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NeutroGeometry and Fractal Geometry

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Abstract. Geometries are structures with certain elements like points, lines, planes, and spaces, among others, that satisfy certain definitions, axioms, properties, and theorems for the total of the elements. NeutroGeometries are geometric structures that meet at least one of these concepts only partially, never for 100% or 0% of the elements. Until now, NeutroGeometries have been developed from the ideas of classical geometries such as Euclidean, Hyperbolic, Elliptic, Mixed (Smarandache) geometries, among others where axiomatization is the basis of their construction. This paper aims to discuss some ideas about the relationship between NeutroGeometries and fractal geometry. This relation is not necessarily obvious; it is mainly established because fractals are structures used to model deterministic chaotic phenomena. The fractal dimension is a numerical value used to measure the complexity of the figure and the maps that represent chaotic phenomena. The more complex the phenomenon, the more unpredictable it becomes and therefore the more uncertain and indeterminate. This indeterminacy is essentially ontological since it deals mostly with natural phenomena. This relationship is proposed in this article for associating the concepts of NeutroGeometry that present degrees of uncertainty or indeterminacy and fractal geometries that model phenomena where unpredictability exists. This idea is reinforced in some works where a direct relationship between entropy and the fractal dimension is demonstrated.

Keywords: NeutroConcept, NeutroGeometry, indeterminacy, unpredictability, fractal geometry, entropy, chaos, fractal dimension.

1 Introduction

Neutrosophy is the branch of philosophy that studies neutrality, understanding in this concept the neutral, the indeterminate, the erroneous, the contradictory, the imprecise, the paradoxical, the unknown, and the inconsistent, among other concepts. This philosophical theory introduced by Professor F. Smarandache has expanded to various parts of knowledge. Especially mathematics has been enriched with concepts such as Neutrosophic Sets in the form of Single-Valued Neutrosophic Sets or Interval-Valued Neutrosophic Sets. These generalize fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and Pythagorean fuzzy sets, among many others.

One of the fields in which Neutrosophy has been presented is Geometry. Several decades ago Smarandache introduced a movement called Paradoxist, where mixed geometric structures were studied. Specifically, the following definition was used [1]:

"An axiom is said to be Smarandachely denied if the axiom behaves in at least two different ways within the same space, i.e., validated and invalidated, or only invalidated but in multiple distinct ways. A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom."

In this type of geometry, emphasis is placed on the non-satisfaction of some axiom. Above all, the controversial Euclid's fifth postulate about the existence of a single parallel that passes through a point outside a straight line. However, within the framework of Neutrosophy, Smarandache defined NeutroGeometry much more recently as part of a series of concepts coming from neutrosophic logic. He defined what he called a NeutroConcept, compared to Concept and AntiConcept. Each of these categories was associated with a triple of values (T, I, F) that represents the triple of Truthfulness, Indeterminacy, and Falseness, respectively, of satisfaction of the concept [2].

A Geometry satisfies each axiom, definition, property, theorem, and proposition, among others, for all elements, that is, with a true degree of satisfaction $(1,0,0)$. The AntiGeometry satisfies it for none in any Concept that is $(0,0,1)$. The elements of the NeutroGeometry satisfy it for the rest of the intermediate values of (T, I, F). The idea of the existence of indeterminacy within the geometric structure is important, although it is enough that a certain degree of truth exists in NeutroGeometry when $I = 0$ [2-4].

The Mixed Geometries or Smarandache Geometries are included within the concept of NeutroGeometry. However, NeutroGeometries have greater logical implications, they are logic applied to Geometry rather than pure geometry modified in its classical axioms as occurs within Mixed Geometries [5].

There is a relatively large amount of research within the field of Mixed Geometries, e.g. the works of H. Iseri who contributed to the Smarandache Manifolds, where geometric structures are composed of "disks" in the shape of equilateral triangles, such that the length of their edges is taken as the unit of measurement within the structure. In these manifolds, all parallelism properties belonging to elliptic, Euclidean, or hyperbolic geometries appear within the same structure [6]. We also owe the creation of a simple geometric model in [7], although it is mathematically poorly founded and developed. More recently, other authors have worked in this line of research such as Carlos Granados [8]. Other works within the field of Smarandache Geometries can be consulted related to finite geometry [9-11] and some real-life cases where Mixed Geometries are satisfied [12]. This last article also outlines some examples of NeutroGeometry that is not Mixed Geometry. We can also consult the development from Euclidean geometry to Smarandache geometry in [13].

