TrF-BWM-Neutrosophic-TOPSIS Strategy under SVNS Environment Approach and Its Application to Select the Most Effective Water Quality Parameter of Aquaponic System

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Abstract: The rapid emergence of aquaponics presents a promising solution for food production in arid regions, heavily reliant on maintaining optimal water quality parameters for overall success. However, monitoring and controlling these parameters can be complex and costly, involving numerous sensors and actuators. Focusing on the most significant parameter for fish growth could alleviate some of this complexity and promote sustainability by ensuring a balanced system. This study introduces a novel fuzzy-based Multiple Criteria Decision Making (MCDM) methodology, combining Trapezoidal Fuzzy with the Best Worst Method (BWM) and Neutrosophic-TOPSIS strategy, to identify the primary water quality parameters impacting fish growth. Through this innovative approach, essential indicators are identified to conduct efficiency analyses. Moreover, the study not only advances scientific understanding but also offers practical guidance for farmers and aquaponic enthusiasts, aiming to foster sustainability and effectiveness in aquaponic effective.

Keywords: BWM, TOPSIS, Neutrosophic set, Aquaponic system, Water quality parameter.

1. Introduction

A major global concern the world is facing is to fulfill the nutritional requirements of an expanding human population, which is expected to reach 10 billion by the year 2050 [1]. The global pandemic, coupled with conflict and climate crisis-related weather patterns has delayed the first Millennium Development Goal which is "eradicate extreme poverty and hunger" [2]. The State of Food Security and Nutrition in the World (SOFI) report estimates that since 2019, an additional 122 million people have experienced hunger. The number of people who are hungry worldwide has plateaued between 2021 and 2022, but there are still numerous places where food crises are getting worse, which has prompted demands for an international effort to address the underlying causes of food insecurity [3]. To meet the additional food demands imposed by the almost 30% population expansion, global food production must rise by as much as 50% [4]. However, the rate of food production has been drastically lowered as a result of natural disasters, climate change, land grabs, declining soil quality, rapid urbanization, unfair trade rules, and other issues [5]. As per the predictions made by [6], current trends in food production will not be able to meet the projected global food demand by 2050, even with the development of high-yielding crop varieties and improved food production techniques. According to [7], by the end of the twenty-first century, climate change alone may be responsible for up to 18% of losses of arable land, which will have a severe impact on the regions that are already facing food insecurity situation. The challenges associated with food production are driving up the demand for innovative practices, systems, and methods in the food production industry.

As a practical response to the food and environmental challenges, aquaponics farming is becoming more widely acknowledged as a way to increase food production rapidly without compromising the environment. Aquaponics is an environment-friendly sustainable method of food production that utilizes the concepts of circular economy and a biological natural system to maximize food production with minimized input and waste. Aquaponics is an integrated approach that uses two fundamental production systems: Aquaculture and Hydroponics. Aquaculture is the breeding of aquatic animals primarily Fish and hydroponics is the cultivation of plants without the use of soil (see **Figure 1**) [8]. In an aquaponic system, the nutrient-rich waste from aquatic animals is transformed into organic fertilizers through microbial activity steered by microorganisms, whereas the hydroponics plants detoxify the water by extracting the nutrients and then allow the fresh, oxygenated water in the fish tank to be recycled [9]. To put it briefly, aquaponics can be defined as an ecosystem in which fish, plants, and microbes coexist as symbionts and contribute to the production of sustainable foods. To support the vital bacteria involved in the nutrient cycle and to maintain the system's integrity, the aquaponic system does not allow any chemical additives and is free of antibiotics. As a result, crops are naturally healthy and essentially grown organically [10]. Being a closed circular system, aquaponics improves labor efficiency and can lead to sustainable output growth, which would boost food security and agricultural profitability [11]. In comparison with the conventional framing, the reduced land consumption and water use, make aquaponics a feasible food production solution for the regions with arid environments and help to develop the economic status with its rapid production. Thus, aquaponics can be seen as an innovative, low-carbon farming technique that is an intense, sustainable, circular, and highly productive farming method [12].

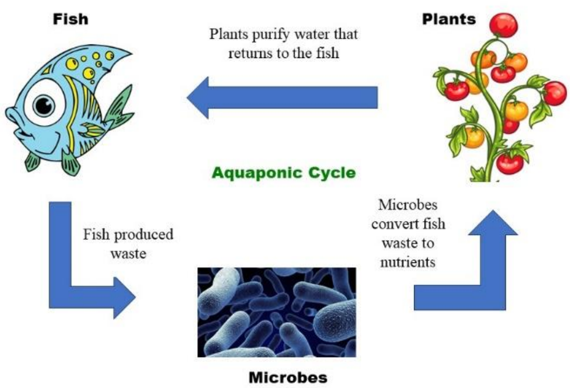

Figure 1: The aquaponic cycle

The aquaponic system operates on a closed-loop water medium, hosting fish, microorganisms, and plants. The physical, chemical, and biological features of the recirculating water play a vital role in the survival of all three living components of aquaponics. Thus, it's crucial to uphold optimal water quality parameters to ensure each component thrives independently. Fish growth rate in aquaponics is a significant trait with implications for ecology, evolution, and conservation. Key water quality parameters affecting fish growth include temperature (T), Dissolved Oxygen (DO) levels, water pH, ammonium concentration, nitrate levels, and many more [13]. Temperature directly influences the metabolic rate of fish which also has an impact on their energy balance and behavior of the body. Additionally, temperature impacts factors such as fish appetite, digestion, energy production, and nutrient absorption within the gastrointestinal tract [14]. According to findings from [15], inadequate water temperature can lead to significant fungal infections in both juvenile and adult fish populations, potentially resulting in the decay of eggs and larvae. Another crucial aspect of recirculating water quality is the level of DO. Fish rely on an adequate concentration of dissolved oxygen in the water for respiration, essential for their survival. The pH level of water also impacts the growth rate of fish, with slightly acidic environments potentially diminishing fish reproduction rates [16]. Ammonia serves as a significant parameter in aquaponics systems too. Even small amounts of ammonia can be highly toxic to fish, and its presence increases significantly in water solutions that become strongly acidic or alkaline [17]. The nitrate level of the water also plays a role in contributing to fish growth. High concentrations of nitrate can be particularly detrimental to fry and juvenile fish, significantly impairing their growth [18]. However, due to system dynamics and environmental factors, these parameters often deviate from ideal values, directly leading to stress, disease, and even death in fish and ultimately affecting the productivity and sustainability of the aquaponics system as a whole [19]. Additionally, the water quality parameters are interconnected to each other. Temperature influences the solubility of oxygen in water. Warmer water holds less dissolved oxygen compared to colder water. This means that as temperature increases, the amount of oxygen available for fish decreases [20]. pH plays a role in the toxicity of ammonia in water. Ammonia exists in two forms: un-ionized ($NH3$) and ionized ($NH4^+$). The proportion of these forms is influenced by pH, with un-ionized ammonia being more toxic to fish. In alkaline conditions (higher pH), more ammonia exists in its toxic form, while in acidic conditions (lower pH), more ammonia is present in its less harmful ionized form [17]. Understanding these interrelationships underscores the importance of holistic system management in aquaponics. The reasons mentioned above clearly indicate that maintaining the optimal values for water quality parameters in an aquaponics system is essential for promoting optimal fish growth by ensuring a stable, well-balanced environment that meets the physiological needs of the fish and supports nutrient cycling for both fish and plant health.

To maintain optimal water quality parameters, artificial control and monitoring systems are frequently deployed in aquaponics setups. These systems employ a network of sensors to gauge various water quality metrics [16]. Actuators are then employed to make necessary adjustments based on the sensor data. For instance, pH levels might require modulation through the addition of acids or bases [21], while oxygen levels can be regulated using aerators or pumps [22]. However, implementing such systems can be intricate due to the multitude of parameters needing supervision and control. This complexity stems from the necessity for numerous sensors and actuators, as well as the integration of these components into a unified system. The hardware complexity presents several challenges. Procuring and installing a large array of sensors and actuators can be costly, particularly for larger aquaponics systems. Additionally, the ongoing expenses associated with maintaining and replacing these components further contribute to the financial burden. The setup and calibration of multiple sensors and actuators can be time-intensive, demanding meticulous adjustment and testing to ensure precision and dependability. Furthermore, the involvement of numerous hardware components necessitates ongoing maintenance to uphold sensor calibration and actuator functionality, potentially increasing the workload for system operators and risking downtime in case of component failures. Lastly, the environmental ramifications of producing and disposing of substantial quantities of sensors and actuators, along with the energy consumption required to power them, raise sustainability concerns. Selecting the most impactful water quality parameter for fish growth could serve as a solution to address these challenges and render aquaponics systems more sustainable. Determining the most influential parameter can lead to a reduction in the number of sensors, thereby simplifying the system, saving time, and lowering costs.

1.1 Literature review

In recent years, various control mechanisms for small-scale aquaponic systems have been proposed, leveraging strategies such as ON-OFF, PID (Proportional Integral Derivative), MPC (Model Predictive Control), and CAE (Controlled Environment Agriculture). Vernandhes et al. (2017) [23] introduced a smart aquaponics monitoring and control system utilizing a sensor network for water quality parameters, managed through a microcontroller. Dutta et al. (2018) [24] and Zamora-Izquierdo et al. (2019) [25] integrated Internet of Things (IoT) technology and sensor networks to regulate water quality parameters. Khaoula et al. (2021) [26] implemented an IoT-based solution for monitoring and controlling water quality and environmental parameters using sensors for water level, temperature, and CO_2, along with actuators. Li et al. (2022) [27] applied PID control to manage dissolved oxygen concentration in aquaponics recirculating water, and Wang et al. (2023) [28] proposed a Controlled Environment Agriculture (CEA) system discussing aspects related to aquaculture, aeroponics, and aquaponics. Channa et al. (2024) [29] explored the integration of Artificial Intelligence and IoT in a smart aquaponics system to monitor and control essential parameters. These studies primarily concentrate on control mechanisms tailored for small-scale aquaponic setups. Some literature also touches upon machine learning methods for yield estimation in aquaponic systems. Debroy and Seban (2022a) [30] and Debroy and Seban (2022a) [31] proposed two prediction methods for fish weight estimation using Artificial Neural Network (ANN) and its hybrid with fuzzy logic (ANFIS), as well as ANN models for predicting tomato biomass in aquaponic systems respectively. Eneh et al. (2023) [32] presented a yield prediction method for aquaponic systems employing various machine learning algorithms. A study on IoT integrated Machine learning based Indoor aquaponics farming is discussed by Rajendiran et al. (2024) [33].

1.2 Research gap

Previous research has predominantly focused on monitoring small-scale smart aquaponics systems, with some studies incorporating IoT-based machine learning methods for predicting yields and controlling system parameters. It's evident from existing literature that the integration of aquaculture and hydroponics in aquaponics systems significantly influences the dependence of fish growth and yield on water quality parameters. However, the complexity of implementing control and monitoring units to maintain optimal values for all these parameters presents challenges, including high costs, time consumption, and maintenance requirements, which can render the system unsustainable. A notable gap in the literature is the absence of a defined methodology for identifying the most significant water quality parameter affecting fish growth. This knowledge gap is crucial as it could help reduce system complexity while maximizing aquaponic system production. Therefore, the key research question arises: **Which water quality parameter holds the most significance for fish growth in aquaponic systems?**

Addressing this question is essential for optimizing aquaponic systems, enhancing sustainability, and improving overall efficiency in food production. By identifying the primary driver of fish growth, aquaponics practitioners can streamline their efforts and resources towards maintaining and optimizing that specific parameter. This not only reduces system complexity but also has the potential to increase fish yields, improve economic returns, and promote environmental sustainability in aquaponic food production.

1.3 Objective of the study

The study aims to explore the correlation between water quality parameters and fish production within aquaponics systems, with the primary objective of identifying the key factor influencing fish growth. Aquaponics is a sustainable farming technique that integrates aquaculture (fish farming) with hydroponics (soilless plant cultivation), creating a symbiotic ecosystem where fish waste serves as a nutrient source for plants, and the plants, in turn, purify the water for the fish. Understanding the water quality parameters is crucial in aquaponics because they directly influence the health and growth of both fish and plants. However, these parameters can fluctuate from their optimal values due to external disturbances. Additionally, these parameters are interrelated and can affect each other, so to maintain the right balance among them is essential. While using actuators to monitor and control these parameters is a potential solution, individually assigning actuators for each parameter would increase system complexity, duration, and costs. Hence, the study seeks to identify the most significant water quality parameter affecting fish growth within aquaponics systems. This involves statistical analyses and modeling techniques to determine which parameter has the strongest correlation or causative relationship with fish productivity. Understanding the primary driver of fish growth allows aquaponics practitioners to prioritize their efforts and resources towards maintaining and optimizing that particular parameter.

