Harmonic Mean Operator Based MADM-Strategy under SVPN-Set Environment

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Abstract: By using single-valued pentapartitioned neutrosophic set (briefly SVPN-set), one can express uncertainty events having incomplete, indeterminate, and erroneous information in practical applications. This study's primary goal is to present a multi-attribute decision-making (briefly MADM) algorithm that operates in the SVPN-set environment using the notion of single-valued pentapartitioned neutrosophic harmonic mean (briefly SVPNHM) operator and single-valued pentapartitioned neutrosophic weighted harmonic mean (briefly SVPNWHM) operator. Further, we provide a numerical illustration here like, "Location Selection for Installation of Outdoor Air Purifier" to validate the effectiveness and rationality of the proposed MADM algorithm.

Keywords: SVPN-set; Indeterminacy; MADM; SVPNHM-operator; SVPNWHM-operator.

1. Introduction:

During 1998, Smarandache [28] presented the idea of neutrosophic set (briefly N-set) and neutrosophic number (briefly N-number), which address uncertain and deficient data in another manner. Two components make up an N-number: a determinate part and an indeterminate part. In this manner, the N-numbers are more effective to manage uncertain and deficient data in certifiable issues. Afterwards, Wang et al. [29] grounded and studied single-valued N-set (briefly SVN-set). MADM and multi-attribute group decision-making (briefly MAGDM) are two critical parts of choice speculations which have been ordinarily applied in numerous logical fields. Till now, many researchers around the globe used SVN-set in their theoretical research [10, 11, 14, 17] and also proposed several MADM and MAGDM techniques [3, 9, 18, 20-23, 26-28] under the SVN-set environment. In the year 2009, Xu [30] presented the notion of harmonic mean operator under the fuzzy environment. Afterwards, Aydin et al. [2] developed a harmonic aggregation operator under trapezoidal Pythagorean fuzzy number environment. A MAGDM approach in fuzzy environment was suggested by Park et al. [25] using generalized fuzzy bonferroni harmonic mean operator. Later, in an intuitionistic fuzzy environment, Zhou et al. [32] presented a MADM method using the normalized weighted bonferroni harmonic mean operator. Thereafter, Mondal et al. [22] proposed a MAGDM strategy based on normalized neutrosophic harmonic mean operator under the SVN-set environment. The principles of SVPN-set and single-valued pentapartitioned neutrosophic number (briefly SVPN-number) were initially introduced in 2020 by Mallick and Pramanik [19]. In the context of the SVPN-set environment, Das et al. [6] came up with a tangent similarity based MADM approach. Afterwards, Das et al. [5] established a MADM algorithm using grey relational analysis method in the context of SVPN-set environment.

The main goal of this research is to demonstrate a novel MADM methodology using the on the SVPNWHM operator. Furthermore, we validate our MADM methodology by providing a real life example, namely "Location Selection for Installation of Outdoor Air Purifier".

The rest of this paper is organized in the following manner:

A few preliminary results on SVPN-set as well as score and accuracy function of SVPN-numbers are shown in section-2. In section-3, two aggregation operators namely, SVPNHM operator and SVPNWHM operator have been introduced. Besides, a unique MADM methodology based on the SVPNWHM operator is presented in section-4. The suggested MADM methodology is demonstrated using a numerical example in section-5, and the outcomes demonstrate their viability and efficacy. In section-6, we provide some closing thoughts and outline some potential areas for further investigation.

2. Preliminaries and Definitions:

The idea of SVPN-set theory was first grounded in 2020 by Mallick and Pramanik [19]. They achieved this by dividing the indeterminacy-membership into three distinct categories: contraction, ignorance and unknown-memberships. Below are the concepts of SVPN-set and SVPN-number.

