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Identification of most impact factors of CIBIL score using Fermatean Neutrosophic Dombi Fuzzy Graphs

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Abstract. Graph theory plays a vital role in modeling real-world scenarios like network security and expert systems. Various extensions of graph theoretical conceptions have been designed for addressing uncertainty in graphical network scenarios. The concept of Fermatean Neutrosophic Dombi Fuzzy Graphs ($FNDF_graphs$) represents a novel and innovative extension in graph theory, combining the principles of Fermatean Neutrosophic fuzzy graphs and the Dombi operator. FNDFGs enhances the representation and analysis of uncertain relationships in a graph also it offers a more comprehensive and flexible approach to modeling uncertainty in graph structures. The main objective of this present research study focused on FNDFGs and their operations. At the end, an algorithm for Fermatean Neutrosophic Dombi fuzzy multi-criteria decision-making is given, which incorporate the concepts of Fermatean Neutrosophic sets and Dombi operations. Furthermore, a numerical example based on the selection of the most suitable CIBIL score application is put forward to illuminate the aptness of the proposed research work.

Keywords: Fermatean Neutrosophic sets; Fermatean Neutrosophic Dombi fuzzy graphs; Dombi; Fermatean fuzzy graphs.

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1. Introduction

Fuzzy set (F_{set}) theory [1] is a mathematical framework for handling uncertainty and partial information, allowing elements to have varying degrees of membership between 0 and 1. This is in contrast to classical set (C_{set}) theory, where membership is binary (0 *or* 1). The F_{set} theory is particularly useful for improving decision-making processes and adapting to real-world scenarios where traditional binary logic falls short. Intuitionistic Fuzzy sets (IF_{set}), introduced by Atanassov [2], extend (F_{set}) by incorporating both membership and non-membership degrees for each element, with their sum not exceeding 1. Neutrosophic sets (N_{set}), developed by Smarandache [4], include truth, falsity, and indeterminacy degrees, allowing their sum to range from 0 to 3, which is an extension of the F_{set} and IF_{set} theory [1, 4, 5]. These extensions enhance applications in artificial intelligence, decision-making, and image analysis by providing more comprehensive measures of uncertainty.

IF_{set} and N_{set} provide robust frameworks for managing imprecision and uncertainty in complex scenarios. Yager [68] created Pythagorean fuzzy sets (PF_{set}) as an extension of IF_{set} by allowing the sum of the squares of membership and non-membership degrees to be less than or equal to 1. This extension provides greater flexibility and better handles uncertainty. Because of this, PF_{set} explain more uncertainty than IF_{set}. Smarandache [4] introduced and subsequently expanded the degree of dependence between components of F_{set} and N_{set}. One special caseknown as the Pythagorean Neutrosophic set (PN_{set}) with independent indeterminacy and dependent truth and falsity is selected from among the three membership functions of N_{set}, subject to the requirement that the sum of the squares for membership, indeterminacy, and non-membership fall between 0 and 2 [28]. Graphs are visuals that show the relationships between objects. Because relationships between objects are more ambiguous, fuzzy graph (F_{graph}) models must be framed rather than regular graphs (R_{graph}), which have the same structure. Kaufmann [9] introduced fuzzy graphs (IF_{graph}) using Zadeh's fuzzy relation. Rosenfeld defined and developed a number of basic and theoretical concepts, including cycles, connectedness, and bridges [10].

The IF_{graph} were introduced by Karunambigai and Parvathi [11], and their further development into the intuitionistic fuzzy hypergraph ($IF_{hypergraph}$)and subsequent exploration of its uses [12] are noteworthy. The qualities of degree and regular SVN_{graph} were also studied [14], while Broumi et al. provided instances and properties of SVN_{graph} [13]. Pythagorean fuzzy graphs PF_{graph} were introduced to the F_{graph} notion in [15]. F_{graph} and PN_{set} were combined to create the recently developed idea of Pythagorean neutrosophic fuzzy graph (PNF_{set}) [1622]. Dombi fuzzy graphs (DF_{graph}) was first introduced by Ashraf et al. [23].

The DF_{graph} were thereafter the subject of much investigation, leading to the creation of the interval valued Dombi fuzzy neutrosophic graph ($IVDFN_{graph}$)[24], the DF_{graph} [19, 25], the PDF_{graph} [25], and the Dombi bipolar fuzzy graph (DBF_{graph}) [27]. Fermatean fuzzy sets (FF_{set}) were first introduced in [36] by allowing them the sum of the cubes of membership and non-membership degrees is less than or equal to 1. Their features were further explored and extended to Fermatean fuzzy graphs (FF_{graph}) in [37, 38, 39]. Fermatean Neutrosophic fuzzy sets (FNF_{set}) were first described in [40] and combine the principles of FNF_{set} with fuzzy graphs, as discussed in [41], providing even more nuanced handling of uncertainty. The study in [37] proposes a new interpretation of the Fermatean neutrosophic Dombi fuzzy network and identifies several by products of its direct, cartesian construction. The Applications of Complex N_{graph} Structures are discussed in [38].

CIBIL, or the Credit Information Bureau (India) Limited, is a prominent credit information company in India, known for providing credit scores, commonly referred to as CIBIL scores. These scores are crucial for lenders in determining an individual's creditworthiness and influence lending decisions. The CIBIL score, ranging from 300 to 900, reflects factors like credit behavior, repayment history, debt-to-income ratio, and credit card usage. A higher score enhances the likelihood of loan approval at favorable terms. CIBIL plays a pivotal role in the Indian financial sector by aiding individuals and businesses in managing their credit health through regular checks and awareness of their credit reports. Maintaining a good CIBIL score is essential for financial well-being and favorable access to credit. Key practices include timely repayment of bills, keeping credit utilization low, having a diverse credit portfolio, limiting new credit applications, regularly checking your credit report for accuracy, maintaining a long credit history, being cautious with joint accounts, managing debt responsibly, updating personal information, using credit wisely, and avoiding settlements or write-offs. Consistent financial discipline and responsible credit management contribute to a positive credit profile over time. In this proposed work, the algorithm of financial decision making problem was developed using the concept of presents FND_{graphs}, which was implemented in the selection of best cibil score application.

This research proposal is organized as follows: Section 2 describes some basic prerequisite material on Pythagorean fuzzy graph (PF_{graph}), Pythagorean neutrosophic fuzzy graph (PNF_{graph}), Dombi fuzzy graph (DF_{graph}), and Pythagorean Dombi fuzzy graph (PDF_{graph}),theory. Section 3 proposes the concept of Fermatean neutrosophic Dombi fuzzy graph ($FNDF_{graph}$) and some basic operations with illustrative numerical examples. In Section 4 we discussed decision-making problems using $FNDF_{graph}$. Section 6 concludes the proposed work with some future directions.

2. Preliminaries

This section describes a brief review of PF_{graph}, PNF_{graph}, and DF_{graph} graph theory, which will be utilized for further development $FNDF_{graph}$.

