical neutrosophic set.

Definition 5. *"Let $\mathfrak{R}$ represent the universal set. The object I= $\{< r, \mu_I(r), \pi_I(r), \vartheta_I(r) > | r \in \mathfrak{R}\}$ known interval-valued neutrosophic spherical fuzzy set (IVNSFS) on universal set, where $\mu_I(r)$, $\pi_I(r)$ and $\vartheta_I(r)$ are interval-valued spherical fuzzy set has the following expression:*

$$0 \le (\mu_I(r)^U)^2 + (\pi_I(r)^U)^2 + (\vartheta_I(r)^U)^2 \le 3.\text{''}$$

Example: Based on the above example, the IVNSFS is defined as:

$$I = \begin{cases} < X([0.98, 0.77\ ], [0.79, 0.67\ ], [0.59, 0.49]) >, \\ < Y([0.44, 0.39], [0.78, 0.66], [0.88, 0.67]) >, \\ < Z([0.99, 0.79], [0.68, 0.55], [0.88, 0.82]\ \ ) >. \end{cases}$$

3. Neutrosophic Cubic Spherical Fuzzy Sets (NCSFSs)

This section of the article presents the new notion of neutrosophic cubic spherical fuzzy sets (NCSFS) with some important basic operations. The definition of NCSFS is represented as follows:

Definition 6. *Consider $\mathfrak{R}$ represent the universal set. A NCSFS in $\mathfrak{R}$ is a set of values $\alpha =< I, A >$, where $I = \{< r, \mu_I(r), \pi_I(r), \vartheta_I(r) > | r \in \mathfrak{R}$ represents an interval valued neutrosophic spherical fuzzy set in $\mathfrak{R}$ and $A = \{< r, \mu_A(x), \pi_A(r), \vartheta_A(r) > | r \in \mathfrak{R}\}$ represent spherical neutrosophic set in $\mathfrak{R}$.*

Example 1. *Suppose $\mathfrak{R} = \{r, t\}$. The set of values $\alpha =< I, A >$ then,*

$\mu_I(r) = \{[0.2, 0.5]\}$, $\mu_A(r) = \{0.6\}$, $\pi_I(r) = \{[0.4, 0.7]\}$, $\pi_A(r) = \{0.7\}$, $\vartheta_I(r) = \{[0.2, 0.4]\}$, $\vartheta_A(r) = \{0.3\}$, then we have α corresponding to r is $\{< \{[0.2, 0.5]\}, \{[0.4, 0.7]\}, \{[0.1, 0.4]\}, \{(0.6,0.7, 0.3)\} >\}$.

$\mu_I(t) = \{[0.1, 0.5]\}$, $\mu_A(y) = \{0.7\}$, $\pi_I(t) = \{[0.3, 0.6]\}$, $\pi_A(y) = \{0.9\}$, $\vartheta_I(t) = \{[0.7, 0.9]\}$, $\vartheta_A(t) = \{0.3\}$, then we have α corresponding to t is $\{< \{[0.1, 0.5]\}, \{[0.3, 0.6]\}, \{[0.7, 0.9]\}, \{(0.7,0.9,0.3)\} >\}$.

Example 2 Assume there is a requirement for staff in the company. Four candidates have applied for the available position. The company's HR division assigns a three-person specialist team to handle hiring decisions. Let $\mathfrak{R} = \{r_1, r_2, r_3, r_4\}$ be a set of four candidates with the following attributes as: E_1 = Education, E_2 = Behavioral Interviews, and E_3 = Experience. Now, based on the expert's rating of the applicant corresponding to the attributes defined in the NCSFS.

Ratings by expert's.

- For, $< r_1$ {Education([0.25, 0.59], [0.54, 0.65], [0.66, 0.78], (0.35,0.72, 0.61))}
  {Behavioral Interviews([0.29, 0.47], [0.65, 0.75], [0.67, 0.86], (0.45,0.73, 0.54))},
  {Experience([0.69, 0.75], [0.42, 0.56], [0.55, 0.67], (0.67,0.53, 0.45))}>

- For, $< r_2$ {Education([0.85, 0.92], [0.25, 0.37], [0.12, 0.45], (0.86,0.51, 0.33))}
  {Behavioral Interviews([0.75, 0.89], [0.34, 0.47], [0.41, 0.54], (0.76,0.37, 0.53))},
  {Experience([0.78, 0.89], [0.24, 0.47], [0.31, 0.44], (0.70,0.47, 0.33))}>

- For, $< r_3$ {Education([0.52, 0.59], [0.54, 0.67], [0.51, 0.64], (0.56,0.57, 0.63))}
  {Behavioral Interviews([0.65, 0.79], [0.54, 0.67], [0.61, 0.74], (0.56,0.55, 0.53))},
  {Experience([0.68, 0.79], [0.34, 04.7], [0.51, 0.64], (0.56,0.57, 0.63)>

- For, $< r_4$ {Education([0.72, 0.85], [0.44, 0.57], [0.41, 0.54], (0.86,0.57, 0.43))}
  {Behavioral Interviews([0.65, 0.79], [0.44, 0.57], [0.41, 0.54], (0.76,0.57, 0.43))},
  {Experience([0.75, 0.89], [0.34, 0.56], [0.41, 0.54], (0.76,0.47, 0.53))}>.

Some Basic Operation on Neutrosophic Cubic Spherical Fuzzy Sets:

We formally define some fundamental operations for understanding and improved readability, considering the previously proposed term, the neutrosophic cubic spherical fuzzy set.

If there are two NCSFSs $\Phi =< I, A >$ and $\Psi =< J, B >$ defined on $\mathfrak{R}$, then we have

$$\Phi \oplus \Psi =$$

$$\Bigg\langle r, \Bigg\{\Bigg[\sqrt{(\mu_I(r)^L)^2 + \left(\mu_J(r)^L\right)^2 - (\mu_I(r)^L)^2\left(\mu_J(r)^L\right)^2}, \sqrt{(\mu_I(r)^U)^2 + \left(\mu_J(r)^U\right)^2 - (\mu_I(r)^U)^2\left(\mu_J(r)^U\right)^2}\Bigg]\Bigg\},$$

$$\Bigg\{\Bigg[\sqrt{(\pi_I(r)^L)^2 + \left(\pi_J(r)^L\right)^2 - (\pi_I(r)^L)^2\left(\pi_J(r)^L\right)^2}, \sqrt{(\pi_I(r)^U)^2 + \left(\pi_J(r)^U\right)^2 - (\pi_I(r)^U)^2\left(\pi_J(r)^U\right)^2}\Bigg]\Bigg\},$$

