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A Novel and an Efficient CODAS Technique to Solve Real-Life MAGDM Problems in Fermatean Neutrosophic Environment

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Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets, and their combinations have been developed to represent uncertain, imprecise, incomplete, and inconsistent information existing in the real world. A neutrosophic set has the potential to be a general framework for uncertainty analysis in various spectra. On the other hand, the idea of Fermatean neutrosophic sets (FNSs) is the hybrid model of Fermatean fuzzy sets and neutrosophic sets, developed to enable the analytical management of ambiguous data from relatively typical real-world decision-making scenarios. In this work, we develop an algorithm to compare the Fermatean neutrosophic set and the neutrosophic set, Fermatean Neutrosophic Weighted Arithmetic Mean (FNWAM) and Fermatean Neutrosophic Weighted Geometric Mean (FNWGM) operators as an accuracy function in addition to the conventional aggregating operators are designed. Further, the CODAS technique for Multiple Attribute Group Decision Making (MAGDM) problems according to the defined operators is proposed. Additionally, to make a clear understanding of the presented study, a university faculty selection problem is studied to illustrate the proposed methodology. Finally, the results of the above two sets prove the authenticity of this study.

Keywords: Fermatean Neutrosophic Set (FNS); Multi-Criteria Decision Making (MCDM); Multiple Attribute Group Decision Making ; Decision Matrix (D-Mx); Negative Ideal Solution (NIS); Positive Ideal Solution (PIS); FNWAM; FNWGM; Score function (SF); Accuracy Function (AF).

1. Introduction

Zadeh proposed his remarkable theory of fuzzy sets (FSs) in 1965 [22] to encounter different types of uncertainties. This novel concept is used successfully in modeling uncertainty in many fields of real life. A fuzzy set is characterized by a membership function μ with the range [0,1]. Further generalization of this fuzzy set was made by K. Atanassov [2] in 1986, which is known as Intuitionistic fuzzy sets (IFS). In IFS, instead of one membership grade, there is also a non-membership grade attached to each element. Further, there is a restriction that the sum of these two grades is less or equal to unity. In IFS the degree of non-belongingness is not independent but it is dependent on the degree of belongingness. IFS can handle imprecise data both complete and incomplete.

Recently a new theory has been introduced which is known as neutrosophic logic and sets. The term neutrosophy means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. Neutrosophic logic was introduced by Florentin Smarandache [17]. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy, and falsity respectively, and which lies between [0, 1], the non-standard unit interval. Neutrosophic sets have been successfully applied in different fields, including decision-making problems [9-10]. In the extent of natural science, operations research, economics, management science, military affairs, and urban planning, NSs have a broad application. They also can be applied to decision-making problems when the ambiguity and complexity of the attributes make the problems impossible to express or value with real numbers.

Since the neutrosophic set is difficult to be directly used in real-life applications. In 2005, Smarandache and Wang et al. [19], proposed the concept of a single-valued neutrosophic set. Here the degree of truth, indeterminacy, and falsity respectively of any element of a neutrosophic set lies in the standard unit interval [0, 1]. The single-valued neutrosophic set is a generalization of the classical set, fuzzy set, intuitionistic fuzzy set paraconsistent sets, etc. and hence has many broad perspectives for dealing with the real-world problems. Singh et.al. have solved many real-life problems in neutrosophic environment that delas with neutrosophic transportation linear programming problems [15], neutrosophic non-linear programming problems [12], fully neutrosophic linear programming problems [16]. Singh and Bhat have proposed novel score and accuracy functions for neutrosophic sets [14]. Also, Singh [13] has proposed a novel Dijkstra algorithm for finding shortest route problem.

In 2020, Senapati and Yager [11] established a new extension of fuzzy sets named Fermatean fuzzy sets. Some important studies on Fermatean fuzzy sets for Multi-Criteria Decision Making (MCDM) problems have been conducted by various researchers like Chinnadurai et.al. [6] used complex Fermatean fuzzy sets to solve MCDM problems, Ganie [7] has solved MCDM problems using distance and knowledge measure in Fermatean fuzzy sets environment. Xu and Shen [20] solved MCDM problems by using similarity measures of Fermatean fuzzy sets for pattern recognition. By extending Fermatean fuzzy sets, Antony and R. Jansi [1] presented the concept of FNSs. FNSs are specific types of neutrosophic sets that are used to model uncertainty, indeterminacy, and incomplete information in decision-making processes [3-5]. It also presents a thorough comparison of the Fermatean neutrosophic set with the neutrosophic sets.

Some significant Highlights of the work

- In addition to the standard aggregating operators, FNWGM and FNWAM operations are created as accuracy functions.
- It is suggested to use the CODAS approach for the Multiple Attribute Group Decision Making issue using the described operators.
- To demonstrate the suggested methods, a faculty selection problem at a university is examined. Ultimately, the findings from the first two sets demonstrate the validity of this research.
- The outcome leads us to the conclusion that, in comparison to neutrosophic sets, FNSs yield more accurate results.

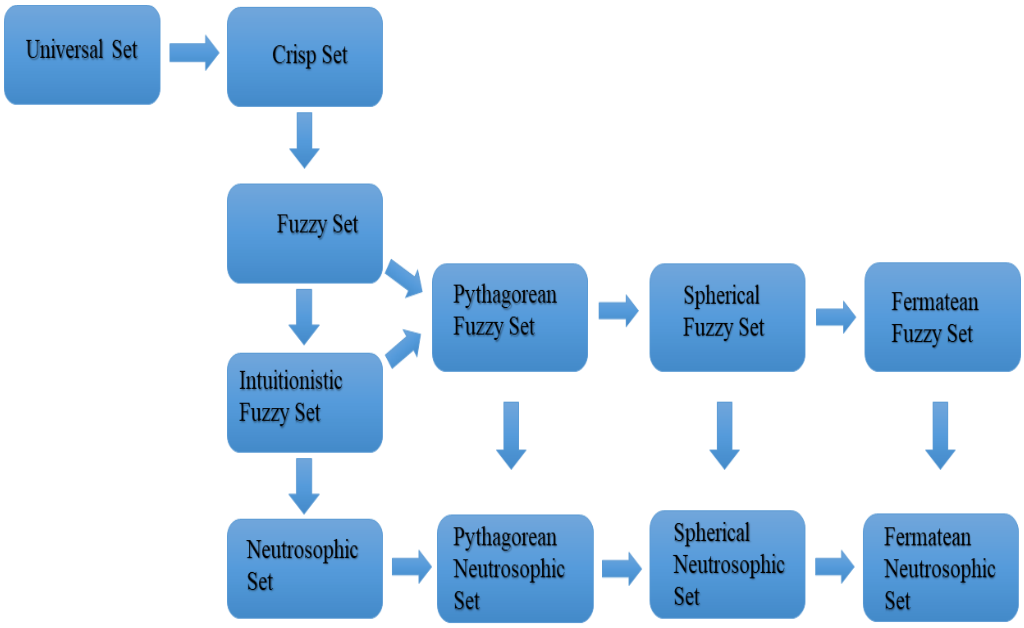

Fig.1 Extensions of FNSs

2. Preliminaries

Some basic definitions are required for better understanding of the proposed work so are presented as follows:

Definition 2.1 [22]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$ where $\mu_A: X \to [0,1]$ is the membership function of the fuzzy set A.