However, a step taken where results are presented in the field of NeutroGeometries beyond the Smarandache Geometries appears in the article [14]. In this article, the author proposes a distance formula within the framework of NeutroGeometry. The mathematical simplicity and at the same time the unreality of the use of distances in classical geometries, whether Euclidean or classical non-Euclidean, becomes evident. This unreality is because the world that geometry tries to model is not homogeneous, and therefore it is not possible to travel from one point to another by the shortest classical distance, since the path may be crossed by obstacles or indeterminacies.

The aforementioned article deals with paths represented in a coordinate system on a two-dimensional surface. The proposed distance formula is valid for any geometry, viz., Euclidean, hyperbolic, elliptic, or mixed. However, the paths proposed to connect one point to another must be rectifiable, that is, there must be a way to measure them. A measure of uncertainty or indeterminacy based on the opinion or experience of a group of people is also proposed and a decision-making algorithm is recommended to solve the problem of moving from one point to another in a practical way.

That is why the idea of distance in this case is associated with a situation of uncertainty or indetermination with a high epistemological component. So, moving from one place to another will depend on people's knowledge of the best path and their cognitive uncertainty or indeterminacy. Another important idea that does not appear, at least explicitly, in the aforementioned works is the relationship of geometry with the dynamic changes of a system. The latter is because these are classical geometries; more emphasis is placed on static realities that do not change over time.

Taking into account all explained so far, it remains to ask if the concept of NeutroGeometry has any relationship with geometry as atypical as fractal since all authors have focused on the idea of associating it with classical geometries. Fractal geometry focuses on the study of the geometric object called fractal which was coined in this way by Benoit Mandelbrot from the Latin word *fractus*, meaning broken or fractured [15]. These objects have been widely studied. Among their characteristics are that they are usually not rectifiable and self-similar. This means that they usually have infinite length even though they occupy a limited space and also have shapes that repeat in the same or similar way at different scales. The visualization of these objects is reminiscent of shapes within nature that cannot be described with the help of classical geometries; however, fractals are usually constructed with simple recurring equations. This is supported by computational development that allows the plotting of these shapes with an adequate resolution.

Associated with the concept of a fractal is the fractal dimension. This is a numerical value of dimension which in fractals are non-integer values. In classical curves or geometric figures, these values are integers, for example, points have dimension 0, rectifiable lines and curves have dimension 1, surfaces have dimension 2, and so on. The fractal dimension has application to study curves that are natural, for example, the crack in a concrete surface, the coastline of a country, and the blood vessel networks of the human body, among many others.

The relationship between fractal forms and the representation of chaotic dynamic systems is known and has been well-studied. These are deterministic phenomena that produce completely different results from small changes in the initial conditions. They are non-random phenomena, since when the same initial value is entered, the results are the same, but on the other hand, a very small change in the initial value gives completely different results. Despite this, they are phenomena that are described with the help of curves included within a limited space.

This paper aims to reflect on and discuss the relationship that may exist between NeutroGeometries and Fractal Geometry. We maintain the thesis that there is a relationship although this has its subtleties. Chaotic phenomena are provided with a high degree of complexity, which is manifested in the fractal dimension of the so-called maps of their graphic representation. In some cases, a direct correlation occurs between the fractal dimension and entropy, for example in the study of the spatial growth of cities. Therefore, it is not difficult to realize that the complexity of natural phenomena implies greater unpredictability and therefore greater uncertainty and indeterminacy. This type of indeterminacy has a primarily ontological nature.

The article is divided into a preliminary section where the most important concepts of NeutroGeometry and fractal geometry are explained. In the following section, we develop the ideas that allow us to link both concepts.

The last section contains the conclusions.

2 Preliminaries

This section contains the basic notions of NeutroGeometry, AntiGeometry, fractal geometry, and fractal dimension.

2.1 NeutroGeometry and AntiGeometry

According to neutrosophic logic, a proposition has a valuation (T, I, F), where T means the degree of truthfulness, I is the degree of indeterminacy and F is the degree of falseness. A Concept is satisfied with valuation according to the triple $(T, I, F) = (1,0,0)$, an AntiConcept is satisfied with the valuation $(T, I, F) = (0,0,1)$, and a NeutroConcept is satisfied for the rest of the possible values of the triple.

Geometry is defined as a geometric structure composed only of Concepts, NeutroGeometry is a geometric structure with at least one NeutroConcept and no AntiConcept, and AntiGeometry is a geometric structure that contains at least one AntiConcept [2, 4].