$$\{[\vartheta_I(r)^L \vartheta_I(r)^L, \vartheta_I(r)^U \vartheta_I(r)^U]\}, \Bigg\{\Bigg(\sqrt{\left(\mu_A(r)\right)^2 + \left(\mu_B(r)\right)^2 - \left(\mu_A(r)\right)^2\left(\mu_B(r)\right)^2}\Bigg),$$

$$\Bigg(\sqrt{\left(\pi_A(r)\right)^2 + \left(\pi_B(r)\right)^2 - \left(\pi_A(r)\right)^2\left(\pi_B(r)\right)^2}\Bigg), \left(\vartheta_A(r)\vartheta_B(r)\right)\Bigg\}\Bigg\rangle.$$

$$\boldsymbol{\Phi}\otimes\boldsymbol{\Psi}=\Big\langle \boldsymbol{r},\{[\mu_I(r)^L\mu_J(r)^L,\mu_I(r)^U\mu_J(r)^U]\},\{[\pi_I(r)^L\pi_J(r)^L,\pi_I(r)^U\pi_J(r)^U]\},$$

$$\left\{\left[\sqrt{(\mu_I(r)^L)^2+\left(\mu_J(r)^L\right)^2-(\mu_I(r)^L)^2\left(\mu_J(r)^L\right)^2},\sqrt{(\mu_I(r)^U)^2+\left(\mu_J(r)^U\right)^2-(\mu_I(r)^U)^2\left(\mu_J(r)^U\right)^2}\right]\right\},\Big\{(\mu_A(r)\mu_B(r)),$$

$$(\pi_A(r)\pi_B(r)),\left(\sqrt{(\vartheta_A(r))^2+(\vartheta_B(r))^2-(\vartheta_A(r))^2(\vartheta_B(r))^2}\right)\Big\}\Big\rangle.$$

$$\boldsymbol{k\Phi}$$

$$=\left\langle\left\{\left[\sqrt{1-(1-(\mu_I(r)^L)^{2(k)})},\sqrt{1-(1-(\mu_I(r)^U)^{2(k)})},\right]\right\},\left\{\left[\sqrt{1-(1-(\pi_I(r)^L)^{2(k)})},\sqrt{1-(1-(\pi_I(r)^U)^{2(k)})},\right]\right\},\right.$$

$$\left.\{[\vartheta_I(r)^{L(k)},\vartheta_I(r)^{U(k)}]\},\left\{\left(\sqrt{1-\left(1-(\mu_A(r))^{2(k)}\right)}\right),\left(\sqrt{1-\left(1-(\pi_A(r))^{2(k)}\right)}\right),(\vartheta_A(r))^{(k)}\right\}\right\rangle.$$

$$\boldsymbol{\Phi}^{k}=$$

$$\left\langle\begin{array}{c}\boldsymbol{r},\{[\mu_I(r)^{L(k)},\mu_I(r)^{U(k)}]\},\{[\pi_I(r)^{L(k)},\pi_I(r)^{U(k)}]\},\left\{\left[\sqrt{1-(1-(\vartheta_I(r)^L)^{2(k)})},\sqrt{1-(1-(\vartheta_I(r)^U)^{2(k)})},\right]\right\},\\ \left\{\left((\mu_A(r))^{(k)}\right),\left((\pi_A(r))^{(k)}\right),\left(\sqrt{1-\left(1-(\vartheta_A(r))^{2(k)}\right)}\right)\right\}\end{array}\right\rangle.$$

where $\boldsymbol{\Phi}^{k}=\boldsymbol{\Phi}\otimes\boldsymbol{\Phi}\otimes\ldots\otimes\boldsymbol{\Phi}(\boldsymbol{k}-\textbf{time})$. For each positive value of *k*, this represents a neutrosophic cubic spherical fuzzy value.

To simplify calculations, consider NCSFS as a neutrosophic cubic spherical fuzzy Number (NCSFN), s.t.

$$\boldsymbol{\Phi}=\langle\{[\mu_I(r_i)^L,\mu_I(r_i)^U],[\pi_I(r_i)^L,\pi_I(r_i)^U],[\vartheta_I(r_i)^L,\vartheta_I(r_i)^U]\},\{(\mu_A(r_i),\pi_A(r_i),\vartheta_A(r_i))\},r_i\in\ \mathfrak{R}\rangle$$

This structure is referred to as neutrosophic cubic spherical fuzzy Number (NCSFN). To create a new algorithm for rating the options based on the recommended aggregation operators, the score, accuracy, and certainty functions over NCSFSs are suitably reframed as follows:

Consider $\boldsymbol{\Phi}=<\boldsymbol{I},\boldsymbol{A}>$, where $\boldsymbol{I}=\{<\boldsymbol{r},\mu_I(r),\pi_I(r),\vartheta_I(r)>\}$ and $\mu_I(r)=[\mu_I(r)^L,\mu_I(r)^U]$,

$\pi_I(r)=[\pi_I(r)^L,\pi_I(r)^U]$, $\vartheta_I(x)=[\vartheta_I(r)^L,\vartheta_I(r)^U]$ and $\boldsymbol{A}=<\boldsymbol{r},\mu_A(x),\pi_A(r),\vartheta_A(r)>$, then

Score function $\boldsymbol{\delta}(\boldsymbol{\Phi})=\frac{1}{2}\left\langle\begin{array}{c}\frac{1}{6}\left(\sqrt{\left((\mu_I(r))^{2^L}+(\mu_I(r))^{2^U}\right)+\left((\pi_I(r))^{2^L}+(\pi_I(r))^{2^U}\right)+\left(1-(\vartheta_I(r))^{2^L}+1-(\vartheta_I(r))^{2^U}\right)}\right)\\+\frac{1}{3}\left(\sqrt{(\mu_A(r))^2+(\pi_A(r))^2+1-(\vartheta_A(r))^2}\right)\end{array}\right\rangle.$

Accuracy function $\aleph(\boldsymbol{\Phi})=\frac{1}{3}\{(\mu_I(r)^L+\mu_I(r)^U)-(\vartheta_I(r)^L+\vartheta_I(r)^U)+\mu_A(r)-\vartheta_A(r)\}.$

Certainty function $\beth(\boldsymbol{\Phi})=\frac{1}{3}\{(\mu_I(r)^L+\mu_I(r)^U)+\mu_A(r)\}.$

Let $\boldsymbol{\Phi}=<\boldsymbol{I},\boldsymbol{A}>$, and $\boldsymbol{\Psi}=<\boldsymbol{J},\boldsymbol{B}>$, are two neutrosophic cubic spherical fuzzy sets (NCSFSs) , we say that

If $\boldsymbol{\delta}(\boldsymbol{\Phi})>\boldsymbol{\delta}(\boldsymbol{\Psi})$ the $\boldsymbol{\Phi}>\boldsymbol{\Psi}$.