Definition 2.2 [2]: Let X be a universe. An intuitionistic fuzzy set A on X can be defined as follows: $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A: X \to [0,1]$ is the degree of membership and and $\nu_A: X \to [0,1]$ is the degree of non- membership of the element x such that $0 \le \mu_A(x) + \nu_A(x) \le 1$ for any $x \in X$.

Definition 2.3 [17]: Let X be a universe set. A Neutrosophic Set (NS) A in X is characterized by a truth membership functionT_A, an indeterminacy membership function I_A and a falsity membership function $F_A$where T_A , I_A and F_A are real standard elements of [0,1]. It can be written as $A = \{\langle X, (T_A(x)) + (I_A(x)) + (F_A(x))\rangle : x \in E, T_A, I_A, F_A \in]^-0, 1^+[\}$. There is no restriction on the sum of $(T_A(x)), (I_A(x))$and $(F_A(x))$ as $0^- \le (T_A(x)) + (I_A(x)) + (F_A(x)) \le 3^+$.

Definition 2.4 [18]: Let $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle | x \in X\}$ are any two neutrosophic sets of a not empty (universe) X on which axioms are defined as follows:

a. $(A) \subseteq (B) iff T_A(x) \le T_B(x); I_A(x) \le I_B(x); and F_A(x) \ge F_B(x)$
b. $(A) = (B) iff T_A(x) = T_B(x); I_A(x) = I_B(x); and F_A(x) = F_B(x) \forall x \in X$
c. $A \cap B = \{\langle x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)\rangle | x \in X\}$ where,
   i. $T_{A\cap B}(x) = min\{T_A(x), T_B(x)\}$
   ii. $I_{A\cap B}(x) = min\{I_A(x), I_B(x)\}$
   iii. $F_{A\cap B}(x) = max\{F_A(x), F_B(x)\}$
d. $A \cup B = \{\langle x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)\rangle | x \in X\}$ where,
   i. $T_{A\cup B}(x) = max\{T_A(x), T_B(x)\}$
   ii. $I_{A\cup B}(x) = max\{I_A(x), I_B(x)\}$
   iii. $F_{A\cup B}(x) = min\{F_A(x), F_B(x)\}$

Let $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ be a neutrosophic set on X , then the complement of the set A ($C(A)$ for short), maybe defined as $C(A) = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$.

Definition 2.5.[19]: Consider X be a set that is not empty (universe). A single-valued neutrosophic set (SVNS) A in X is a neutrosophic set which is of the form

$$A = \{\langle x: (T_A(x)) + (I_A(x)) + (F_A(x))\rangle, x \in X\}$$

that is characterized by the degree of membership (namely $(T_A(x))$, the degree of indeterminacy (namely $(I_A(x))$ and the degree of non-membership (namely $(F_A(x))$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$ such that $0 \le (T_A(x)) + (I_A(x)) + (F_A(x)) \le 3$, for all $x \in X$, respectively. For X, $SVNS(X)$ denotes the collection of all single valued neutrosophic sets of X.

Definition 2.6. [21]: Let X be a non-empty set (universe). A Pythagorean set A on X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$, $0 \le T_A(x) + I_A(x) + F_A(x) \le 2$, for every x in X. $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership. Here $T_A(x)$ *and* $F_A(x)$ are dependent components and $I_A(x)$ is an independent component.

Definition 2.7. [8]: Consider X be a set that is not empty (universe). A Pythagorean Neutrosophic Set (PNS) A on X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, where $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy, $F_A(x)$ is the degree of non- membership, and $T_A(x), I_A(x), F_A(x) \in [0,1]$, $0 \le (T_A(x))^2 + (I_A(x))^2 + (F_A(x))^2 \le 2$, for every $x \in X$. Here $T_A(x)$ *and* $F_A(x)$ are dependent components and $I_A(x)$ is an independent.

3. Fermatean Neutrosophic set (FNS) [11]

The cube sum of the parameters in FNS can range between 0 and 2, and it is possible to define each of them individually between 0 and 1 independently. In this section, the explanation of FNS and overview of Fermatean distance measurement, arithmetic operation and aggregation and de-neutrosophication processes are provided.

Definition 3.1: Let X be a non-empty set (universe). A Fermatean neutrosophic set (FNS) $\tilde{S}$ on X is an object of the form:

$\tilde{S} = \{< s, (T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s)) > | s \in X\}$ (1) where,

$T_{\tilde{S}}(s), I_{\tilde{S}}(s), F_{\tilde{S}}(s): X \to [0,1]$, $0 \le T_{\tilde{S}}{}^3(s) + F_{\tilde{S}}{}^3(s) \le 1, 0 \le I_{\tilde{S}}{}^3(s) \le 1$ then

$0 \le T_{\tilde{S}}{}^3(s) + I_{\tilde{S}}{}^3(s) + F_{\tilde{S}}{}^3(s) \le 2 \ \forall \ s \in X$ (2)

and $T_{\tilde{S}}(s)$ is the degree of membership, $I_{\tilde{S}}(s)$ is the degree of inderminancy and $F_{\tilde{S}}(s)$ is the degree of non-membership. Here $T_{\tilde{S}}(s)$ and $F_{\tilde{S}}(s)$ are dependent components and $I_{\tilde{S}}(s)$ is an independent component.