Definition 2.1.[19] Consider Ü be a non-null set. Then *H*, a SVPN-set over Ü is defined by

$$Đ=\{(ȇ, Ť_{Đ}(ȇ), Č_{Đ}(ȇ), Ğ_{Đ}(ȇ), Ǔ_{Đ}(ȇ), Ň_{Đ}(ȇ)) : ȇ \in Ü\},$$

where $Ť_{Đ}(ȇ)$, $Č_{Đ}(ȇ)$, $Ğ_{Đ}(ȇ)$,$Ǔ_{Đ}(ȇ)$ and $Ň_{Đ}(ȇ)$ are the truth, contradiction, ignorance, unknown and falsity membership function lies in the interval [0,1] such that

$$0 \leq Ť_{Đ}(ȇ) + Č_{Đ}(ȇ) + Ğ_{Đ}(ȇ) + Ǔ_{Đ}(ȇ) + Ň_{Đ}(ȇ) \leq 5, \forall ȇ \in Ü.$$

For each $ȇ \in Ü$, $[Ť_{Đ}(ȇ), Č_{Đ}(ȇ), Ğ_{Đ}(ȇ), Ǔ_{Đ}(ȇ), Ň_{Đ}(ȇ)]$ is referred to as a SVPN-number.

Definition 2.2.[19] Assume that Ü be a non-empty set. Suppose that Đ={(ể, Ť$_{Đ}$(ê), Č$_{Đ}$(ê), Ğ$_{Đ}$(ê), Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)): ể∈Ü} and Q={(ể, Ť$_{Q}$(ê), Č$_{Q}$(ê), Ğ$_{Q}$(ê), Ǔ$_{Q}$(ê), Ň$_{Q}$(ê)): ể∈Ü} be two SVPN-sets defined over Ü. Then, *Đ⊆Q* if and only if Ť$_{Đ}$(ê)≤ Ť$_{Q}$(ê), Č$_{Đ}$(ê)≤Č$_{Q}$(ê), Ğ$_{Đ}$(ê)≥Ğ$_{Q}$(ê), Ǔ$_{Đ}$(ê)≥Ǔ$_{Q}$(ê), Ň$_{Đ}$(ê)≥Ň$_{Q}$(ê), ∀ ể∈Ü.

Definition 2.3.[19] Assume that Ü be a non-empty set. Suppose that Đ={(ể, Ť$_{Đ}$(ê), Č$_{Đ}$(ê), Ğ$_{Đ}$(ê), Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)): ể∈Ü} and Q={(ể, Ť$_{Q}$(ê), Č$_{Q}$(ê), Ğ$_{Q}$(ê), Ǔ$_{Q}$(ê), Ň$_{Q}$(ê)): ể∈Ü} be two SVPN-sets defined over Ü. Then, their intersection is defined by *Đ∩Q* = {(ể, min{Ť$_{Đ}$(ê),Ť$_{Q}$(ê)}, min{Č$_{Đ}$(ê),Č$_{Q}$(ê)}, max{Ğ$_{Đ}$(ê),Ğ$_{Q}$(ê)}, max{Ǔ$_{Đ}$(ê),Ǔ$_{Q}$(ê)}, max{Ň$_{Đ}$(ê),Ň$_{Q}$(ê)}): ể∈Ü}.

Definition 2.4.[19] Assume that Ü be a non-empty set. Suppose that Đ={(ể, Ť$_{Đ}$(ê), Č$_{Đ}$(ê), Ğ$_{Đ}$(ê), Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)): ể∈Ü} and Q={(ể, Ť$_{Q}$(ê), Č$_{Q}$(ê), Ğ$_{Q}$(ê), Ǔ$_{Q}$(ê), Ň$_{Q}$(ê)): ể∈Ü} be two SVPN-sets defined over Ü. Then, their union is defined by *Đ∪Q* = {(ể, max{Ť$_{Đ}$(ê),Ť$_{Q}$(ê)}, max{Č$_{Đ}$(ê),Č$_{Q}$(ê)}, min{Ğ$_{Đ}$(ê),Ğ$_{Q}$(ê)}, min{Ǔ$_{Đ}$(ê),Ǔ$_{Q}$(ê)}, min{Ň$_{Đ}$(ê),Ň$_{Q}$(ê)}): ể∈Ü}.

Definition 2.5.[19] Assume that Ü be a non-empty set. Suppose that Đ={(ể, Ť$_{Đ}$(ê), Č$_{Đ}$(ê), Ğ$_{Đ}$(ê), Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)): ể∈Ü} be a SVPN-set defined over Ü. Then, the complement of *Đ* is *Đ*^{c} = {(ể, Ň$_{Đ}$(ê), Ǔ$_{Đ}$(ê), 1-Ğ$_{Đ}$(ê), Č$_{Đ}$(ê), Ť$_{Đ}$(ê)): ể ∈Ü}.