If $\boldsymbol{\delta(\Phi) = \delta(\Psi)}$ and $\aleph(\boldsymbol{\Phi}) > \aleph(\boldsymbol{\Psi})$ then $\boldsymbol{\Phi > \Psi}$.

If $\boldsymbol{\delta(\Phi) = \delta(\Psi)}$, $\aleph(\boldsymbol{\Phi}) = \aleph(\boldsymbol{\Psi})$ and $\beth(\boldsymbol{\Phi}) > \beth(\boldsymbol{\Psi})$ then $\boldsymbol{\Phi > \Psi}$.

If $\boldsymbol{\delta(\Phi) = \delta(\Psi)}$, $\aleph(\boldsymbol{\Phi}) = \aleph(\boldsymbol{\Psi})$ and $\beth(\boldsymbol{\Phi}) = \beth(\boldsymbol{\Psi})$ then $\boldsymbol{\Phi = \Psi}$.

The value of score function ranges from 0 and 1, i.e., $\boldsymbol{\delta(\Phi) \in [0, 1]}$, the value the accuracy function ranges from -1 and 1, i.e., $\aleph(\boldsymbol{\Phi}) \in [\boldsymbol{-1, 1}]$ and the value of the certainty function ranges from 0 and 1, i.e., $\beth(\boldsymbol{\Phi}) \in [\boldsymbol{0, 1}]$.

4. Exponential Operational Laws in NCSFSs

For NCSFSs $\Phi =< I, A >$ and a scalar $e > 0$, the exponential operation of Φ is expressed as:

$$e^{\Phi} = \begin{cases} \left\langle \begin{array}{c} r, \left\{\left[(e)^{\sqrt{1-(\mu_I(r)^L)^2}}, (e)^{\sqrt{1-(\mu_I(r)^U)^2}}\right]\right\}, \left\{\left[(e)^{\sqrt{1-(\pi_I(r)^L)^2}}, (e)^{\sqrt{1-(\pi_I(r)^U)^2}}\right]\right\}, \\ \left\{\left[\sqrt{1-(e)^{2(\vartheta_{I(r)}{}^L)}}, \sqrt{1-(e)^{2(\vartheta_{I(r)}{}^U)}}\right]\right\}, \left\{(e)^{\sqrt{1-(\mu_{A(r)})^2}}\right\}, \left\{(e)^{\sqrt{1-(\pi_A(r))^2}}\right\}, \left\{\sqrt{1-(e)^{2(\vartheta_{A(r)})}}\right\} \end{array} \right\rangle; e \in (0,1) \\ \left\langle \begin{array}{c} r, \left\{\left[\left(\frac{1}{e}\right)^{\sqrt{(\mu_I(r)^L)^2}}, \left(\frac{1}{e}\right)^{\sqrt{(\mu_I(r)^U)^2}}\right]\right\}, \left\{\left[\left(\frac{1}{e}\right)^{\sqrt{(\pi_I(r)^L)^2}}, \left(\frac{1}{e}\right)^{\sqrt{(\pi_I(r)^U)^2}}\right]\right\}, \\ \left\{\left[\sqrt{1-\left(\frac{1}{e}\right)^{2(\vartheta_I(r)^L)}}, \sqrt{1-\left(\frac{1}{e}\right)^{2(\vartheta_I(r)^U)}}\right]\right\}, \left\{\left(\frac{1}{e}\right)^{\sqrt{1-(\mu_{A(r)})^2}}\right\}, \left\{\left(\frac{1}{e}\right)^{\sqrt{1-(\pi_A(r))^2}}\right\}, \left\{\sqrt{1-\left(\frac{1}{e}\right)^{2(\vartheta_A(r))}}\right\} \end{array} \right\rangle; e \geq 1 \end{cases} \quad (1)$$

If $\Phi > \beta$, then $e^{\Phi} > e^{\Psi}$.

Theorem 1. Consider $\Phi =< I, A >$, $\Psi =< J, B >$ and $\Upsilon =< K, C >$ be three NCSFSs, a saclar $e, e_1 \in (0,1)$, and $\lambda > 0$, then

i. $e^{\Phi} \oplus e^{\Psi} = e^{\Psi} \oplus e^{\Phi}$
ii. $e^{\Phi} \otimes e^{\Psi} = e^{\Psi} \otimes e^{\Phi}$
iii. $(e^{\Phi} \oplus e^{\Psi}) \oplus e^{\Upsilon} = e^{\Phi}(\oplus e^{\Psi} \oplus e^{\Upsilon})$
iv. $(e^{\Phi} \otimes e^{\beta}) \otimes e^{\Upsilon} = e^{\Phi} \otimes (e^{\Psi} \otimes e^{\Upsilon})$
v. $\lambda(e^{\Phi} \oplus e^{\Psi}) = \lambda e^{\Phi} \oplus \lambda e^{\Psi}$
vi. $(e^{\Phi} \otimes e^{\Psi})^{\lambda} = (e^{\Phi})^{\lambda} \otimes (e^{\Psi})^{\lambda}$
vii. $e^{\Phi} \otimes e_1{}^{\Phi} = (ee_1)^{\Phi}$.

Proof.1. $e^{\Phi} \oplus e^{\Psi} =$

$$\left\langle \begin{array}{c} \left\{\left[(e)^{\sqrt{1-(\mu_I(r)^L)^2}}, (e)^{\sqrt{1-(\mu_I(r)^U)^2}}\right]\right\}, \left\{\left[(e)^{\sqrt{1-(\pi_I(r)^L)^2}}, (e)^{\sqrt{1-(\pi_I(r)^U)^2}}\right]\right\}, \\ \left\{\left[\sqrt{1-(e)^{2(\vartheta_{I(r)}{}^L)}}, \sqrt{1-(e)^{2(\vartheta_I(r)^U)}}\right]\right\}, \left\{(e)^{\sqrt{1-(\mu_{A(r)})^2}}\right\}, \left\{(e)^{\sqrt{1-(\pi_A(r))^2}}\right\}, \left\{\sqrt{1-(e)^{2(\vartheta_A(r))}}\right\} \end{array} \right\rangle$$

$$\oplus \left\langle \begin{array}{c} \left\{\left[(e)^{\sqrt{1-(\mu_J(r)^L)^2}}, (e)^{\sqrt{1-(\mu_J(r)^U)^2}}\right]\right\}, \left\{\left[(e)^{\sqrt{1-(\pi_J(r)^L)^2}}, (e)^{\sqrt{1-(\pi_J(r)^U)^2}}\right]\right\}, \\ \left\{\left[\sqrt{1-(e)^{2(\vartheta_{J(r)}{}^L)}}, \sqrt{1-(e)^{2(\vartheta_J(r)^U)}}\right]\right\}, \left\{(e)^{\sqrt{1-(\mu_{B(r)})^2}}\right\}, \left\{(e)^{\sqrt{1-(\pi_{B(r)})^2}}\right\}, \left\{\sqrt{1-(e)^{2(\vartheta_B(r))}}\right\} \end{array} \right\rangle$$