Specifically, NeutroGeometry is a geometric structure composed of elements such as points, lines, planes, spaces, hyperspaces, and so on, where some definition, axiom, theorem, and property, among others, is a NeutroConcept. That is, it fulfills that there is a NeutroAxiom or a NeutroProperty or a NeutroDefinition, and so on. The intuitive idea to define NeutroGeometry is that the physical world in which we live is not homogeneous and contains indeterminations [2, 4]. For example, although the straight line is the shortest one between two points in a city, it is not always possible to follow the straight path because there may be construction elements such as buildings that can prevent our passage. In this way, the shortest path would be a broken line following the traffic.

It is also clear that Mixed Geometries or Smarandache Geometries are NeutroGeometries. In them, the Smarandachely denied axioms are partially fulfilled. An example that can be consulted is the model shown in Figure 1.

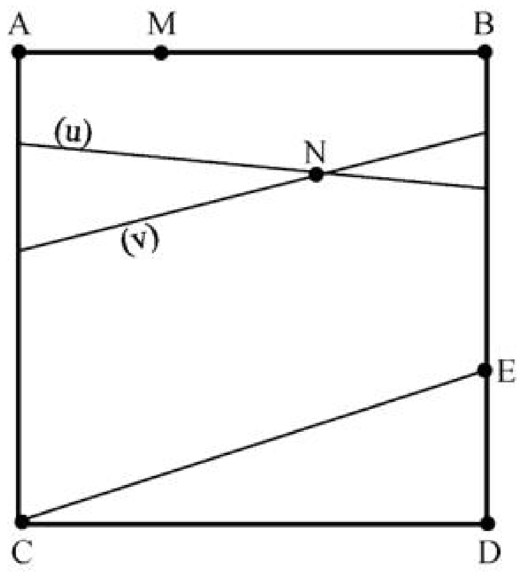

Figure 1: Bhattacharya's model of Smarandache Geometry. See [7].

Figure 1 shows a model made up of a rectangle where the points are the Euclidean points contained in the interior of the rectangle and those on the border. The lines go from one point on the edge AC to another point on the edge BD. If the line CE is taken, it does not have any parallel that passes through any point inside the segment DE. It has at least two parallel lines (u and v) that pass through point N and a single parallel that passes through point M. This model is one of the first to appear in a mixed geometry.

Figure 2 shows another model where a NeutroGeometry is not a Smarandache Geometry.

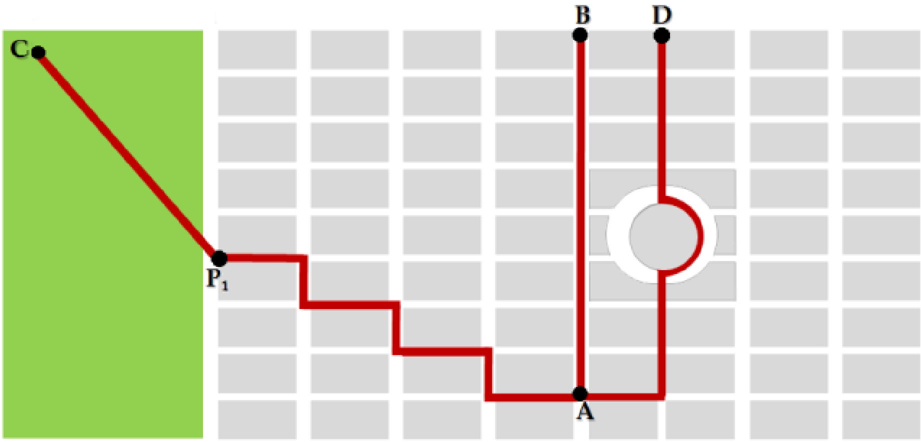

Figure 2: Euclidean model of an urban area showing a park (green area), blocks (gray rectangles), roundabout (gray circle), and paths (red lines). Source: [12].

Formally, the elements in Figure 2 correspond to Euclidean geometry; among the proposed paths, only AB and P_1C can be joined in straight Euclidean lines. There is no geometry defined in the literature where AD is a line in the sense of geodesic. The line AC presents a valid path for Taxicab Geometry in the section AP_1 and the Euclidean geometry in the section P_1C.

Specifically, to travel on the path AC shown in the figure, the shortest line (geodesic) is to follow the path marked in red. However, this path is not valid in both Euclidean geometry and Taxicab Geometry. This is why the property being a geodesic is a NeutroProperty in this example. This is the shortest path in real life, but it is not a line in Euclidean and Taxicab geometries, only partially.

These drawbacks show the rigidity of geometries to provide us with solutions to real-life problems because they are based on axioms that prioritize cognitive simplicity over applicability to daily life problems. However, the search for a more general solution is obtained by sacrificing the simplicity of the classical models. An example of this is the distance proposal for NeutroGeometry as explained below [14].

