Operations with n-Refined Literal Neutrosophic Numbers using the Identification Method and the n-Refined AH-Isometry

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Abstract

If we know the determinate and sub-indeterminate part(s) of an argument $a_0 + a_1 I_1 + a_2 I_2 + ... + a_n I_n$ how do we similarly find the determinate and sub-indeterminate part(s) of a function (or operation) of this argument $f(a_0 + a_1 I_1 + a_2 I_2 + ... + a_n I_n)$? The AH-Isometry is designed to do just that.

The function *f* may be of one or more variables (arguments), it may also be some unary or n-ary operation. Real examples are presented in this paper of such arguments and functions.
We now present for the first time the most general form, the n-Refined AH-Isometry.

Keywords: Indeterminacy, Sub-Indeterminacies, n-Refined Literal and Numerical Neutrosophic Numbers, Identification Method, AH-Isometry, n-Refined Neutrosophic AH-Isometry

1. Introduction

The literal indeterminacy (I) was for the first time refined/split into literal sub-indeterminacies (I_1, I_2, …, I_n) by Smarandache [1] in 2015, who defined a multiplication law of these literal sub-indeterminacies to be able to build the Refined I-Neutrosophic Algebraic Structures.

The AH-Isometry [2] was firstly introduced by Mohammad Abobala and Ahmad Hatip in 2021, where AH stands for Abobala-Hatip.

The 2-Refined Neutrosophic AH-Isometry [3] was introduced by Celik and Hatip in 2022. Many papers [5-10] were published about this fundamental AH-Isometry in the rings of neutrosophic literal numbers.

We now introduce for the first time the n-Refined Neutrosophic AH-Isometry.

2. Operations with n-Refined Literal Neutrosophic Numbers

Let's have the following two n-Refined Literal Neutrosophic Numbers:

$N_1 = a_0 + a_1 I_1 + a_2 I_2 + ... + a_m I_m$ where $a_0, a_1, a_2, ..., a_m$ are real or complex numbers,

and similarly

$N_2 = b_0 + b_1 I_1 + b_2 I_2 + ... + b_m I_m$ where $b_0, b_1, b_2, ..., b_m$ are real or complex numbers,

The first three are Straightforward Operations:

2.1. Addition:

$$N_1 + N_2 = (a_0 + b_0) + (a_1 + b_1) I_1 + (a_2 + b_2) I_2 + ... + (a_m + b_m) I_m$$

2.2. Subtraction:

$$N_1 - N_2 = (a_0 - b_0) + (a_1 - b_1) I_1 + (a_2 - b_2) I_2 + ... + (a_m - b_m) I_m$$

2.3. Scalar Multiplication:

Let c be a real or complex number. Then:

$$c \cdot N_1 = c \cdot a_0 + c \cdot a_1 I_1 + c \cdot a_2 I_2 + ... + c \cdot a_m I_m$$

2.4. Multiplication:

In many previous applications the easiest way of multiplication (intersection) of sub-indeterminacies was defined as:

$I_j \cdot I_k = I_{\min\{j,k\}}$ for all $i, j \in \{1, 2, ..., m\}$.

Consequently, $(I_j)^2 = (I_j)^r = I_j$, for any integer $r \geq 1$,

and

$$I_{j_1} \cdot I_{j_2} \cdot ... \cdot I_{j_p} = I_{\min\{j_1, j_2, ..., j_p\}}$$

also

$\sqrt[s]{I_j} = I_j$, for integer $s \geq 1$, since, raising both sides to the s-power, one gets: $(\sqrt[s]{I_j})^s = (I_j)^s = I_j$.

2.5. Negative Exponent:

$(I_j)^{-m}, m > 0,$ is undefined.

2.6. Remark

The multiplication and division of the literal or numerical sub-indeterminacies ($I_a \cdot I_k \;\; or \;\; I_a \div I_k$) are defined by experts depending on each specific application.
These applications on the literal sub-indeterminacies do not, in general, coincide with the same applications on the numerical sub-indeterminacies).

2.7. Division by Identification Method of 2-Refined Neutrosophic Literal Numbers:

Let's divide $N_1 \div N_2$ by the identification method.

The determinate part of the 2-refined literal neutrosophic number $N_1 = a_0 + a_1 I_1 + a_2 I_2$ is a_0, while a_1 is its first sub-indeterminate part, and a_2 is its second sub-indeterminate part.

And, similarly, the determinate part of the 2-refined literal neutrosophic number $N_2 = b_0 + b_1 I_1 + b_2 I_2$ is b_0, while b_1 is its first sub-indeterminate part, and b_2 is its second sub-indeterminate part.