Table 1. Literature review of basic Preliminaries

Typical Reference	**Definition**
S.Naz et aS.[15]	$\mathbf{PF_{graph}}: G = (\mathscr{M}, \mathscr{N})$ with $\mathscr{M} = \left\langle \tilde{\lambda}_{\mathscr{M}}, \tilde{\beta}_{\mathscr{M}} \right\rangle$ is a PF_{set} in $\mathscr{Z}$ with $0 \leq \tilde{\alpha}^2_{\mathscr{N}}(s^*) + \tilde{\beta}^2_{\mathscr{N}}(s^*) \leq 1 \ \forall \ s \in \mathscr{Z}$ and $\mathscr{N} = \left\langle \tilde{\lambda}_{\mathscr{N}}, \tilde{\beta}_{\mathscr{N}} \right\rangle$ is a PF_{set} in $\mathscr{Q} : \mathscr{Z} \times \mathscr{Z}$ such that $\tilde{\lambda}_{\mathscr{M}}(s^*, t^*) \leq (\tilde{\lambda}_{\mathscr{M}}(s^*) \wedge \tilde{\lambda}_{\mathscr{M}}(t^*))$, $\tilde{\beta}_{\mathscr{N}}(s^*, t^*) \geq (\tilde{\beta}_{\mathscr{M}}(s^*) \vee \tilde{\beta}_{\mathscr{N}}(t^*))$ and $0 \leq \tilde{\alpha}^2_{\mathscr{N}}(s^*, t^*) + \tilde{\beta}^2_{\mathscr{N}}(s^*, t^*) \leq 1$ $\forall \ s^*, t^* \in \mathscr{Z}$.
D.Ajay et al.[19]	**Dombi′s family** : $\mathbf{t-norm}: \frac{1}{1+[(\frac{1-\tilde{g_1}}{g_1})^{\tilde{\gamma}}+(\frac{1-g_2}{g_2})^{\tilde{\gamma}}]^{\frac{1}{\tilde{\gamma}}}}, \tilde{\gamma} > 0.$ $\mathbf{t-conorm}: \frac{1}{1+[(\frac{1-g_1}{g_1})^{-\tilde{\gamma}}+(\frac{1-g_2}{g_2})^{-\tilde{\gamma}}]^{\frac{1}{-\tilde{\gamma}}}}, \tilde{\gamma} > 0.$ $\mathbf{negation}: 1 - g_1.$ $\mathbf{T-operators}: \mathscr{T}(g_1, g_2) = \frac{g_1 g_2}{g_1 + g_2 - g_1 g_2}$ and $\mathscr{S}(g_1, g_2) = \frac{g_1 + g_2 - 2g_1 g_2}{1 - g_1 g_2}.$ which is obtained by taking $\tilde{\gamma} = 1$, in Dombi family of t-norms and t-conorms.
D.Ajay et al.[16]	$\mathbf{PNF_{graph}}$:$G = (\mathscr{M}, \mathscr{N})$, where $\mathscr{M} = \{a_1, a_2, \cdots, a_n\}$ such that $\tilde{\lambda}_{\mathscr{M}}, \tilde{\beta}_{\mathscr{M}}, \tilde{\gamma}_{\mathscr{M}}$ are from $\mathscr{M}$ to $[0,1]$ with $0 \leq \tilde{\lambda}^2_{\mathscr{M}}(a_i) + \tilde{\beta}^2_{\mathscr{M}}(a_i) + \tilde{\gamma}^2_{\mathscr{M}}(a_i) \leq 2 \ \forall a_i \in \mathscr{M}$ and $\tilde{\lambda}_{\mathscr{N}}, \tilde{\beta}_{\mathscr{N}}, \tilde{\gamma}_{\mathscr{N}}$ are from $\mathscr{A} \times \mathscr{A}$ to $[0,1]$ such that $\tilde{\lambda}_{\mathscr{N}}(a_i, a_j) \leq (\tilde{\lambda}_{\mathscr{M}}(a_i) \wedge \tilde{\lambda}_{\mathscr{M}}(a_j))$,$\tilde{\beta}_{\mathscr{N}}(a_i, a_j) \leq (\tilde{\beta}_{\mathscr{M}}(a_i) \wedge \tilde{\beta}_{\mathscr{M}}(a_j))$ and $\tilde{\gamma}_{\mathscr{N}}(a_i, a_j) \leq (\tilde{\gamma}_{\mathscr{M}}(a_i) \vee \tilde{\gamma}_{\mathscr{M}}(a_j))$ with $0 \leq \tilde{\lambda}^2_{\mathscr{N}}(a_i, a_j) + \tilde{\beta}^2_{\mathscr{N}}(a_i, a_j) + \tilde{\gamma}^2_{\mathscr{N}}(a_i, a_j) \leq 2 \ \forall a_i, a_j \in \mathscr{M} \times \mathscr{M}$.
P.Chellamani et al.[17]	$\mathbf{DF_{graph}}$: $G = (\mathscr{M}, \mathscr{N})$, where $\mathscr{M} : \mathscr{Z} \to [0,1]$ is contained in $\mathscr{Z}$ and $\mathscr{N} : \mathscr{Z} \times \mathscr{Z} \to [0,1]$ is a symmetric fuzzy relation on $\mathscr{M}$ such that $\tilde{\lambda}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)} \ \forall \ s^*, t^* \in \mathscr{Z}$.
Akram et al. [39]	$\mathbf{PDF_{graph}}$: $\tilde{\lambda}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}$, $\tilde{\beta}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - 2\tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)}{1 - \tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)}$ and $0 \leq \tilde{\lambda}^2_{\mathscr{N}}(s^*, t^*) + \tilde{\beta}^2_{\mathscr{N}}(s^*, t^*) \leq 1 \ \forall \ s^*, t^* \in \mathscr{Z}$.

3. **Fermatean Neutrosophic Dombi Fuzzy Graphs**

$FNDF_{graph}$ is a sophisticated mathematical framework that combines Fermatean fuzzy sets, Dombi operators, and neutrosophic logic. They provide a flexible and nuanced way to model complex systems with uncertain, vague, and contradictory information. This approach is valuable in fields such as decision-making and artificial intelligence, where precise handling of uncertain data is crucial.

Definition 3.1. $FNDF_{graph}$ is an ordered pair $G = (\mathscr{M}, \mathscr{N})$ with $\mathscr{M} = \left\langle \tilde{\lambda}_{\mathscr{M}}, \tilde{\beta}_{\mathscr{M}}, \tilde{\gamma}_{\mathscr{M}} \right\rangle$ is a FN_{set} in $\mathscr{X}$ and $\mathscr{N} = \left\langle \tilde{\lambda}_{\mathscr{N}}, \tilde{\beta}_{\mathscr{N}}, \tilde{\gamma}_{\mathscr{N}} \right\rangle$ is a FN_{set} in $\mathscr{Q} : \mathscr{X} \times \mathscr{X}$ such that

$$\tilde{\lambda}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*) \cdot \tilde{\lambda}_{\mathscr{M}}(s^*, t^*)},$$
$$\tilde{\beta}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*) \cdot \tilde{\beta}_{\mathscr{M}}(s^*, t^*)},$$
$$\tilde{\gamma}_{\mathscr{N}}(s^*, t^*) \leq \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)},$$
and $0 \leq \tilde{\alpha}^3_{\mathscr{N}}(s^*, t^*) + \tilde{\beta}^3_{\mathscr{N}}(s^*, t^*) + \tilde{\gamma}^3_{\mathscr{N}}(s^*, t^*) \leq 3$ for all $s^*, t^* \in \mathscr{X}$.

Definition 3.2. Let $\bar{\mathscr{N}} = \left\{ \left((s^*, t^*), \tilde{\lambda}_{\mathscr{N}}(s^*, t^*), \tilde{\beta}_{\mathscr{N}}(s^*, t^*), \tilde{\gamma}_{\mathscr{N}}(s^*, t^*) \right) / s^*, t^* \in \mathscr{E} \right\}$ be a $FNDF_{edge\ set}$ in $FNDF_{graph}$ then the order and size of $FNDF_{graph}$ is defined by

$(i).\bar{O}(FNDF_{graph}) = \left(\sum_{s^* \mathscr{X}} \tilde{\lambda}_{\mathscr{M}}(s^*), \sum_{s^* \in \mathscr{X}} \tilde{\beta}_{\mathscr{M}}(s^*), \sum_{s^* \in \mathscr{X}} \tilde{\gamma}_{\mathscr{M}}(s^*) \right),$

$(ii).\bar{S}(FNDF_{graph}) = \left(\sum_{s^*, t^* \in \mathscr{E}} \tilde{\lambda}_{\mathscr{N}}(s^*, t^*), \sum_{s^*, t^* \in \mathscr{E}} \tilde{\beta}_{\mathscr{N}}(s^*, t^*), \sum_{s^*, t^* \in \mathscr{E}} \tilde{\gamma}_{\mathscr{N}}(s^*, t^*) \right)$

Definition 3.3. Let $\bar{\mathscr{N}} = \left\{ \left((s^*, t^*), \tilde{\lambda}_{\mathscr{N}}(s^*, t^*), \tilde{\beta}_{\mathscr{N}}(s^*, t^*), \tilde{\gamma}_{\mathscr{N}}(s^*, t^*) \right) / s^*, t^* \in \mathscr{E} \right\}$ be a $FNDF_{edge\ set}$ in $FNDF_{graph}$ then the degree and total degree of vertex $s^* \in \mathscr{X}$ is defined by $\bar{D}_{FNDF_{graph}}(s^*) = \left(\bar{D}_{\tilde{\alpha}}(s^*), \bar{D}_{\tilde{\beta}}(s^*), \bar{D}_{\tilde{\gamma}}(s^*) \right)$, and $(\bar{T}\bar{D})_{FNDF_{graph}}(s^*) = \left((\bar{T}\bar{D})_{\tilde{\alpha}}(s^*), (\bar{T}\bar{D})_{\tilde{\beta}}(s^*), (\bar{T}\bar{D})_{\tilde{\gamma}}(s^*) \right)$, where