Definition 1 ([14])**.** Given (G, d_G) is a metric space in a geometric structure. The *NeutroGeometric distance* between x_1 and x_2 for $x_1, x_2 \in X$ is defined as:

$$d_{NG}(x_1, x_2) = inf\left\{\frac{L(p)}{\varepsilon(p)} : p \text{ is a rectifiable path from } x_1 \text{ to } x_2\right\} \qquad (1)$$

Where $\varepsilon(p)$ is the passability function of p.

This formula explains that if we want to calculate the neutrosophic distance that joins the points x_1 and x_2 in a plane corresponding to a certain geometry (Euclidean, hyperbolic, elliptic, mixed, taxicab, another one) having a coordinate system and a metric space associated with the base geometry, then the length of all the rectifiable paths that join both points are divided by a function called passability that measures the uncertainty or indeterminacy that joins both points along the chosen path.

This formula is a decision-making method where the distance is sought taking into account what is the shortest path in the classical sense and also the real possibility of passing through this path taking into account the knowledge of the locals or those people who know the place. It is therefore an uncertainty and indeterminacy that are epistemological.

The example that appears in the article is the following:

Example 1 ([14])**.** Suppose we are in a boat and we want to navigate a river whose bed has irregularities so that the left bank has calm waters and the right bank has turbulent waters in a certain section. However, we wish to go from point A to point B on the right side of the river. Furthermore, the path of the river itself is sinuous, see Figure 3.

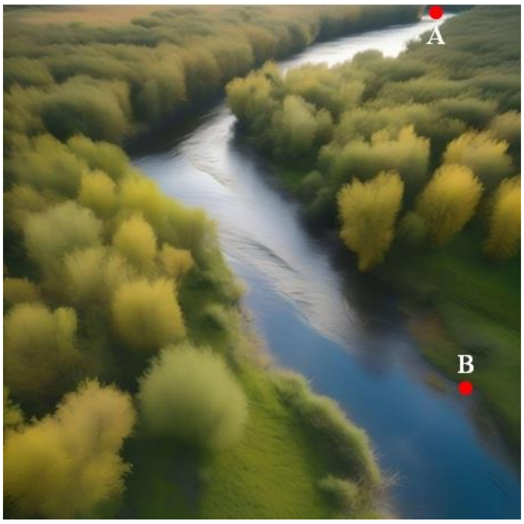

Figure 3: Picture of the river of the example. We need to go from point A to point B. This image was generated by an AI tool. Source: [14].

That is why the shortest path that passes on the right is impassable and hence $\varepsilon(p_1) = 0$. The path that goes through the center has some areas with a percentage of danger, let us say $\varepsilon(p_2) = 0.66667$. While the path that goes left and then turns right is the longest, and it is completely safe, therefore $\varepsilon(p_3) = 1$, see Figure 4.

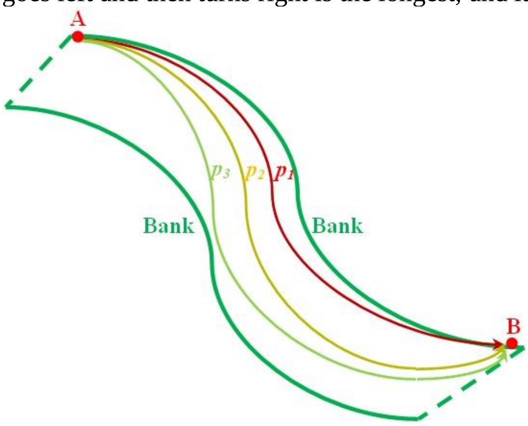

Figure 4: Map of the river showing the three paths, p_1, p_2, and p_3. Source: [14].

Let us suppose that the Euclidean length of each path from A to B is viz., $l(p_1) = 1.5\ km$, $l(p_2) = 1.9\ km$ and $l(p_3) = 2.1\ km$. To go on the shortest path in the Euclidean distance, i.e. a straight line we would have to navigate sections of the river, then go overland in a swampy area, then navigate another section, and so on, which is not practical.

The NeutroGeometric lengths of the three paths in the example are, $l_{NG}(p_1) = \frac{1.5}{0}\ km = \infty\ km$, $l_{NG}(p_2) = \frac{1.9}{0.66667}\ km = 2.85\ km$, while $l_{NG}(p_3) = \frac{2.1}{1}\ km = 2.1 km$, therefore we prefer the path p_3 to make the crossing, even though it is the longest according to Euclidean geometry.