Definition 3.2. Let $\tilde{A} = \{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\}$ and $\tilde{B} = \{\langle x, T_{\tilde{B}}(x), I_{\tilde{B}}(x), F_{\tilde{B}}(x)\rangle | x \in X\}$ are any two Fermatean neutrosophic sets (FNSs). The basic operational laws of FNSs are defined below:

$$\tilde{A} \oplus \tilde{B} = \left\{ \left(T_{\tilde{A}}^{\ 3} + T_{\tilde{B}}^{\ 3} - T_{\tilde{A}}^{\ 3} T_{\tilde{B}}^{\ 3}\right)^{\frac{1}{3}}, I_{\tilde{A}} I_{\tilde{B}}, \left(\left(1 - T_{\tilde{B}}^{\ 3}\right) F_{\tilde{A}}^{\ 3} + \left(1 - I_{\tilde{A}}^{\ 3}\right) F_{\tilde{B}}^{\ 3} - F_{\tilde{A}}^{\ 3} F_{\tilde{B}}^{\ 3}\right)^{\frac{1}{3}} \right\} \tag{3}$$

$$\tilde{A} \otimes \tilde{B} = \left\{ (T_{\tilde{A}} T_{\tilde{B}}), \left(I_{\tilde{A}}^3 + I_{\tilde{B}}^3 - I_{\tilde{A}}^3 I_{\tilde{B}}^3\right)^{\frac{1}{3}}, \left(\left(1 - I_{\tilde{B}}^3\right) F_{\tilde{A}}^3 + \left(1 - I_{\tilde{A}}^3\right) F_{\tilde{B}}^3 - F_{\tilde{A}}^3 F_{\tilde{B}}^3\right)^{\frac{1}{3}} \right\} \tag{4}$$

$$\lambda \bullet \tilde{A} = \left\{ \left(1 - \left(1 - T_{\tilde{A}}^{\ 3}\right)^{\lambda}\right)^{\frac{1}{3}}, I_{\tilde{A}}^{\ \lambda}, \left(\left(1 - T_{\tilde{A}}^{\ 3}\right)^{\lambda} - \left(1 - T_{\tilde{A}}^{\ 3} - F_{\tilde{A}}^{\ 3}\right)^{\lambda}\right)^{\frac{1}{3}} \right\} \tag{5}$$

$$\tilde{A}^{\lambda} = \left\{ T_{\tilde{A}}^{\ \lambda}, \left(1 - \left(1 - I_{\tilde{A}}^{\ 3}\right)^{\lambda}\right)^{\frac{1}{3}}, \left(\left(1 - I_{\tilde{A}}^{\ 3}\right)^{\lambda} - \left(1 - I_{\tilde{A}}^{\ 3} - F_{\tilde{A}}^{\ 3}\right)^{\lambda}\right)^{\frac{1}{3}} \right\} \lambda \ge 0 \tag{6}$$

Definition 3.3. For any two FNSa $\widetilde{M} = (T_{\widetilde{M}}, I_{\widetilde{M}}, F_{\widetilde{M}})$and $\widetilde{N} = (T_{\widetilde{N}}, I_{\widetilde{N}}, F_{\widetilde{N}})$, some scalar operational laws are defined. Let $\lambda, \lambda_1, \lambda_2 > 0$are some scalars, the operational laws are as follows:

1. $\widetilde{M} \oplus \widetilde{N} = \widetilde{N} \oplus \widetilde{M}$ (7)
2. $\widetilde{M} \otimes \widetilde{N} = \widetilde{M} \otimes \widetilde{N}$ (8)
3. $\lambda(\widetilde{M} \oplus \widetilde{N}) = \lambda\widetilde{M} \oplus \lambda\widetilde{N}$ (9)
4. $\lambda_1\widetilde{M} \oplus \lambda_2\widetilde{M} = (\lambda_1 + \lambda_2)\widetilde{M}$ (10)
5. $(\widetilde{M} \otimes \widetilde{N})^{\lambda} = \widetilde{M}^{\lambda} \otimes \widetilde{N}^{\lambda}$ (11)
6. $\widetilde{M}^{\lambda_1} \otimes \widetilde{M}^{\lambda_2} = \widetilde{M}^{\lambda_1 + \lambda_2}$ (12)

Definition 3.4. Let $\tilde{A}$ be a set defined on universe X for the weight vector, $= (z_1, z_2, z_3, \ldots . z_n)$; $z_j \in [0,1]; \sum_{j=1}^{n} z_j \le 2$. A Fermatean neutrosophic weighted arithmetic mean

(FNWAM) is defined as:

$$FNWAM_z(\tilde{A}_1, \tilde{A}_2, \dots \tilde{A}_n) = z_1\tilde{A}_1 + z_2\tilde{A}_2 + z_3\tilde{A}_3 + \dots + z_n\tilde{A}_n$$

$$\left\{\left[1 - \prod_{j=1}^{n}\left(1 - T_{\tilde{A}}{}^{3}\right)^{z_j}\right]^{\frac{1}{3}}, \left[\prod_{j=1}^{n}(I_{\tilde{A}})^{z_j}\right]^{\frac{1}{3}}, \left[\prod_{j=1}^{n}\left(1 - T_{\tilde{A}}{}^{3}\right)^{z_j} - \prod_{j=1}^{n}\left(1 - T_{\tilde{A}}{}^{3} - F_{\tilde{A}}{}^{3}\right)^{z_j}\right]^{\frac{1}{3}}\right\} \tag{13}$$

Definition 3.5. Let $\tilde{A}$ be a set defined on universe X for the weight vector, $= (z_1, z_2, z_3, \dots . z_n)$; $z_j \in [0,1]; \sum_{j=1}^{n} z_j \leq 2$. A Fermatean neutrosophic weighted geometric mean (FNWGM) is defined as

$$FNWGM_z(\tilde{A}_1, \tilde{A}_2, \dots \tilde{A}_n) = \tilde{A}_1{}^{z_1} + \tilde{A}_2{}^{z_2} + \tilde{A}_3{}^{z_3} + \dots + \tilde{A}_n{}^{z_n}$$

$$\left\{\prod_{j=1}^{n} T_{\tilde{A}}{}^{z_j}, \left[1 - \prod_{j=1}^{n}\left(1 - I_{\tilde{A}}{}^{3}\right)^{z_j}\right]^{\frac{1}{3}}, \prod_{j=1}^{n}\left(1 - I_{\tilde{A}}{}^{3}\right)^{z_j} - \prod_{j=1}^{n}\left(1 - I_{\tilde{A}}{}^{3} - F_{\tilde{A}}{}^{3}\right)^{z_j}\right\} \tag{14}$$

Definition 3.6. Let $\tilde{S} = \{\langle x, T_{\tilde{S}}(x), I_{\tilde{S}}(x), F_{\tilde{S}}(x)\rangle | s \in X\}$ on X be any FNS, then a score function (SF) and accuracy function (AF) for FNS classification are defined as:

$$Score(\tilde{S}) = \left(T_{ijw} - F_{ijw}\right)^3 - \left(I_{ijw} - F_{ijw}\right)^3 \tag{15}$$

$$Accuracy(\tilde{S}) = T_{\tilde{S}}{}^3 + I_{\tilde{S}}{}^3 + F_{\tilde{S}}{}^3 \tag{16}$$