$(i).\bar{D}_{\tilde{\alpha}}(s^*) = \sum_{s^*, t^* \neq s^* \in \mathscr{X}} \tilde{\lambda}_{\mathscr{N}}(s^*, t^*) = \sum_{s^*, t^* \neq s \in \mathscr{X}} \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)},$

$\bar{D}_{\tilde{\beta}}(s^*) = \sum_{s^*, t^* \neq s^* \in \mathscr{X}} \tilde{\beta}_{\mathscr{N}}(s^*, t^*) = \sum_{s^*, t^* \neq s \in \mathscr{X}} \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)},$

$\bar{D}_{\tilde{\gamma}}(s^*) = \sum_{s^*, t^* \neq s^* \in \mathscr{X}} \tilde{\gamma}_{\mathscr{N}}(s^*, t^*) = \sum_{s^*, t^* \neq s \in \mathscr{X}} \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}$

$(ii).(\bar{T}\bar{D})_{\tilde{\alpha}}(s^*) = \sum_{s^*, t^* \neq s^* \in \mathscr{X}} \tilde{\lambda}_{\mathscr{N}}(s^*, t^*) + \tilde{\lambda}_{\mathscr{M}}(s^*) =$
$\sum_{s^*, t^* \neq s^* \in \mathscr{X}} \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} + \tilde{\lambda}_{\mathscr{M}}(s^*),$

$(\bar{T}\bar{D})_{\tilde{\beta}}(s^*) = \sum_{s^*, t^* \neq s^* \in \mathscr{X}} \tilde{\beta}_{\mathscr{N}}(s^*, t^*) + \tilde{\beta}_{\mathscr{M}}(s^*) =$
$\sum_{s^*, t^* \neq s^* \in \mathscr{X}} \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)} + \tilde{\beta}_{\mathscr{M}}(s^*),$

$(\bar{T}\bar{D})_{\tilde{\gamma}}(s^*) = \sum_{s^*,t^* \neq s^* \in \mathscr{Z}} \tilde{\gamma}_{\mathscr{N}}(s^*,t^*) + \tilde{\gamma}_{\mathscr{M}}(s^*) =$
$\sum_{s^*,t^* \neq s^* \in \mathscr{Z}} \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)} + \tilde{\gamma}_{\mathscr{M}}(s^*).$

Definition 3.4. The complement of a $FNDF_{graph}$ $G = (\mathscr{M}, \mathscr{N})$ is a $FNDF_{graph}$ $G^C = (\mathscr{M}^C, \mathscr{N}^C)$ which is defined by

$(i).\tilde{\lambda}^C_{\mathscr{M}}(s^*) = \tilde{\lambda}_{\mathscr{M}}(s^*)$, $\tilde{\beta}^C_{\mathscr{M}}(s^*) = \tilde{\beta}_{\mathscr{M}}(s^*)$ and $\tilde{\gamma}^C_{\mathscr{M}}(s^*) = \tilde{\gamma}_{\mathscr{M}}(s^*)$.

$(ii).\tilde{\lambda}^C_{\mathscr{N}}(s^*,t^*) =$
$$\begin{cases} \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)} & if\ \tilde{\lambda}_{\mathscr{N}}(s^*,t^*) = 0, \\ \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\tilde{\lambda}_{\mathscr{M}}(t^*)} - \tilde{\lambda}_{\mathscr{N}}(s^*,t^*) & if\ 0 < \tilde{\lambda}_{\mathscr{N}}(s^*,t^*) \leq 1 \end{cases}$$

$(iii).\tilde{\beta}^C_{\mathscr{N}}(s^*,t^*) =$
$$\begin{cases} \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)} & if\ \tilde{\beta}_{\mathscr{N}}(s^*,t^*) = 0, \\ \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\tilde{\beta}_{\mathscr{M}}(t^*)} - \tilde{\beta}_{\mathscr{N}}(s^*,t^*) & if\ 0 < \tilde{\beta}_{\mathscr{N}}(s^*,t^*) \leq 1 \end{cases}$$

$(iv).\tilde{\gamma}^C_{\mathscr{N}}(s^*,t^*) =$
$$\begin{cases} \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\tilde{\gamma}_{\mathscr{M}}(s^*)\tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\tilde{\gamma}_{\mathscr{M}}(t^*)} & if\ \tilde{\gamma}_{\mathscr{N}}(s^*,t^*) = 0, \\ \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\tilde{\gamma}_{\mathscr{M}}(s^*)\tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\tilde{\gamma}_{\mathscr{M}}(t^*)} - \tilde{\gamma}_{\mathscr{N}}(s^*,t^*) & if\ 0 < \tilde{\gamma}_{\mathscr{N}}(s^*,t^*) \leq 1 \end{cases}$$

Theorem 3.5. *If $G = (\mathscr{M}, \mathscr{N})$ is a $FNDF_{graph}$, then $(G^C)^C = G$.*

Proof. Consider G as a $FNDF_{graph}$. By definition, $(FNDF_{graph})^C$, we have
$(\tilde{\lambda}^C_{\mathscr{M}})^C(s^*) = \tilde{\lambda}^C_{\mathscr{M}}(s^*) = \tilde{\lambda}_{\mathscr{M}}(s^*)$, $(\tilde{\beta}^C_{\mathscr{M}})^C(s^*) = \tilde{\beta}^C_{\mathscr{M}}(s^*) = \tilde{\beta}_{\mathscr{M}}(s^*)$, $(\tilde{\gamma}^C_{\mathscr{M}})^C(s^*) = \tilde{\gamma}^C_{\mathscr{M}}(s^*) = \tilde{\gamma}_{\mathscr{M}}(s^*)$, for all $s^* \in \mathscr{Z}$.

If $\tilde{\lambda}_{\mathscr{N}}(s^*,t^*) = 0$, $\tilde{\beta}_{\mathscr{N}}(s^*,t^*) = 0$, $\tilde{\gamma}_{\mathscr{N}}(s^*,t^*) = 0$, then

$(\tilde{\lambda}^C_{\mathscr{M}})^C(s^*) =$
$$\frac{(\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C}{\tilde{\lambda}^C_{\mathscr{M}}(s^*) + \tilde{\lambda}^C_{\mathscr{M}}(t^*) - (\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C} = \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} = \tilde{\lambda}_{\mathscr{N}}(s^*,t^*),$$
$(\tilde{\beta}^C_{\mathscr{M}})^C(s^*) =$
$$\frac{(\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*))^C}{\tilde{\beta}^C_{\mathscr{M}}(s^*) + \tilde{\beta}^C_{\mathscr{M}}(t^*) - (\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*))^C} = \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)} = \tilde{\beta}_{\mathscr{N}}(s^*,t^*),$$
$(\tilde{\gamma}^C_{\mathscr{M}})^C(s^*) =$
$$\frac{\tilde{\gamma}^C_{\mathscr{M}}(s^*) + \tilde{\gamma}^C_{\mathscr{M}}(t^*) - 2(\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*))^C}{1 - (\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*))^C} = \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)} =$$
$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)$.

If $0 < \tilde{\lambda}_{\mathscr{N}}(s^*,t^*), \tilde{\beta}_{\mathscr{N}}(s^*,t^*), \tilde{\gamma}_{\mathscr{N}}(s^*,t^*) \leq 1$, then

$$(\tilde{\lambda}^C_{\mathscr{N}})^C(s^*,t^*) = \frac{(\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C}{\tilde{\lambda}^C_{\mathscr{M}}(s^*) + \tilde{\lambda}^C_{\mathscr{M}}(t^*) - (\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C} - \tilde{\lambda}^C_{\mathscr{N}}(s^*,t^*)$$

$$= \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} - \left[\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} - \tilde{\lambda}_{\mathscr{N}}(s^*,t^*)\right] = \tilde{\lambda}_{\mathscr{N}}(s^*,t^*),$$

$$(\tilde{\beta}^C_{\mathscr{N}})^C(s^*,t^*) = \frac{(\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*))^C}{\tilde{-}\mathscr{M}^C(s^*) + \tilde{\lambda}^C_{\mathscr{M}}(t^*) - (\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*))^C} - \tilde{\beta}^C_{\mathscr{N}}(s^*,t^*)$$

$$= \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)} - \left[\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*) + \tilde{\beta}_{\mathscr{M}}(t^*) - \tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)} - \tilde{\beta}_{\mathscr{N}}(s^*,t^*)\right] = \tilde{\beta}_{\mathscr{N}}(s^*,t^*),$$

$$(\tilde{\gamma}^C_{\mathscr{M}})^C(s^*) = \frac{\tilde{\gamma}^C_{\mathscr{M}}(s^*) + \tilde{\gamma}^C_{\mathscr{M}}(t^*) - 2(\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*))^C}{1 - (\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*))^C} - \tilde{\gamma}^C_{\mathscr{N}}(s^*,t^*)$$

$$= \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)} - \left[\frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)} - \tilde{\gamma}_{\mathscr{N}}(s^*,t^*)\right] = \tilde{\gamma}_{\mathscr{N}}(s^*,t^*)$$

$\forall s^*, t^* \in \mathscr{Z}$.