Definition 2.6.[28] Assume that ể=[Ť$_{Đ}$(ê), Č$_{Đ}$(ê),Ğ$_{Đ}$(ê),Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)] and ῄ=[Ť$_{Đ}$(ῄ), Č$_{Đ}$(ῄ),Ğ$_{Đ}$(ῄ),Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ῄ)] be two different SVPN-numbers over Ü. Then,

(*i*) ể⊆ῄ if and only if Ť$_{Đ}$(ê) ≤Ť$_{Đ}$(ῄ), Č$_{Đ}$(ê) ≤Č$_{Đ}$(ῄ), Ğ$_{Đ}$(ê) ≥Ğ$_{Đ}$(ῄ), Ǔ$_{Đ}$(ê) ≥Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ê) ≥Ň$_{Đ}$(ῄ);

(*ii*) ể = ῄ if and only if Ť$_{Đ}$(ê) = Ť$_{Đ}$(ῄ), Č$_{Đ}$(ê) = Č$_{Đ}$(ê), Ğ$_{Đ}$(ê) = Ğ$_{Đ}$(ῄ), Ǔ$_{Đ}$(ê) = Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ê) = Ň$_{Đ}$(ῄ);

(*iii*) μ. ể = [1-(1-Ť$_{Đ}$(ê))^{μ}, 1-(1-Č$_{Đ}$(ê))^{μ}, (Ğ$_{Đ}$(ê))^{μ}, (Ǔ$_{Đ}$(ê))^{μ}, (Ň$_{Đ}$(ê))^{μ}], where μ>0;

(i*v*) ê^{μ} = [(Ť$_{Đ}$(ê))^{μ}, (Č$_{Đ}$(ê))^{μ}, 1-(1-Ğ$_{Đ}$(ê))^{μ}, 1-(1-Ǔ$_{Đ}$(ê))^{μ}, 1-(1-Ň$_{Đ}$(ê))^{μ}], where μ>0;

(*v*) ể+ῄ = [Ť$_{Đ}$(ê)+ Ť$_{Đ}$(ῄ)-Ť$_{Đ}$(ê).T$_{Đ}$(ῄ), Č$_{Đ}$(ê)+C$_{Đ}$(ῄ)-Č$_{Đ}$(ê).Č$_{Đ}$(ῄ), Ğ$_{Đ}$(ê).Ğ$_{Đ}$(ῄ), Ǔ$_{Đ}$(ê).Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ê).Ň$_{Đ}$(ῄ)];

(*vi*) ể. ῄ = [Ť$_{Đ}$(ê).Ť$_{Đ}$(ῄ), Č$_{Đ}$(ê).C$_{Đ}$(ῄ), Ğ$_{Đ}$(ê)+Ğ$_{Đ}$(ῄ)-Ğ$_{Đ}$(ê).Ğ$_{Đ}$(ῄ), Ǔ$_{Đ}$(ê)+Ǔ$_{Đ}$(ῄ)-Ǔ$_{Đ}$(ê).Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ê)+ Ň$_{Đ}$(ῄ)-Ň$_{Đ}$(ê).Ň$_{Đ}$(ῄ)].

Definition 2.7. [28] Suppose that ể=[Ť$_{Đ}$(ê),Č$_{Đ}$(ê),Ğ$_{Đ}$(ê),Ǔ$_{Đ}$(ê),Ň$_{Đ}$(ê)] be a SVPN-number over Ü. Then,

(*i*) score function is defined as follows:

$$S(ê) = \frac{[Ť_{Đ}(ể)+Č_{Đ}(ể)+1-Ğ_{Đ}(ể)+1-Ǔ_{Đ}(ể)+1-Ň_{Đ}(ể)]}{5} \quad (1)$$