Definition 3.7. Let $\tilde{A} = \{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\}$ and $\tilde{B} = \{\langle x, T_{\tilde{B}}(x), I_{\tilde{B}}(x), F_{\tilde{B}}(x)\rangle | x \in X\}$ are any two FNSs. For comparing any two FNSs, a comparison method is developed as follows:

- If $Score(\tilde{S})_{\tilde{A}} < Score(\tilde{S})_{\tilde{B}}$ then $\tilde{A} < \tilde{B}$
- If $Score(\tilde{S})_{\tilde{A}} > Score(\tilde{S})_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$

If $Score(\tilde{S})_{\tilde{A}} = Score(\tilde{S})_{\tilde{B}}$ then check $Accuracy(\tilde{S})$ in the next step (17)

- If $Accuracy(\tilde{S})_{\tilde{A}} > Accuracy(\tilde{S})_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$
- If $Accuracy(\tilde{S})_{\tilde{A}} < Accuracy(\tilde{S})_{\tilde{B}}$ then $\tilde{A} < \tilde{B}$
- If $Accuracy(\tilde{S})_{\tilde{A}} = Accuracy(\tilde{S})_{\tilde{B}}$ then $\tilde{A} = \tilde{B}$

4. Proposed CODA'S technique for MAGDM

A Decision Matrix (D-Mx) with entries that represent the assessment scores of every choice in relation to every criterion in a neutrosophic environment can be used to represent a multi-criteria decision making (MCDM) problem. Suppose that $S = \{s_1, s_2, s_3, \dots . s_m\} (m \geq 2)$ represents distinct collection of m possible options, and $K = \{K_1, K_2, K_3, \dots . K_n\}$ be the weight vector derived from every requirement that meet $0 \leq z_j \leq 1$ and $\sum_{j=1}^{n} z_j \leq 2$.

Step 1. Let D-Mx s use the linguistic terms (LT) listed in Table 1 to complete the assessment matrices for decisions and criteria.

Table 1. Terms used in linguistics and their associated Spherical Neutrosophic Number

LT		(T, I, F)
Probably More Significant	PMS	(0.95, 0.9, 0.4)
Extremely Significant	ES	(0.9, 0.8, 0.4)
High Priority	HP	(0.8, 0.9, 0.5)
Relatively Greater Significance	RGS	(0.7, 0.9, 0.6)
Equally Important	EI	(0.6, 0.8, 0.6)
Very Minimal Significance	VMS	(0.6, 0.8, 0.7)
Low Priority	LP	(0.5, 0.7, 0.8)

extremely low significant	ELS	(0.5, 0.5, 0.9)
Definitely Not Important	DNI	(0.4, 0.5, 0.95)

Step 2. Aggregate the outcomes reached by D-Mx using FNWAM. Aggregate the DMs' Fermatean Neutrosophic linguistic judgements of the selection criteria. Assemble neutrosophic D-Mx based on Decision Makers' views. Indicate the alternative's evaluation value $S_i(i = 1,2,\dots m)$ with respect to criterion $K_j(j = 1,2\dots n)$ by $K_j(\tilde{S}_i) = (T_{ij}, I_{ij}, F_{ij})$ and $D = \left(K_j(\tilde{S}_i)\right)_{m\times n}$ is a Fermatean Neutrosophic Decision Matrix (FN D-Mx). D-Mx for MCDM problem using FNS, $D = \left(K_j(\tilde{S}_i)\right)_{m\times n}$ must be put together as shown in equation (18).

$$D = \left(K_j(\tilde{S}_i)\right)_{m\times n} = \begin{pmatrix} (\tilde{T}_{11}, \tilde{I}_{11}, \tilde{F}_{11}) & (\tilde{T}_{12}, \tilde{I}_{12}, \tilde{F}_{12}) & \dots & (\tilde{T}_{1n}, \tilde{I}_{1n}, \tilde{F}_{1n}) \\ (\tilde{T}_{21}, \tilde{I}_{21}, \tilde{F}_{21}) & (\tilde{T}_{22}, \tilde{I}_{22}, \tilde{F}_{22}) & \dots & (\tilde{T}_{2n}, \tilde{I}_{2n}, \tilde{F}_{2n}) \\ \vdots & \vdots & \dots & \vdots \\ (\tilde{T}_{m1}, \tilde{I}_{m1}, \tilde{F}_{m1}) & (\tilde{T}_{m2}, \tilde{I}_{m2}, \tilde{F}_{m2}) & \dots & (\tilde{T}_{mn}, \tilde{I}_{mn}, \tilde{F}_{mn}) \end{pmatrix} \tag{18}$$

Step 3. Following the determination of the alternative ratings and the weights assigned to the criteria, the aggregated weighted FN D-Mx is built using multiplication equation and then the aggregated weighted FN D-Mx can be defined as follows:

$$D = \left(K_j(\tilde{S}_{iz})\right)_{m\times n}$$

$$= \begin{pmatrix} (T_{11z}, I_{11z}, F_{11z}) & (T_{12z}, I_{12z}, F_{12z}) & \dots & (T_{1nz}, I_{1nz}, F_{1nz}) \\ (T_{21z}, I_{21z}, F_{21z}) & (T_{22z}, I_{22z}, F_{22z}) & \dots & (T_{2nz}, I_{2nz}, F_{2nz}) \\ \vdots & \vdots & \dots & \vdots \\ (T_{m1z}, I_{m1z}, F_{m1z}) & (T_{m2z}, I_{m2z}, F_{m2z}) & \dots & (T_{mnz}, I_{mnz}, F_{mnz}) \end{pmatrix} \tag{19}$$

Step 4. De-neutrosophication of the aggregated weighted D-Mx is done by utilizing below given equation,

$$Score\left(K_j(\tilde{S}_{iz})\right) = \left(T_{ijz} - F_{ijz}\right)^3 - \left(I_{ijz} - F_{ijz}\right)^3 \tag{20}$$

de-neutrosophication of the aggregated weighted D-Mx is done.