Hence, the $((FNDF_{graph})^C)^C$ is a $FNDF_{graph}$ itself. □

Definition 3.6. A homomorphism of $FNDF_{graph}$ $h : G_1 \to G_2$ with $G_1 = (\mathscr{M}_1, \mathscr{N}_1)$ and $G_2 = (\mathscr{M}_2, \mathscr{N}_2)$ is a map $h : \mathscr{Z}_1 \to \mathscr{Z}_2$ satisfying

$(i).\tilde{\lambda}_{\mathscr{M}_1}(s^*) \leq \tilde{\lambda}_{\mathscr{M}_2}(h(s^*))$,

$\tilde{\beta}_{\mathscr{M}_1}(s^*) \leq \tilde{\beta}_{\mathscr{M}_2}(h(s^*))$,

$\tilde{\gamma}_{\mathscr{M}_1}(s^*) \leq \tilde{\gamma}_{\mathscr{M}_2}(h(s^*))$.

$(ii).\tilde{\lambda}_{\mathscr{N}_1}(s^*,t^*) \leq \tilde{\lambda}_{\mathscr{N}_2}(h(s^*)h(t^*))$,

$\tilde{\beta}_{\mathscr{N}_1}(s^*,t^*) \leq \tilde{\beta}_{\mathscr{N}_2}(h(s^*)h(t^*))$,

$\tilde{\gamma}_{\mathscr{N}_1}(s^*,t^*) \leq \tilde{\gamma}_{\mathscr{N}_2}(h(s^*)h(t^*))\ \forall\ s^* \in \mathscr{Z}_1, (s^*,t^*) \in \mathscr{E}_1$.

Definition 3.7. An isomorphism of $FNDF_{graph}$ $h : G_1 \to G_2$ with $G_1 = (\mathscr{M}_1, \mathscr{N}_1)$ and $G_2 = (\mathscr{M}_2, \mathscr{N}_2)$ is a bijective mapping $h : \mathscr{Z}_1 \to \mathscr{Z}_2$ satisfying

$(i).\tilde{\lambda}_{\mathscr{M}_1}(s^*) = \tilde{\lambda}_{\mathscr{M}_2}(h(s^*))$,

$\tilde{\beta}_{\mathscr{M}_1}(s^*) = \tilde{\beta}_{\mathscr{M}_2}(h(s^*))$,

$\tilde{\gamma}_{\mathscr{M}_1}(s^*) = \tilde{\gamma}_{\mathscr{M}_2}(h(s^*))$.

$(ii).\tilde{\lambda}_{\mathscr{N}_1}(st) = \tilde{\lambda}_{\mathscr{N}_2}(h(s^*)h(t^*))$,

$\tilde{\beta}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\beta}_{\mathscr{N}_2}(h(s^*)h(t^*)),$

$\tilde{\gamma}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\gamma}_{\mathscr{N}_2}(h(s^*)h(t^*)) \ \forall \ s^* \in \mathscr{Z}_1, s^*, t^* \in \mathscr{E}_1.$

Definition 3.8. A weak isomorphism $FNDF_{graph}$ $h : G_1 \to G_2$ with $G_1 = (\mathscr{M}_1, \mathscr{N}_1)$ and $G_2 = (\mathscr{M}_2, \mathscr{N}_2)$ is a bijective homomorphism $h : \mathscr{Z}_1 \to \mathscr{Z}_2$ satisfying

(i).h is a homomorphism.

(ii).$\tilde{\lambda}_{\mathscr{M}_1}(s^*) = \tilde{\lambda}_{\mathscr{M}_2}(h(s^*)),$

$\tilde{\beta}_{\mathscr{M}_1}(s^*) = \tilde{\beta}_{\mathscr{M}_2}(h(s^*)),$

$\tilde{\gamma}_{\mathscr{M}_1}(s^*) = \tilde{\gamma}_{\mathscr{M}_2}(h(s^*)),\ s^* \in \mathscr{Z}_1.$

Definition 3.9. A co-weak isomorphism $FNDF_{graph}$ $h : G_1 \to G_2$ with $G_1 = (\mathscr{M}_1, \mathscr{N}_1)$ and $G_2 = (\mathscr{M}_2, \mathscr{N}_2)$ is a bijective homomorphism $h : \mathscr{Z}_1 \to \mathscr{Z}_2$ satisfying

(i).h is a homomorphism.

(ii).$\tilde{\lambda}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\lambda}_{\mathscr{N}_2}(h(s^*)h(t^*)),$

$\tilde{\beta}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\beta}_{\mathscr{N}_2}(h(s^*)h(t^*)),$

$\tilde{\gamma}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\gamma}_{\mathscr{N}_2}(h(s^*)h(t^*)) \ \forall \ s^* \in \mathscr{Z}_1, (s^*, t^*) \in \mathscr{E}_1.$

Definition 3.10. A $FNDF_{graph}$ $G = (\mathscr{M}, \mathscr{N})$ is said to be self-complement if $G^C \cong G$.

Proposition 3.11. *If $G = (\mathscr{M}, \mathscr{N})$ is a self-complementary $FNDF_{graph}$, then*

$$\sum\nolimits_{s^* \neq t} \tilde{\lambda}_{\bar{\mathscr{N}}}(s^*, t^*) = \frac{1}{2} \sum\nolimits_{s^* \neq t^*} \frac{\tilde{\lambda}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\lambda}_{\bar{\mathscr{M}}}(t^*)}{\tilde{\lambda}_{\bar{\mathscr{M}}}(s^*) + \tilde{\lambda}_{\bar{\mathscr{M}}}(t^*) - \tilde{\lambda}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\lambda}_{\bar{\mathscr{M}}}(s^*)},$$
$$\sum\nolimits_{s \neq t^*} \tilde{\beta}_{\bar{\mathscr{N}}}(s^*, t^*) = \frac{1}{2} \sum\nolimits_{s^* \neq t^*} \frac{\tilde{\beta}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\beta}_{\bar{\mathscr{M}}}(t^*)}{\tilde{\beta}_{\bar{\mathscr{M}}}(s^*) + \tilde{\beta}_{\bar{\mathscr{M}}}(t^*) - \tilde{\beta}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\beta}_{\bar{\mathscr{M}}}(t^*)},$$
$$\sum\nolimits_{s^* \neq t} \tilde{\gamma}_{\bar{\mathscr{N}}}(s^*, t^*) = \frac{1}{2} \sum\nolimits_{s^* \neq t^*} \frac{\tilde{\gamma}_{\bar{\mathscr{M}}}(s^*) + \tilde{\gamma}_{\bar{\mathscr{M}}}(t^*) - 2\ \tilde{\gamma}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\gamma}_{\bar{\mathscr{M}}}(t^*)}{1 - \tilde{\gamma}_{\bar{\mathscr{M}}}(s^*)\ \tilde{\gamma}_{\bar{\mathscr{M}}}(t^*)}.$$

Proof. Let G be a self-complementary $FNDF_{graph}$. Then there exists an isomorphism h : $\mathscr{Z}_1 \to \mathscr{Z}_2$ such that

$\tilde{\lambda}^C_{\mathscr{M}}(h(s^*)) = \tilde{\lambda}_{\mathscr{M}}(s^*),\ \tilde{\beta}^C_{\mathscr{M}}(h(s^*)) = \tilde{\beta}_{\mathscr{M}}(s^*),\ \tilde{\gamma}^C_{\mathscr{M}}(h(s^*)) = \tilde{\gamma}_{\mathscr{M}}(s^*).\ \forall\ s^* \in \mathscr{Z}_1$