$$=\left\langle\left\{\left[(e)^{\sqrt{1-(\mu_I(r)^L)^2}}+(e)^{\sqrt{1-(\mu_J(r)^L)^2}}-(e)^{\sqrt{1-(\mu_I(r)^L)^2}}(e)^{\sqrt{1-(\mu_J(r)^L)^2}},\ (e)^{\sqrt{1-(\mu_I(r)^U)^2}}+(e)^{\sqrt{1-(\mu_J(r)^U)^2}}-\right.\right.\right.$$

$$\left.\left.(e)^{\sqrt{1-(\mu_I(r)^U)^2}}(e)^{\sqrt{1-(\mu_J(r)^U)^2}}\right]\right\},\left\{\left[(e)^{\sqrt{1-(\pi_I(r)^L)^2}}+(e)^{\sqrt{1-(\pi_J(r)^L)^2}}-(e)^{\sqrt{1-(\pi_I(r)^L)^2}}(e)^{\sqrt{1-(\pi_J(r)^L)^2}},\right.\right.$$

$$(e)^{\sqrt{1-(\pi_I(r)^U)^2}}+(e)^{\sqrt{1-(\pi_J(r)^U)^2}}-$$

$$\left.\left.(e)^{\sqrt{1-(\pi_I(r)^U)^2}}(e)^{\sqrt{1-(\pi_J(r)^U)^2}}\right]\right\},\left\{\left[\left(\sqrt{1-(e)^{2(\vartheta_I(r)^L)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_J(r)^L)}}\right),\left(\sqrt{1-(e)^{2(\vartheta_I(r)^U)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_J(r)^U)}}\right)\right]\right\},$$

$$\left\{(e)^{\sqrt{1-(\mu_{A(r)})^2}}+(e)^{\sqrt{1-(\mu_{B(r)})^2}}-(e)^{\sqrt{1-(\mu_{A(r)})^2}}(e)^{\sqrt{1-(\mu_{B(r)})^2}}\right\},\left\{(e)^{\sqrt{1-(\pi_A(r))^2}}+(e)^{\sqrt{1-(\pi_{B(r)})^2}}-\right.$$

$$\left.\left.(e)^{\sqrt{1-(\pi_A(r))^2}}(e)^{\sqrt{1-(\pi_{B(r)})^2}}\right\},\left\{\sqrt{1-(e)^{2(\vartheta_{A(r)})}}\sqrt{1-(e)^{2(\vartheta_{B(r)})}}\right\}\right\rangle$$

$$=\left\langle\left\{\left[(e)^{\sqrt{1-(\mu_J(r)^L)^2}}+(e)^{\sqrt{1-(\mu_I(r)^L)^2}}-(e)^{\sqrt{1-(\mu_J(r)^L)^2}}(e)^{\sqrt{1-(\mu_I(r)^L)^2}},(e)^{\sqrt{1-(\mu_J(r)^U)^2}}+\ (e)^{\sqrt{1-(\mu_I(r)^U)^2}}-\right.\right.\right.$$

$$\left.\left.(e)^{\sqrt{1-(\mu_J(r)^U)^2}}(e)^{\sqrt{1-(\mu_I(r)^U)^2}}\right]\right\},\left\{\left[\left(\sqrt{1-(e)^{2(\vartheta_{J(r)}{}^L)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_{I(r)}{}^L)}}\right),\left(\sqrt{1-(e)^{2(\vartheta_J(r)^U)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_{I(r)}{}^U)}}\right)\right]\right\},$$

$$\left\{(e)^{\sqrt{1-(\mu_{B(r)})^2}}+(e)^{\sqrt{1-(\mu_A(r))^2}}-(e)^{\sqrt{1-(\mu_B(r))^2}}(e)^{\sqrt{1-(\mu_A(r))^2}}\right\},\left\{(e)^{\sqrt{1-(\pi_{B(r)})^2}}+(e)^{\sqrt{1-(\pi_{A(r)})^2}}-\right.$$

$$\left.\left.(e)^{\sqrt{1-(\pi_{B(r)})^2}}(e)^{\sqrt{1-(\pi_{A(r)})^2}}\right\},\left\{\sqrt{1-(e)^{2(\vartheta_{B(r)})}}\sqrt{1-(e)^{2(\vartheta_{A(r)})}}\right\}\right\rangle=e^{\Psi}\oplus e^{\Phi}.$$

Similarly, the properties (ii), (iii) and (iv) can be easily proved with the help to defined basic operations on NCSFS.

Proof. v. $\lambda(e^{\Phi}\oplus e^{\Psi})=\lambda\left\langle\left\{\left[(e)^{\sqrt{1-(\mu_I(r)^L)^2}}+(e)^{\sqrt{1-(\mu_J(r)^L)^2}}-\right.\right.\right.$

$$(e)^{\sqrt{1-(\mu_I(r)^L)^2}}(e)^{\sqrt{1-(\mu_J(r)^L)^2}},\ (e)^{\sqrt{1-(\mu_I(r)^U)^2}}+\ (e)^{\sqrt{1-(\mu_J(r)^U)^2}}-$$

$$\left.\left.(e)^{\sqrt{1-(\mu_I(r)^U)^2}}(e)^{\sqrt{1-(\mu_J(r)^U)^2}}\right]\right\},\left\{\left[(e)^{\sqrt{1-(\pi_I(r)^L)^2}}+(e)^{\sqrt{1-(\pi_J(r)^L)^2}}-(e)^{\sqrt{1-(\pi_I(r)^L)^2}}(e)^{\sqrt{1-(\pi_J(r)^L)^2}},\right.\right.$$

$$(e)^{\sqrt{1-(\pi_I(r)^U)^2}}+(e)^{\sqrt{1-(\pi_J(r)^U)^2}}-$$

$$\left.\left.(e)^{\sqrt{1-(\pi_I(r)^U)^2}}(e)^{\sqrt{1-(\pi_J(r)^U)^2}}\right]\right\},\left\{\left[\left(\sqrt{1-(e)^{2(\vartheta_{I(r)}{}^L)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_{J(r)}{}^L)}}\right),\left(\sqrt{1-(e)^{2(\vartheta_I(r)^U)}}\right)\left(\sqrt{1-(e)^{2(\vartheta_J(r)^U)}}\right)\right]\right\},$$