2.2 Fractal Geometry

Fractal Geometry is due to the mathematician Benoît Mandelbrot who introduced it in the mid-1970s [15]. Fractals are geometric objects that have irregularity as a characteristic. They are usually curves or surfaces that are not differentiable at any point and also have infinite lengths (they are not rectifiable). Another characteristic is that they are self-similar, that is, when the scale of a portion of the object is increased, a figure equal or similar to the entire object is obtained, and this occurs at any scale. This self-similarity can be exact as in classical fractals, approximate as in nature where the parts are approximately equal to the whole, and statistical self-similarity where some self-similarity characteristics are preserved.

A classic example of a fractal is the Koch curve; Figure 5 shows the construction of the so-called Koch snowflake.

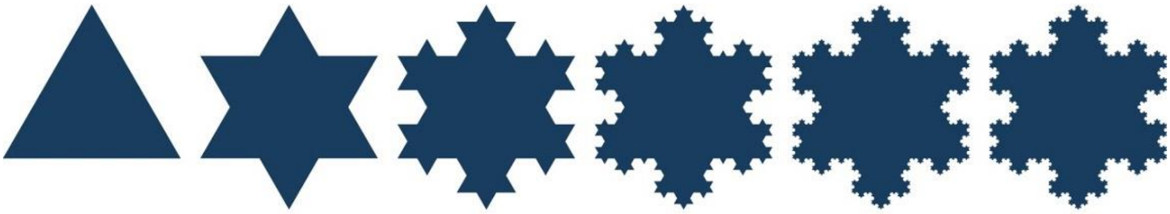

Figure 5: Koch snowflakes fractal. Source: http://www.pixabay.com.

To construct this fractal, an equilateral triangle is taken as seen in the figure. From each of its edges, the segment that is in the middle third is eliminated and replaced by two edges of an equilateral triangle. For the new edges, the process is repeated infinitely. From this process, a bounded object is obtained, but one that has infinite length and is also self-similar.

These objects cannot be described with elements of classical geometry and are reminiscent of elements of nature such as broccoli, cauliflower, and trees, among others. For example, see Figure 6 with the details of a broccoli and Figure 7 contains an artificially generated tree using code in the Python language.

Figure 6: Broccoli showing self-similarity properties. Source: http://www.pixabay.com.

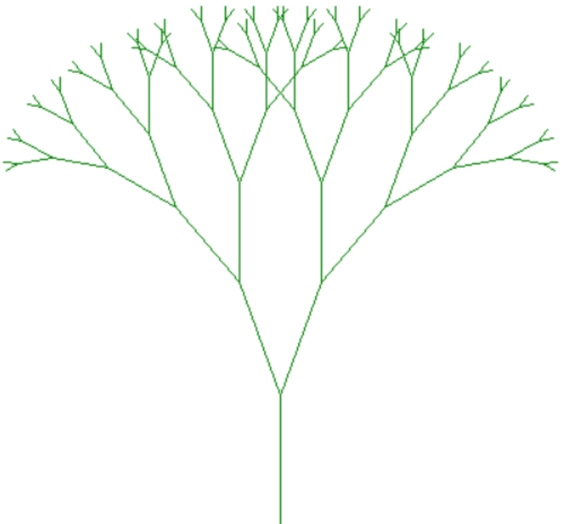

Figure 7: Tree generated using Python code.

An important concept within this theory is the fractal dimension. While classical objects have an integer topological dimension, the fractal dimension can be real, not integer. For example, the Koch curve has a fractal dimension of $d_f = \frac{\ln(4)}{\ln(3)} \approx 1.2619$. This number means that the curve has more complexity than a smooth curve in the two-dimensional space such as a straight line in Euclidean geometry, however, it is not so complex as to occupy a complete two-dimensional space, for example, a square that has topological dimension 2.

The number above is obtained from the general formula:

$$d_f = \frac{\ln m}{\ln r} \qquad (2)$$

Where r is the scale factor, while m is the minimum number of copies of this square needed to cover the object. For example, in the Koch snowflake fractal, 4 squares with a scale factor of 3 are needed to cover the entire figure [16, 17]. This is used when the figure satisfies the self-similarity property.

There is more than one definition for the fractal dimension. A simple one is the Box Dimension, which is calculated from Equation 3 [17].

$$d_B = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln(1/\varepsilon)} \qquad (3)$$

If the limit exists, where $N(\varepsilon)$ is the minimum D-dimensional ε side cubes needed to cover the D-dimensional figure.