$\tilde{\lambda}^C_{\mathscr{M}}(h(s^*))(h(t^*)) = \tilde{\lambda}_{\mathscr{M}}(s^*),\ \tilde{\beta}^C_{\mathscr{M}}(h(s^*))(h(t^*)) = \tilde{\beta}_{\mathscr{M}}(s^*),\ \tilde{\gamma}^C_{\mathscr{M}}(h(s^*))(h(t^*)) = \tilde{\gamma}_{\mathscr{M}}(s^*, t^*).\ \forall$ $(s^*, t^*) \in \mathscr{E}_1.$

By definition of complement of G, we have

$$\tilde{\lambda}^C_{\mathscr{M}}(h(s^*))(h(t^*)) = \frac{(\tilde{\lambda}_{\mathscr{M}}(h(s^*))\ \tilde{\lambda}_{\mathscr{M}}(h(t^*)))^C}{\tilde{\lambda}^C_{\mathscr{M}}(h(s^*)) + \tilde{\lambda}^C_{\mathscr{M}}(h(t^*)) - (\tilde{\lambda}_{\mathscr{M}}(h(s^*))\ \tilde{\lambda}_{\mathscr{M}}(h(t^*)))^C} - \tilde{\lambda}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\tilde{\lambda}_{\mathscr{M}}(s^*, t^*) = \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} - \tilde{\lambda}_{\mathscr{N}}(h(s^*)h(t^*))$$

$$\sum_{s^*\neq t^*}\tilde{\lambda}_{\mathscr{M}}(s^*,t^*)=\sum_{s^*\neq t^*}\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}-\sum_{s\neq t}\tilde{\lambda}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\sum_{s^*\neq t^*}\tilde{\lambda}_{\mathscr{M}}(s^*,t^*)+\sum_{s^*\neq t^*}\tilde{\lambda}_{\mathscr{N}}(h(s^*)h(t^*))=\sum_{s^*\neq t^*}\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}$$
$$2\sum_{s^*\neq t^*}\tilde{\lambda}_{\mathscr{M}}(s^*,t^*)=\sum_{s^*\neq t^*}\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}$$
$$\sum_{s^*\neq t^*}\tilde{\lambda}_{\mathscr{M}}(s^*,t^*)=\frac{1}{2}\sum_{s^*\neq t^*}\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}$$

Similarly,

$$\tilde{\beta}^C_{\mathscr{M}}(h(s^*))(h(t^*))=\frac{(\tilde{\beta}_{\mathscr{M}}(h(s^*))\ \tilde{\beta}_{\mathscr{M}}(h(t^*)))^C}{\tilde{\beta}^C_{\mathscr{M}}(h(s^*))+\tilde{\beta}^C_{\mathscr{M}}(h(t^*))-(\tilde{\beta}_{\mathscr{M}}(h(s^*))\ \tilde{\beta}_{\mathscr{M}}(h(t^*)))^C}-\tilde{\beta}_{\mathscr{N}}(h(s^*)h(t^*)),$$
$$\tilde{\beta}_{\mathscr{M}}(s^*,t^*)=\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}-\tilde{\beta}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\sum_{s^*\neq t^*}\tilde{\beta}_{\mathscr{M}}(s^*,t^*)=\sum_{s^*\neq t^*}\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}-\sum_{s\neq t}\tilde{\beta}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\sum_{s\neq t}\tilde{\beta}_{\mathscr{M}}(s^*,t^*)+\sum_{s^*\neq t^*}\tilde{\beta}_{\mathscr{N}}(h(s^*)h(t^*))=\sum_{s^*\neq t^*}\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}$$
$$2\sum_{s^*\neq t^*}\tilde{\beta}_{\mathscr{M}}(s^*,t^*)=\sum_{s^*\neq t^*}\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}$$
$$\sum_{s^*\neq t^*}\tilde{\beta}_{\mathscr{M}}(s^*,t^*)=\tfrac{1}{2}\sum_{s^*\neq t^*}\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}$$

$$\tilde{\gamma}^C_{\mathscr{M}}(h(s^*)h(t^*))=\frac{\tilde{\gamma}^C_{\mathscr{M}}(h(s^*))+\tilde{\gamma}^C_{\mathscr{M}}(h(t^*))-2\tilde{\gamma}^C_{\mathscr{M}}(h(s^*))\ \tilde{\gamma}^C_{\mathscr{M}}(h(t^*))}{1-\tilde{\gamma}^C_{\mathscr{M}}(h(s^*))\ \tilde{\gamma}^C_{\mathscr{M}}(h(t^*))}-\tilde{\gamma}^C_{\mathscr{M}}(h(s^*)h(t^*))$$
$$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}-\tilde{\gamma}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\sum_{s\neq t}\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\sum_{s^*\neq t^*}\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}-\sum_{s\neq t}\tilde{\gamma}_{\mathscr{N}}(h(s^*)h(t^*))$$
$$\sum_{s\neq t^*}\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)+\sum_{s\neq t}\tilde{\gamma}_{\mathscr{N}}(h(s^*)h(t^*))=\sum_{s^*\neq t^*}\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}$$
$$2\sum_{s^*\neq t^*}\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\sum_{s\neq t}\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}$$
$$\sum_{s^*\neq t^*}\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\frac{1}{2}\sum_{s\neq t}\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}.\ \square$$

Proposition 3.12. *Let $G=(\mathscr{M},\mathscr{N})$ be a $FNDF_{graph}$. If*

$$\tilde{\lambda}_{\mathscr{N}}(s^*,t^*)=\frac{1}{2}\left(\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}\right),$$
$$\tilde{\beta}_{\mathscr{N}}(s^*,t^*)=\frac{1}{2}\left(\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}\right),$$

$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\right)\ \forall\ s^*,t^* \in \mathscr{Z}$ *then G is self-complementary.*

Proof. Let G be a $FNDF_{graph}$ that satisfies

$$\tilde{\lambda}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}\right),$$
$$\tilde{\beta}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}\right),$$
$$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\right)\ \forall\ s^*,t^* \in \mathscr{Z}.$$

Then clearly, $I : \mathscr{Z} \to \mathscr{Z}$ is the identify mapping represents an isomorphism from G to G^C satisfying the condition:

$\tilde{\lambda}^C_{\mathscr{M}}(s^*) = \tilde{\lambda}^C_{\mathscr{M}}(I(s^*))$, $\tilde{\beta}^C_{\mathscr{M}}(s^*) = \tilde{\beta}^C_{\mathscr{M}}(I(s^*))$, and $\tilde{\gamma}^C_{\mathscr{M}}() = \tilde{\gamma}^C_{\mathscr{M}}(I(s^*))\ \forall\ s \in \mathscr{Z}$.

The membership grade of an edge (s^*,t^*) is given by

$$\tilde{\lambda}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}\right)\ \forall\ s^*,t^* \in \mathscr{Z}.$$

we have $\tilde{\lambda}^C_{\mathscr{N}}(I(s^*)I(t^*)) = \tilde{\lambda}^C_{\mathscr{N}}(s^*,t^*)$

$$= \frac{(\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C}{\tilde{\lambda}^C_{\mathscr{M}}(s^*)+\tilde{\lambda}^C_{\mathscr{M}}(t^*)-(\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C} - \tilde{\lambda}_{\mathscr{N}}(s^*,t^*)$$
$$= \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)} - \frac{1}{2}\left(\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}\right)$$
$$= \frac{1}{2}\left(\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}\right) = \tilde{\lambda}_{\mathscr{N}}(s^*,t^*).$$

In similar way, the indeterminacy grade of an edge (s^*,t^*) is

$$\tilde{\beta}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}\right)\ \forall\ s^*,t^* \in \mathscr{Z}.$$

we have $\tilde{\beta}^C_{\mathscr{N}}(I(s^*)I(t^*)) = \tilde{\beta}^C_{\mathscr{N}}(s^*,t^*)$

$$= \frac{(\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*))^C}{\tilde{\beta}^C_{\mathscr{M}}(s^*)+\tilde{\beta}^C_{\mathscr{M}}(t^*)-(\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*))^C} - \tilde{\lambda}_{\mathscr{N}}(s^*,t^*)$$
$$= \frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)} - \frac{1}{2}\left(\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}\right)$$
$$= \frac{1}{2}\left(\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}\right) = \tilde{\beta}_{\mathscr{N}}(s^*,t^*).$$

$$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*) = \frac{1}{2}\left(\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\right)\ \forall\ s^*,t^* \in \mathscr{Z}.$$