$$\left\{(e)^{\sqrt{1-\mu_{A(r)}{}^2}}+(e)^{\sqrt{1-\mu_B(r)^2}}-(e)^{\sqrt{1-\mu_A(r)^2}}(e)^{\sqrt{1-\mu_{B(r)}{}^2}}\right\},\left\{(e)^{\sqrt{1-\pi_{A(r)}{}^2}}+(e)^{\sqrt{1-\pi_{B(r)}{}^2}}-\right.$$

$$\left.\left.(e)^{\sqrt{1-\pi_{A(r)}{}^2}}(e)^{\sqrt{1-\pi_{B(r)}{}^2}}\right\},\left\{\sqrt{1-(e)^{2(\vartheta_{A(r)})}}\sqrt{1-(e)^{2(\vartheta_{B(r)})}}\right\}\right\rangle$$

$$= \left\langle \left\{ \left[1 - \left(1 - \left((e)^{\sqrt{1-(\mu_I(r)^L)^2}} + (e)^{\sqrt{1-(\mu_J(r)^L)^2}} - (e)^{\sqrt{1-(\mu_I(r)^L)^2}} (e)^{\sqrt{1-(\mu_J(r)^L)^2}} \right) \right)^{\lambda}, 1 - \left(1 - \left((e)^{\sqrt{1-(\mu_I(r)^U)^2}} + (e)^{\sqrt{1-(\mu_J(r)^U)^2}} - (e)^{\sqrt{1-(\mu_I(r)^U)^2}} (e)^{\sqrt{1-(\mu_J(r)^U)^2}} \right) \right)^{\lambda} \right] \right\}, \left\{ \left[1 - \left(1 - \left((e)^{\sqrt{1-(\pi_I(r)^L)^2}} + (e)^{\sqrt{1-(\pi_J(r)^L)^2}} - (e)^{\sqrt{1-(\pi_I(r)^L)^2}} (e)^{\sqrt{1-(\pi_J(r)^L)^2}} \right) \right)^{\lambda}, 1 - \left(1 - \left((e)^{\sqrt{1-(\pi_I(r)^U)^2}} + (e)^{\sqrt{1-(\pi_J(r)^U)^2}} - (e)^{\sqrt{1-(\pi_I(r)^U)^2}} (e)^{\sqrt{1-(\pi_J(r)^U)^2}} \right) \right)^{\lambda} \right] \right\}, \left\{ \left[\left(\left(1 - \sqrt{1-(e)^{2(\vartheta_{I(r)}{}^L)}} \right) \left(1 - \sqrt{1-(e)^{2(\vartheta_J(r)^L)}} \right) \right)^{\lambda}, \left(\left(1 - \sqrt{1-(e)^{2(\vartheta_I(r)^U)}} \right) \left(1 - \sqrt{1-(e)^{2(\vartheta_{J(r)}{}^U)}} \right) \right)^{\lambda} \right] \right\}, \left\{ 1 - \left(1 - \left((e)^{\sqrt{1-(\mu_{A(r)})^2}} + (e)^{\sqrt{1-(\mu_{B(r)})^2}} - (e)^{\sqrt{1-(\mu_A(r))^2}} (e)^{\sqrt{1-(\mu_{B(r)})^2}} \right) \right)^{\lambda} \right\}, \left\{ 1 - \left(1 - \left((e)^{\sqrt{1-(\pi_A(r))^2}} + (e)^{\sqrt{1-(\pi_{B(r)})^2}} - (e)^{\sqrt{1-(\pi_A(r))^2}} (e)^{\sqrt{1-(\pi_{B(r)})^2}} \right) \right)^{\lambda} \right\}, \left\{ \left(\left(1 - \sqrt{1-(e)^{2(\vartheta_{A(r)})}} \right) \left(1 - \sqrt{1-(e)^{2(\vartheta_B(r))}} \right) \right)^{\lambda} \right\} \right\rangle.$$

$$\lambda e^{\Phi} \oplus \lambda e^{\Psi} = \left\langle \left\{ \left[1 - \left(1 - (e)^{\sqrt{1-(\mu_I(r)^L)^2}} \right)^{\lambda}, 1 - \left(1 - (e)^{\sqrt{1-(\mu_I(r)^U)^2}} \right)^{\lambda} \right] \right\}, \left\{ \left[1 - \left(1 - (e)^{\sqrt{1-(\pi_I(r)^L)^2}} \right)^{\lambda}, 1 - \left(1 - (e)^{\sqrt{1-(\pi_I(r)^U)^2}} \right)^{\lambda} \right] \right\}, \left\{ \left[\left(1 - \sqrt{1-(e)^{2(\vartheta_I(r)^L)}} \right)^{\lambda}, \left(1 - \sqrt{1-(e)^{2(\vartheta_I(r)^U)}} \right)^{\lambda} \right] \right\}, \left\{ 1 - \left(1 - (e)^{\sqrt{1-(\mu_{A(r)})^2}} \right)^{\lambda} \right\}, \left\{ 1 - \left(1 - (e)^{\sqrt{1-(\pi_A(r))^2}} \right)^{\lambda} \right\}, \left\{ \left(1 - \sqrt{1-(e)^{2(\vartheta_A(r))}} \right)^{\lambda} \right\} \right\rangle \oplus \left\langle \left\{ \left[1 - \left(1 - (e)^{\sqrt{1-(\mu_J(r)^L)^2}} \right)^{\lambda}, 1 - \left(1 - (e)^{\sqrt{1-(\mu_J(r)^U)^2}} \right)^{\lambda} \right] \right\}, \left\{ \left[1 - \left(1 - (e)^{\sqrt{1-(\pi_J(r)^L)^2}} \right)^{\lambda}, 1 - \left(1 - (e)^{\sqrt{1-(\pi_J(r)^U)^2}} \right)^{\lambda} \right] \right\}, \left\{ \left[\left(1 - \sqrt{1-(e)^{2(\vartheta_J(r)^L)}} \right)^{\lambda}, \left(1 - \sqrt{1-(e)^{2(\vartheta_{J(r)}{}^U)}} \right)^{\lambda} \right] \right\}, \left\{ 1 - \left(1 - (e)^{\sqrt{1-(\mu_{B(r)})^2}} \right)^{\lambda} \right\}, \left\{ 1 - \left(1 - (e)^{\sqrt{1-(\pi_{B(r)})^2}} \right)^{\lambda} \right\}, \left\{ \left(1 - \sqrt{1-(e)^{2(\vartheta_{B(r)})}} \right)^{\lambda} \right\} \right\rangle$$

$$=\Bigg\langle\Bigg\{\Bigg[1-\Big(1-\Big((e)^{\sqrt{1-(\mu_I(r)^L)^2}}+(e)^{\sqrt{1-(\mu_J(r)^L)^2}}-(e)^{\sqrt{1-(\mu_I(r)^L)^2}}(e)^{\sqrt{1-(\mu_J(r)^L)^2}}\Big)\Big)^{\lambda},1-\Big(1$$