The findings of this study will not only contribute to the scientific understanding of aquaponics systems but also offer practical guidance to farmers and enthusiasts. Armed with knowledge of the most critical water quality parameters and their effects, farmers can refine their system management practices to achieve higher fish yields, improved economic returns, and enhanced sustainability in aquaponic food production. Ultimately, the research endeavors to facilitate the development of efficient and environmentally friendly aquaponics systems capable of meeting the increasing demand for nutritious, locally produced seafood.

This study aims to pioneer the creation of a hybrid Multi-Criteria Decision-Making (MCDM) model designed to identify the most crucial water quality parameter influencing fish growth in aquaponic systems. The proposed approach, named TrF-BWM-Neutrosophic-TOPSIS under SVNS Environment, integrates various decision-making techniques into a unified framework, a novel concept not explored in existing literature. In this hybrid model, criteria weights are determined in a trapezoidal fuzzy environment through the BWM method. Subsequently, the ranking of alternatives is refined using TOPSIS within a Neutrosophic fuzzy environment. This comprehensive methodology offers a unique perspective on optimizing decision-making processes in aquaponic systems by leveraging diverse analytical tools synergistically.

Compared to AHP [34], BWM [35] offers several advantages by requiring fewer paired comparisons and maintaining higher consistency in its pairwise comparison matrix. This improved consistency significantly enhances problem-solving efficacy compared to AHP. However, despite these benefits, BWM also presents certain drawbacks such as potential ambiguity in human evaluations and inaccuracies in criterion-related facts [36]. To address these limitations, BWM can be adapted to handle fuzzy numbers instead of crisp values, leading to more robust ranking conclusions in real-world scenarios. Guo and Zhao (2017) [37] introduced a fuzzy BWM technique termed triangular fuzzy MCDM, which allows for estimating the prospective performance of decision-evaluation groups regarding each criterion using triangular or trapezoidal fuzzy numbers [38]. There exists a relationship between trapezoidal and triangular fuzzy numbers, with a triangular fuzzy number emerging when a trapezoidal fuzzy number equals one of the two most promising values at least once [39]. Researchers have determined that trapezoidal fuzzy numbers offer advantages over triangular fuzzy numbers in designing procedures [40]. The trapezoidal fuzzy BWM, introduced by Majumder et al. [40] in 2022, represents a significant advancement in this field, providing a more nuanced approach to decision-making under uncertainty.

In this study, the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method, as outlined in the primary reference [41], is employed to determine the optimal option. The process of assigning criteria weights is governed by TrF-FOCUM [42], facilitating this determination. The rationale for selecting TOPSIS lies in its user-friendly nature and its ability to meet both qualitative and quantitative requirements flexibly. By assessing each option based on its best and worst results, TOPSIS contributes to a more robust ranking outcome. Moreover, it can accommodate cost-benefit considerations, making it suitable for scenarios where performance-cost interaction is significant. While previous research has extensively explored fuzzy and intuitionistic fuzzy MCDM problems, the increasing recognition of ambiguity's role in MCDM complexities underscores the necessity of incorporating neutrosophic sets. Neutrosophic sets address environments characterized by uncertainty, indeterminacy, and inconsistency within the MCDM methodology. Despite the attention given to fuzzy and intuitionistic fuzzy MCDM challenges, incorporating indeterminacy into the realm of MCDM difficulties is considered essential. In 2018, Biswas et al. [43] developed TOPSIS strategy for trapezoidal neutrosophic number environment. In 2023, Neutrosophic BWM-TOPSIS was developed by [44] to determine alternative rankings, marking a significant advancement in addressing the complexities of decision-making under uncertainty.

1.4 Advantage and novelty:

Numerous benefits come from the hybrid technique that combines Neutrosophic-TOPSIS strategy for alternative level evaluation in a Single Valued Neutrosophic (SVN) environment with Trapezoidal Fuzzy with the Best Worst Method (BWM) for criteria level evaluation.

- The hybrid method can handle various forms of uncertainty in SVN contexts by employing both Trapezoidal Fuzzy logic and Neutrosophic logic. While Neutrosophic logic addresses uncertainty at the alternative level, Trapezoidal Fuzzy logic addresses uncertainty at the criteria level, offering a thorough handling of ambiguity throughout the decision-making process.
- The Best Worst Method (BWM) enables decision-makers to accurately represent subjective preferences by methodically assessing criteria according to their relative significance. This guarantees that the evaluation process appropriately reflects the preferences of decision-makers, resulting in more trustworthy and significant conclusions.
- By offering a systematic approach to determining the significance of criteria, the BWM improves the clarity of the decision-making process. The weighting and ranking of the criteria are easily understood by decision-makers, which promotes intuitive decision-making.
- The neutrosophic-TOPSIS technique enhances the resilience of the decision-making process to ambiguity by explicitly considering degrees of truth, indeterminacy, and falsehood at the alternative level. This ensures the reliability of decision outcomes even in the presence of unclear or ambiguous facts.
- By merging the advantages of BWM, the TOPSIS method, and Trapezoidal Fuzzy BWM, the hybrid technique facilitates effective multi-criteria decision-making. Decision-makers can take uncertainties and personal preferences into account while methodically evaluating alternatives based on a variety of factors.
- In SVN contexts, the hybrid technique enhances the quality and reliability of decision outputs by integrating both Neutrosophic and Trapezoidal Fuzzy logic. By taking into account both objective standards and subjective preferences, decision-makers tend to make more robust and well-informed choices.

2. Initial preparations

2.1. TrFN (Trapezoidal fuzzy number)

Ever since Zadeh introduced fuzzy [45] sets in 1965, researchers have devised numerous types of fuzzy numbers, including triangular, trapezoidal, and Gaussian variants [46]. The subsequent summary offers a succinct outline of key definitions pertinent to fuzzy sets and TFNs (Triangular Fuzzy Numbers).

Definition 1: A fuzzy set, symbolized as $U=\{(u,\tau_u(u)):u\in\Re\}$, is defined by a set of ordered pairs, where $\tau_u(u)\in[0,1]$ indicates the membership mapping of object g within the fuzzy set.

Definition 2: TrFN is presented by $\tilde{O}=(\theta_1,\theta_2,\theta_3,\theta_4)$, where $\tilde{O}$'s membership mapping $\tau_{\tilde{O}}$ is presented by [47].

$$\tau_{\tilde{O}}=\begin{cases}0,\ u<\theta_1\\ \dfrac{u-\theta_1}{\theta_2-\theta_1},\ \theta_1\le u\le\theta_2\\ 1,\ \theta_2\le u\le\theta_3\\ \dfrac{\theta_3-u}{\theta_3-\theta_4},\ \theta_3\le u\le\theta_4\\ 0,\ u>\theta_4\end{cases}$$

Here, θ_1 and θ_4 denote parameters defining the range of potential assessments; $\tilde{O}$ represents the lower bound, θ_1 signifies the upper limit of θ_4, and $[\theta_1,\theta_4]$ corresponds to the interval of $\tilde{O}$.

When the two most favorable values of a Trapezoidal Fuzzy Number (TrFN) align, it transforms into a TFN, indicating that TFNs are a particular instance of TrFNs. TrFNs find utility in pessimistic or conservative contexts, where exclusive membership is attributed to a restricted subset of the discourse universe. On the other hand, in optimistic or tolerant scenarios, TrFNs assign full membership to a wider subset of the discourse universe. By fostering tolerance and optimism, TrFNs adeptly manage events [48]. Under the assumption that $\tilde{O}_1=(\theta_1^{/},\theta_2^{/},\theta_3^{/},\theta_4^{/})$ and $\tilde{O}_2=(\theta_1^{//},\theta_2^{//},\theta_3^{//},\theta_4^{//})$ [49] are the two positive TrFNs, the operational principles are as follows.

$$\tilde{O}_1\oplus\tilde{O}_2=(\theta_1^{/},\theta_2^{/},\theta_3^{/},\theta_4^{/})\oplus(\theta_1^{//},\theta_2^{//},\theta_3^{//},\theta_4^{//})=(\theta_1^{/}+\theta_1^{//},\theta_2^{/}+\theta_2^{//},\theta_3^{/}+\theta_3^{//},\theta_4^{/}+\theta_4^{//})$$

$$\tilde{O}_1\otimes\tilde{O}_2=(\theta_1^{/},\theta_2^{/},\theta_3^{/},\theta_4^{/})\otimes(\theta_1^{//},\theta_2^{//},\theta_3^{//},\theta_4^{//})=(\theta_1^{/}\theta_1^{//},\theta_2^{/}\theta_2^{//},\theta_3^{/}\theta_3^{//},\theta_4^{/}\theta_4^{//})$$

$$\tilde{O}_1/\tilde{O}_2=(\theta_1^{/},\theta_2^{/},\theta_3^{/},\theta_4^{/})/(\theta_1^{//},\theta_2^{//},\theta_3^{//},\theta_4^{//})=(\theta_1^{/}/\theta_4^{//},\theta_2^{/}/\theta_3^{//},\theta_3^{/}/\theta_2^{//},\theta_4^{/}/\theta_1^{//})$$

$$\mu\tilde{O}_1=(\mu\theta_1^{/},\mu\theta_2^{/},\mu\theta_3^{/},\mu\theta_4^{/});\mu\in\Re^{+}$$

$$\tilde{O}_1^{-1}=(1/\theta_4^{/},1/\theta_3^{/},1/\theta_2^{/},1/\theta_1^{/})$$

Definition 3: Defuzzification refers to the method of converting a fuzzy number, which represents uncertainty or imprecision, into a single precise value within the crisp domain. This process is crucial for obtaining a concrete decision or action based on fuzzy logic outputs. It typically involves applying an inverse transformation to the fuzzy output, which could be a fuzzy set or a fuzzy number, to derive a precise value that best represents the fuzzy input. By doing so, defuzzification enables clearer and more actionable outcomes from fuzzy logic-based systems. Considering $\tilde{O}_1 = \left(\theta_1^{/}, \theta_2^{/}, \theta_3^{/}, \theta_4^{/}\right)$ is a TrFN, the associated crisp value $\Re\left(\tilde{O}_1\right)$ can be computed using the equation given below:

$$\Re\left(\tilde{O}_1\right) = \frac{\theta_1^{/} + 2\theta_2^{/} + 2\theta_3^{/} + \theta_4^{/}}{6}$$

2.2. Preliminary of Single Valued Neutrosophic Set (SVNS)

The foundation of Neutrosophic Sets [50] was laid by Prof. Smarandache, who introduced the concept. Subsequently, [51] pioneered the idea of Single-Valued Neutrosophic Set (SVNS) to address situations marked by uncertainty and incomplete information. The overview of SVNS was presented by Pramanik [52] to address situations marked by uncertainty and incomplete information.

The subsequent definition characterizes an SVNS Δ over a predetermined set Y :

$$\Delta = \left\{\left(b, \mathrm{A}_Z(b), \mathrm{B}_Z(b), \mathrm{M}_Z(b)\right) : b \in Y\right\}$$

where $\mathrm{A}_Z : R \to \{0,1\} \cup (0,1)$, $\mathrm{B}_Z : R \to \{0,1\} \cup (0,1)$, $\mathrm{M}_Z : R \to \{0,1\} \cup (0,1)$ and so $0 \le \mathrm{A}_Z(b) + \mathrm{B}_Z(b) + \mathrm{M}_Z(b) \le 3$. If an SVNS Δ over a given set Y , we refer to the triplet $(\mathrm{A}_Z(b), \mathrm{B}_Z(b), \mathrm{M}_Z(b))$ as a Single-Valued Neutrosophic Number (SVNN).