This dimension is used not only in fractals but also in real objects, a classic example is the box dimension of the coasts of Great Britain [18]. There are phenomena where its fractal dimension is used as a measurement, since classical measurements such as length or area, among others, do not make sense for these objects. Among these cases is the spatial growth of cities over time or the crack in the concrete or wall, see Figures 8 and 9 [19].

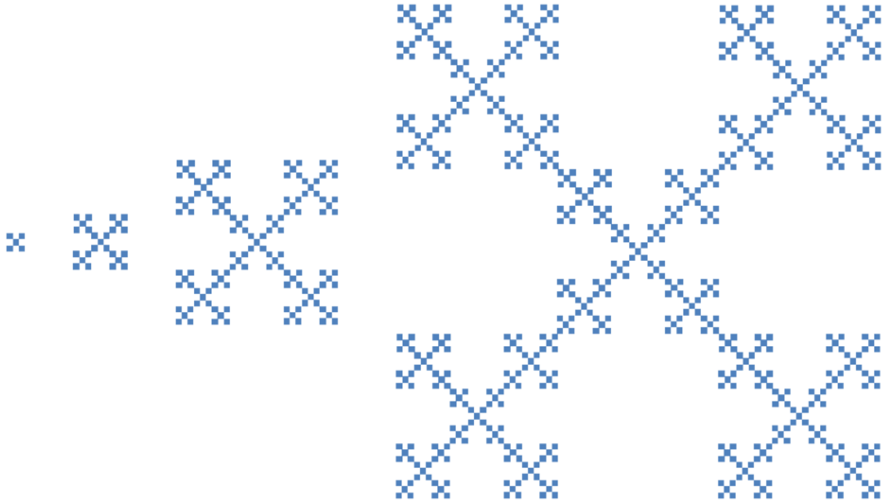

Figure 8: Self-similar fractal used to model the spatial growth of a city. Source: The authors.

Figure 9: Crack in the wall. It is a figure that cannot be represented by classical geometries. Source: http://www.pixabay.com.

3 NeutroGeometry and Fractal Geometry

Perhaps when the idea of creating a theory of NeutroGeometry arose, the possibility of associating this concept with fractal geometry was not thought of. However, there are common points between these two geometries. The NeutroGeometries arose from the Paradoxist movement where only two concepts opposed to each other that co-existed in the same structure were taken into account, mainly the concept of axiom, and within the axioms the fifth postulate of Euclid and its variants of non-Euclidean geometries. However, fractal geometry is not represented axiomatically; therefore it does not make sense to talk about Mixed or Smarandache fractal geometries in this context.

From a formal point of view, both theories can be combined with the self-similarity property. There are curves in the plane that are not self-similar, we have as an example any known smooth curve, a very simple one is the circumference. At the opposite extreme, we have curves that have exact self-similarity such as classic fractals like the one seen in Figure 5. It is also known that there are intermediate curves, where the self-similarity is partial as occurs in several fractals, or even on surfaces like mountains, self-similarity is said to be statistical, since only some parts are somewhat similar to the whole. That is why self-similarity is a Concept that has an AntiConcept (not self-similarity) and a NeutroConcept (approximate or statistical self-similarity).

As for the measure of the complexity of the fractal figure, which is the fractal dimension, which is taken due to the lack of others such as length or area, there is no single measure either. The Box Dimension is just one of them and does not always coincide with the other definitions of fractal dimension.

There are:

$d_T, d_{HB}, d_{MB}, d_E, d_C$,
Where:
d_T: is the topological dimension that is always an integer,
d_{HB}: is the Haussdorff-Besicovitch dimension,
d_{MB}: is the Minkowski-Bouligand or box-counting dimension that had been explained previously,
d_P: is the packaging dimension,
d_C: is the dimension of the Euclidean space that contains the figure and is an integer.

Therefore, if d_f is the fractal dimension of the object, then $d_f \in [d_T, d_C]$ or more specifically for fractal objects we have $d_f \in [min\{d_{HB}, d_{MB}, d_P\}, max\{d_{HB}, d_{MB}, d_P\}]$. That is, if there is a measurement called "fractal dimension" of a fractal object, it must be contained in this last interval. However, in general, it is fulfilled $d_f \in [d_T, d_C]$. Also, any fractal dimension is always calculated approximately.

This is reinforced by the definition of multifractal dimension, because some objects have a self-similarity that does not hold equally for all scales and it is necessary to define a series of fractal dimensions instead of a single numerical value, to describe the object [17].