$$-\Big((e)^{\sqrt{1-(\mu_I(r)^U)^2}}+(e)^{\sqrt{1-(\mu_J(r)^U)^2}}-(e)^{\sqrt{1-(\mu_I(r)^U)^2}}(e)^{\sqrt{1-(\mu_J(r)^U)^2}}\Big)\Big)^{\lambda}\Bigg]\Bigg\},\Bigg\{\Bigg[1-\Big(1$$

$$-\Big((e)^{\sqrt{1-(\pi_I(r)^L)^2}}+(e)^{\sqrt{1-(\pi_J(r)^L)^2}}-(e)^{\sqrt{1-(\pi_I(r)^L)^2}}(e)^{\sqrt{1-(\pi_J(r)^L)^2}}\Big)\Big)^{\lambda},1$$

$$-\Big(1-\Big((e)^{\sqrt{1-(\pi_I(r)^U)^2}}+(e)^{\sqrt{1-(\pi_J(r)^U)^2}}-(e)^{\sqrt{1-(\pi_I(r)^U)^2}}(e)^{\sqrt{1-(\pi_J(r)^U)^2}}\Big)\Big)^{\lambda}\Bigg]\Bigg\},\Bigg\{s\Bigg[\Bigg(\Bigg(1$$

$$-\sqrt{1-(e)^{2(\vartheta_I(r)^L)}}\Big)\Big(1-\sqrt{1-(e)^{2(\vartheta_J(r)^L)}}\Big)\Big)^{\lambda},\Big(\Big(1-\sqrt{1-(e)^{2(\vartheta_I(r)^U)}}\Big)\Big(1-\sqrt{1-(e)^{2(\vartheta_J(r)^U)}}\Big)\Big)^{\lambda}\Bigg]\Bigg\},\Bigg\{1$$

$$-\Big(1-\Big((e)^{\sqrt{1-(\mu_{A(r)})^2}}+(e)^{\sqrt{1-(\mu_{B(r)})^2}}-(e)^{\sqrt{1-(\mu_{A(r)})^2}}(e)^{\sqrt{1-(\mu_{B(r)})^2}}\Big)\Big)^{\lambda}\Bigg\},\Bigg\{1-\Big(1$$

$$-\Big((e)^{\sqrt{1-(\pi_A(r))^2}}+(e)^{\sqrt{1-(\pi_{B(r)})^2}}-(e)^{\sqrt{1-(\pi_{A(r)})^2}}(e)^{\sqrt{1-(\pi_{B(r)})^2}}\Big)\Big)^{\lambda}\Bigg\},\Bigg\{\Bigg(\Big(1-\sqrt{1-(e)^{2(\vartheta_{A(r)})}}\Big)\Big(1$$

$$-\sqrt{1-(e)^{2(\vartheta_B(r))}}\Big)\Bigg)^{\lambda}\Bigg\}\Bigg\rangle.$$

From equation (3) and (4), we get $\lambda(e^{\Phi}\oplus e^{\Psi})=\lambda e^{\Phi}\oplus\lambda e^{\Psi}$.

Similarly, the properties (vi) and (vii) can be easily proved with the help of defined basic operations on NCSFSs.

5. Exponential Aggregation Operators for NCSFSs

Definition 7. Consider $\{\Phi_j=\langle I_j,A_j\rangle\}$ be a accumulation of NCSFSs with $e_j\in(0,1)$ be the real number. The neutrosophic cubic spherical fuzzy weighted exponential aggregation (NCSFWEA) operator is expressed as follows:

$$\text{NCSFWEA}(\Phi_1,\Phi_2\cdots\Phi_n)=\underset{j=1}{\overset{n}{\otimes}}\left(e_j\right)^{\Phi_j}; \tag{5}$$

where $\{\Phi_j=\langle I_j,A_j\rangle\}$ are the exponential weighted vector of attribute values $e_j\in(0,1)$.

Theorem 2. Suppose $\{\Phi_j=\langle I_j,A_j\rangle\}$ be a accumulation of NCSFSs and $e_j\in(0,1)$ be the real number, the NCSF weighted exponential aggregation operator is

NCSFWEA$(\Phi_1,\Phi_2\cdots\Phi_n)=$

$$\left\langle\begin{array}{c}\left\{\left[\prod_{j=1}^{n}(e_j)^{\sqrt{1-\left(\mu_{I_{(j)}}{}^{L}\right)^2}},\prod_{j=1}^{n}(e_j)^{\sqrt{1-\left(\mu_{I_{(j)}}{}^{U}\right)^2}}\right]\right\},\left\{\left[\prod_{j=1}^{n}(e_j)^{\sqrt{1-\left(\pi_{I_{(j)}}{}^{L}\right)^2}},\prod_{j=1}^{n}(e_j)^{\sqrt{1-\left(\pi_{I_{(j)}}{}^{U}\right)^2}}\right]\right\},\\ \left\{\left[\sqrt{1-\prod_{j=1}^{n}(e)^{2\left(\vartheta_{I_{(j)}}{}^{L}\right)}},\sqrt{1-\prod_{j=1}^{n}(e)^{2\left(\vartheta_{I}{}^{U}\right)}}\right]\right\},\left\{\prod_{j=1}^{n}(e)^{\sqrt{1-\mu_{A_{(j)}}{}^{2}}}\right\},\left\{\prod_{j=1}^{n}(e)^{\sqrt{1-\pi_{A_{(j)}}{}^{2}}}\right\},\left\{\sqrt{1-\prod_{j=1}^{n}(e)^{2\left(\vartheta_{A_{(j)}}\right)}}\right\}\end{array}\right\rangle; \tag{6}$$

where, $\{\Phi_j=\langle I_j,A_j\rangle\}$ are the exponential weighted vector of attributes value $e_j\in(0,1)$. Moreover NCSFWEA$(\Phi_1,\Phi_2\cdots\Phi_n)$ is also a NCSFSs.

Proof. Theorem proof is based on the mathematical induction principle.