Mandal & Basu (2019) [53] introduced a novel scoring function aimed to address MADM challenges within the SVNS framework, outlined as follows:
The scoring mechanism is established through the following steps:

(i) Let's assume Θ represents the origin in a three-dimensional space, and $\Omega = \left(h_\theta, d_\theta, \upsilon_\theta\right)$, denoted as an SVNN, signifies a point within this space. Perform a translation of this point into Ω to arrive at $\Lambda = \left(h_\varpi, d_\varpi, \upsilon_\varpi\right)$. Here $h_\varpi = h_\theta + \xi,\ d_\varpi = d_\theta + \xi,\ \upsilon_\varpi = \upsilon_\theta + \xi,$ where $\xi > 0,$ each representing υ_ϖ, a real number that remains distinct and unchanging throughout the specific problem, play a crucial role. Now, let's contemplate another point, $\Lambda^{/} = \left(h_\varpi, -d_\varpi, -\upsilon_\varpi\right)$, which is the result of reflecting $\Lambda = \left(h_\varpi, d_\varpi, \upsilon_\varpi\right)$ across the *x-axis*.

(ii) Locate the score function, which is $N_1(\Lambda) = \cos\vartheta$, with ϑ representing the angle between $\Theta\Lambda$ and $\Theta\Lambda^{/}$, and Θ denoting the origin.

(iii) If the score values for two distinct SVNNs, $\Lambda_1 = \left(h_{\varpi_1}, d_{\varpi_1}, \upsilon_{\varpi_1}\right)$ and $\Lambda_2 = \left(h_{\varpi_2}, d_{\varpi_2}, \upsilon_{\varpi_2}\right)$, denoted as $N_1(\Lambda_1)$ and $N_1(\Lambda_2)$ respectively, are equal, determine $\Lambda_1^{**} = \left(h_{\varpi_1}, -d_{\varpi_1}, -\sqrt{\upsilon_{\varpi_1}}\right)$ and $\Lambda_2^{**} = \left(h_{\varpi_2}, -d_{\varpi_2}, -\sqrt{\upsilon_{\varpi_2}}\right)$ respectively for the corresponding translated points $\Lambda_1^{*} = \left(h_{\varpi_1^*}, d_{\varpi_1^*}, \upsilon_{\varpi_1^*}\right)$ and $\Lambda_2^{*} = \left(h_{\varpi_2^*}, d_{\varpi_2^*}, \upsilon_{\varpi_2^*}\right)$ where, $h_{\varpi_1^*} = h_{\varpi_1} + \xi$, $d_{\varpi_1^*} = d_{\varpi_1} + \xi$, $\upsilon_{\varpi_1^*} = \upsilon_{\varpi_1} + \xi$ and $h_{\varpi_2^*} = h_{\varpi_2} + \xi, d_{\varpi_2^*} = d_{\varpi_2} + \xi, \upsilon_{\varpi_2^*} = \upsilon_{\varpi_2} + \xi$.

(iv) Determine $\cos\phi$ and $\cos\lambda$, where ϕ represents the angle between $\Theta\Lambda_2^{*}$ and $\Theta\Lambda_2^{**}$, and λ signifies the angle between $\Theta\Lambda_2^{*}$ and $\Theta\Lambda_2^{**}$, with Θ denoting the origin.

(v) The score mapping $N_2(\Lambda_1) = \cos\phi$ as well as $N_2(\Lambda_2) = \cos\lambda$.

3. TrF-BWM-Neutrosophic-TOPSIS Strategy under SVNS Environment Approach

The TrF-BWM-Neutrosophic-TOPSIS Strategy under the SVNS Environment approach [54, 55] comprises two main phases. In the initial phase, the TrF-BWM method is employed to determine the weight or priority value (PV) of the criteria. Following this, in the second phase, the Neutrosophic-TOPSIS Strategy under the SVNS Environment is utilized to establish the ranking of alternatives. **Figure 2** provides a visual representation of the computational steps integral to this approach.

3.1 Phase I: Trapezoidal Fuzzy Best-Worst Method (TrF BWM)

The TrFBWM process entails five distinct steps:

Step 1: Selection of Criteria: MCDM strategies utilize selection criteria to identify the primary indicators. The letters $\Gamma = \{\Gamma_{\wp} : \wp = 1(1)\Psi\}$ and $M = \{M_{\sigma} : \sigma = 1(1)\Phi\}$ represent the criteria and indicators, respectively. The set of elements referred to as Γ influences the performance of each indicator.

Step 2: Selecting the Best and Worst Criteria: In this process, the Decision Maker (DM) identifies the optimal (best) and least favorable (worst) criteria. Through various surveys, such as expert opinions, literature reviews, and media analyses, the highest and lowest-ranking criteria is identified by the DM. Γ_ε and Γ_ω represent the best and worst standards, respectively. This is followed by $\Gamma_\varepsilon \in \Gamma$ and $\Gamma_\omega \in \Gamma$.

Step 3: Relative Assessment: Developing a pair-wise comparison matrix is deemed superfluous. Step 3 provides further detail on the pair-wise comparisons undertaken in sub-steps 1 and 2. The criteria undergo assessment utilizing linguistic metrics of relevance, employing trapezoidal fuzzy pair-wise comparisons. **Table 1** showcases trapezoidal fuzzy measurements of importance for each item within the linguistic domain. During the pair-wise comparisons of criteria conducted by the Decision Maker (DM), "EI" signifies equally important, "UMS" denotes upper measure scale, and "Intermediate" indicates the intermediate scale.

Table 1: Assessments relying on linguistic measures of importance.

List of linguistic phrases defined	Essential fuzziness measures for the trapezoidal shape (χ,τ,ρ,ξ)	SI
UMS	$\left(8,\frac{17}{2},9,9\right)$	9
Intermediate scale	$\left(\delta-1,\delta-\frac{1}{2},\delta+\frac{1}{2},\delta+1\right)$	$\delta=2(1)8$
EI	$(1,1,1,1)$	1

Sub-Step 1: Comparison of Best Criteria (BC) to all criteria:

In this step, comparison of each criterion of Set Γ with the best criteria of Set $\Gamma_\varepsilon \in \Gamma$ is performed. This phase is detailed in **Table 2**. In this scenario, both $\tilde{\partial}_{\varepsilon\wp}\ (\wp=1(1)\Psi)$ and $\tilde{\partial}_{\varepsilon\varepsilon}=\left(8,\frac{17}{2},9,9\right)$ have the option to select one action from **Table 1**.

Table 2: The criterion that most closely aligns with each pair of criteria.

BC (Best Criteria)	Γ_1	Γ_2	...	Γ_Ψ
Γ_ε	$\tilde{\partial}_{\varepsilon 1}$	$\tilde{\partial}_{\varepsilon 2}$	...	$\tilde{\partial}_{\varepsilon\Psi}$

Sub-Step 2: Every criterion is evaluated in relation to the worst criteria (WC):

Table 3 presents the comparison of each criterion in Set Γ with the worst criterion in Set $\Gamma_\omega \in \Gamma$. Additionally, the measures of significance from **Table 1** are selected by $\tilde{\partial}_{\wp\omega}\ (\wp=1(1)\Psi)$ as well as $\tilde{\partial}_{\omega\omega}=(1,1,1,1)$.

Table 3: Comparing every criterion with the most rigorous ones.

WC (Worst Criteria)	Γ_{ω}
Γ_1	$\tilde{\partial}_{1\omega}$
Γ_2	$\tilde{\partial}_{2\omega}$
$\vdots$	$\vdots$
Γ_{Ψ}	$\tilde{\partial}_{\wp\omega}$

Step 4: Evaluation of PV of trapezoidal fuzzy:

This step entails identifying the optimal fuzzy trapezoidal priority values. Let $\tilde{P}_1, \tilde{P}_2, ..., \tilde{P}_{\wp}$ represent the decision variables in optimization approaches, and let $\tilde{P}_1, \tilde{P}_2, ..., \tilde{P}_{\wp}$ denote the priority values for the corresponding $\Gamma_1, \Gamma_2, ..., \Gamma_{\wp}$ criteria. The respective priority values for criteria Γ_{ε} and Γ_{ω} are $\tilde{P}_{\varepsilon}$ and $\tilde{P}_{\omega}$.

As per equations (1) and (2), the optimal trapezoidal fuzzy priority value is characterized by two ratios, denoted as $\frac{\tilde{P}_{\varepsilon}}{\tilde{P}_{\wp}}$ and $\frac{\tilde{P}_{\wp}}{\tilde{P}_{\omega}}$ $(\wp = 1(1)\Psi)$.

$$\frac{\tilde{P}_{\varepsilon}}{\tilde{P}_{\wp}} - \tilde{\partial}_{\varepsilon\wp} = 0 \quad (1)$$

and $$\frac{\tilde{P}_{\wp}}{\tilde{P}_{\omega}} - \tilde{\partial}_{\wp\omega} = 0, \forall \wp = 1(1)\Psi \quad (2)$$

For all *i*, the situation remains consistent if the distances between $\left|\frac{\tilde{P}_{\varepsilon}}{\tilde{P}_{\wp}} - \tilde{\partial}_{\varepsilon\wp}\right|$ and $\left|\frac{\tilde{P}_{\wp}}{\tilde{P}_{\omega}} - \tilde{\partial}_{\wp\omega}\right|$, $\forall \wp = 1(1)\Psi$ are minimized or maximized. A trapezoidal fuzzy set comprises each TrF priority value. $\tilde{P}_{\wp} = \left(\chi_{\tilde{P}_{\wp}}, \tau_{\tilde{P}_{\wp}}, \rho_{\tilde{P}_{\wp}}, \xi_{\tilde{P}_{\wp}}\right), (\wp = 1(1)\Psi)$ should be considered when selecting an optimal indicator. In this context, the problem can be formulated as follows:

$$\left.\begin{aligned} &\min_{u}\max\left\{\left|\frac{\tilde{P}_{\varepsilon}}{\tilde{P}_{\wp}}-\tilde{\partial}_{\varepsilon\wp}\right|,\left|\frac{\tilde{P}_{\wp}}{\tilde{P}_{\omega}}-\tilde{\partial}_{\wp\omega}\right|\right\}\\ &\text{Subject to}\sum_{\wp=1}^{\Psi}\Re\left(\tilde{P}_{\wp}\right)=1\\ &\qquad\chi_{\tilde{P}_{\wp}}\le\tau_{\tilde{P}_{\wp}}\le\rho_{\tilde{P}_{\wp}}\le\xi_{\tilde{P}_{\wp}},\forall\,\wp=1(1)\Psi\\ &\qquad\chi_{\tilde{P}_{\wp}}\ge 0,\forall\,\wp=1(1)\Psi \end{aligned}\right\} \quad (3)$$

wher $\tilde{P}_{\varepsilon}=\left(\chi_{\tilde{P}_{\varepsilon}},\tau_{\tilde{P}_{\varepsilon}},\rho_{\tilde{P}_{\varepsilon}},\xi_{\tilde{P}_{\varepsilon}}\right)$, $\tilde{P}_{\wp}=\left(\chi_{\tilde{P}_{\wp}},\tau_{\tilde{P}_{\wp}},\rho_{\tilde{P}_{\wp}},\xi_{\tilde{P}_{\wp}}\right)$, $\tilde{P}_{\omega}=\left(\chi_{\tilde{P}_{\omega}},\tau_{\tilde{P}_{\omega}},\rho_{\tilde{P}_{\omega}},\xi_{\tilde{P}_{\omega}}\right)$, $\tilde{\partial}_{\varepsilon\wp}=\left(\chi_{\tilde{\partial}_{\varepsilon\wp}},\tau_{\tilde{\partial}_{\varepsilon\wp}},\rho_{\tilde{\partial}_{\varepsilon\wp}},\xi_{\tilde{\partial}_{\varepsilon\wp}}\right)$,

and $\tilde{\partial}_{\omega\wp}=\left(\chi_{\tilde{\partial}_{\omega\wp}},\tau_{\tilde{\partial}_{\omega\wp}},\rho_{\tilde{\partial}_{\omega\wp}},\xi_{\tilde{\partial}_{\omega\wp}}\right)$.