So, we have $\tilde{\gamma}^C_{\mathscr{M}}(I(s^*)I(t^*)) = \tilde{\gamma}^C_{\mathscr{N}}(s^*,t^*) = \frac{\tilde{\gamma}^C_{\mathscr{M}}(s^*)+\tilde{\gamma}^C_{\mathscr{M}}(t^*)-2\tilde{\gamma}^C_{\mathscr{M}}(s^*)\ \tilde{\gamma}^C_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}^C_{\mathscr{M}}(s^*)\ \tilde{\gamma}^C_{\mathscr{M}}(t^*)} - \tilde{\gamma}^C_{\mathscr{M}}(s^*,t^*)$

$= \frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)} - \frac{1}{2}\left(\frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\right)$
$= \frac{1}{2}\left(\frac{\tilde{\gamma}_{\mathscr{M}}(s^*) + \tilde{\gamma}_{\mathscr{M}}(t^*) - 2\ \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(s^*)}{1 - \tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\right) = \tilde{\gamma}_{\mathscr{N}}(s^*, t^*)$
Since the conditions of isomorphism $\tilde{\lambda}^C_{\mathscr{N}}(I(s^*)I(s^*)) = \tilde{\lambda}_{\mathscr{N}}(s^*, t^*), \tilde{\beta}^C_{\mathscr{N}}(I(s^*)I(t^*)) = \tilde{\beta}_{\mathscr{N}}(s^*, t^*)$ and $\tilde{\gamma}^C_{\mathscr{N}}(I(s^*)I(t^*)) = \tilde{\gamma}_{\mathscr{N}}(s^*, t^*)$ are satisfied by I, $G = (\mathscr{M}, \mathscr{N})$ is self-complementary. □

Proposition 3.13. *If $G_1 = (\mathscr{M}_1, \mathscr{N}_1)$ and $G_2 = (\mathscr{M}_2, \mathscr{N}_2)$ are two isomorphic $FNDF_{graphs}$, then their complements are also isomorphic.*

Proof. Suppose G_1 and G_2 are two isomorphic $FNDF_{graphs}$. Then by definition of isomorphism, there exists a bijective mapping $h : \mathscr{Z}_1 \to \mathscr{Z}_2$ that satisfies
$\tilde{\lambda}_{\mathscr{M}_1}(s^*) = \tilde{\lambda}_{\mathscr{M}_2}(h(s^*))$, $\tilde{\beta}_{\mathscr{M}_1}(s^*) = \tilde{\beta}_{\mathscr{M}_2}(h(s^*))$ and $\tilde{\gamma}_{\mathscr{M}_1}(s^*) = \tilde{\gamma}_{\mathscr{M}_2}(h(s^*))\ \forall\ s \in \mathscr{Z}_1$,
$\tilde{\lambda}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\lambda}_{\mathscr{N}_2}(h(s^*)h(t^*))$, $\tilde{\beta}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\beta}_{\mathscr{N}_2}(h(s^*)h(t^*))$ and $\tilde{\gamma}_{\mathscr{N}_1}(s^*, t^*) = \tilde{\gamma}_{\mathscr{N}_2}(h(s^*)h(t^*))$. $\forall\ s^*, t^* \in \mathscr{E}_1$.
From definition of $FNDF'_{graph}s$ complement, (s^*, t^*) is
$\tilde{\lambda}^C_{\mathscr{N}_1}(s^*, t^*) = \frac{\tilde{\lambda}_{\mathscr{M}_1}(s^*)\ \tilde{\lambda}_{\mathscr{M}_1}(t^*)}{\tilde{\lambda}_{\mathscr{M}_1}(s^*) + \tilde{\lambda}_{\mathscr{M}_1}(t^*) - \tilde{\lambda}_{\mathscr{M}_1}(s^*)\ \tilde{\lambda}_{\mathscr{M}_1}(t^*)} - \tilde{\lambda}_{\mathscr{N}_1}(s^*, t^*)$
$= \frac{\tilde{\lambda}_{\mathscr{M}_2}(h(s^*))\ \tilde{\lambda}_{\mathscr{M}_2}(h(t^*))}{\tilde{\lambda}_{\mathscr{M}_2}(h(s^*)) + \tilde{\lambda}_{\mathscr{M}_2}(h(t^*)) - \tilde{\lambda}_{\mathscr{M}_2}(h(s^*))\ \tilde{\lambda}_{\mathscr{M}_2}(h(t^*))} - \tilde{\lambda}_{\mathscr{N}_2}(h(s^*)h(t^*))$
$= \tilde{\lambda}^C_{\mathscr{N}_1}(h(s^*)h(t^*))$.

Similarly, $\tilde{\beta}^C_{\mathscr{N}_1}(s^*, t^*) = \frac{\tilde{\beta}_{\mathscr{M}_1}(s^*)\ \tilde{\beta}_{\mathscr{M}_1}(t^*)}{\tilde{\beta}_{\mathscr{M}_1}(s^*) + \tilde{\beta}_{\mathscr{M}_1}(t^*) - \tilde{\beta}_{\mathscr{M}_1}(s^*)\ \tilde{\beta}_{\mathscr{M}_1}(t^*)} - \tilde{\beta}_{\mathscr{N}_1}(s^*, t^*)$
$= \frac{\tilde{\beta}_{\mathscr{M}_2}(h(s^*))\ \tilde{\beta}_{\mathscr{M}_2}(h(t^*))}{\tilde{\beta}_{\mathscr{M}_2}(h(s^*)) + \tilde{\beta}_{\mathscr{M}_2}(h(t^*)) - \tilde{\beta}_{\mathscr{M}_2}(h(s^*))\ \tilde{\beta}_{\mathscr{M}_2}(h(t^*))} - \tilde{\beta}_{\mathscr{N}_2}(h(s^*)h(t^*))$
$= \tilde{\beta}^C_{\mathscr{N}_1}(h(s^*)h(t^*))$.

Also, is,
$\tilde{\gamma}^C_{\mathscr{N}_1}(s^*, t^*) = \frac{\tilde{\gamma}_{\mathscr{M}_1}(s^*) + \tilde{\gamma}_{\mathscr{M}_1}(t^*) - 2\tilde{\gamma}_{\mathscr{M}_1}(s^*)\ \tilde{\gamma}_{\mathscr{M}_1}(t^*)}{1 - \tilde{\gamma}_{\mathscr{M}_1}(s^*)\ \tilde{\gamma}_{\mathscr{M}_1}(t^*)} - \tilde{\gamma}_{\mathscr{N}_1}(s^*, t^*)$
$= \frac{\tilde{\gamma}_{\mathscr{M}_2}(h(s^*)) + \tilde{\gamma}_{\mathscr{M}_2}(h(t^*)) - 2\ \tilde{\gamma}_{\mathscr{M}_2}(h(s^*))\ \tilde{\gamma}_{\mathscr{M}_2}(h(t^*))}{1 - \tilde{\gamma}_{\mathscr{M}_2}(h(s^*))\ \tilde{\gamma}_{\mathscr{M}_2}(h(t^*))} - \tilde{\gamma}_{\mathscr{N}_2}(h(s^*)h(t^*))$
$= \tilde{\gamma}^C_{\mathscr{N}_1}(h(s^*)h(t^*))$.

Hence, the complements are isomporhic to each other and the converse also true.□

Definition 3.14. A $FNDF_{graph}$ is complete if
$\tilde{\lambda}_{\mathscr{N}}(s^*, t^*) = \frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*) + \tilde{\lambda}_{\mathscr{M}}(t^*) - \tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}$,

$$\tilde{\beta}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)},$$
$$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\ \forall\ s^*,t^*\in\mathscr{Z}.$$

The above mentioned properties are satisfied for the $FNDF_{graph}$ in Example 1, thus the $FNDF_{graph}$ is a complete $FNDF_{graph}$.

Definition 3.15. A $FNDF_{graph}$ is said to be strong if

$$\tilde{\lambda}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)}{\tilde{\lambda}_{\mathscr{M}}(s^*)+\tilde{\lambda}_{\mathscr{M}}(t^*)-\tilde{\lambda}_{\mathscr{M}}(s^*)\ \tilde{\lambda}_{\mathscr{M}}(t^*)},$$
$$\tilde{\beta}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)}{\tilde{\beta}_{\mathscr{M}}(s^*)+\tilde{\beta}_{\mathscr{M}}(t^*)-\tilde{\beta}_{\mathscr{M}}(s^*)\ \tilde{\beta}_{\mathscr{M}}(t^*)},$$
$$\tilde{\gamma}_{\mathscr{N}}(s^*,t^*)=\frac{\tilde{\gamma}_{\mathscr{M}}(s^*)+\tilde{\gamma}_{\mathscr{M}}(t^*)-2\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}{1-\tilde{\gamma}_{\mathscr{M}}(s^*)\ \tilde{\gamma}_{\mathscr{M}}(t^*)}\ \forall\ s^*,t^*\in\mathscr{E}.$$

4. **Numerical Approach**

The concept of $FNDF_{graphs}$ as a novel approach to decision-making which is suitable for handling uncertainties and imprecise information, from the real-world scenarios. The selection of the most suitable CIBIL score (Credit Information Bureau (India) Limited) applications is crucial for individuals and businesses seeking financial products. In this section, we discussed how the proposed methodology was applied to the selection of CIBIL score applications.