6. Methodology for MCDM problem under NCSFSs

This section presents a methodology for solving the MCDM problem based on proposed exponential aggregation operators. The process is then demonstrated using a numerical example. The steps listed below are:

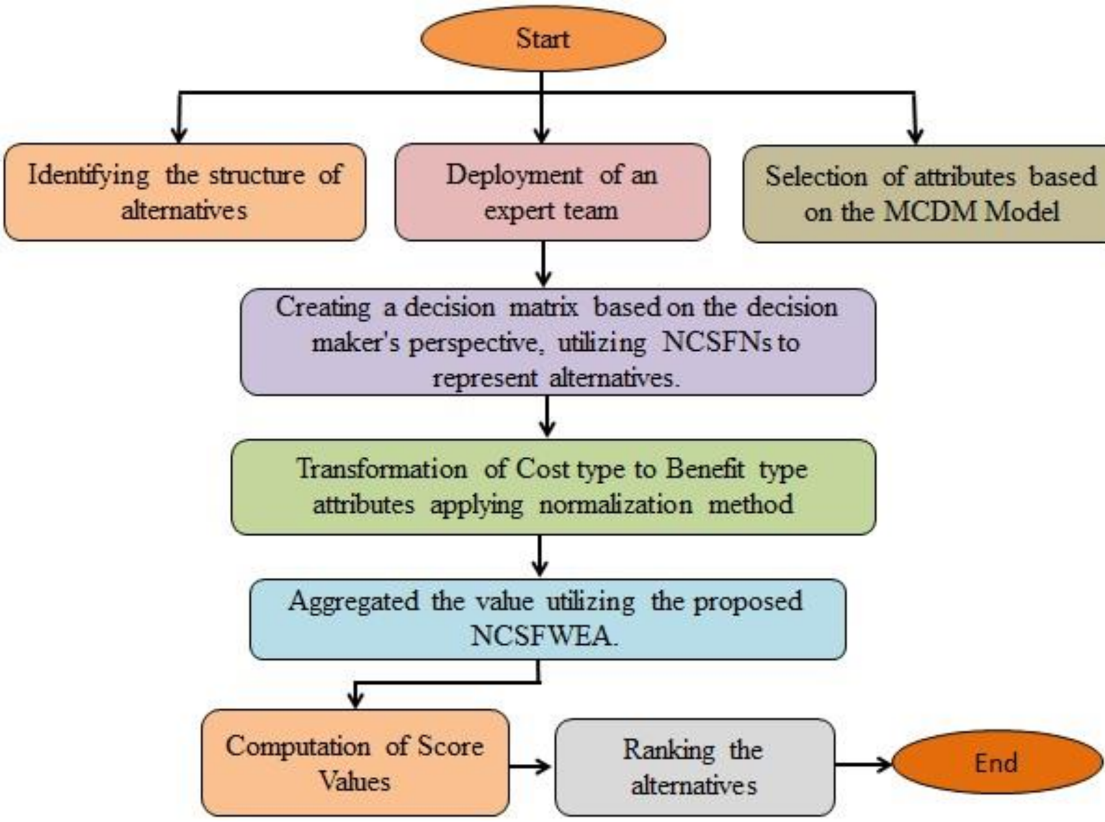

Fig 2: Methodology procedure for addressing MCDM problem under NCSFSs.

Step1. Consider an issue of decision-making challenge with n alternatives, i.e. $\mathfrak{I}_1, \mathfrak{I}_2, \cdots \mathfrak{I}_n$ and m attributes, i.e. $E_1, E_2, \cdots E_n$. The NCSFSs Φ_j is used as a weight vector to the attributes E_j $(j = 1,2, \cdots m)$. Then experts evaluate the presented alternatives according to a set of attributes and give their choice of values based on the spherical fuzzy information indicated by $e_{ij} < i = 1,2, \cdots n >$ and $< j = 1,2, \cdots n >$ and $0 \leq e_{ij} \leq 1$. Next, depending on the choice values, a decision matrix $D = (e_{ij})$, $< i = 1,2, \cdots n >$ and $< j = 1,2, \cdots n >$ is presented, where e_{ij} indicates the choice of alternatives $\mathfrak{I}_i$ that correspond to attribute E_j.

$$D = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1m} \\ e_{21} & e_{22} & \cdots & e_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \cdots & e_{nm} \end{bmatrix}$$

Step 2. The attributes are usually divided into two groups: benefit type and cost type. If the MCDM attributes are of the same kind, choice values do not require normalization. If the MCDM attributes are of distinct kinds, then the following formula can be utilized to convey choice values from the benefit type to the cost type

$$e_{ij} = \begin{cases} e_{ij}\ ; & j \in \text{ benifit type} \\ e_{ij}{}^{c}; & j \in \text{ cost type} \end{cases}.$$

Step 3. Now aggregated the value by using the proposed neutrosophic cubic spherical fuzzy weighted exponential aggregation operators (NCSFWEA) for each alternative $\mathfrak{J}_{1,}\mathfrak{J}_{2,} \cdots \mathfrak{J}_{n}$.

Step 4. We compute the scoring value $\delta(\varepsilon_i)$; $i = 1,2, \cdots n$ for each alternative $\mathfrak{J}_i; i = 1,2, \cdots n$ by utilizing the scoring function formula.

Step 5. And at last, ranking the alternative with the most favorable score value. If the score values of the two different options are identical, apply the accuracy function formula for ranking; otherwise, use the certainty function formula.

Selection of solid waste disposal sites under the proposed NCSFWEA with numerical illustration.

We implement the suggested NCSFWEA to a waste disposal site selection task that has five alternatives i.e. sites $\mathfrak{J}_{1,}$ $\mathfrak{J}_{2,}\mathfrak{J}_{3},\mathfrak{J}_{4}$ $,\mathfrak{J}_{5}$ and four attributes E_1 = Distance to residential areas; E_2 = savings in cost; E_3 = ground depth; E_4 =slope . Expert opinions are expressed in the structure of a decision matrix D = $(e_{ij})_{5\times 3}$, whose elements show the choice of alternative $\mathfrak{J}_i$ (i = $1, \ldots, 5$) over the attribute E_j (j = $1, 2, 3,4$)

$$D = \begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.5 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.8 & 0.2 & 0.2 & 0.3 \\ 0.7 & 0.2 & 0.2 & 0.4 \\ 0.8 & 0.4 & 0.5 & 0.6 \end{bmatrix}.$$

For each attribute, the exponential neutrosophic cubic spherical fuzzy weights are given as follows:

$\omega_1 = \langle \{[0.4, 0.8]\}, \{[0.2, 0.5]\}, \{[0.5,0.9]\}, \{(0.9,0.8,0.3)\} \rangle$,

$\omega_2 = \langle \{[0.2, 0.6]\}, \{[0.3, 0.7]\}, \{[0.4,0.8]\}, \{(0.7,0.5,0.2)\} \rangle$,

$\omega_3 = \langle \{[0.4, 0.6]\}, \{[0.2, 0.4]\}, \{[0.5,0.9]\}, \{(0.8,0.9,0.1)\} \rangle$,

$\omega_4 = \langle \{[0.3, 0.5]\}, \{[0.4, 0.7]\}, \{[0.2,0.6]\}, \{(0.5,0.7,0.3)\} \rangle$.