The process of transforming problem (3) into a non-linear optimization problem is as follows. The non-linear adapted form of equation (3) is expressed as follows:

$$\left.\begin{aligned} &\min\ \tilde{\Delta}\\ &\text{Subject to}\left|\frac{\tilde{P}_{\varepsilon}}{\tilde{P}_{\wp}}-\tilde{\partial}_{\varepsilon\wp}\right|\le\tilde{\Delta}\\ &\qquad\left|\frac{\tilde{P}_{\wp}}{\tilde{P}_{\omega}}-\tilde{\partial}_{\wp\omega}\right|\le\tilde{\Delta}\\ &\qquad\sum_{i=1}^{n}\Re\left(\hat{w}_{i}\right)=1\\ &\qquad\chi_{\tilde{P}_{\wp}}\le\tau_{\tilde{P}_{\wp}}\le\rho_{\tilde{P}_{\wp}}\le\xi_{\tilde{P}_{\wp}},\forall\,\wp=1(1)\Psi\\ &\qquad\chi_{\tilde{P}_{\wp}}\ge 0,\forall\,\wp=1(1)\Psi \end{aligned}\right\} \quad (4)$$

where, $\tilde{\Delta}=\left(\chi_{\tilde{\Delta}},\tau_{\tilde{\Delta}},\rho_{\tilde{\Delta}},\xi_{\tilde{\Delta}}\right)$

Assume, $\Delta=\chi_{\tilde{\Delta}}$ becomes $\tilde{\Delta}=(\Delta,\Delta,\Delta,\Delta)$. Consequently, equation (4) can be reformulated as: a

$$\left.\begin{aligned} &\min\ \left(\Delta,\Delta,\Delta,\Delta\right)\\ &\text{Subject to}\left|\frac{\left(\chi_{\tilde{P}_{\varepsilon}},\tau_{\tilde{P}_{\varepsilon}},\rho_{\tilde{P}_{\varepsilon}},\xi_{\tilde{P}_{\varepsilon}}\right)}{\left(\chi_{\tilde{P}_{\wp}},\tau_{\tilde{P}_{\wp}},\rho_{\tilde{P}_{\wp}},\xi_{\tilde{P}_{\wp}}\right)}-\left(\chi_{\tilde{\partial}_{\varepsilon\wp}},\tau_{\tilde{\partial}_{\varepsilon\wp}},\rho_{\tilde{\partial}_{\varepsilon\wp}},\xi_{\tilde{\partial}_{\varepsilon\wp}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\ &\qquad\left|\frac{\left(\chi_{\tilde{P}_{\wp}},\tau_{\tilde{P}_{\wp}},\rho_{\tilde{P}_{\wp}},\xi_{\tilde{P}_{\wp}}\right)}{\left(\chi_{\tilde{P}_{\omega}},\tau_{\tilde{P}_{\omega}},\rho_{\tilde{P}_{\omega}},\xi_{\tilde{P}_{\omega}}\right)}-\left(\chi_{\tilde{\partial}_{\omega\wp}},\tau_{\tilde{\partial}_{\omega\wp}},\rho_{\tilde{\partial}_{\omega\wp}},\xi_{\tilde{\partial}_{\omega\wp}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\ &\qquad\sum_{\wp=1}^{\Psi}\Re\left(\tilde{P}_{\wp}\right)=1\\ &\qquad\chi_{\tilde{P}_{\wp}}\le\tau_{\tilde{P}_{\wp}}\le\rho_{\tilde{P}_{\wp}}\le\xi_{\tilde{P}_{\wp}},\forall\,\wp=1(1)\Psi\\ &\qquad\chi_{\tilde{P}_{\wp}}\ge 0,\forall\,\wp=1(1)\Psi \end{aligned}\right\} \quad (5)$$

The solution to equation (5) yields the optimal trapezoidal fuzzy priority value $\left(\tilde{P}_1*, \tilde{P}_2*, ..., \tilde{P}_\wp*\right)$, where $\tilde{P}_\wp* = \left(\chi_{\tilde{P}_\wp*}, \tau_{\tilde{P}_\wp*}, \rho_{\tilde{P}_\wp*}, \xi_{\tilde{P}_\wp*}\right)$, $\forall \wp = 1(1)\Psi$ signifies the optimal trapezoidal fuzzy priority value.

Step 5: Verify TrF-BWM in CR:

Considering the Consistency Ratio (CR), pairwise comparisons hold the highest significance. Pairwise comparisons remain consistent if $\tilde{\partial}_{\varepsilon\wp} \times \tilde{\partial}_{\wp\varpi} - \tilde{\partial}_{\varepsilon\varpi} = 0$ is true. A pairwise comparison matrix with $\tilde{\partial}_{\varepsilon\wp} \times \tilde{\partial}_{\wp\varpi} - \tilde{\partial}_{\varepsilon\varpi} > or < 0$ indicates inconsistency. The maximum inequality in $\tilde{\Delta}$ arises when $\tilde{\partial}_{\varepsilon\varpi}$ equals $\tilde{\partial}_{\varepsilon\wp}$ and $\tilde{\partial}_{\wp\varpi}$ .As $\left(\frac{\tilde{P}_\varepsilon}{\tilde{P}_\wp}\right) \times \left(\frac{\tilde{P}_\wp}{\tilde{P}_\varpi}\right) - \left(\frac{\tilde{P}_\varepsilon}{\tilde{P}_\varpi}\right) = 0$ represents the largest inequality, it can be interpreted as follow:

$$\left(\tilde{\partial}_{\varepsilon\wp} - \tilde{\Delta}\right) \times \left(\tilde{\partial}_{\wp\varpi} - \tilde{\Delta}\right) - \left(\tilde{\partial}_{\varepsilon\varpi} - \tilde{\Delta}\right) = 0 \qquad (6)$$

To ascertain the maximum possible level of fuzzy consistency, utilize $\tilde{\partial}_{\varepsilon\wp} = \tilde{\partial}_{\wp\varpi} = \tilde{\partial}_{\varepsilon\varpi}$. As a result, the equation (6) is modified to

$$\tilde{\Delta}^2 - \left(1 + 2\tilde{\partial}_{\varepsilon\varpi}\right)\tilde{\Delta} + \left(\tilde{\partial}_{\varepsilon\varpi}^2 - \tilde{\partial}_{\varepsilon\varpi}\right) = 0 \qquad (7)$$

where $\tilde{\Delta} = \left(\chi_{\tilde{\Delta}}, \tau_{\tilde{\Delta}}, \rho_{\tilde{\Delta}}, \zeta_{\tilde{\Delta}}\right)$ and $\tilde{P}_{\varepsilon\varpi} = \left(\chi_{\varepsilon\varpi}, \tau_{\varepsilon\varpi}, \rho_{\varepsilon\varpi}, \zeta_{\varepsilon\varpi}\right)$ are both TrFS

In terms of trapezoidal fuzzy values, $\tilde{P}_{\varepsilon\varpi} = \left(\chi_{\varepsilon\varpi}, \tau_{\varepsilon\varpi}, \rho_{\varepsilon\varpi}, \zeta_{\varepsilon\varpi}\right)$ can reach a maximum of $\left(8, \frac{17}{2}, 9, 9\right)$. The consistency index (CI) of TrFS is calculated as 13.77 according to equation (7). Consequently, $\max\left\{\chi_{\varepsilon\varpi}, \tau_{\varepsilon\varpi}, \rho_{\varepsilon\varpi}, \zeta_{\varepsilon\varpi}\right\}$ values exceeding 9 are prohibited. It is feasible to derive the CI for TrFS through a similar procedure. **Table 4** displays the CI values for TrFS.

Table 4: CI for TrFS.

(χ, τ, ρ, ξ)	**CI**
$(1,1,1,1)$	3
$\left(8, \frac{17}{2}, 9, 9\right)$	13.77
$\delta = 7$	12.58
$\delta = 5$	10
$\delta = 3$	7.37

Step 6: Crisp priority's value:

Finally, convert the ideal PV $\left(\tilde{P}_1*, \tilde{P}_2*, ..., \tilde{P}_{\wp}*\right)$ where $\tilde{P}_{\wp}* = \left(\chi_{\tilde{P}_{\wp}*}, \tau_{\tilde{P}_{\wp}*}, \rho_{\tilde{P}_{\wp}*}, \xi_{\tilde{P}_{\wp}*}\right)$, $\forall \wp = 1(1)\Psi$ into its crisp value using formula (8).

$$\Re(\tilde{P}_{\wp}*) = \frac{\chi_{\tilde{P}_{\wp}*} + 2\tau_{\tilde{P}_{\wp}*} + 2\rho_{\tilde{P}_{\wp}*} + \xi_{\tilde{P}_{\wp}*}}{6}, \forall \wp = 1(1)\Psi \quad (8)$$

Step 7: PVs with normalized values: Equation (9) is employed to standardize the priority values of each criterion.

$$\tilde{P}_{\wp}* = \frac{\Re(\tilde{P}_{\wp}*)}{\sum_{\wp=1}^{\Psi} \Re(\tilde{P}_{\wp}*)}, \forall \wp = 1(1)\Psi \quad (9)$$

3.2 Phase II: Neutrosophic-TOPSIS Strategy under SVNS Environment

Consider the set of alternative $E = \{\alpha_\gamma : \gamma = 1(1)\vartheta\}$, $\gamma \ge 1$ and $H = \{\sigma_\theta : \theta = 1(1)\eta\}$, $\theta \ge 2$ be the set of attributes with weights $\Re(\mu_\theta^*)$, $\theta = 1(1)\eta$ respectively.

Decision-makers assign ratings to the $\alpha_\gamma, \gamma = 1(1)\vartheta$ alternatives based on the attributes $\sigma_\theta, \theta = 1(1)\eta$, which are represented using an SVNN. Let's assume the rating for the γ^{th} attribute concerning the θ^{th} alternative is presented as follows:

$\alpha_\gamma^* = \left(\kappa_\theta, A_{\alpha_\kappa}(\sigma_\theta), B_{\alpha_\kappa}(\sigma_\theta), M_{\alpha_\kappa}(\sigma_\theta)\right)$, $\gamma = 1(1)\vartheta$, where, $0 \le A_{\alpha_\kappa}(Z_\varphi) + B_{\alpha_\kappa}(Z_\sigma) + M_{\alpha_\kappa}(Z_\varphi) \le 3$. Here $\left(A_{\gamma\theta}, B_{\gamma\theta}, M_{\gamma\theta}\right)$ is denoted as an SVNN. $\alpha_{\gamma\theta}^*$, ($\gamma = 1(1)\vartheta$ and $\theta = 1(1)\eta$), where θ represents the number of attributes and γ represents the number of alternatives. Determine the decision matrix based on the ratings:

$$\nabla^* = \left[\alpha_{\gamma\theta}^*\right]_{\vartheta\times\eta}$$

Now, the TOPSIS method is encapsulated as follows:

Step 1: The score-matrix $\nabla = \left[\alpha_{\gamma\theta}\right]_{\vartheta\times\eta}$, ($\gamma = 1(1)\vartheta$ and $\theta = 1(1)\eta$)is acquired from the decision matrix $\nabla^* = \left[\alpha_{\gamma\theta}^*\right]_{\vartheta\times\eta}$ utilizing the following described in the preliminary section: i.e.,

$\alpha_{\gamma\theta} = \mathrm{N}_1\left(\left[\alpha^*_{\gamma\theta}\right]_{\vartheta\times\eta}\right)$.

Step 2: Determination of normalized decision matrix $\Upsilon = \left[\upsilon_{\gamma\theta}\right]_{\vartheta\times\eta}$

where, $\upsilon_{\gamma\theta} = \frac{\alpha_{\gamma\theta}}{\sqrt{\sum_{\theta=1}^{\eta}\alpha_{\gamma\theta}}}$,............(10); $\gamma = 1(1)\vartheta$

Step 3: Calculate the weighted normalized decision matrix $\Xi = \left[\tau_{\gamma\theta}\right]_{\vartheta\times\eta}$

where $\tau_{\gamma\theta} = \Re(\tilde{\mu}^*_{\theta}) \otimes \upsilon_{\gamma\theta}$, $\gamma = 1(1)\vartheta$ and $\theta = 1(1)\eta$

Step 4: Identify the Neutrosophic Positive Ideal Solution (NPIS) and Neutrosophic Negative Ideal Solution (NNIS), denoted by ρ^+ as well as ρ^- respectively,

$\rho^+ = \left\{\tau^+_1, \tau^+_2, ..., \tau^+_\eta\right\}$, where, $\tau^+_\theta = \max_{\theta} \tau_{\gamma\theta}, \theta = 1(1)\eta$

$\rho^- = \left\{\tau^-_1, \tau^-_2, ..., \tau^-_\eta\right\}$, where, $\tau^-_\theta = \min_{\theta} \tau_{\gamma\theta}, \theta = 1(1)\eta$

Step 5: Calculate the distance of each alternative between NPIS as well as NNIS using the equations (11) and (12) which are given below:

$$\hbar^+_\gamma = \sqrt{\sum_{\theta=1}^{\eta}\left(\tau_{\gamma\theta} - \rho^+\right)^2}, \gamma = 1(1)\vartheta \qquad (11)$$

$$\hbar^-_\gamma = \sqrt{\sum_{\theta=1}^{\eta}\left(\tau_{\gamma\theta} - \rho^-\right)^2}, \gamma = 1(1)\vartheta \qquad (12)$$

Step 6: Evaluate the performance score for each alternative using the equation (13):

$$\aleph_\gamma = \hbar^-_\gamma \Big/ \left(\hbar^+_\gamma + \hbar^-_\gamma\right), \gamma = 1(1)\vartheta \qquad (13)$$

Step 7: Rank the alternatives according to their performance scores. The alternative with the highest performance score will be given the top ranking, while the alternative with the lowest performance score will be assigned the lowest ranking.