(*ii*) accuracy function is defined as follows:

$$A(ê) = \frac{[Ť_{Đ}(ể)+Č_{Đ}(ể)-Ň_{Đ}(ể)]}{3} \quad (2)$$

Definition 2.8.[28] Let ể=[Ť$_{Đ}$(ê), Č$_{Đ}$(ê),Ğ$_{Đ}$(ê),Ǔ$_{Đ}$(ê), Ň$_{Đ}$(ê)] and ῄ=[Ť$_{Đ}$(ῄ), Č$_{Đ}$(ῄ),Ğ$_{Đ}$(ῄ),Ǔ$_{Đ}$(ῄ), Ň$_{Đ}$(ῄ)] be two SVPN-numbers defined over Ü. Then,

(*i*) *S*(ê) > *S*(ῄ) ⇒ ể >ῄ;

(*ii*) *S*(ê) = *S*(ῄ), *A*(ê) > *A*(ῄ) ⇒ ể>ῄ;

(*iii*) *S*(ê) = *S*(ῄ), *A*(ê) = *A*(ῄ), T$_{K}$(ê)>T$_{K}$(ῄ) ⇒ ể>ῄ.

Proposition 2.1.[28] The accuracy and score functions of a SVPN-number are bounded.

Proposition 2.2.[28] The accuracy and score functions of a SVPN-number are monotone.

3. Single-Valued Pentapartitioned Neutrosophic Harmonic Mean Operator:

Here, we introduce the notions of SVPNHM operator and SVPNWHM operator under the SVPN-set environment.

Definition 3.1. Suppose that ȇt=[ŤK(ȇt),ČK(ȇt),ǦK(ȇt),ǓK(ȇt),ŇK(ȇt)] (t=1, 2, 3, …., y) be the family of SVPN-numbers over Ü. Then, the SVPNHM operator is defined as follows:

$$\text{SVPNHM}\ (ȇ_1,\ ȇ_2,\ldots,ȇ_y) = \frac{y}{\sum_{t=1}^{y}\frac{1}{ȇ_t}}, \quad (3)$$

where $\frac{1}{v_t}$ = [1/(ŤR(ȇ)+1), 1/(ČR(ȇ)+1), 1/(ǦR(ȇ)+1), 1/(ǓR(ȇ)+1), 1/(ŇR(ȇ)+1)] (4)

Example 3.1. Let u = (0.6, 0.2, 0.6, 0.2, 0.7) and v = (0.7, 0.2, 0.1, 0.4, 0.5) be two SVPN-numbers over Ü. Then, SVPNHM (u, v) = $2.\frac{1}{(\frac{1}{u}+\frac{1}{v})}$ = (0.86, 0.79, 0.27, 0.34, 0.36), which is also a SVPN-number.

Definition 3.2. Suppose that ȇt=[ŤK(ȇt),ČK(ȇt),ǦK(ȇt),ǓK(ȇt),ŇK(ȇt)] (t=1, 2, 3, …., y) be the family of SVPN-numbers over Ü. Then, the SVPNWHM operator is defined as follows:

$$\text{SVPNWHM}\ (ȇ_1,\ ȇ_2,\ldots,ȇ_y) = \frac{y}{\sum_{t=1}^{y}\frac{\delta_t}{ȇ_t}}\ , \quad (5)$$

where δ_t is the corresponding weight of ȇt (t=1,2,…, y), such that $\delta_t \in [0,1]$ and $\sum_{t=1}^{y}\delta_t = 1$.

Example 3.2. Let u=(0.8, 0.2, 0.7, 0.4, 0.3) and v=(0.6, 0.5, 0.4, 0.3, 0.2) be two SVPN-numbers over Ü. Let δ_1=0.5 and δ_2=0.4 be the corresponding weighted value of u and v.

Then, SVPNWHM (u, v) = $2.\frac{1}{(\frac{\delta_1}{u}+\frac{\delta_2}{v})} = 2.\frac{1}{(\delta_1.\ (\frac{1}{u}) + \delta_2.\ (\frac{1}{v}))}$ = (0.32, 0.34, 0.81, 0.85, 0.80), which is also a SVPN-number.

4. SVPNWHM Operator Based MADM Strategy under the SVPN-set Environment:

This section developed a MADM technique using SVPNWHM operator under the SVPN-set environment.