4.1 Algorithm for CIBIL Score Application Selection:

Below is a algorithm for MCDM aimed at selecting the most suitable CIBIL score application.

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S1: Input attributes $A=\{a_1,a_2,\cdots,a_k\}$ and factors $F=\{f_1,f_2,\cdots,f_n\}$ with weight vector $W=\{w_1,w_2,\cdots,w_n\}$ and construct $FF_{relation}$ $L^{(g)}=(l^{(g)}_{pq})_{k\times k}$ to each criterion.

S2: Aggregate all $l^{(g)}_{pq}=(\tilde{\alpha}^{(g)}_{pq},\tilde{\beta}^{(g)}_{pq},\tilde{\gamma}^{(g)}_{pq})$ $(p,q=1,2,\cdots,k)$ regarding criteria $F_p(p=1,2,3,4)$ and get $L=(l_{pq})_{k\times k}$, where $l_{pq}=(\tilde{\lambda}_{pq},\tilde{\beta}_{pq},\tilde{\gamma}_{pq})$ is the value assigned for the alternative a_p over a_q with respect to all the considered criteria F_l by using Fermatean Neutrosophic Dombi fuzzy weighted arithmetic averaging ($FNDFWAA$) operator given by

$$l_{pq}=FNDFWAA(l^{(1)}_{pq},l^{(2)}_{pq},\cdots,l^{(n)}_{pq},)=$$
$$\sqrt{1-\frac{1}{1+\left[\sum_{j=1}^{n}w_j\left(\frac{(\tilde{\beta}^{j}_{pq})^3}{1-(\tilde{\beta}^{g}_{pq})^3}\right)^{\rho}\right]^{\frac{1}{\rho}}}},\ \sqrt{1-\frac{1}{1+\left[\sum_{j=1}^{n}w_j\left(\frac{(\tilde{\alpha}^{j}_{pq})^3}{1-(\tilde{\beta}^{g}_{pq})^3}\right)^{\rho}\right]^{\frac{1}{\rho}}}},\ \frac{1}{1+\left[\sum_{j=1}^{n}w_j\left(\frac{1-(\tilde{\gamma}^{j}_{pq})}{(\tilde{\gamma}^{g}_{pq})}\right)^{\rho}\right]^{\frac{1}{\rho}}}$$

S3: Draw $FNFD_{graphs}$ based on L.

S4: Draw the $FNFPD_{graphs}$ based on the condition $\tilde{\lambda}_{pq}\geq 0.5$ $(l,p=1,2,\cdots,k)$.

S5: Compute $out-d(A_i)$ $(i=1,2,\cdots,k)$ for each A_i in the $FNFPD_{graphs}$.

S6: Arrangement of the alternatives based on the diminishing value of the membership degrees of $out-d(A_i)$.

S7: The optimal alternative is the alternative with the maximum membership degree of $out - d(A_i)$.

4.2 Choosing the most suitable CIBIL score application:

Any individual's creditworthiness can be represented numerically through their CIBIL scores. Its importance lies in its impact on various financial aspects, including loan approval, interest rates, credit card offers, negotiating power, rental approvals, employment opportunities, insurance premiums, and access to financial products. A good CIBIL score reflects financial discipline and enhances an individual's ability to secure favorable terms in financial transactions. It serves as a tool for lenders to assess credit risk and is a crucial element in financial planning and responsible financial behavior. Regular monitoring and maintenance of a healthy CIBIL score are essential for achieving financial stability and flexibility.

Selecting the most suitable CIBIL score application involves considering several factors to ensure it meets a person needs and provides accurate and valuable credit information. CRISIL (Credit Rating Information Services of India Limited) (A_1), ICRA (Investment Information and Credit Rating Agency) Limited(A_2), CARE (Credit Analysis and Research Limited) (A_3), India Ratings and Research Pvt. Ltd.(A_4), and Brickwork Ratings India Pvt Ltd. (A_5), are indeed the five major credit rating agencies or credit bureaus operating in India. These agencies play a crucial role in providing credit reports and credit scores, which are used by lenders to assess the creditworthiness of individuals and businesses.

Several factors contribute to the calculation of the CIBIL score, and understanding these factors is crucial for maintaining a healthy credit profile. The five major factors that influence the CIBIL score are Payment HistoryF_1, Credit Utilization Ratio F_2, Length of Credit History F_3, Types of Credit F_4, and New Credit and Inquiries F_5.

$W = (0.3, 0.1, 0.1, 0.3, 0.2)$presents preferable information $L^{(g)} = l_{pq}^{(g)})_{5\times 5}$ $(g = 1, 2, 3, 4, 5)$. Here $l_{pq}^{(g)} = (\tilde{\alpha}_{pq}^{(g)}, \tilde{\beta}_{pq}^{(g)}, \tilde{\gamma}_{pq}^{(g)})$ is the Fermatean Neutrosophic number assigned by decision-making expert. Also, The degree to each CIBIL score application A_l are $\tilde{\alpha}_{pq}^{(g)}, \tilde{\beta}_{pq}^{(g)}$ and $\tilde{\gamma}_{pq}^{(g)}$ by either preferred or not preferred over the application A_p regarding the given criteria. $L^{(g)} = (l_{pq}^{(g)})_{5\times 5}$ are given tables (I - V).

Table 1. Comparision for Factor I

$L^{(1)}$	A_1	A_2	A_3	A_4	A_5
A_1	(.5,.4,.5)	(.7,.4,.2)	(.7,.5,.1)	(.4,.2,.5)	(.4,.2,.4)
A_2	(.2,.4,.7)	(.5,.4,.5)	(.6,.5,.4)	(.7,.2,.6)	(.5,.2,.5)
A_3	(.1,.5,.7)	(.4,.5,.6)	(.5,.4,.5)	(.7,.4,.3)	(.4,.1,.5)
$A4$	(.5,.2,.4)	(.6,.2,.7)	(.3,.4,.7)	(.5,.4,.5)	(.4,.2,.5)
$A5$	(.4,.2,.4)	(.5,.2,.5)	(.5,.1,.4)	(.5,.2,.4)	(.5,.4,.5)

Table 2. Comparision for Factor II

$L^{(2)}$	A_1	A_2	A_3	A_4	A_5
A_1	(.5,.4,.5)	(.8,.8,.5)	(.7,.3,.4)	(.8,.4,.7)	(.8,.5,.6)
A_2	(.5,.8,.8)	(.5,.4,.5)	(.7,.4,.5)	(.3,.4,.6)	(.7,.6,.5)
A_3	(.4,.3,.7)	(.5,.4,.7)	(.5,.4,.5)	(.7,.7,.6)	(.8,.7,.8)
A_4	(.7,.4,.8)	(.6,.4,.3)	(.6,.7,.7)	(.5,.4,.5)	(.8,.8,.6)
A_5	(.6,.5,.8)	(.5,.6,.7)	(.8,.7,.8)	(.6,.8,.8)	(.5,.4,.5)

Table 3. Comparision for Factor III

$L^{(3)}$	A_1	A_2	A_3	A_4	A_5
A_1	(.5,.4,.5)	(.8,.6,.5)	(.7,.4,.6)	(.9,.7,.4)	(.7,.5,.5)
A_2	(.5,.6,.8)	(.5,.4,.5)	(.8,.6,.6)	(.9,.8,.6)	(.6,.8,.5)
A_3	(.6,.4,.7)	(.6,.6,.8)	(.5,.4,.5)	(.8,.4,.7)	(.7,.8,.9)
A_4	(.4,.7,.9)	(.6,.8,.9)	(.7,.4,.8)	(.5,.4,.5)	(.8,.6,.7)
A_5	(.5,.5,.7)	(.5,.8,.6)	(.9,.8,.7)	(.7,.6,.8)	(.5,.4,.5)