4. Methodology

This research endeavors to determine the principal indicators that are essential for carrying out an efficiency analysis through the application of a novel fuzzy-based MCDM methodology. The methodology comprises two fundamental stages: the implementation of the MCDM technique and the validation of the proposed model. The intricate process is shown schematically in **Figure 2**.

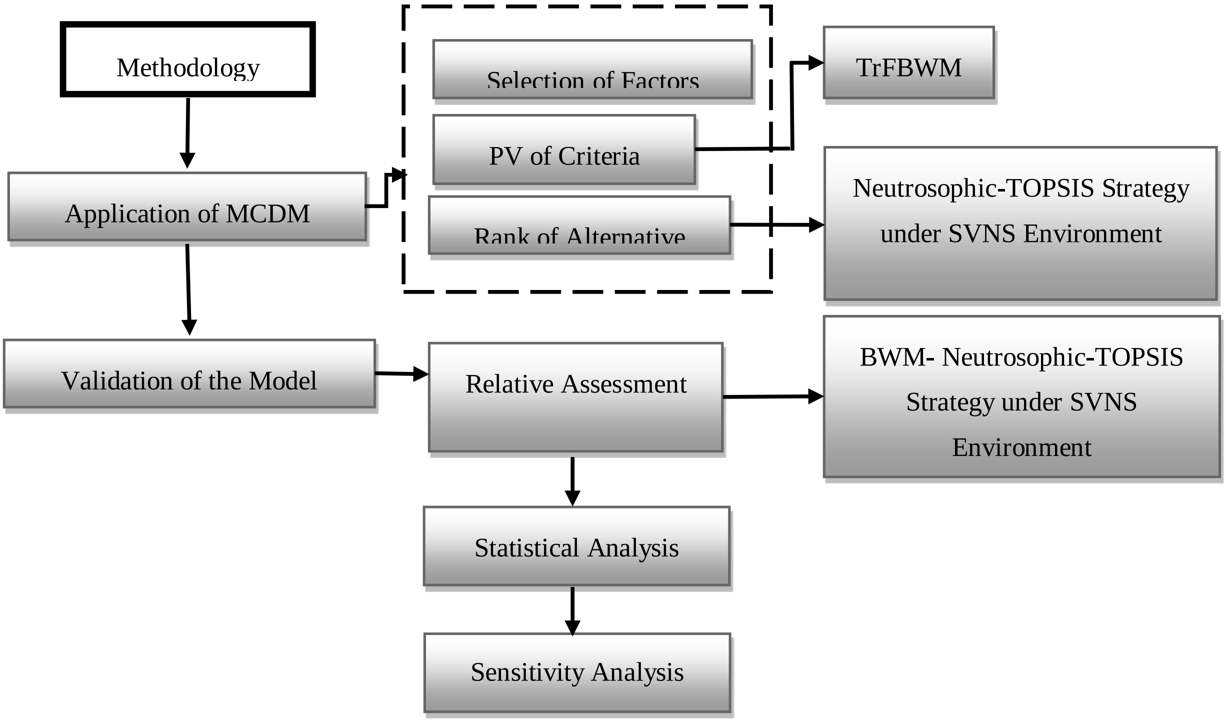

Figure 2: Schematic diagram of Methodology.

4.1 Application of MCDM

The purpose of the following section is to evaluate the PV of each criterion and alternative. Consequently, this phase encompasses three components: the identification of factors, the utilization of TrFBWM, and the application of Neutrosophic-TOPSIS strategy [54, 55] under SVNS environment. The decision hierarchy of the issue is depicted in **Figure 3**.

Step 1: Selection of Factors:

A thorough study of the relevant literature is reviewed for the selection of criteria and alternatives followed by the selection of a panel of specialists and stakeholders. **Table 5** displays all the chosen criteria and alternatives under examination.

Table 5: The selected alternatives and criteria for the consideration of this study.

Criteria	Alternative	Definition
Literature Review (Γ_1)	Nitrate (M_1)	Nitrate levels in aquaponic recirculating water can have several effects on fish growth. Elevated nitrate levels can lead to stress in fish and may impair their growth rates over time. High nitrate concentrations may affect water quality parameters such as pH and alkalinity, which can in turn impact fish metabolism. Changes in

		water chemistry can disrupt metabolic processes, leading to reduced growth rates in fish [18].
Aquaponics Experts (Γ_2)	Temperature (M_2)	The temperature in aquaponics refers to the measurement of the heat level within the aquaponic system, which encompasses both the water and the surrounding environment. Maintaining an optimal temperature is crucial for the overall health and productivity of the aquaponics living components, especially fish, within the system. Temperature directly influences the metabolic rate, oxygen demand, efficiency of feed conversion, digestive efficiency, of the fish. Additionally, it effects the susceptibility of the fish to disease. Overall, maintaining a stable and suitable temperature range within the aquaponics system is essential for promoting optimal fish growth [13-15, 20].
Aquaponics Field Workers (Γ_3)	pH (M_3)	Extreme pH levels can be harmful to fish. Very low or very high pH can cause stress to fish, leading to reduced appetite, impaired growth, and increased susceptibility to diseases. pH also effects the respiration process of fish. Optimal pH levels support efficient respiration, ensuring adequate oxygen supply for growth [16,17].
	Ammonia (M_4)	Ammonia is released into the water through fish waste and decomposing organic matter. High levels of ammonia are toxic to fish, causing stress, tissue damage, and even death. Ammonia toxicity can impair fish growth by affecting metabolic processes and reducing appetite [17].
	Turbidity (M_5)	Turbidity in aquaponics system is the haziness of the recirculating water due to Total Suspended Solids (TSS). High turbidity levels can interfere with fish feeding behaviour. Elevated levels of turbidity can disrupt fish feeding behavior and induce stress in fish. Furthermore, it can compromise the respiratory process and increase the likelihood of fish diseases [56].
	Dissolved Oxygen (M_6)	Fish rely on dissolved oxygen in the water for respiration. Adequate oxygen levels are essential to support the metabolic processes involved in growth. Insufficient oxygen can lead to stress, reduced growth rates, and even mortality in fish [16, 20].

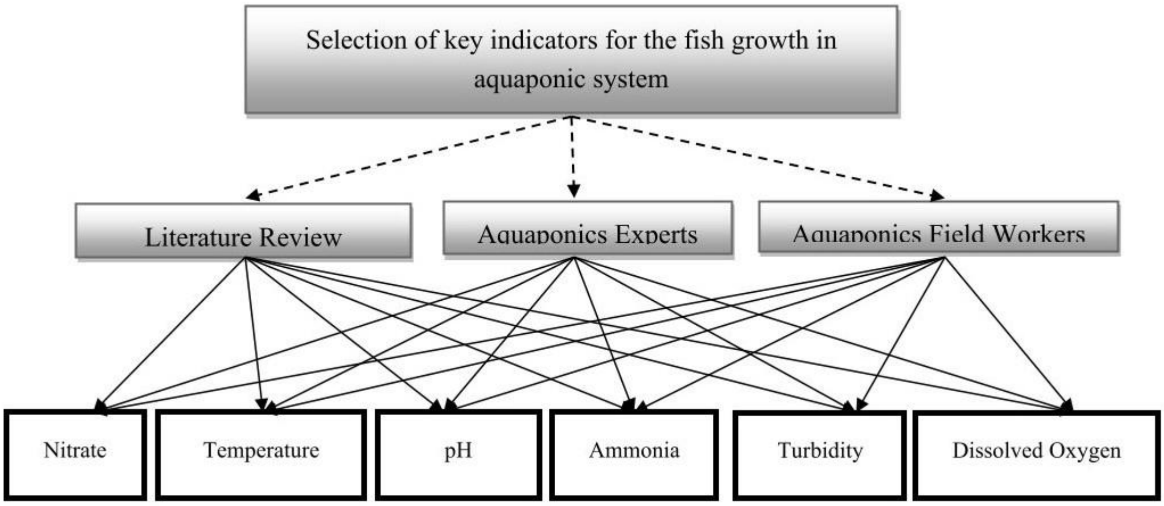

Figure 3: Hierarchical structure of the decision-making problem.

Step 2: Application of TrFBWM (Model I):

Initially, the process involves identifying the most favorable and least favorable criteria according to the consensus among the majority of specialists. Subsequently, pairwise evaluations for each criterion are conducted, as outlined in **Table 6**. Following this, additional pairwise comparisons are made utilizing the least favorable criteria, as demonstrated in **Table 7**. Equation (3) embodies this mathematical formulation after constructing the pairwise comparison matrix.

Table 6: Table illustrating the comparison of each criterion with the best criteria (BC).

	Γ_1	Γ_2	Γ_3
Γ_1 (Best Criteria)	$(1,1,1,1)$	$\left(2,\frac{5}{2},\frac{7}{2},4\right)$	$\left(6,\frac{13}{2},\frac{15}{2},8\right)$

Table 7: Table illustrating the comparison of the worst criteria (WC) with each set of criterion.

	Γ_3 (Worst Criteria)
Γ_1	$\left(6,\frac{13}{2},\frac{15}{2},8\right)$
Γ_2	$\left(4,\frac{9}{2},\frac{11}{2},6\right)$
Γ_3	$(1,1,1,1)$

$$\left.\begin{aligned}
&\min\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\text{Subject to}\left|\frac{\left(\chi_{\tilde{P}_\varepsilon},\tau_{\tilde{P}_\varepsilon},\rho_{\tilde{P}_\varepsilon},\xi_{\tilde{P}_\varepsilon}\right)}{\left(\chi_{\tilde{P}_1},\tau_{\tilde{P}_1},\rho_{\tilde{P}_1},\xi_{\tilde{P}_1}\right)}-\left(\chi_{\tilde{\partial}_{\varepsilon 1}},\tau_{\tilde{\partial}_{\varepsilon 1}},\rho_{\tilde{\partial}_{\varepsilon 1}},\xi_{\tilde{\partial}_{\varepsilon 1}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\left|\frac{\left(\chi_{\tilde{P}_\varepsilon},\tau_{\tilde{P}_\varepsilon},\rho_{\tilde{P}_\varepsilon},\xi_{\tilde{P}_\varepsilon}\right)}{\left(\chi_{\tilde{P}_2},\tau_{\tilde{P}_2},\rho_{\tilde{P}_2},\xi_{\tilde{P}_2}\right)}-\left(\chi_{\tilde{\partial}_{\varepsilon 2}},\tau_{\tilde{\partial}_{\varepsilon 2}},\rho_{\tilde{\partial}_{\varepsilon 2}},\xi_{\tilde{\partial}_{\varepsilon 2}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\left|\frac{\left(\chi_{\tilde{P}_\varepsilon},\tau_{\tilde{P}_\varepsilon},\rho_{\tilde{P}_\varepsilon},\xi_{\tilde{P}_\varepsilon}\right)}{\left(\chi_{\tilde{P}_3},\tau_{\tilde{P}_3},\rho_{\tilde{P}_3},\xi_{\tilde{P}_3}\right)}-\left(\chi_{\tilde{\partial}_{\varepsilon 3}},\tau_{\tilde{\partial}_{\varepsilon 3}},\rho_{\tilde{\partial}_{\varepsilon 3}},\xi_{\tilde{\partial}_{\varepsilon 3}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\left|\frac{\left(\chi_{\tilde{P}_1},\tau_{\tilde{P}_1},\rho_{\tilde{P}_1},\xi_{\tilde{P}_1}\right)}{\left(\chi_{\tilde{P}_\omega},\tau_{\tilde{P}_\omega},\rho_{\tilde{P}_\omega},\xi_{\tilde{P}_\omega}\right)}-\left(\chi_{\tilde{\partial}_{\omega 1}},\tau_{\tilde{\partial}_{\omega 1}},\rho_{\tilde{\partial}_{\omega 1}},\xi_{\tilde{\partial}_{\omega 1}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\left|\frac{\left(\chi_{\tilde{P}_2},\tau_{\tilde{P}_2},\rho_{\tilde{P}_2},\xi_{\tilde{P}_2}\right)}{\left(\chi_{\tilde{P}_\omega},\tau_{\tilde{P}_\omega},\rho_{\tilde{P}_\omega},\xi_{\tilde{P}_\omega}\right)}-\left(\chi_{\tilde{\partial}_{\omega 2}},\tau_{\tilde{\partial}_{\omega 2}},\rho_{\tilde{\partial}_{\omega 2}},\xi_{\tilde{\partial}_{\omega 2}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\left|\frac{\left(\chi_{\tilde{P}_3},\tau_{\tilde{P}_3},\rho_{\tilde{P}_3},\xi_{\tilde{P}_3}\right)}{\left(\chi_{\tilde{P}_\omega},\tau_{\tilde{P}_\omega},\rho_{\tilde{P}_\omega},\xi_{\tilde{P}_\omega}\right)}-\left(\chi_{\tilde{\partial}_{\omega 3}},\tau_{\tilde{\partial}_{\omega 3}},\rho_{\tilde{\partial}_{\omega 3}},\xi_{\tilde{\partial}_{\omega 3}}\right)\right|\le\left(\Delta,\Delta,\Delta,\Delta\right)\\
&\sum_{i=1}^{3}\Re\left(\tilde{P}_i\right)=1\\
&0\le\chi_{\tilde{P}_i}\le\tau_{\tilde{P}_i}\le\rho_{\tilde{P}_i}\le\xi_{\tilde{P}_i}\le 1,\ i=1,2,3\\
&\Delta\ge 0
\end{aligned}\right\}\qquad(14)$$