The following results are obtained by aggregating the various choice values of each aspect using the aggregation operator NCSFWEA given in equation (6):

$$\varepsilon_1 = \left\langle \begin{matrix} \{[0.015962,0.035322]\}, \{[0.014039,0.02555]\}, \{[0.989016,0.999682]\}, \\ \{(0.064241,0.066373,0.915998)\} \end{matrix} \right\rangle$$

$$\varepsilon_2 = \left\langle \begin{matrix} \{[0.06051,0.10971]\}, \{[0.057486,0.091746]\}, \{[0.950405,0.995559]\}, \\ \{(0.14997,0.138816,0.887293)\} \end{matrix} \right\rangle$$

$$\varepsilon_3 = \left\langle \begin{matrix} \{[0.012216,0.023479]\}, \{[0.011863,0.023286]\}, \{[0.986267,0.999668]\}, \\ \{(0.038582,0.045545,0.91558)\} \end{matrix} \right\rangle$$

$$\varepsilon_4 = \left\langle \begin{matrix} \{[0.014222,0.027802]\}, \{[0.013548,0.027662]\}, \{[0.986252,0.999632]\}, \\ \{(0.046699,0.051627,0.906982)\} \end{matrix} \right\rangle$$

$$\varepsilon_5 = \left\langle \begin{matrix} \{[0.108082,0.155078]\}, \{[0.106454,0.1576]\}, \{[0.918333,0.98791]\}, \\ \{(0.199907,0.203039,0.782001)\} \end{matrix} \right\rangle.$$

Now, use the scoring function to calculate the scoring value; $\delta(\varepsilon_1) = 0.063832$, $\delta(\varepsilon_2) = 0.095898$, $\delta\ (\varepsilon_3) = 0.063736$, $\delta(\varepsilon_4) = 0.066565$, $\delta(\varepsilon_5) = 0.133046$.

Lastly, depending on the outcomes of the score function, we rank the alternatives with the highest value

$$\delta(\varepsilon_5) > \delta(\varepsilon_2) > \delta(\varepsilon_4) > \delta(\varepsilon_1) > \delta(\varepsilon_3).$$

As a result, alternative $\delta(\varepsilon_5)$ is the most suitable.

7. Results & Discussions

- Finally, we can state that the suggested neutrosophic cubic spherical fuzzy set is a new and supportable combination of SNS and IVSNS. By enhancing the area of uncertainty compoment this gives more felxiblity to experts to give their opinion without any restiction.
- In the existing theory of neutrosophic sets, including the SVNSS [13], SVNHFS [23] and IVNHS [24] have limitations due to the restriction on the constraint.
- Exponential aggregation operators under NCSFSs have shown effectiveness in addressing industrial issues, reducing infectious diseases, and disposing of garbage properly. This improves disposal efficiency and decreases contamination risks by enhancing waste management systems.
- As a result of developing NCSFS and their aggregation operator, this research contributes significantly to the theoretical advancement of fuzzy set theory.
- NCSFS and its exponential aggregation operators need complex mathematical formulations and computations, resulting in significant computational draws in large-scale decision-making. Future studies should concentrate on creating algorithms or approximation techniques that are more effective to minimize computational time and resource use.
- The performance of the proposed aggregate operators may be sensitive to parameter selection. A detailed sensitivity analysis will be required to understand this impact and establish guidelines for practical applications.
- The study focuses primarily on the mathematical and theoretical aspects of NCSFS. Integrating these concepts with other interdisciplinary approaches such as machine learning, artificial intelligence, and big data analytics may enhance their applicability and effectiveness in complex decision-making scenarios.

8. Conclusions & Scope for Future Work

This article introduces a new notion of neutrosophic cubic spherical fuzzy sets (NCSFSs), which completely combines SNSs and IVSNSs using fundamental algebraic operations. The development of exponential operational laws under NCSFSs provided vital information, which has been subsequently expanded to establish exponential aggregation operators with distinguishing characteristics. Using these advancements, a methodology based on aggregation operators has been developed to solve Multiple Criteria Decision Making (MCDM) problems in a neutrosophic spherical environment while considering expert opinions. Finally, the proposed theory's practical application has been demonstrated by its effective use in determining the best site for solid waste disposal, as illustrated by a numerical example. This thorough examination demonstrates the flexibility and suitability of neutrosophic cubic spherical fuzzy sets in addressing actual-world decision-making obstacles. Future developments of this theory could include hybrid weighted aggregating operators, which are helpful in multiple types of decision-making processes, including TOPSIS, VIKOR, WASPAS, and AHP, among others. It could also be applied to other real-world decision-making problems using the newly developed operators.

List of Symbols and Notations Used in this paper

Symbols\Notations	Description
$\mathfrak{R}$	Universal set
$\in$	Belongs to
∇	Spherical Fuzzy set
$\mu(r)$	Membership degree
$\pi(r)$	Indeterminancy/hesitation degree
$\nu(r)$	Non-membership degree
$\mu(r)^U$	Upper membership degree
$\pi(r)^U$	Upper indeterminancy degree
$\nu(r)^U$	Upper non-membership degree
I'	Interval-valued spherical fuzzy set
$\mathcal{N}$	Neutrosophic set
A	Spherical neutrosophic set
I	Interval-valued neutrosophic spherical fuzzy set
NCSFSs	Neutrosophic cubic spherical fuzzy sets
SNS	Spherical neutrosophic set
IVNSFS	Interval-valued neutrosophic spherical fuzzy set
MCDM	Multi-criteria decision making
PFNs	Pythagorean fuzzy numbers
SWM	Solid waste management
FS	Fuzzy set
IVFS	Interval-valued fuzzy set
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
NS	Neutrosophic set
SS	Soft set
SVNS	Single-valued neutrosophic set
HFS	Hesitant fuzzy set
PFS	Picture fuzzy set
OPFS	Ortho-pair fuzzy set
SFS	Spherical fuzzy set
SVNHFS	Single-valued neutrosophic hesitant fuzzy set
INS	Interval neutrosophic set

HSS	Hypersoft set
PSVNHF	Probabilistic single-valued neutrosophic hesitant fuzzy
PINHFWA	Probabilistic interval neutrosophic hesitant fuzzy weighted averaging
PINHFWG	Probabilistic interval neutrosophic hesitant fuzzy weighted geometric
NHFS	Neutrosophic hesitant fuzzy set
LCA	Life cycle assessment
TSFHSSs	T-sperical fuzzy hypersoft sets
n-HSNMs	n-hyperspherical neutrosophic matrices
EL	Exponential-logarithm
NCSFWEAO	Neutrosophic cubic spherical fuzzy weighted exponential aggregation operator

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