However, the relationship between NeutroGeometry and fractality is especially noticeable in the representation of deterministic chaos through fractal theory. A system is chaotic when it exhibits aperiodic behavior and is sensitive to initial conditions; therefore it is not predictable [17]. That is, despite it is not random, for small changes in the initial conditions we obtain large changes in the results. We are going to illustrate this idea with the help of the logistic equation shown in (4).

$$x_{n+1} = rx_n(1 - x_n) \qquad (4)$$

This equation represents the growth of a population dynamically over time, note that it is a recurring equation.

The behavior of this equation depends on the parameter r. It has been studied that:

For $r < 1$ it is fulfilled $x_n \to 0$ when $n \to \infty$,
For $1 < r < 3$ it is fulfilled $x_n \to a \neq 0$ when $n \to \infty$, see Figure 10.
For $r > 3$ the graph oscillates between two values (Figure 11), four values (Figure 12), and so on until reaching a chaotic state (Figure 13).

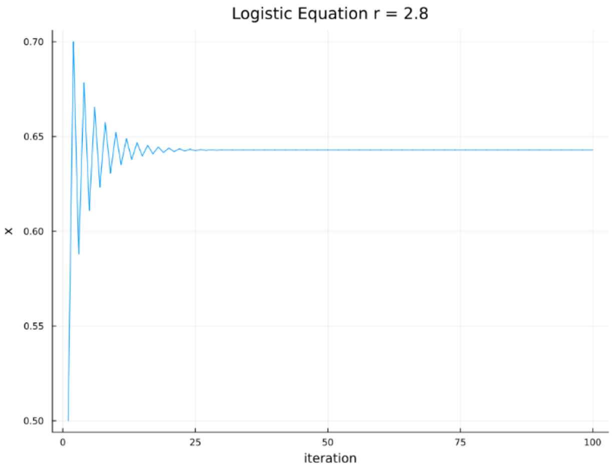

Figure 10: Convergence of the logistic equation for $r = 2.8$. See [16, 17].

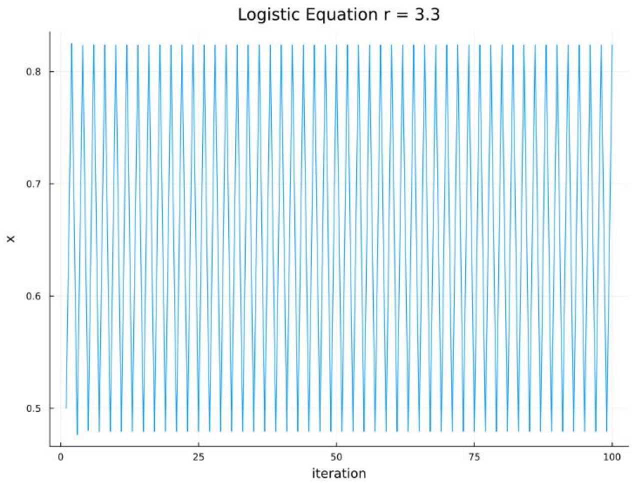

Figure 11: Convergence of the logistic equation for $r = 3.3$. See [16, 17].

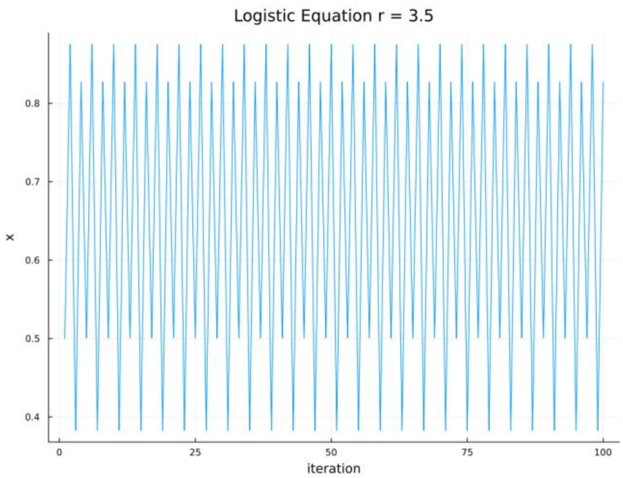

Figure 12: Convergence of the logistic equation for $r = 3.5$. See [16, 17].

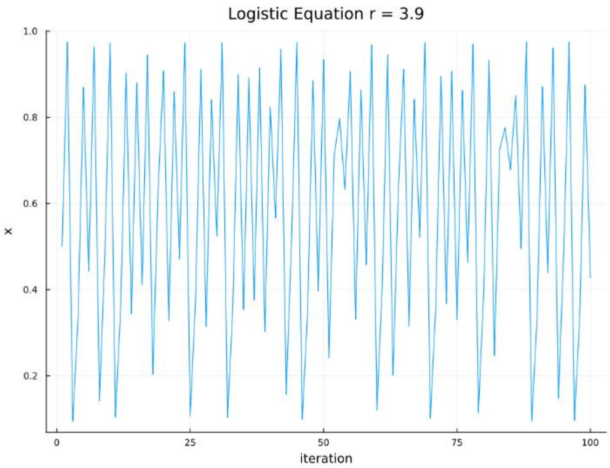

Figure 13: Convergence of the logistic equation for $r = 3.9$. See [16, 17].