Denote $(a_0 + a_1 I_1 + a_2 I_2) \div (b_0 + b_1 I_1 + b_2 I) = \dfrac{a_0 + a_1 I_1 + a_2 I_2}{b_0 + b_1 I_1 + b_2 I_2} \equiv x + y I_1 + z I_2$

where $a_0, a_1, a_2, b_0, b_1, b_2$ are known real numbers, while x, y, z are unknown real numbers we need to find out.
We know that

$$I_1 \cdot I_1 = I_1$$
$$I_2 \cdot I_2 = I_2$$
$$I_1 . I_2 = I_2 . I_1 = I_1$$

where I_1, I_2 are literal sub-indeterminacies.
By identification ($\equiv$) method, we have:

$$\frac{a_0 + a_1 I_1 + a_2 I_2}{b_0 + b_1 I_1 + b_2 I_2} \equiv x + y I_1 + z I_2$$

or $a_0 + a_1 I_1 + a_2 I_2 \equiv (b_0 + b_1 I_1 + b_2 I_2) \cdot (x + y I_1 + z I_2)$

Whence:

$$a_0 + a_1 I_1 + a_2 I_2 \equiv b_0 x + b_0 y I_1 + b_0 z I_2 + b_1 I_1 x + b_1 I_1 y I_1 + b_1 I_1 z I_2 + b_2 I_2 x + b_2 I_2 y I_1 + b_2 I_2 z I_2$$
$$\equiv b_0 x + b_0 y I_1 + b_0 z I_2 + b_1 I_1 x + + b_1 I_1 y + b_1 I_1 z + b_2 I_2 x + b_2 I_1 y + b_2 I_2 z$$
$$\equiv b_0 x + (b_0 y + b_1 x + + b_1 y + b_1 z) I_1 + (b_0 z + b_2 x + b_2 z) I_2$$

We get a linear system of *3* equations with *3* variables *x*, *y*, *z*:

$$a_0 = b_0 x$$
$$a_1 = b_0 y + b_1 x + +b_1 y + b_1 z = b_1 x + (b_0 + b_1) y + b_1 z$$
$$a_2 = b_0 z + b_2 x + b_2 z = b_2 x + (b_0 + b_2) z$$

that we need to solve. It has no solution: if $b_0 = 0$ and $a_0 \neq 0$,

or if $b_0 + b_2 = 0$ and $a_0 + a_2 \neq 0$, or if $b_0 + b_1 + b_2 = 0$ and $a_0 + a_1 + a_2 \neq 0$.

2.8. Division of 2-Refined Literal Neutrosophic Numbers using the AH-Isometry

Let $R[I_1, I_2] = \{a + bI_1 + cI_2; a, b, c \in R\}$, where R is the set of real numbers, be the **2-Refined Literal Neutrosophic Numbers Ring**.

Let $f : R[I_1, I_2] \times R[I_1, I_2] \rightarrow R[I_1, I_2]$ be a 2-refined literal neutrosophic function (or operation) of two variables.

Then

$$f(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2) =$$
$$= f(a_0, b_0) + [f(a_0 + a_1 + a_2, b_0 + b_1 + b_2) - f(a_0 + a_2, b_0 + b_2)]I_1 + [f(a_0 + a_2, b_0 + b_2) - f(a_0, b_0)]I_2$$

$$= \frac{a_0}{b_0} + [\frac{a_0 + a_1 + a_2}{b_0 + b_1 + b_2} - \frac{a_0 + a_2}{b_0 + b_2}]I_1 + [\frac{a_0 + a_2}{b_0 + b_2} - \frac{a_0}{b_0}]I_2$$

which works when all denominators $b_0, b_0 + b_2, b_0 + b_1 + b_2$ are different from zero.

If $b_0 = 0$ and $a_0 = 0$, or if $b_0 + b_2 = 0$ and $a_0 + a_2 = 0$, or if $b_0 + b_1 + b_2 = 0$ and $a_0 + a_1 + a_2 = 0$,

it may be possible to get some solutions, depending on each application.

Let $g : R[I_1, I_2] \rightarrow R[I_1, I_2]$ be a 2-refined literal neutrosophic function (or operation) of one variable. Using the AH-Isometry, we get:

$$g(a_0 + a_1 I_1 + a_2 I_2) = g(a_0) + [g(a_0 + a_1 + a_2) - g(a_0 + a_2)]I_1 + [g(a_0 + a_2) - g(a_0)]I_2 \quad (*)$$

3. **Example of 2-Refined AH-Isometry**:

Calculate the natural logarithm: $L = \ln(5 + 2I_1 - 3I_2)$.