In a MADM problem, suppose that, Θ be the set of n alternatives namely Θ1, Θ2,…, Θn, and Ǻ be the set of m attributes namely Ǻ1, Ǻ2,…, Ǻm. Assume that, the decision maker gives his/her evaluation information for each alternative Θi (i=1, 2, …, n) corresponding to the attributes Ǻj (j=1, 2, ……, m) by using the SVPN-sets as follows:

$$\Theta_i=\{(Ǻ_j,Ť_{\Theta_i}(Ǻ_j),Č_{\Theta_i}(Ǻ_j),Ǧ_{\Theta_i}(Ǻ_j),Ǔ_{\Theta_i}(Ǻ_j),Ň_{\Theta_i}(Ǻ_j)): Ǻ_j \in Ǻ\},$$

where $0 \le Ť_{\Theta_i}(Ǻ_j) + C_{\Theta_i}(Ǻ_j) + Ǧ_{\Theta_i}(Ǻ_j) + Ǔ_{\Theta_i}(Ǻ_j) + Ň_{\Theta_i}(Ǻ_j) \le 5$, for each i=1,2,…, n, and j=1,2,….., m.

For convenience,

Θ_{ij}=($\check{T}_{ij}$,$\check{C}_{ij}$,$\check{G}_{ij}$,$\check{U}_{ij}$,$\check{N}_{ij}$) denotes the SVPN-number ($\check{T}_{\Theta_i}$(Ấ$_j$), $\check{C}_{\Theta_i}$(Ấ$_j$), $\check{G}_{\Theta_i}$(Ấ$_j$), $\check{U}_{\Theta_i}$(Ấ$_j$), $\check{N}_{\Theta_i}$(Ấ$_j$)) in the SVPN-set Θ_i(i=1,2,…,n).

Then, the single-valued pentapartitioned neutrosophic decision matrix is denoted by $D=\left(A_{ij}\right)_{n\times m}$.

Our suggested MADM methodology consists of the subsequent steps:

Step-1: *Start*

Step-2: *Formation of the Decision Matrix*

By utilizing the assessment data of each alternative against each attribute, the decision maker constructed the decision matrix in this step.

Step-3: *Calculate the Weights of Each Attribute*

In this step, the decision maker can calculate the weight of each attribute by using the compromise function, which is described as follows:

Compromise Function:

For each attribute $\hat{w}_j$(j=1, 2, ….,m), the compromise function is defined by:

$$\Omega_j=\sum_{i=1}^{n}(3+\check{T}_{\Theta_i}(Ấ_j)+\check{C}_{\Theta_i}(Ấ_j)-\check{G}_{\Theta_i}(Ấ_j)-\check{U}_{\Theta_i}(Ấ_j)-\check{N}_{\Theta_i}(Ấ_j))/5 \quad (6)$$

The j^{th} attribute's weights are then determined as follows:

$$\hat{w}_j=\frac{\Omega_j}{\sum_{j=1}^{m}\Omega_j} \quad (7)$$

Here, $\sum_{j=1}^{m}\hat{w}_j=1$.

Step-4. *Calculation of Aggregate Value using the SVPNWHM Operator*

In this stage, the decision maker can use eq. (4) and eq. (5) to calculate the aggregate value of all the attributes with respect to the corresponding alternative.

Step-5. *Determine the Score Value of Each Alternative with respect to the Aggregate Value*

Using equations (1) and (2), the decision maker calculates each alternative's score and accuracy value in relation to the aggregate values of all alternatives in this stage.

Step-6. *Ranking of the Alternatives*

In this stage, the decision-maker uses Definition 2.8 to assign a rank to each choice.

Step-7. *End*

The flow chart of our proposed MADM strategy was shown by the following flow chart:

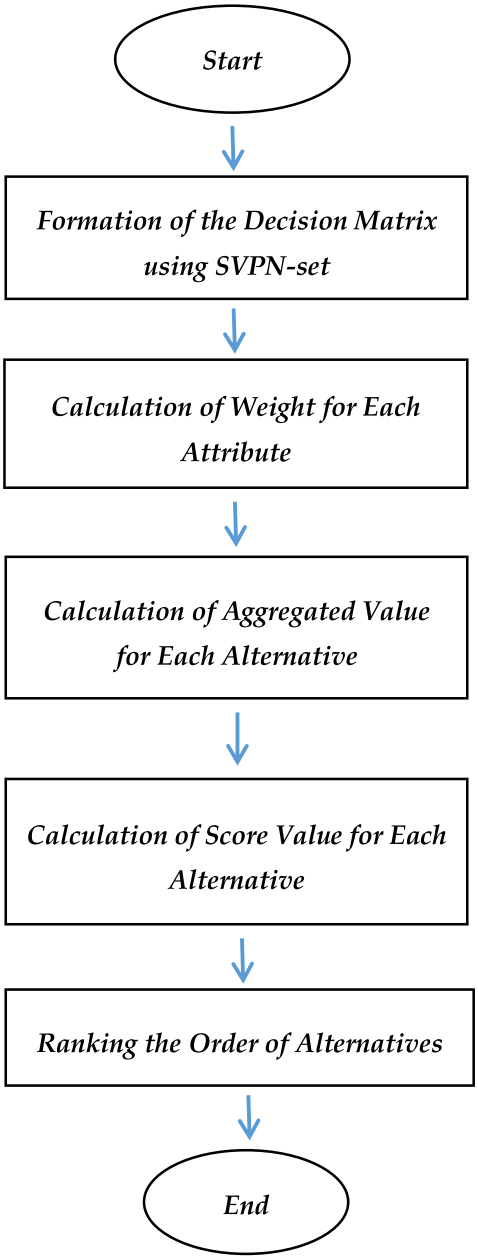

Figure-1

5. Validation of the Proposed MADM-Strategy:

Example 5.1. "*Location Selection for the Installation of Outdoor Air Purifier*".

Suppose that, the state government of any state (of India) wants to install an Air Purifier in a particular area to supply the fresh air in the environment to neutralize the air concentration. Let the government initiates to select the appropriate place for the installation of Air Purifier. After the initial screening, the decision makers select four alternatives (areas) $\breve{A}_1$, $\breve{A}_2$ and $\breve{A}_3$, for further evaluation.

The appropriate and scientific process that makes the fresh air from pollute air is done by air purifier. The process must improve the quality of air in the environment. Air pollution means where we meet the undesired concentration of impurities in air. After the filtering Air by purifier it remove dust, airborne bacteria, airborne particles, neutralize smoke, and many other particles.

The following are the stages of the air purifier treatment plants:

(*a*) Absorbed pollute Air,

(*b*) Removed visible particle by using pad filter and pleated filter,

(*c*) Carbon filtering it removes odors and capture gases and organic compound from smoke,

(*d*) Capture microscope particles,

(*e*) Supply fresh air,

(*f*) Changing filter.

The organic and inorganic components are filtered naturally. To increase the air quality at indoor normally used air purifier, there are few natural way have noticed some of them increase Vegetation, Salt lamp, Active charcoal, House plant, Essential oil like oregano, cinnamon , rosemary. Bacteria and other harmful micro-pollutants are removed by the using … plates. Sometimes, it is very difficult to choose a geographical area for the installation of air purifier treatment plant because of its hierarchical nature. Till now, many researchers around the globe presented their models for the selection of land for the installation of air purifier treatment plant. Based on a review of the accumulated scientific evidence some guidelines were well documented in WHO manual (WHO 2005) on four most common air pollutants- particulate matter , ozone , nitrogen dioxide and sulfur dioxide for maintaining the quality of natural air . So, the location of installation of air purifier treatment plant has also decide the type of air treatment process, steps of treatment procedures to be implemented in the pollute air treatment plant for supply of fresh air in desired level of purity. That is why the selection of location for installation or relocation of air purifier treatment plant can be treated as a MADM problem. The selection of attribute is a high factor for identifying the location for the installation of the air purifier treatment plant. For the selection of location for the installation of air purifier treatment plant, the decision maker need to choose some attributes so that they can easily select the suitable location.

In [4], Choudhury et al. have selected four attributes namely (*i*) Technological Constraints, (*ii*) Environmental Constraints, (*iii*) Economical Constraints, (*iv*) Political Constraints, for the selection of geographical land for the installation of surface water treatment plant.

Here, we have also selected the above mentioned four attributes to select the suitable location for the installation of Air Purifier.

1. Technological Constrains (Θ_1):

Among many parameter technological constraints is one of the most important factor for the air purifier treatment like machinery selection, Engineering process, use of electrify, air quality index level (briefly AQI), different components level in air etc. Technology can help to maintain the all factors including the animal health and environment. This is why we considering technical constrain is the first criterion in this study.