The equation (14) presents an optimization problem that is nonlinear in fuzzy form. The crisp Mathematical expression of equation (14) is represented by equation (15).

$$\left.\begin{aligned}
&\min \Delta \\
&\text{Subject to } \chi_{\tilde{P}_\varepsilon} - 2\xi_{\tilde{P}_2} \le \Delta\xi_{\tilde{P}_2}, \chi_{\tilde{P}_\varepsilon} - 82\xi_{\tilde{P}_2} \ge -\Delta\xi_{\tilde{P}_2} \\
&\tau_{\tilde{P}_\varepsilon} - \frac{5}{2}\rho_{\tilde{P}_2} \le \Delta\rho_{\tilde{P}_2}, \tau_{\tilde{P}_\varepsilon} - \frac{5}{2}\rho_{\tilde{P}_2} \ge -\Delta\rho_{\tilde{P}_2} \\
&\rho_{\tilde{P}_\varepsilon} - \frac{7}{2}\tau_{\tilde{P}_2} \le \Delta\tau_{\tilde{P}_2}, \rho_{\tilde{P}_\varepsilon} - \frac{7}{2}\tau_{\tilde{P}_2} \ge -\Delta\tau_{\tilde{P}_2} \\
&\xi_{\tilde{P}_\varepsilon} - 4\chi_{\tilde{P}_2} \le \Delta\chi_{\tilde{P}_2}, \xi_{\tilde{P}_\varepsilon} - 4\chi_{\tilde{P}_2} \ge -\Delta\chi_{\tilde{P}_2} \\
&\chi_{\tilde{P}_\varepsilon} - 6\xi_{\tilde{P}_3} \le \Delta\xi_{\tilde{P}_3}, \chi_{\tilde{P}_\varepsilon} - 6\xi_{\tilde{P}_3} \ge -\Delta\xi_{\tilde{P}_3} \\
&\tau_{\tilde{P}_\varepsilon} - \frac{13}{2}\rho_{\tilde{P}_3} \le \Delta\rho_{\tilde{P}_3}, \tau_{\tilde{P}_\varepsilon} - \frac{13}{2}\rho_{\tilde{P}_3} \ge -\Delta\rho_{\tilde{P}_3} \\
&\rho_{\tilde{P}_\varepsilon} - \frac{15}{2}\tau_{\tilde{P}_3} \le \Delta\tau_{\tilde{P}_3}, \rho_{\tilde{P}_\varepsilon} - \frac{15}{2}\tau_{\tilde{P}_3} \ge -\Delta\tau_{\tilde{P}_3} \\
&\xi_{\tilde{P}_\varepsilon} - 8\chi_{\tilde{P}_3} \le \Delta\chi_{\tilde{P}_3}, \xi_{\tilde{P}_\varepsilon} - 8\chi_{\tilde{P}_3} \ge -\Delta\chi_{\tilde{P}_3} \\
&\chi_{\tilde{P}_2} - 4\xi_{\tilde{P}_\omega} \le \Delta\xi_{\tilde{P}_\omega}, \chi_{\tilde{P}_2} - 4\xi_{\tilde{P}_\omega} \ge -\Delta\xi_{\tilde{P}_\omega} \\
&\tau_{\tilde{P}_2} - \frac{9}{2}\rho_{\tilde{P}_\omega} \le \Delta\rho_{\tilde{P}_\omega}, \tau_{\tilde{P}_2} - \frac{9}{2}\rho_{\tilde{P}_\omega} \ge -\Delta\rho_{\tilde{P}_\omega} \\
&\rho_{\tilde{P}_2} - \frac{11}{2}\tau_{\tilde{P}_\omega} \le \Delta\tau_{\tilde{P}_\omega}, \rho_{\tilde{P}_2} - \frac{11}{2}\tau_{\tilde{P}_\omega} \ge -\Delta\tau_{\tilde{P}_\omega} \\
&b_{\tilde{P}_2} - 6\chi_{\tilde{P}_\omega} \le \Delta\chi_{\tilde{P}_\omega}, b_{\tilde{P}_2} - 6\chi_{\tilde{P}_\omega} \ge -\Delta\chi_{\tilde{P}_\omega} \\
&\sum_{i=1}^{3}\left(\frac{\chi_{\tilde{P}_i} + 2\tau_{\tilde{P}_i} + 2\rho_{\tilde{P}_i} + \xi_{\tilde{P}_i}}{6}\right) = 1 \\
&0 \le \chi_{\tilde{P}_i} \le \tau_{\tilde{P}_i} \le \rho_{\tilde{P}_i} \le \xi_{\tilde{P}_i} \le 1,\ i = 1,2,3 \\
&\Delta \ge 0
\end{aligned}\right\} \quad (15)$$

The relative relevance of each criterion is determined through PVs and the TrF-BWM approach. The optimal trapezoidal fuzzy PVs for three criteria, denoted as Γ_1, Γ_2 and Γ_3, can be evaluated utilizing the solutions to equation (15), which are $\tilde{P}*_1$ = (0.544, 0.582, 0.660, 0.679), $\tilde{P}*_2$ = (0.229, 0.267, 0.345, 0.373), $\tilde{P}*_3$ = (0.075, 0.077, 0.077, 0.077), and Δ = (1.036, 1.036, 1.036, 1.036).
Subsequently, employing equation (8), the precise PVs for $\Re(\tilde{P}*_1)$ =0.618, $\Re(\tilde{P}*_2)$ =0.304, and $\Re(\tilde{P}*_3)$ = 0.076 are determined. The normalised PVs for each criterion are given in equation (9) and are $\tilde{P}*_1$=0.059, $\tilde{P}*_2$=0.281, and $\tilde{P}*_3$ = 0.660.Since $\tilde{P}_{\varepsilon\wp} = \tilde{P}_{\varepsilon 3} = \left(6, \frac{13}{2}, \frac{15}{2}, 8\right)$, so CI is 12.58 in this case that gives CR = 0.544/12.58 i.e., 0.0432. As a result, the model indicates that the most significant criterion is Γ_1.

Step 3: Neutrosophic-TOPSIS Strategy under SVNS Environment (Model II):

The decision maker employs SVNNs to evaluate alternatives based on their attributes, resulting in the creation of decision **Matrix-1**.

Matrix-1: Decision Matrix

$$\nabla = \begin{matrix} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{matrix} \begin{matrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ \left[\begin{matrix} (0.72,0.25,0.23) & (0.72,0.3,0.13) & (0.74,0.17,0.02) \\ (0.99,0.13,0.3) & (1,0.24,0.04) & (0.73,0.17,0.11) \\ (0.77,0.16,0.27) & (0.81,0.35,0.21) & (1,0.24,0.1) \\ (0.71,0.16,0.2) & (0.7,0.31,0.14) & (0.73,0.42,0.05) \\ (0.77,0.42,0.18) & (0.97,0.14,0.11) & (0.91,0.11,0.03) \\ (0.95,0.15,0.26) & (0.26,0.89,0.22) & (0.79,0.34,0.07) \end{matrix}\right] \end{matrix}$$

Matrix ∇ is obtained by translating the entries of each entry. Each entry in matrix ∇ is increased by 0.01 across all components, resulting in **Matrix-2**.

Matrix-2: Translation of ∇

$$\begin{matrix} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{matrix} \begin{matrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ \left[\begin{matrix} (0.73,0.26,0.24) & (0.73,0.31,0.14) & (0.75,0.18,0.03) \\ (1,0.14,0.31) & (1.01,0.25,0.05) & (0.74,0.18,0.12) \\ (0.78,0.17,0.28) & (0.82,0.36,0.22) & (1.01,0.25,0.11) \\ (0.72,0.17,0.21) & (0.71,0.32,0.15) & (0.74,0.43,0.06) \\ (0.78,0.43,0.19) & (0.98,0.15,0.12) & (0.92,0.12,0.04) \\ (0.96,0.16,0.27) & (0.9,0.23,0.03) & (0.8,0.35,0.08) \end{matrix}\right] \end{matrix}$$

The subsequent stage involves determining the score matrix through the application of the score function. **Matrix-3** represents the score matrix denoted as ∇^*. The score value of

$$\mathrm{N}_1(0.73,0.26,0.24) = \frac{0.73\times0.73+0.26\times(-0.26)+0.24\times(-0.24)}{\sqrt{0.73^2+0.26^2+0.24^2}\sqrt{0.73^2+(-0.26)^2+(-0.24)^2}} = 0.61951$$

Matrix-3: Score Matrix

$$\nabla^* = \begin{matrix} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{matrix} \begin{matrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ \left[\begin{matrix} 0.61951 & 0.643231 & 0.888217 \\ 0.792596 & 0.880195 & 0.842530 \\ 0.700153 & 0.581373 & 0.863706 \\ 0.753128 & 0.602861 & 0.487841 \\ 0.467084 & 0.926 & 0.962894 \\ 0.806881 & 0.875434 & 0.664715 \end{matrix}\right] \end{matrix}$$

The Normalized Decision Matrix is determined by using equation (10) on matrix ∇^*. As shown in **Matrix-4**, the Normalized Decision Matrix decision matrix is denoted by Υ.

$$\upsilon_{11} = \frac{0.61951}{\sqrt{0.61951^2+0.792596^2+0.700153^2+0.753128^2+0.467084^2+0.806881^2}} = 0.361389$$

Matrix-4: Decision Matrix with Normalization

$$\Upsilon = \begin{matrix} & \Gamma_1 & \Gamma_2 & \Gamma_3 \\ M_1 & 0.361389 & 0.343146 & 0.452573 \\ M_2 & 0.462358 & 0.46956 & 0.429294 \\ M_3 & 0.408431 & 0.310147 & 0.440084 \\ M_4 & 0.439334 & 0.32161 & 0.24857 \\ M_5 & 0.272472 & 0.493996 & 0.490623 \\ M_6 & 0.470691 & 0.46702 & 0.338692 \end{matrix}$$

After establishing the weights of the criteria, the weighted normalized decision matrix is calculated. This involves multiplying each criteria weight by the corresponding element in its respective row of the matrix Υ. Denoted as matrix Ξ, it is represented by **Matrix-5.**