To represent the Logistic Map, each value of the parameter r is plotted against the convergence points of Equation 4 for this parameter. This is represented in Figure 14.

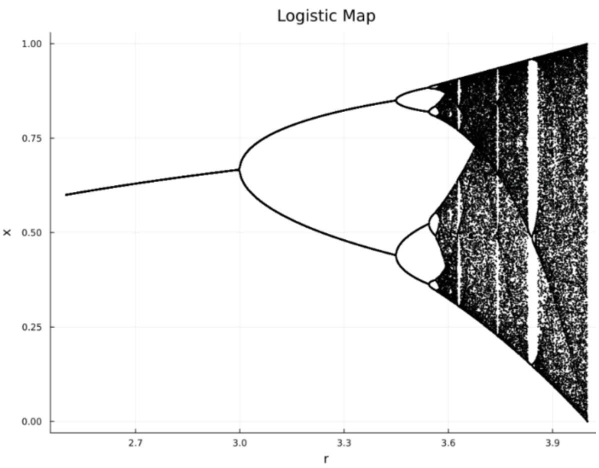

Figure 14: Logistic Map. Plot of parameter vs. point(s) of convergence. See [16, 17].

Figure 14 represents the Logistics Map. It is not difficult to notice that it presents some characteristics of approximate self-similarity. This map requires multifractal dimension theory to calculate its complexity.

We can also notice that for $r = 2.8$ its result is completely predictable, however, for some value $r > 3$ the value begins to bifurcate, obtaining indeterminate prediction results. The larger the r value becomes, the greater the indeterminacy and unpredictability of the system, see Figures 10-12 and Figure 14. This is manifested to a greater degree for certain values close to $r = 4$ which is where the system becomes chaotic, Figures 13 and 14. This is a classic example of the relationship between fractal geometry and the behavior of nonlinear dynamical systems.

This could also be an example where Neutrosophy that studies indeterminacy corresponds totally with the idea of fractal geometry and nonlinear dynamic systems and chaos [20].

This last idea is reinforced by some studies where the fractal dimension is associated with entropy. Entropy is the measure of order of the system [19]. In some phenomena such as the spatial growth of cities or the agglomeration of cities, a positive correlation has been found between the fractal dimension and entropy. This means that in these cases the complexity of urban spatial growth can be measured both with the help of entropy and with the fractal dimension. We can also intuitively link these concepts when it becomes evident that indeterminacy, which is an important component of Neutrosophy, is typical of complex systems. A complex system is understood to be one where very different results are obtained for small variations in the initial conditions. So we can appreciate the relationship between NeutroGeometry[21] and Fractal Geometry[22] when the indeterminacy is intrinsic to the system or what is the same when it is an ontological indetermination [23, 24].

Conclusion

NeutroGeometry is a logical approach to geometric systems based on neutrosophic logic. A NeutroGeometry contains at least one NeutroConcept and never an AntiConcept. However, this idea originally intended for classical geometric systems, such as Euclidean geometry, classical non-Euclidean geometries, or mixed Smarandache geometries, did not take into account other less orthodox geometries such as fractal geometry. Fractal geometry does not respond to any specific axiom, although it is composed of objects called fractals that are immersed in a Euclidean line, plane, space, or hyperspace and are composed of points or segments of points. The objective of this article was to reflect and discuss the relationship that may exist between the ideas of NeutroGeometry, Neutrosophy, and fractal geometry or the phenomena in which it is applied such as nonlinear or chaotic systems. On the one hand, self-similarity properties are NeutroProperties in some cases, when it is approximate or statistical. On the other hand, if it is considered that there is an "intrinsic" fractal dimension of an object, this can only be determined accurately when it is understood as a number within an indeterminate interval since there are several definitions of fractal dimension that do not always coincide each other.

Perhaps the most interesting relation is in the application of fractal geometry to model nonlinear dynamic systems or chaotic systems. These systems can be indeterminate or unpredictable, which falls within the field of study of Neutrosophy. In turn, the maps of some of these systems, such as the logistic map, present self-similarity properties typical of fractal objects. Although this self-similarity may not be exact.

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