Table 4. Comparision for Factor IV

$L^{(4)}$	A_1	A_2	A_3	A_4	A_5
A_1	(.5,.4,.5)	(.9,.7,.5)	(.6,.3,.4)	(.7,.5,.6)	(.8,.3,.6)
A_2	(.5,.7,.9)	(.5,.4,.5)	(.9,.6,.5)	(.6,.4,.7)	(.7,.6,.6)
A_3	(.4,.3,.6)	(.5,.6,.9)	(.5,.4,.5)	(.9,.6,.4)	(.6,.7,.8)
A_4	(.6,.5,.7)	(.7,.4,.6)	(.4,.6,.9)	(.5,.4,.5)	(.9,.7,.6)
A_5	(.6,.3,.8)	(.6,.6,.7)	(.8,.7,.6)	(.6,.7,.9)	(.5,.4,.5)

Table 5. Comparision for Factor V

$L^{(5)}$	A_1	A_2	A_3	A_4	A_5
A_1	(.5,.4,.5)	(.8,.5,.4)	(.6,.6,.8)	(.6,.7,.8)	(.9,.4,.6)
A_2	(.4,.5,.8)	(.5,.4,.5)	(.8,.7,.7)	(.7,.5,.8)	(.8,.6,.5)
A_3	(.8,.6,.6)	(.7,.7,.8)	(.5,.4,.5)	(.8,.5,.4)	(.8,.6,.7)
A_4	(.8,.7,.6)	(.8,.5,.7)	(.4,.5,.8)	(.5,.4,.5)	(.8,.8,.7)
A_5	(.6,.4,.9)	(.5,.6,.8)	(.7,.6,.8)	(.7,.8,.8)	(.5,.4,.5)

With the purpose to complete the grouped $l_{pq} = (\tilde{\lambda}_{pq}, \tilde{\beta}_{pq}, \tilde{\gamma}_{pq})$ $(p, q = 1, 2, 3, 4, 5)$ of the cibil score application A_l over A_p regarding all considered factors $l^{(g)}(g = 1, 2, 3, 4, 5)$, the FND-FWAA operator is defined as

$$l_{pq} = FNDFWAA(l_{pq}^{(1)}, l_{pq}^{(2)}, \cdots, l_{pq}^{(n)},) = \sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\tilde{\beta}_{pq}^{j})^3}{1-(\tilde{\beta}_{pq}^{g})^3}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\tilde{\alpha}_{pq}^{j})^3}{1-(\tilde{\beta}_{pq}^{g})^3}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{1-(\tilde{\gamma}_{pq}^{j})}{(\tilde{\gamma}_{pq}^{g})}\right)^{\rho}\right]^{\frac{1}{\rho}}}$$

Dombi's t-norm and t-conorm are obtained when $\rho = 1$. and the values are shown in Table VI.

Table VI. Combined Fermatean Neutrosophic fuzzy relation

L	A_1	A_2	A_3	A_4	A_5
A_1	(.368, .253, .5)	(.762, .503, .33)	(.534, .323, .226)	(.614, .416, .574)	(.712, .225, .514)
A_2	(.284, .503, .793)	(.368, .253, .5)	(.746, .457, .556)	(.617, .482, .663)	(.573, .463, .527)
A_3	(.457, .323, .648)	(.416, 457, .741)	(.368, .253, .5)	(.756, .4, .391)	(.566, .52, .668)
A_4	(.532, .416, .573)	(.579, .374, .604)	(.332, .4, .782)	(.368, .253, .5)	(.745, .578, .591)
A_5	(.409, .224, .619)	(.394, .463, .63)	(.675, .52, .566)	(.487, .578, .632)	(.368, .253, .5)

TheFND_{graphs} according to L, is in Figure 3.

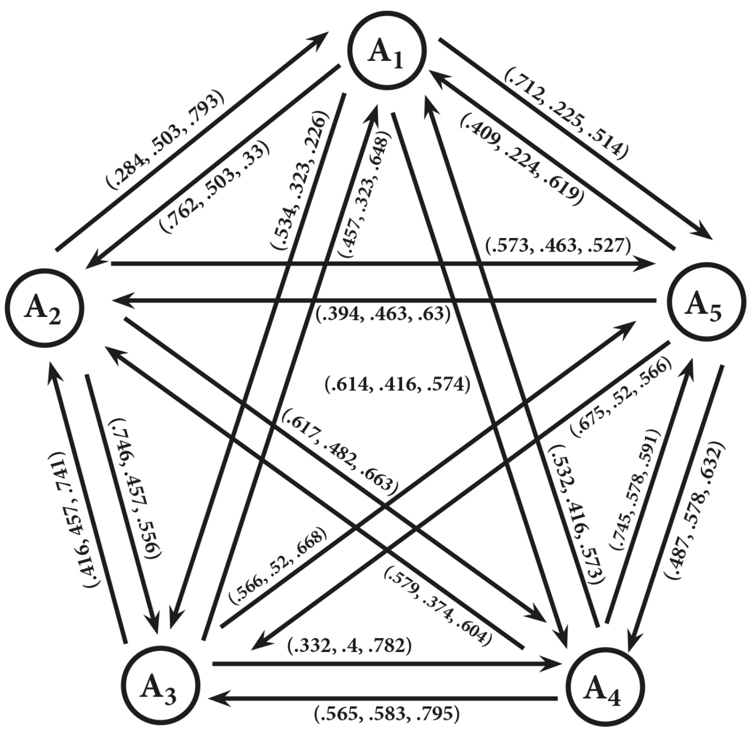

FIGURE 1. PND_{graphs}

We consider the condition of $\tilde{\lambda}_{pq} \geq 0.5$ $(l, p = 1, 2, 3, 4, 5)$ a partial directed graph is drawn in Figure 4.

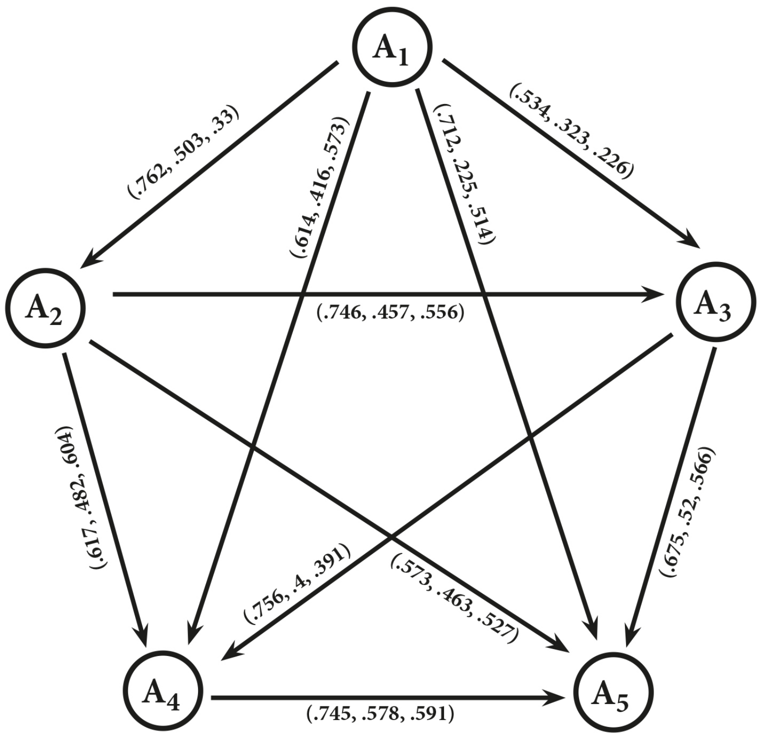

FIGURE 2. $PDPND_{graphs}$

The out-degrees $out - d(A_l)$ $(l = 1, 2, 3, 4, 5)$ are computed as
$out - d(A_1) = (2.622, 1.467, 1.643)$
$out - d(A_2) = (2.698, 1.905, 2.017)$
$out - d(A_3) = (2.711, 1.7, 1.739)$
$out - d(A_4) = (2.732, 1.876, 2.159)$
$out - d(A_5) = (2.705, 1.786, 2.198)$
Based on the above computation, the optimal ranking order is

$$A_4 \succ A_3 \succ A_5 \succ A_2 \succ A_1$$

So, A_4 is the best application for money transferring.

5. Conclusion

This work presents a concept of $FNDF_{graph}$. A few definitions and properties of this novel $FNDF_{graph}$ model have been described along with its introduction. The concept of $FNDF_{graph}$ offers a novel way to study and analyze systems with complex relationships and dynamic properties. The $FNDF_{graph}$ concept can be further developed to study the functioning of both bipolar and $FNDF_{graph}$, as well as some practical applications.

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