Step 5. Using SF acquired in Step 4, find the Fermatean Neutrosophic positive ideal solution (FN-PIS) as follows:

$$S^* = \left\{K_j, \max_i \left\langle Score\left(K_j(S_{iz})\right)\right\rangle | j = 1,2,\dots n\right\} \tag{21}$$

$$S^* = \{\langle K_1, (T_1{}^*, I_1{}^*, F_1^*)\rangle, \langle K_2, (T_2{}^*, I_2{}^*, F_2^*)\rangle, \dots \langle K_n, (T_n{}^*, I_n{}^*, F_n^*)\rangle\}$$

and Fermatean Neutrosophic negative ideal solution (FN-NIS) as follows:

$$S^- = \left\{K_j, \min_i \left\langle Score\left(K_j\left(S_{iz}\right)\right)\right\rangle | j = 1,2,\dots n\right\} \tag{22}$$

$$S^- = \{\langle K_1, (T_1{}^-, I_1{}^-, F_1^-)\rangle, \langle K_2, (T_2{}^-, I_2{}^-, F_2^-)\rangle, \dots \langle K_n, (T_n{}^-, I_n{}^-, F_n^-)\rangle\}$$

Step 6. The distances between alternative S_i, FN-NIS, and FN-PIS is calculated, accordingly.
For the FN-NIS:

$$D(S_i, S^-) = \sqrt{\frac{1}{3}\sum_{i=1}^{n}\left(\left({T_{S_i}}^3 - {T_{S^-}}^3\right)^2 + \left({I_{S_i}}^3 - {I_{S^-}}^3\right)^2 + \left({F_{S_i}}^3 - {F_{S^-}}^3\right)^2\right)} \tag{23}$$

For the FN-PIS:

$$D(S_i, S^*) = \sqrt{\frac{1}{3}\sum_{i=1}^{n}\left(\left({T_{S_i}}^3 - {T_{S^*}}^3\right)^2 + \left({I_{S_i}}^3 - {I_{S^*}}^3\right)^2 + \left({F_{S_i}}^3 - {F_{S^*}}^3\right)^2\right)} \quad (24)$$

Step 7. Calculate the minimum and maximum distances to the FN-NIS and FN-PIS, respectively.

$$D_{max}(S_i, S^-) = \max_{i \leq i \leq m} (S_i, S^-) \quad (25)$$

$$D_{min}(S_i, S^*) = \min_{i \leq i \leq m} (S_i, S^*). \quad (26)$$

Step 8. Compute the revised proximity ratio using equation given below:

$$\xi(S_i) = \frac{D(S_i, S^-)}{D_{max}(S_i, S^-)\ \frac{D(S_i, S^*)}{D_{min}(S_i, S^*)}} \quad (27)$$

In equation (27) the subtraction's second element must be at least equal to its first element, then only the result is zero or negative. So, Equation (27) is altered to Equation (28) to obtain result as zero

$$\xi(S_i) = \frac{D(S_i, S^*)}{D_{min}(S_i, S^*)\ \frac{D(S_i, S^-)}{D_{max}(S_i, S^-)}} \quad (28)$$

Step 9. Determine the best solution by rating the alternatives in the best possible order. The alternatives are organized according to the rising closeness ratio.

5. Application on Fermatean Neutrosophic set

Consider the evaluation of university professors for tenure and promotions. The criteria used at some universities are teaching, research, service and social participation. Weights must be determined for these criteria, and the candidates must also be evaluated with regard to each criterion. Let us consider any four criteria, say Teaching ($K1$), Research ($K2$), Service ($K3$), Social Participation ($K4$), and ten faculty members ($S1$, $S2$, $S3$, $S4$, $S5$, $S6$, $S7$, $S8$, $S9$, $S10$). Let an evaluation committee of 03 decision makers (ĐM1, ĐM2, and ĐM3) is constituted who has an experience with the university faculty selection. The weights of the D-Mx, are considered as 0.4, 0.5 and 0.3 representing their level of experience respectively.

Firstly, the judgements made by the decision-makers with regard to the objective are compiled using the language phrases as presented in Section 4, Step-1, then onwards every decision is rendered from Tables 2 to Table 4, as given below:

Table 2. Decisions of ĐM1

ĐM1	(K_1)	(K_2)	(K_3)	(K_4)
S_1	ES	HP	EI	RGS
S_2	PMS	ES	HP	EI
S_3	LP	VMS	ES	ELS
S_4	EI	ELS	LP	DNI
S_5	VMS	ELS	HP	HP
S_6	LP	EI	ES	HP
S_7	HP	RGS	RGS	ELS
S_8	VMS	LP	HP	EI
S_9	LP	VMS	HP	PMS
S_{10}	PMS	RGS	HP	VMS

Table 3. Decisions of ĐM2

ĐM2	(K_1)	(K_2)	(K_3)	(K_4)
S_1	PMS	HP	LP	PMS
S_2	VMS	LP	HP	EI
S_3	HP	RGS	RGS	ELS
S_4	ELS	EI	LP	ES
S_5	ES	HP	EI	RGS
S_6	PMS	ES	HP	EI
S_7	VMS	ELS	HP	HP
S_8	LP	EI	ES	HP
S_9	LP	VMS	HP	ELS
S_{10}	VMS	ES	ELS	DNI

Table 4. Decisions of ĐM3

ĐM3	(K_1)	(K_2)	(K_3)	(K_4)
S_1	HP	ES	PMS	RGS

S_2	VMS	PMS	ES	VMS
S_3	VMS	ELS	HP	HP
S_4	LP	EI	ES	HP
S_5	VMS	LP	HP	EI
S_6	LP	VMS	HP	PMS
S_7	HP	RGS	RGS	ELS
S_8	ELS	EI	LP	ES
S_9	ES	HP	EI	RGS
S_{10}	PMS	ES	HP	EI

The significance levels of the DMs are considered when combining these judgements utilizing the FNWAM and FNWGM operators. The decision matrices shown in Tables 5 and 6 are obtained.

Table 5. FND-Mx by using FNWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.27312 0.8406786 0.4130087	0.864289 0.850639 0.47017	0.807666 0.74141 0.629188	0.87753 0.88123 0.5294
S_2	0.20744 0.8019901 0.5747402	0.883933 0.741409 0.54257	0.864289 0.85064 0.470173	0.63267 0.76508 0.69
S_3	0.12813 0.7692854 0.6704603	0.664564 0.704771 0.75931	0.849157 0.84068 0.502306	0.65805 0.51921 0.853
S_4	0.07608 0.735039 0.7249439	0.603416 0.633957 0.78606	0.73719 0.67845 0.704012	0.83928 0.65676 0.6858
S_5	0.18699 0.765082 0.567264	0.713852 0.646009 0.75498	0.774253 0.83083 0.548577	0.68754 0.85064 0.6477
S_6	0.21868 0.7390773 0.5906023	0.824727 0.765082 0.53741	0.87323 0.84068 0.460059	0.85704 0.83083 0.5568
S_7	0.15471 0.8308349 0.5935959	0.671578 0.656833 0.78389	0.782646 0.88123 0.565592	0.71385 0.58398 0.8056
S_8	0.07036 0.7156687 0.7658741	0.603416 0.725289 0.70888	0.854821 0.7705 0.533136	0.82992 0.81149 0.545
S_9	0.13354 0.6784459 0.7040125	0.706234 0.792599 0.66539	0.80032 0.85064 0.529397	0.83358 0.65683 0.7063
S_{10}	0.25783 0.8308349 0.4856471	0.888984 0.80199 0.45308	0.757056 0.65683 0.723123	0.56883 0.60485 0.8754