Matrix-5: Weighted Normalized Decision Matrix

$$\Xi = \begin{matrix} & \Gamma_1 & \Gamma_2 & \Gamma_3 \\ M_1 & 0.361389\times 0.61823 & 0.343146\times 0.304791 & 0.452573\times 0.076977 \\ M_2 & 0.462358\times 0.61823 & 0.46956\times 0.304791 & 0.429294\times 0.076977 \\ M_3 & 0.408431\times 0.61823 & 0.310147\times 0.304791 & 0.440084\times 0.076977 \\ M_4 & 0.439334\times 0.61823 & 0.32161\times 0.304791 & 0.24857\times 0.076977 \\ M_5 & 0.272472\times 0.61823 & 0.493996\times 0.304791 & 0.490623\times 0.076977 \\ M_6 & 0.470691\times 0.61823 & 0.46702\times 0.304791 & 0.338692\times 0.076977 \end{matrix}$$

$$= \begin{matrix} & \Gamma_1 & \Gamma_2 & \Gamma_3 \\ M_1 & 0.223422 & 0.104588 & 0.034838 \\ M_2 & 0.285844 & 0.143118 & 0.033046 \\ M_3 & 0.252505 & 0.09453 & 0.033877 \\ M_4 & 0.27161 & 0.098024 & 0.019134 \\ M_5 & 0.16845 & 0.150566 & 0.037767 \\ M_6 & 0.29099 & 0.142344 & 0.026072 \end{matrix}$$

Subsequently, ascertain NPIS and NNIS using the formulas $\tau^{+}_{\theta} = \max_{\theta} \tau_{\gamma\theta}, \theta = 1(1)6$ and $\tau^{-}_{\theta} = \min_{\theta} \tau_{\gamma\theta}, \theta = 1(1)6$ values, respectively.

$$\tau^{-}_{1} = \min\{0.223422, 0.285844, 0.252505, 0.27161, 0.16845, 0.290996\} = 0.16845$$

So, $\rho^{+} = \{\tau^{+}_{1}, \tau^{+}_{2}, \tau^{+}_{3}\} = \{0.290996, 0.150565, 0.037767\}$

$$\rho^{-} = \{\tau^{-}_{1}, \tau^{-}_{2}, \tau^{-}_{3}\} = \{0.16845, 0.09453, 0.019134\}$$

$$\tau^{+}_{1} = \max\{0.223422, 0.285844, 0.252505, 0.27161, 0.16845, 0.290996\} = 0.290996$$

Subsequently, calculate the distance between each alternative using NPIS and NNIS formulas (11) and (12). **Table 8** presents the distances between alternatives computed via NPIS and NNIS.

$$\hbar_1^+ = \sqrt{\left(\tau_{11} - \rho_1^+\right)^2 + \left(\tau_{12} - \rho_2^+\right)^2 + \left(\tau_{13} - \rho_3^+\right)^2}$$
$$= \sqrt{\left(0.223422 - 0.290996\right)^2 + \left(0.104588 - 0.150565\right)^2 + \left(0.034838 - 0.037767\right)^2}$$
$$= 0.081785$$

$$\hbar_1^- = \sqrt{\left(\tau_{11} - \rho_1^-\right)^2 + \left(\tau_{12} - \rho_2^-\right)^2 + \left(\tau_{13} - \rho_3^-\right)^2}$$
$$= \sqrt{\left(0.223422 - 0.16845\right)^2 + \left(0.104588 - 0.09453\right)^2 + \left(0.034838 - 0.019134\right)^2}$$
$$= 0.058048$$

Table 8: NPIS and NNIS distances from each alternative.

	Value		**Value**
$\hbar_1^+$	0.081785	$\hbar_1^-$	0.058048
$\hbar_2^+$	0.010213	$\hbar_2^-$	0.12781
$\hbar_3^+$	0.068093	$\hbar_3^-$	0.085338
$\hbar_4^+$	0.059022	$\hbar_4^-$	0.103219
$\hbar_5^+$	0.122545	$\hbar_5^-$	0.059052
$\hbar_6^+$	0.014296	$\hbar_6^-$	0.131725

Compute the performance score for each alternative utilizing formula (13). **Table 9** illustrates the performance scores for all alternatives. Arrange the alternatives in increasing order based on their performance scores and assign ranks correspondingly.

$$\aleph_1 = \hbar_1^- \Big/ \left(\hbar_1^+ + \hbar_1^-\right)$$
$$= 0.058048 \Big/ \left(0.081785 + 0.058048\right)$$
$$= 0.415126$$

Table 9: Performance scores of alternative.

Alternatives	Performance scores	Rank
M1	$\aleph_1 = 0.415126$	5
M2	$\aleph_2 = 0.926008$	1
M3	$\aleph_3 = 0.556197$	4
M4	$\aleph_4 = 0.636206$	3
M5	$\aleph_5 = 0.325182$	6
M6	$\aleph_6 = 0.902095$	2

4.2 Result from comparative study

Experts have been consulted to ascertain the vectors suitable for employment in the BWM-Neutrosophic-TOPSIS Strategy under the SVNS Environment, as well as the most and least significant aspects. Through expert consensus, it has been determined that Γ_1 holds the highest significance, whereas, Γ_3 is regarded as the least significant criterion. The best-to-others vector is outlined in **Table 10** and the worst-to-others vector is delineated in **Table 11**.

Table 10: Comparison of best criteria with other criteria.

	Γ_1	Γ_2	Γ_3
Γ_1 (Best Criteria)	1	2	6

Table 11: To the worst criteria, there are other criteria.

	Γ_3 (Worst Criteria)
Γ_1	6
Γ_2	4
Γ_3	1

The nonlinear mathematical model (16) can also be used to determine the weight of each criterion.

$$\min \Delta$$

$$s.t. \left|\frac{P_1}{P_2} - 2\right| < \Delta$$

$$\left|\frac{P_1}{P_3} - 6\right| < \Delta$$

$$\left|\frac{P_2}{P_3} - 4\right| < \Delta \quad (16)$$

$$\sum_{j=1}^{3} P_j = 1$$
$$P_j \geq 0, \text{ for all } j = 1(1)3$$

Utilizing the equation (10) on matrix $\nabla *$, the Normalized Decision Matrix using can be determined. As shown in **Matrix-6**, the Normalized Decision Matrix decision matrix is denoted by Υ.

Matrix-6: Normalized Decision Matrix

$$\Upsilon = \begin{matrix} & \Gamma_1 & \Gamma_2 & \Gamma_3 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{matrix} \begin{bmatrix} 0.361389 & 0.343146 & 0.452573 \\ 0.462358 & 0.46956 & 0.429294 \\ 0.408431 & 0.310147 & 0.440084 \\ 0.439334 & 0.32161 & 0.24857 \\ 0.272472 & 0.493996 & 0.490623 \\ 0.470691 & 0.46702 & 0.338692 \end{bmatrix}$$

After the criteria weights have been established, next step is to determine the weighted normalized decision matrix. This matrix is created by multiplying each criterion weight by the corresponding element in the associated matrix Υ row. This resulting weighted normalized decision matrix (**Matrix-7**) is denoted by matrix Ξ.

Matrix-7: Weighted Normalized Decision Matrix

$$\Xi = \begin{matrix} & \Gamma_1 & \Gamma_2 & \Gamma_3 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{matrix} \begin{bmatrix} 0.213548 & 0.109183 & 0.041143 \\ 0.273212 & 0.149405 & 0.039027 \\ 0.241346 & 0.098683 & 0.040008 \\ 0.259607 & 0.102331 & 0.022597 \\ 0.161006 & 0.15718 & 0.044602 \\ 0.278136 & 0.148597 & 0.03079 \end{bmatrix}$$

Next step is to identify NPIS and NNIS using the formulas $\tau^{+}_{\theta} = \max_{\theta} \tau_{\gamma\theta}, \theta = 1(1)6$ and $\tau^{-}_{\theta} = \min_{\theta} \tau_{\gamma\theta}, \theta = 1(1)6$ values, respectively.

So, $\rho^{+} = \{\tau^{+}_{1}, \tau^{+}_{2}, \tau^{+}_{3}\} = \{0.278135, 0.15718, 0.044602\}$

$\rho^{-} = \{\tau^{-}_{1}, \tau^{-}_{2}, \tau^{-}_{3}\} = \{0.161006, 0.098683, 0.0225972\}$

Next, use formulae (11) and (12) to determine each alternative's distance from both NPIS and NNIS. **Table 12** shows the distances for the weights of each alternative from NPIS and NNIS.

Table 12: Distances of each alternative between NPIS as well as NNIS.

	Value		Value
$\hbar_1^-$	0.080544	$\hbar_1^-$	0.0567
$\hbar_2^-$	0.01076	$\hbar_2^-$	0.124229
$\hbar_3^-$	0.069257	$\hbar_3^-$	0.082205
$\hbar_4^-$	0.061936	$\hbar_4^-$	0.098668
$\hbar_5^-$	0.11713	$\hbar_5^-$	0.062499
$\hbar_6^-$	0.016262	$\hbar_6^-$	0.127585

Use formula (13) to determine the evaluation score for each option. The performance scores for each alternative by proposed method and current method are shown in **Figure 4**.

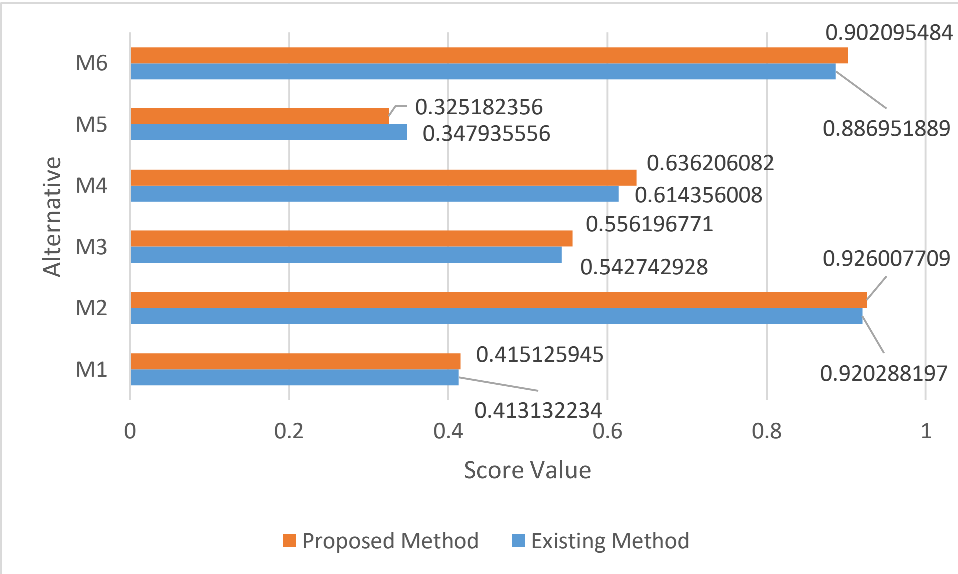

Figure 4: Comparative study.

4.3 Statistical Analysis

The rankings generated by the two methods can be compared using the Spearman correlation coefficient, which quantifies the linear association between two variables. This coefficient ranges from -1 to 1: -1 indicates no linear correlation, 0 signifies no linear correlation, and 1 denotes the complete absence of linear correlation. To assess the relationship between variables on interval scales, Pearson's correlation coefficient [56] can be utilized, as illustrated in equation (17).

$$\rho(\varpi, \iota) = \frac{\operatorname{cov}(\varpi, \iota)}{\sigma_\varpi \sigma_\iota} \tag{17}$$

γ as well as ς are covariant in $\operatorname{cov}(\gamma, \varsigma)$. SD is represented by γ as well as ς in both σ_γ as well as σ_ς.

The Pearson correlation coefficient is important for assessing the absence of a perfect correlation between two variables when it deviates from a value of 1, as given in equation (18).

$$\left.\begin{array}{l} H_0 : -\infty < \rho \leq 0 \\ H_1 : 0 < \rho < \infty \end{array}\right\} \quad 18)$$

Pearson correlation coefficient alongside Student's *t-distribution* with degrees of freedom $\square -1$, is presented by equation (19).

$$t = \rho\left(\frac{\square - 2}{1-\rho^2}\right)^{\frac{1}{2}} \quad (19)$$

The null hypothesis should be rejected if t (equation (19)) suppresses $t_\alpha(\square - 2)$. The Pearson correlation coefficient ρ falls within the range of□ .