2. Environmental Constrains (Θ_2):

Environmental constrains is another essential parameter for air purifier treatment plant. In case of place selection in highly pollutant and population area is a difficult task. Outside of city are not suitable since the consumption air purifier needed to install in extremely pollute places. But some time hospital, stadium, school area has to decide for installing air purifier for minimizes the other issues. The work of conventional smoke tower plants clean polluted air and minimizes air pollution.

3. Economical Constrains (Θ_3):

For the air treatment plant economical constrains is the one of the most important factor to execute the installation plane. The cost of distribution of air from the treatment to the environment and other expenditure like, machinery, labor, plates and other technology used to minimize. The use of the technological and geographical selection can help the economical constrains.

4. Political Constrains (Θ_4):

The political constrains is another basic factor for the proper installation of air purifier. The balancing of clean air management economic rationale of demand management is the work of political constrains.

By using the decision makers' evaluation information about the alternatives against each attributes we get the following decision matrix:

Table-1

	$\acute{A}_1$	$\acute{A}_2$	$\acute{A}_3$	$\acute{A}_4$
Θ_1	(0.8,0.6,0.2,0.3,0.7)	(0.6, 0.4,0.1,0.5,0.6)	(0.5,0.5,0.7,0.4,0.3)	(0.8, 0.5,0.4,0.2,0.4)
Θ_2	(0.8, 0.6,0.4,0.2,0.5)	(0.7, 0.2,0.4,0.6,0.7)	(0.9, 0.3,0.4,0.3,0.8)	(0.7, 0.2,0.3,0.6,0.4)
Θ_3	(0.9, 0.5,0.7,0.2,0.4)	(0.6, 0.2,0.4,0.7,0.4)	(0.5, 0.4,0.4,0.3,0.7)	(0.8, 0.2,0.4,0.6,0.5)

By using eq. (6) and eq. (7), we get the weight vector $\hat{w}= (\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4)$ = (0.2900302, 0.2205438, 0.2356495, 0.2537764) for the previously mentioned four attributes corresponding to the four alternatives.

Now, the decision maker can find the aggregate value of all the attributes with respect to the corresponding alternative by using eq. (4) and eq. (5). The aggregate value is given in the following table.

Table-2

	Aggregate Value
SVPNWHM (Θ_1: $\acute{A}_1$, $\acute{A}_2$, $\acute{A}_3$, $\acute{A}_4$)	(0.99771196, 0.70732183, 0.02562288, 0.15405526, 0.19856120)
SVPNWHM (Θ_2: $\acute{A}_1$, $\acute{A}_2$, $\acute{A}_3$, $\acute{A}_4$)	(0.99820335, 0.75082793, 0.02466152, 0.15405526, 0.39608476)
SVPNWHM (Θ_3: $\acute{A}_1$, $\acute{A}_2$, $\acute{A}_3$, $\acute{A}_4$)	(0.99686998, 0.68982898, 0.05410631, 0.17649619, 0.19856120)

Now, we have, $S(\Theta_1)$=0.8653589, $S(\Theta_2)$=0.8348459, $S(\Theta_3)$=0.8515071, by using the eq. (1). This implies, $S(\Theta_2)< S(\Theta_3)< S(\Theta_1)$. Hence, Θ_1 is the most suitable location for the installation of the outdoor Air purifier.

6. Conclusions:

In this study, we have grounded the notion of SVPNHM operator, SVPNWHM operator, and provide some appropriate examples on them. Then, we have proposed a SVPNWHM operator based MADM technique under the SVPN-set environment. Finally, we have given a real world numerical example namely "Location Selection for the Installation of Outdoor Air Purifier" demonstrate the practical significance of our recommended MADM methodology.

The recommended MADM methodology can also be utilized for dealing with the other decision-making problems such as brick selection [20], surface water treatment plant location selection [4], logistic center location selection [26, 27], tender selection [9], etc.

Furthermore, the proposed MADM technique is expected to open up new avenues for investigation in the SVPN-Set setting.

Conflict of Interest: The authors affirm that they have no conflicts of interest.

Authors Contribution: All authors contributed equally to the preparation of this article.

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