We know about the argument $5 + 2I_1 - 3I_2$ that its determinate part is *5*, its first sub-indeterminate part is $2I_1$, and his second sub-indeterminate part is $-3I_2$.

But we do not know the determinate part, nor the first and second sub-indeterminate parts, of the whole expression *L*. The AH-Isometry helps us to find them.

The function $g = ln$ (natural logarithm), $a_0 = 5$, $a_1 = 2$, $a_2 = -3$ and we substitute them into the above (*) relationship.

$L = ln(5) + [\, ln(5+2+(-3)) - ln(5+(-3))\,]I_1 + [\, ln(5+(-3)) - ln(5)\,]I_2$

$= ln(5) + [\, ln(4) - ln(2)]I_1 + [\, ln(2)-ln(5)\,]I_2$

$= ln(5) + ln(4/2)\cdot I_1 + ln(2/5)\cdot I_2$

$= ln(5) + ln(2)\cdot I_1 + ln(0.4)\cdot I_2$

Therefore, the determinate part of L is $ln(5)$, the first sub-indeterminate part of L is $ln(2)$, and the second sub-indeterminate part of L is $ln(0.4)$.

4. Examples of 2-Refined Numerical Neutrosophic Numbers

First Example:

$$N_1 = 5 + 4I_1 - 2I_2 = \quad 5 + 4\cdot(0.\overline{35}) - 2\cdot(0.\overline{127})$$

where the determinate part of N_1 is 5, while the numerical sub-indeterminacies parts are:

$I_1 = 0.\overline{35} = \quad 0.353535 \ldots$ (infinitely many decimals),

$I_2 = 0.\overline{127} = \quad 0.127127127 \ldots$ (infinitely many decimals).

Second Example:

$N_2 = -2 + e + 3\pi = -2 + 1\cdot I_1 - 3I_2$, where

$I_1 = e = 2.7182818 \ldots$ and

$I_2 = \quad \pi = 3.1415926 \ldots$

5. Example of 4-Refined Numerical Neutrosophic Numbers:

$$N_4 = 8 + 2\sqrt[2]{7} - 6\sqrt[3]{111} + \sqrt[4]{54} - \sqrt[5]{4}$$
$$\equiv 8 + 2I_1 - 6I_2 + I_3 - I_4$$

where

$I_1 = \sqrt[2]{7} = 2.645751 \ldots$

$I_2 = \quad \sqrt[3]{111} = 4.805895 \ldots$

$I_3 = \quad \sqrt[4]{54} = 2.710806 \ldots$

$I_4 = \quad \sqrt[5]{4} = 1.319507 \ldots$

6. More Examples

Zeina and Abobala [4] have defined the literal neutrosophic ring:

$R[I] = \{a + bI; a, b \in R\}$, where I = literal indeterminacy, with $I^2 = I$, where a is the determinate part of the neutrosophic number $a + bI$, while bI is its indeterminate part.

A function, $f : R[I] \to R[I]$, by using the AH-Isometry, can be calculated as:

$f(a+bI) = f(a) + [f(a+b) - f(a)]I$

For example, $\sin(\frac{\pi}{3}+0.5I)=\sin(\frac{\pi}{3})+[\sin(\frac{\pi}{3}+0.5)-\sin(\frac{\pi}{3})]I$, where the determinate part is $\sin(\frac{\pi}{3})$ and indeterminate part is $[\sin(\frac{\pi}{3}+0.5)-\sin(\frac{\pi}{3})]I$.

Into the original expression $\sin(\frac{\pi}{3}+0.5I)$ it is not clear what is the determinate part and what is the indeterminate part. The AH-Isometry helped us distinguish between these parts.

7. **Theorem of Isomorphism**:

Let $R_n(I)=\{a_0+\sum_{i=1}^{n}a_iI_i;\ \ a_i\in R\}$ be the n-refined neutrosophic real ring.
Define:

$$f: R_n(I)\to R\times R\times..\times R\ \ (n+1\ \ times)$$
$$f(a_0+\sum_{i=1}^{n}a_iI_i)=(a_0,\sum_{i=0}^{n}a_i,\sum_{i\neq 1}^{n}a_i,\sum_{i\neq 1,2}^{n}a_i,\sum_{i\neq 1,2,3}^{n}a_i,\dots,a_0+a_n)$$

Then *f* is a ring isomorphism.

Proof:

It is clear that *f* is well-defined mapping and preserves addition operation. We must prove that it keeps multiplication.