Table 6. FN D-Mx by using FNWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)

S_1	0.8739484 0.9167447 0.0386399	0.7925994 0.92232 0.0145842	0.5676264 0.949417 0.36485	0.7593298 0.9369043 0.2109124
S_2	0.6510442 0.8480513 0.1041311	0.6675735 0.8557976 0.0717245	0.7925994 0.9734057 0.2443212	0.5417283 0.8362945 0.3582132
S_3	0.5815384 0.8638603 0.0528844	0.5539882 0.8515889 0.0591803	0.750193 0.9714403 0.1814646	0.5930127 0.8901439 0.2742413
S_4	0.4682055 0.7684854 0.1290153	0.503627 0.8087295 0.0814144	0.5192132 0.9249037 0.4498819	0.6149959 0.8301354 0.4744039
S_5	0.6634789 0.874405 0.6335135	0.5505845 0.8266696 0.6951435	0.6625804 0.8962461 0.5247078	0.4962624 0.9089642 0.559875
S_6	0.5999856 0.8874495 0.6464972	0.6634789 0.8326255 0.590631	0.8019901 0.9028368 0.4818561	0.6976357 0.8962461 0.5059124
S_7	0.6625804 0.9219875 0.5657301	0.5508757 0.8549678 0.6874469	0.6968088 0.9249327 0.5412399	0.5505845 0.8071637 0.7209706
S_8	0.494528 0.8460034 0.7318585	0.503627 0.8061146 0.679085	0.7047714 0.8593818 0.5812094	0.7064448 0.8814948 0.5187233
S_9	0.5192132 0.822129 0.7468214	0.5905588 0.8642892 0.634654	0.701821 0.9089642 0.5055983	0.5262504 0.8549678 0.6617572
S_{10}	0.7472779 0.9219875 0.5356048	0.7969539 0.8732296 0.5607912	0.6048504 0.8549678 0.6508325	0.4423193 0.7570557 0.8065158

Now, to display the important weights of the language phrases used to express the criteria determined by DMs are given below in the Table 7

Table 7. The weights assigned to each criterion

Criteria	DM1	DM2	DM3
(K_1)	LP	VMS	HP
(K_2)	RGS	EI	RGS
(K_3)	PMS	RGS	HP
(K_4)	HP	HP	VMS

As the weights assigned to the criteria and evaluations of the substitutions have been determined, then the aggregated weighted Fermatean Neutrosophic choice matrices are constructed using Equation (4), as illustrated in Tables 8 and 9 given below:

Table 8. Weighted FN D-Mx according to FNWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.599732 0.9696	0.5532716 0.97272	0.50402 0.9661945	0.4858238 0.9798257

	0.2378533	0.2228067	0.2522586	0.1877903
S_2	0.446768 0.9593 0.2620644	0.4659976 0.95282 0.2926157	0.7037833 0.9803937 0.1935042	0.4335559 0.9566684 0.2858825
S_3	0.399071 0.9577 0.2640716	0.3867098 0.95739 0.2639464	0.6661288 0.9791279 0.1960074	0.4011101 0.9347745 0.3467808
S_4	0.351964 0.9409 0.3182602	0.3515553 0.93619 0.3394726	0.4610319 0.9525171 0.2989162	0.4921935 0.9537692 0.2893229
S_5	0.128316 0.9052 0.4315658	0.4983021 0.90528 0.425867	0.6874925 0.9647806 0.2634483	0.5502502 0.9610388 0.2674005
S_6	0.150066 0.8958 0.4516889	0.5756981 0.93358 0.3706279	0.7753783 0.9666839 0.2596971	0.6859033 0.956238 0.2925744
S_7	0.106169 0.9301 0.3636541	0.4687926 0.90769 0.4157295	0.6949454 0.9747206 0.2182882	0.5713099 0.9043526 0.4271843
S_8	0.048287 0.8875 0.4542645	0.4212124 0.92369 0.3804751	0.7590325 0.9535271 0.306038	0.6642016 0.9516355 0.3096972
S_9	0.091641 0.8746 0.487558	0.4929843 0.94067 0.3318198	0.7106386 0.9686287 0.24833	0.6671278 0.9180629 0.3994349
S_{10}	0.176931 0.9301 0.3733505	0.6205529 0.94313 0.3482501	0.6722225 0.9343263 0.3579231	0.4552487 0.9081359 0.4132314

Table 9. Weighted FN D-Mx according to FNWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	0.599732 0.9696 0.2378533	0.5532716 0.97272 0.2228067	0.50402 0.9661945 0.2522586	0.4858238 0.9798257 0.1877903
S_2	0.446768 0.9593 0.2620644	0.4659976 0.95282 0.2926157	0.7037833 0.9803937 0.1935042	0.4335559 0.9566684 0.2858825
S_3	0.399071 0.9577 0.2640716	0.3867098 0.95739 0.2639464	0.6661288 0.9791279 0.1960074	0.4011101 0.9347745 0.3467808
S_4	0.351964 0.9409 0.3182602	0.3515553 0.93619 0.3394726	0.4610319 0.9525171 0.2989162	0.4921935 0.9537692 0.2893229
S_5	0.455301 0.9474 0.3038595	0.3843338 0.9497 0.2936334	0.5883338 0.9777741 0.2052041	0.3971687 0.9756863 0.2079718
S_6	0.41173 0.9526 0.2826554	0.4631394 0.95131 0.3033269	0.7121217 0.9791279 0.2004881	0.5583317 0.9724287 0.2288113

S_7	0.454685 0.9668 0.2342095	0.3845371 0.95743 0.2624053	0.6187267 0.983725 0.169682	0.4406437 0.9506175 0.3007638
S_8	0.339362 0.9361 0.3291924	0.3515553 0.94422 0.3171253	0.625797 0.9703515 0.2379528	0.5653818 0.9686945 0.2450548
S_9	0.356301 0.9268 0.3560827	0.4122377 0.96002 0.2602199	0.6231772 0.9803937 0.1920993	0.4211687 0.9621 0.2618756
S_{10}	0.512807 0.9668 0.2385146	0.5563112 0.96252 0.2609107	0.5370728 0.9694799 0.2376378	0.3539969 0.939153 0.3321644

Now, the SF for the Tables 8 and Table 9, are calculated using Equation (19), and are illustrate below in Tables 10 and 11:

Table 10. SF according to FNWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	-0.102696624	-0.0970618	-0.07354798	-0.1199294
S_2	-0.108987271	-0.0440079	-0.12045131	-0.0528184
S_3	-0.152202584	-0.0254067	-0.11598152	-0.0127956
S_4	-0.203787216	-0.0242385	-0.05676854	-0.0322167
S_5	-0.116205997	-0.018027	-0.12084457	-0.0906297
S_6	-0.106870937	-0.0514848	-0.11371195	-0.0901128
S_7	-0.130865281	-0.0199341	-0.14220281	-0.0168308
S_8	-0.227415649	-0.0339416	-0.08437937	-0.0856538
S_9	-0.177373815	-0.0485612	-0.12900768	-0.0305925
S_{10}	-0.1011057	-0.0712032	-0.05027061	-0.0208083

Table 11. SF according to FNWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S_1	-0.132904692	-0.1490711	-0.11516239	-0.1799611
S_2	-0.077655645	-0.0714967	-0.17934191	-0.0863273
S_3	-0.071635948	-0.0738806	-0.16560057	-0.0407079
S_4	-0.050781853	-0.0501121	-0.06871214	-0.0640134
S_5	-0.053462748	-0.0610407	-0.18135355	-0.1538215
S_6	-0.061277311	-0.0732582	-0.17167781	-0.1544827
S_7	-0.102629021	-0.0730339	-0.1970089	-0.0607866

S_8	-0.048977168	-0.0596674	-0.13723765	-0.1384089
S_9	-0.041574044	-0.0845966	-0.19297989	-0.0903405
S_{10}	-0.10844757	-0.10184	-0.121563	-0.0490283

The FN-PIS and FN-NIS corresponding to the highest and worst scores are shown in Tables 12 and 13.

Table 12. FN-PIS and FN-NIS according to FNWAM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Best)	0.176931 0.9103 0.5100636	0.4983021 0.88301 0.633333	0.6722225 0.9180678 0.5439011	0.5266505 0.8747414 0.6542648
S^- (Worst)	0.048287 0.8596 0.6503624	0.6033144 0.94205 0.4795027	0.6949454 0.9656181 0.4191644	0.7023054 0.9577875 0.4414136

Table 13. FN-PIS and FN-NIS according to FNWGM operator

Alternatives	(K_1)	(K_2)	(K_3)	(K_4)
S^* (Great)	0.356301 0.9062 0.6296002	0.3515553 0.91746 0.5813109	0.4610319 0.9381152 0.529159	0.4011101 0.9163859 0.5938768
S^- (Poor)	0.599732 0.9603 0.4452952	0.5532716 0.96327 0.4308737	0.6231772 0.9730824 0.3814316	0.4858238 0.9725006 0.407416

Based on Equations (23) and Equation (24), the distance between the option S_{i} from both the FN-PIS and FN-NIS can be determined and hence are presented in the Tables 14 and in Table 15 respectively.

Table 14. Distance to PIS and NIS according to FNWAM operator

Alternatives	$D(S_i, S^*)$	$D(S_i, S^-)$
S_1	-0.0001984	-0.000719
S_2	0.0001634	-0.0021034
S_3	0.0001311	-0.0001512
S_4	0.0009971	4.081E-05
S_5	-0.0006797	-0.0009046
S_6	0.0004388	0.0002226
S_7	-0.0001534	4.069E-05
S_8	0.0005392	-0.0007033
S_9	0.0008378	5.726E-05
S_{10}	-3.289E-05	-0.0026617

Table 15. Distance to PIS and NIS according to FNWGM operator

Alternatives	$D(S_i, S^*)$	$D(S_i, S^-)$
S_1	0.0385384	-0.0003446
S_2	0.0845744	-0.0003869
S_3	0.062182	-0.000897
S_4	0.0256772	-0.0024915
S_5	0.0197199	-0.0008921
S_6	0.0905429	-0.0006766
S_7	0.0296447	-0.0008057
S_8	0.0451872	-0.0022975
S_9	0.0396455	-0.0012789
S_{10}	0.0401722	0.0004508

Using Tables 14 and Table 15, the maximum and minimum distances is evaluated to the FN-NIS and FN-PIS, respectively, and the closeness ratios are computed using Equation (28), as shown in Tables 16 and 17. Also, the aggregation operators determine how the ranks differ. So, according to the FNWAM operator, the closeness ratio for each alternative shows that the best option is $S7$, and over all ranking is $S7 > S3 > S9 > S4$ >S4>S8>S1>S5>S2>S10 as shown in Table 16.

Table 16. Every alternative's closeness ratio according to the FNWAM operator

Alternatives	Closeness Ratio	Rank
S_1	3.5219025	7
S_2	9.2088361	9
S_3	0.4863661	2
S_4	1.6503041	5
S_5	5.0637916	8
S_6	1.6455789	4
S_7	0.0428936	1
S_8	2.3661877	6
S_9	1.4898353	3
S_{10}	12.005712	10

And, the closest alternative, according to the proximity ratios based on the FNWGM operator, $S10$, and overall ranking is $S10 > S1 > S5 > S7$ >S9>S2>S3>S6>S4>S8, as shown in the Table 17.

Table 17. Closeness ratio of each alternative according to FNWGM operator

Alternatives	Closeness Ratio	Rank
S_1	2.7187086	2
S_2	5.1470364	6
S_3	5.1430573	7
S_4	6.828937	9
S_5	2.9789264	3
S_6	6.0923355	8
S_7	3.2905556	4
S_8	7.3879469	10
S_9	4.8473876	5
S_{10}	1.0371401	1

6. Comparative analysis among NS and FNS

In this section, comparison between the Neutrosophic set and Fermatean neutrosophic set is presented. Here, initially linguistics values are considered for neutrosophic set which are related to Fermatean neutrosophic value that is truth and indeterminacy values are same and false values are different form Fermatean neutrosophic set.

Table 18. Terms used in linguistics and their associated Neutrosophic Number

LT		(T, I, F)
Probably More Significant	PMS	(0.95, 0.9, 0.05)
Extremely Significant	ES	(0.9, 0.8, 0.1)
High Priority	HP	(0.8, 0.9, 0.2)
Relatively Greater Significance	RGS	(0.7, 0.9, 0.3)
Equally Important	EI	(0.6,0.8, 0.4)
Very Minimal Significance	VMS	(0.6,0.7,0.4)
Low Priority	LP	(0.5, 0.7,0.5)
Extremely Low Significant	ELS	(0.5,0.6, 0.5)
Definitely Not Important	DNI	(0.4, 0.5, 0.6)

Using the proposed algorithm of Section 4, closeness ratio of each alternative is computed according to the Neutrosophic Weighted Arithmetic Mean and Neutrosophic Weighted Geometric Mean operator, and then the maximum and minimum distances to the Neutrosophic Negative Ideal Solutions and Neutrosophic Positive Ideal Solutions, and the closeness ratios are computed using Equation (28), as shown in Tables 19 and 20 below:

Table 19. Every alternative's closeness ratio according to the NWAM operator

Alternatives	Closeness Ratio	Rank
S_1	7.814948	5
S_2	0.328438	1
S_3	9.972041	7
S_4	28.63632	10
S_5	6.517592	3
S_6	6.972054	4
S_7	9.011573	6
S_8	25.02782	9
S_9	21.91097	8
S_{10}	1.034024	2

Table 20. Closeness ratio of each alternative according to NWGM operator

Alternatives	Closeness Ratio	Rank
S_1	0.1892229	2
S_2	0.5188592	4
S_3	1.3105063	7
S_4	2.122334	10
S_5	1.4066268	9
S_6	0.2431025	3
S_7	1.1300182	6
S_8	1.3662806	8
S_9	1.0490239	5
S_{10}	0.1534107	1

7. Results and Discussions

In the Fermatean Neutrosophic CODAS approach, the outputs tend to be closer together compared to those produced by traditional neutrosophic logic. Here, the role of the false membership function is minimal. In the linguistic table, FNS contains a greater variety of values in the falsity function. This diversity in the falsity function alters the priority of selection. In contrast, in neutrosophic sets, truth and false functions are interdependent, restricting the sum of truth and false functions to be less than or equal to one. Consider a faculty selection problem where a candidate has ten years of teaching service, but only at the college level. For university-level teaching, they might face challenges. In this case, truth value of 0.8 and a false value of 0.4 is assigned to the lack of teaching service. Such nuanced evaluations are possible within FNSs. These adjustments can significantly impact ranking in the selection process. This method proves particularly useful and yields accurate results, especially in the medical field. In the comparison below, the assessment of the Neutrosophic Weighted Arithmetic Mean (NWAM) and Neutrosophic Weighted Geometric Mean (NWGM) against the FNWAM and FNWGM are presented.

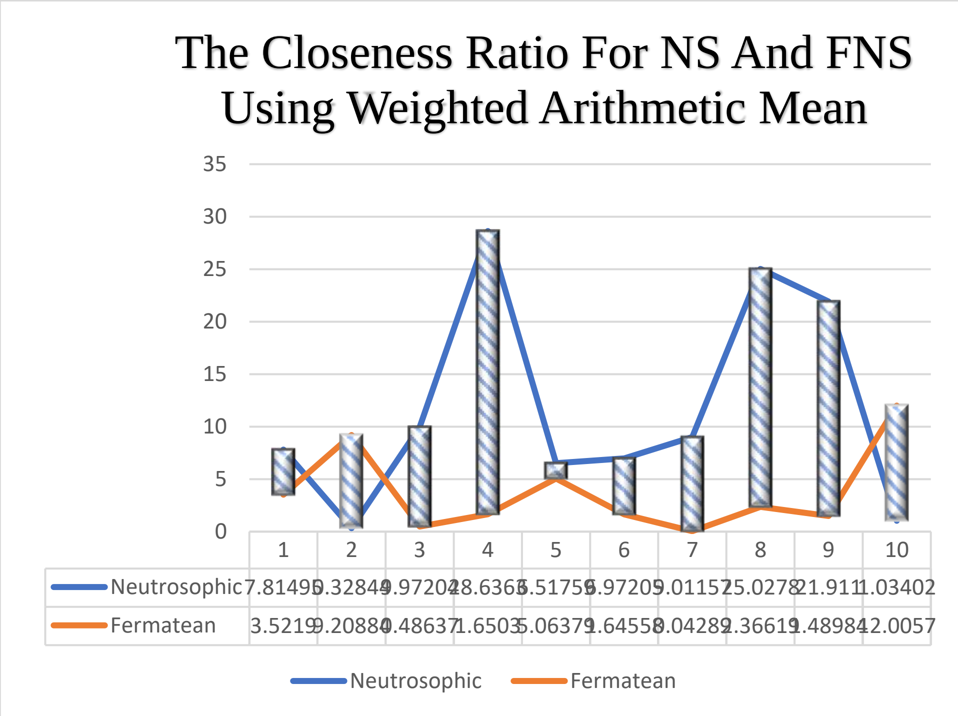

Fig.3 The closeness ratio for NS and FNS using Weighted Arithmetic Mean

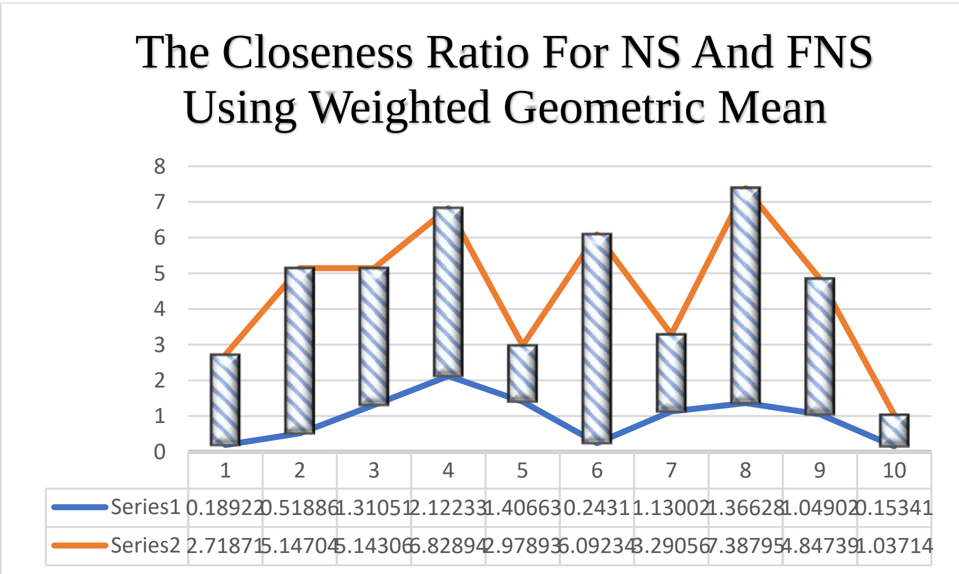

Fig.4 The closeness ratio for NS and FNS using Weighted Geometric Mean

8. Conclusions

This paper clearly states that the proposed technique helps to understand that while both types of sets-NS and FNS, address uncertainty, though they differ in their representation, conditions, and applications. On applying the proposed CODAS technique on a real-life problem, it is concluded that the Fermatean neutrosophic technique are providing more reliable results when compared to neutrosophic set in particular cases. This can be inferred from the obtained results that the neutrosophic sets are more general, while FNSs offer a more structured approach to handling uncertainty with specific constraints and functions. Furthermore, it is believed that these FNSs can be readily applied to real-life problems to determine the optimal solutions.

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