Both methods produce rankings, which are evaluated using the Spearman correlation coefficient. If the rankings are identical, resulting a Spearman correlation coefficient of 1, there is no need for further hypothesis testing. However, if the rankings differ, a hypothesis test can be conducted to validate the Spearman correlation coefficients, as detailed in equation (17). To compare the proposed approach with BWM-TOPSIS strategy [44] under SVNS environment weights, the analysis will utilize the Pearson correlation coefficient, as outlined in **Table 13**. Notably, there is a discernible correlation between the proposed PV models and the currently established PV models, as supported by $\alpha = 0.05$ [58]. The analysis findings suggest that $t = \rho\left(\frac{\square - 2}{1-\rho^2}\right)^{\frac{1}{2}}$ should outperform $t_{0.05}(\square - 2)$. A hypothesis test is carried out to confirm a positive correlation between the attributes of the existing and proposed methods.

Table 13: t-Test: Paired Two Sample for Means.

	Proposed Method (Variable 1)	**Existing Method (Variable 2)**
Mean	0.626802	0.620901
Variance	0.061233	0.056849
Observations ρ	6	6
Pearson Correlation	0.9986	
Hypothesized Mean Difference α	0	
Df	5	
t Stat	0.920422	
P(T<=t) one-tail	0.199791	
t Critical one-tail	2.015048	
P(T<=t) two-tail	0.399583	
t Critical two-tail	2.570582	

4.4 Sensitivity Analysis

To explore the effect of altering weight coefficient values, different scenarios are examined, each consisting of a distinct set. Sensitivity analysis is employed to assess the predominant criterion using equation (20) and determine the sensitivity of the criterion PVs ($\Re(\tilde{P}_{\wp})$). Additionally, the progression of the dominant criterion is investigated.

$$\tilde{P}'_{\wp} = \tilde{P}_{\wp}\left(\frac{1-r\tilde{P}_{\varepsilon}}{1-\tilde{P}_{\varepsilon}}\right),\ \wp = 1(1)3 \qquad (20)$$

When $P'_{\wp}$ represents the initial value of the criterion, denoted, $\tilde{P}_{\wp}\,(\wp = 1(1)\Psi)$; $\tilde{P}_{\varepsilon}$, it signifies the criterion's starting point. $r_{\wp} \in (0,1) \cup \{0,1\}$, on the other hand, represents the adjusted value.

Equation (20) is employed to generate 25 distinct scenarios for this study. Across these scenarios, the variable r has the capacity to take on random values ranging from 0 to 1. **Figure 5** and **6** illustrate the outcomes of the sensitivity analysis conducted seperately for each alternative of $\hbar_{\gamma}^{+}$ and $\hbar_{\gamma}^{-}$. **Figure 7** shows the results of the sensitivity analysis. As shown in the **Figure 7**, "Temperature" is the most sensitive parameter in each case.

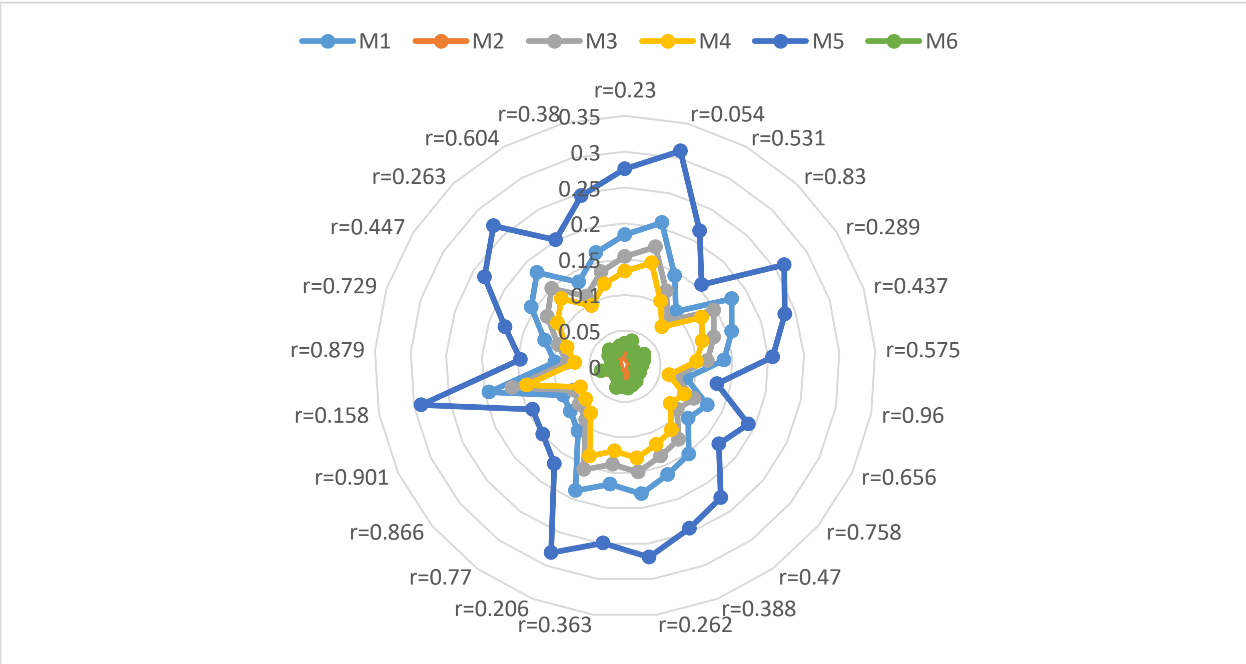

Figure 5: Results of the sensitivity analysis for $\hbar_{\gamma}^{+}$.

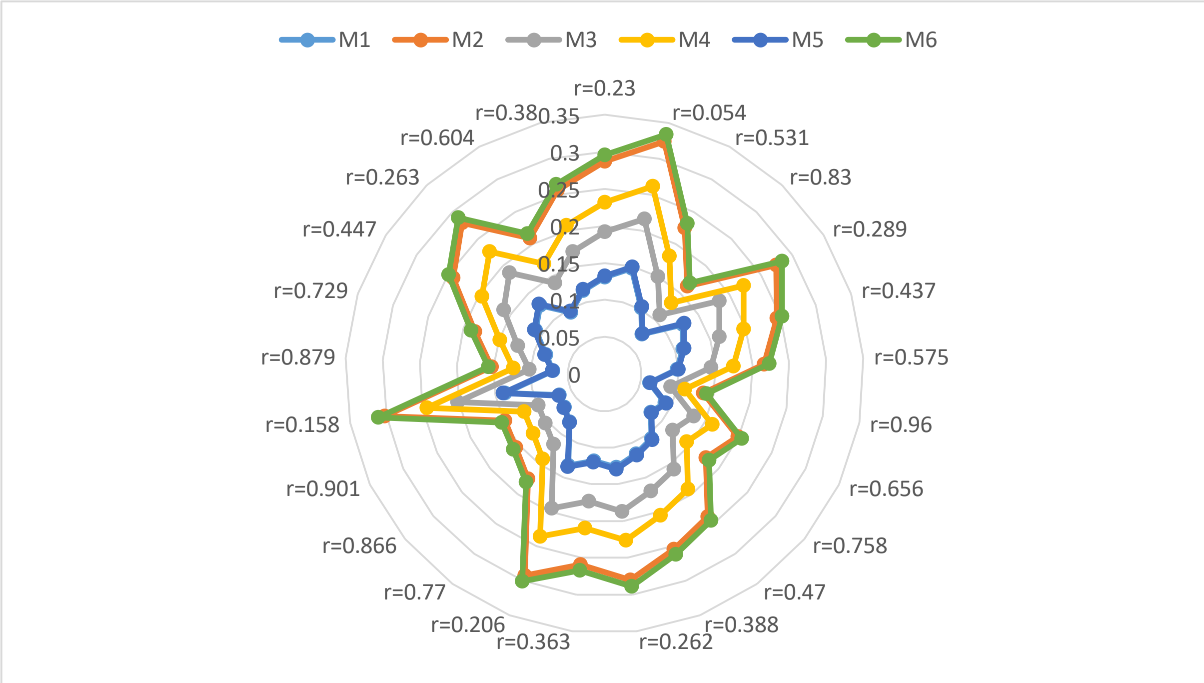

Figure 6: Results of the sensitivity analysis for $\hbar_{\gamma}^{-}$.

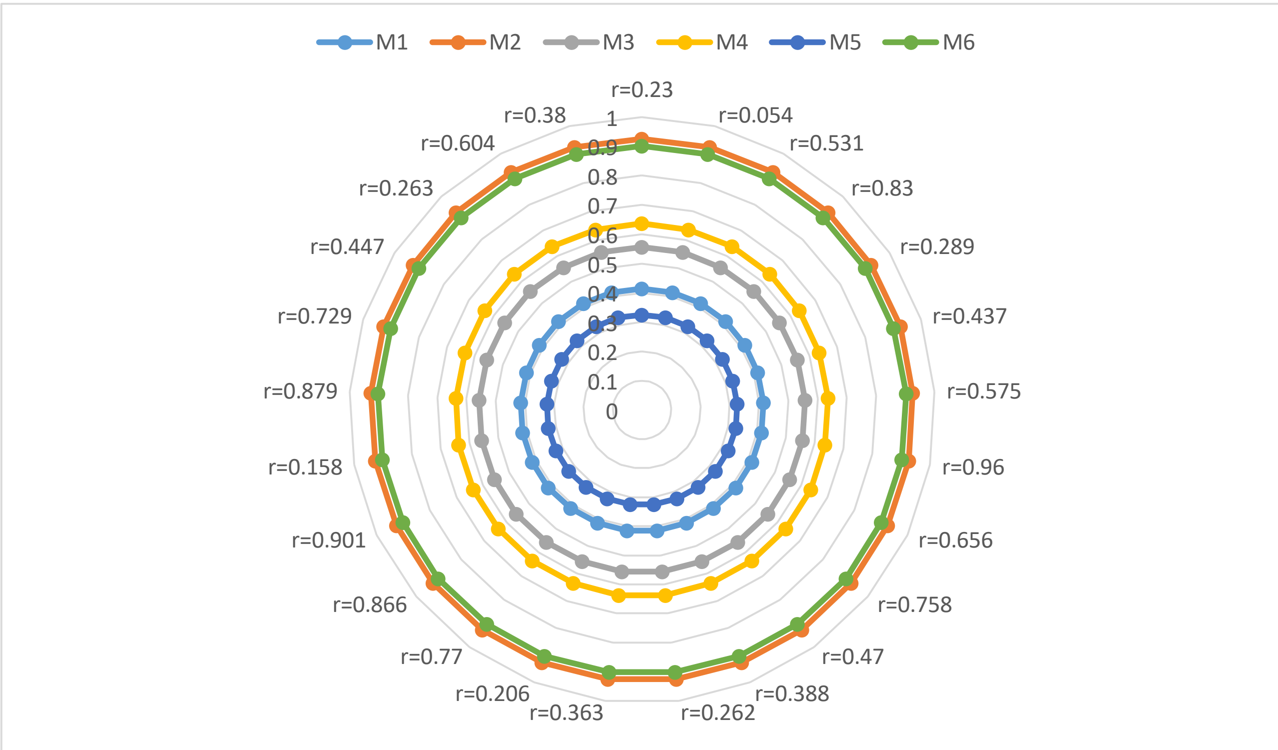

Figure 7: Results of the sensitivity analysis.

5. Conclusions

The objective of the present study was to enhance a hybrid MCDM method namely TrF-BWM-Neutrosophic-TOPSIS Strategy under SVNS Environment Approach. The method was applied to identify the most significant water quality parameter for fish growth within the aquaponic system. For this, the study selected three criteria and six alternatives through literature reviews. According to the results of the proposed method "Temperature" is the most significant factor, followed by "Dissolved oxygen".

The result was validated by comparing the proposed method with the BWM- Neutrosophic-TOPSIS Strategy under SVNS Environment Approach. The results of the proposed method, supported by the existing MCDM technique, are for the majority of significant alternatives. Statistical support is also provided for the results of the proposed method. Also, the model was validated by using Sensitivity Analysis and Scenario Analysis.

The current paradigm's heavy dependence on the method of development, as well as its features, is one of its main drawbacks. The results of the mathematical framework can shift when fresh variables or different approaches are used. Moreover, this method can be extended to aid in the selection of the most suitable fish species for aquaponic system, aiming to maximize overall system production.

In a nutshell, by pinpointing the primary factor driving fish growth, aquaponics practitioners can focus their efforts and resources on maintaining and optimizing that particular parameter. This not only simplifies system management but also carries the potential to boost fish yields, enhance economic returns, and advance environmental sustainability in aquaponic food production.

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Authors' Contribution: Each author has made an equal contribution in developing the paper.

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