Assume that A = $a_0+\sum_{i=1}^{n}a_iI_i$, B = $b_0+\sum_{i=1}^{n}b_iI_i$, then for n = 2, according to the refined AH-isometry the theorem holds see [3]. Assume that it is true for k, we prove for k+1.

Assume that A = $a_0+\sum_{i=1}^{k+1}a_iI_i$, B = $b_0+\sum_{i=1}^{k+1}b_iI_i$

$A\cdot B=(a_0+\sum_{i=1}^{k}a_iI_i+a_{k+1}I_{k+1})(b_0+\sum_{i=1}^{k}b_iI_i+b_{k+1}I_{k+1})=(a_0+\sum_{i=1}^{k}a_iI_i)(b_0+\sum_{i=1}^{k}b_iI_i)+(a_{k+1}I_{k+1})(b_0+\sum_{i=1}^{k}b_iI_i)+(a_{k+1}I_{k+1})(b_{k+1}I_{k+1})+(a_0+\sum_{i=1}^{k}a_iI_i)(b_{k+1}I_{k+1})\ =$

$(a_0+\sum_{i=1}^{k}a_iI_i)(b_0+\sum_{i=1}^{k}b_iI_i)+a_{k+1}b_0I_{k+1}+\sum_{i=1}^{k}b_ia_{k+1}I_i+a_{k+1}b_{k+1}I_{k+1}+(a_0b_{k+1}I_{k+1}+\sum_{i=1}^{k}b_{k+1}a_iI_i)=(a_0+\sum_{i=1}^{k}a_iI_i)(b_0+\sum_{i=1}^{k}b_iI_i)+\sum_{i=1}^{k}b_ia_{k+1}I_i+(a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1})I_{k+1}+\sum_{i=1}^{k}a_ib_{k+1}I_i$,

put T = $a_0+\sum_{i=1}^{k}a_iI_i, S=b_0+\sum_{i=1}^{k}b_iI_i$, then

$A\cdot B=TS+\sum_{i=1}^{k}b_ia_{k+1}I_i+(a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1})I_{k+1}+\sum_{i=1}^{k}a_ib_{k+1}I_i$.

On the other hand, we have:

$f(A\cdot B)=f(TS)+f(\sum_{i=1}^{k}b_ia_{k+1}I_i)+f((a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1})I_{k+1})+f(\sum_{i=1}^{k}a_ib_{k+1}I_i)\ =$

$=f(T)f(S)+(0,\sum_{i=1}^{k}b_i\,a_{k+1},\sum_{\neq 1}^{n}b_i\,a_{k+1},\dots,b_ka_{k+1})+(0,a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1},a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1},\dots.,a_{k+1}b_0+a_{k+1}b_{k+1}+a_0b_{k+1})+(0,\sum_{i=1}^{k}a_i\,b_{k+1},\sum_{i\neq 1}^{k}a_i\,b_{k+1},\dots,a_kb_{k+1})\ =$

$$\left(a_0,\sum_{i=0}^{k}a_i,\sum_{i\neq 1}^{k}a_i,\sum_{i\neq 1,2}^{k}a_i,\sum_{i\neq 1,2,3}^{k}a_i,\dots,a_0+a_k\right).(b_0,\sum_{i=0}^{k}b_i,\sum_{i\neq 1}^{k}b_i,\sum_{i\neq 1,2}^{k}b_i,\sum_{i\neq 1,2,3}^{k}b_i,\dots,b_0+b_k)+$$

$(0, \ a_{k+1}, \dots, \ a_{k+1}) \cdot \left(0, \sum_{i=1}^{k} b_i, \sum_{i\neq 0,1}^{k} b_i, \sum_{i\neq 0,1,2}^{k} b_i, \sum_{i\neq 0,1,2,3}^{k} b_i, \dots, b_k\right) + (0, a_{k+1}b_0 + a_{k+1}b_{k+1} + a_0 b_{k+1}, a_{k+1}b_0 + a_{k+1}b_{k+1} + a_0 b_{k+1}, \dots, a_{k+1}b_0 + a_{k+1}b_{k+1} + a_0 b_{k+1}) + (0, b_{k+1}, \dots, \ b_{k+1}) \cdot \left(0, \sum_{i=1}^{k} a_i, \sum_{i\neq 0,1}^{k} a_i, \sum_{i\neq 0,1,2}^{k} a_i, \sum_{i\neq 0,1,2,3}^{k} a_i, \dots, a_k\right) =$

$$\left(a_0, \sum_{i=0}^{k} a_i, \sum_{i\neq 1}^{k} a_i, \sum_{i\neq 1,2}^{k} a_i, \sum_{i\neq 1,2,3}^{k} a_i, \dots, a_0 + a_k\right) \cdot \left(b_0, \sum_{i=0}^{k} b_i, \sum_{i\neq 1}^{k} b_i, \sum_{i\neq 1,2}^{k} b_i, \sum_{i\neq 1,2,3}^{k} b_i, \dots, b_0 + b_k\right) +$$

$(\ 0, \ a_{k+1}, \dots, \ a_{k+1} \ ) \ \cdot(\ 0, \sum_{i=1}^{k} b_i + b_0 + b_{k+1}, \sum_{i\neq 0,1}^{k} b_i + b_0 + b_{k+1}, \sum_{i\neq 0,1,2}^{k} b_i + b_0 + b_{k+1}, \dots b_0 + b_k + b_{k+1}) + (\ 0, \ a_0, \dots, \ a_0) \cdot(\ 0, \ b_{k+1}, \dots, \ b_{k+1}) =$
$\left(a_0, \sum_{i=0}^{k+1} a_i, \sum_{i\neq 1}^{k+1} a_i, \sum_{i\neq 1,2}^{k+1} a_i, \sum_{i\neq 1,2,3}^{k+1} a_i, \dots, a_0 + a_k\right) \cdot$
$(b_0, \sum_{i=0}^{k+1} b_i, \sum_{i\neq 1}^{k+1} b_i, \sum_{i\neq 1,2}^{k+1} b_i, \sum_{i\neq 1,2,3}^{k+1} b_i, \dots, b_0 + b_k)) = f(A) \cdot f(B).$

Thus, f is a ring isomorphism, and it is called the ***n-Refined Neutrosophic AH-Isometry****.*

8. Examples of 3- and 4-Refined Neutrosophic Isomorphisms

For *n = 3*, the *3*-refined neutrosophic case, the formula of *f* is
$f(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3) = (a_0, a_0 + a_1 + a_2 + a_3, a_0 + a_2 + a_3, a_0 + a_3).$

For *n = 4*, the *4*-refined neutrosophic case, the formula of *f* is

$$f(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 + a_4 I_4) = (a_0, a_0 + a_1 + a_2 + a_3 + a_4, a_0 + a_2 + a_3 + a_4, a_0 + a_3 + a_4, a_0 + a_4).$$

9. Inverse AH-Isomorphism

The formula of inverse isomorphism is:

$f^{-1}: R \times R \times \dots \times R \ \ (n+1 \ \ times) \to R_n(I)$ such that*:*

$$f^{-1}(a_0, a_1, a_2, a_3 \dots a_n) = a_0 + I_1(a_1 - a_2) + I_2(a_2 - a_3) + I_3(a_3 - a_4) + \dots + \ I_n(a_n - a_0).$$

Application:

Find the inverse of the following *3*-refined neutrosophic real number $x = 1 + I_1 + I_2 + I_3$.
Now, we will see how the n-refined AH-isometry is very powerful in simplifying hard computations.

First, we compute: $f(x) = (1,4,3,2)$, $[f(x)]^{-1} = (1, \frac{1}{4}, \frac{1}{3}, \frac{1}{2})$, hence:

$$x^{-1} = f^{-1}[[f(x)]^{-1}] = 1 + \left(\frac{1}{4} - \frac{1}{3}\right) I_1 + \left(\frac{1}{3} - \frac{1}{2}\right) I_2 + \left(\frac{1}{2} - 1\right) I_3 = 1 + \left(-\frac{1}{12}\right) I_1 + \left(-\frac{1}{6}\right) I_2 + \left(-\frac{1}{2}\right) I_3.$$

By an easy computation, we can see:

$$x \cdot x^{-1} = 1 + \left(-\frac{1}{12} - \frac{1}{12} - \frac{1}{12} - \frac{1}{12} - \frac{1}{6} - \frac{1}{2} + 1\right) I_1 + \left(-\frac{1}{6} - \frac{1}{6} - \frac{1}{6} - \frac{1}{2} + 1\right) I_2 + \left(-\frac{1}{2} - \frac{1}{2} + 1\right) I_3 = 1.$$

10. Conclusion

We extended for the first time the Neutrosophic AH-Isometry to n-Refined Neutrosophic AH-Isometry, to find the determinate part of a function, as well as its multiple sub-indeterminate parts.

Many real examples of n-refined neutrosophic numbers, that have a determinate part and many types of sub-indeterminacies, were presented.

The AH-Isometry is designed to find the determinate part and sub-indeterminate parts of a n-refined neutrosophic function of the form $f(a_0 + a_1 I_1 + a_2 I_2 + ... + a_n I_n)$.

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