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Evaluative Analysis of the Incorporation of ICT in the Solving of Mathematical Problems in Primary Education Students with the Help of Neutrosophic Statistics on Fuzzy Data

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Abstract. This paper aims to determine the influence of incorporating Information and Communication Technologies (ICT) in solving mathematical problems in students of the Primary Educational Institution N° 70656 Ricardo Palma Puno, Peru. To collect the information, a pre-test and a post-test were applied in the variable "resolution of mathematical problems" to an experimental group that received classes with the help of the use of ICT. The test was applied to a control group of the same age that followed the traditional method of learning mathematics. Teachers were asked to evaluate children using a linguistic scale because it is simpler and more reliable. These values are associated with a triple of fuzzy numbers, to which t-tests are applied to compare the results of the pre-test with the post-test, and also between the experimental group and the control group. The triple represents the truthfulness, indeterminacy, and falseness. For processing, the theory of statistical methods for fuzzy data was generalized to the neutrosophic framework. The results show that the incorporation of ICT in the experimental group has significantly influenced the learning of mathematical problem-solving in the case of the skills of subtracting and dividing. In general, it can be stated that the incorporation of ICT in the resolution of mathematical problems of addition, subtraction, multiplication, and division in primary education has positive effects.

Keywords: Primary education, mathematics teaching, fuzzy data statistics, fuzzy numbers, random fuzzy numbers, t-test, Neutrosophic Statistics, Neutrosophic Probability, Interval-Valued Neutrosophic Sets.

1 Introduction

The application of technology is part of our existence and is a natural phenomenon in the development of humanity. The advancement of science and technology allowed teachers to give pedagogical use to technological instruments in the teaching-learning process.

Currently, extraordinary use is made of tools and devices that improve the interrelation of information and communication between people, the need for their use goes from lesser to greater. This phenomenon has meant a metamorphosis in the way people relate today.

Information and Communications Technologies (ICT) have necessarily become a novel communication option due to the form of changes in the processing and access to information. Therefore, the priorities of using the inclusion of ICT in education as policies at a global level have gained much importance since the seventies of the 20th century and with predominance today, definitely more frequently in recent years in education.

The use of technology appears in various fields and for different purposes. Teachers must know how ICT works, they must be able to face the challenges in the management of technologies, and they must update themselves, investigate, recreate, and be ready to interact in an information society. Immersion in the field of ICT is part of the job of an education professional. It is also important for the social insertion of the child, as well as of young people and adults in the circumstances faced by the information society. Youth are massively involved in the incorporation of ICT as a great opportunity for change in the process of communication transformation. The use of the latest generation mobile devices is a powerful part of the youth population. Educational agents who use technologies open new, more active, and useful spaces compared to those who do not use them or who are passive users.

The use of the Internet to communicate digitally is common today; it is a successful and fast way to communicate asynchronously using personal smartphones. Especially in the time of the COVID-19 pandemic, classes between teachers and students at all educational levels were possible mostly with the use of mobile phones.

There are many advantages of using ICT in education today. Its incorporation into the educational field has generated a more appropriate way to work and learn, above all, applications have been created and many forms of access and use of technological tools for learning have been expanded, which has been positive for the massification of the knowledge. ICT allows the freedom to work from anywhere. As in the field of health, ICT has a high potential to support educational interventions in the promotion of healthy habits and lifestyles, as well as in the prevention of health risk factors. These are more interactive and collaborative tools, they allow students to learn more entertainingly, be better trained, and communicate with each other. Furthermore, according to research, the contribution of ICT to learning is evident when they are used in a collaborative and participatory way.

However, there are also challenges and disadvantages of using ICT. There is inequality in its use by members of society; those individuals with a higher economic level take advantage of it more. There is also the limitation of knowledge of how to use it, especially for those people born in the analog era. The use of ICT can become a mental problem for those people who consume it compulsively; the individual can even isolate himself from direct human contact, although he (she) has the feeling of connection with the rest of the world.

This article aims to determine the influence of the incorporation of Information and Communication Technologies in the learning of mathematical problem-solving in students of the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru. The results obtained from the study can suggest what happens in other schools in the province and the country. For this purpose, the evaluation of basic mathematical skills was collected from 70 boys and girls from the institution for subsequent statistical study as an experimental group and from 71 boys and girls from a control group.

This study was carried out by comparing the results between the group of students who were allowed to use ICT in their learning and the control group who traditionally received classes. Members of each group were randomly selected with simple random sampling. The results between the two groups were compared using an unpaired Student's t-test [1, 2]. To complement, comparisons were also made between the changes obtained by the experimental group before and after using ICT in classes.

It was determined not to use numerical data in the evaluations as is done traditionally, but rather values according to a linguistic scale. Each value of this scale is associated with a triple of fuzzy numbers that are processed [3], one is used to determine truthfulness, another to determine indeterminacy and a third represents falseness. This procedure is justified because natural language is a form more similar to human beings than the numerical scale. Therefore, the theory of what is known as fuzzy data statistics [4-7] is generalized to the neutrosophic field. The idea is to extend the definition of Fuzzy Random Number to the neutrosophic field, where the statistical tests that are carried out for triangular fuzzy numbers are repeated for two more triangular fuzzy numbers. With this procedure, accuracy is gained when indeterminacy is incorporated.

This generalization takes us to the field of Neutrosophic Sets. Since these are numerical intervals, we would have particular cases of Interval-Valued Neutrosophic Sets [8]. In this case, we would have intervals of classic probabilities, another interval where there are undetermined probabilities and a third interval of probabilities that do not hold.

The article is divided into a Materials and Methods section, where the basic notions of fuzzy data statistics are explained, which includes the theory of fuzzy numbers, some notions of neutrosophic sets, and also the Student's t-test. The following section contains the results of the study carried out. The article finishes with a section dedicated to conclusions.

2 Materials and Methods

2.1 Basic Notions of Fuzzy Random Numbers

Fuzzy random numbers are the basis of the theory presented in this article. Below are the formal definitions.

Definition 1. ([7, 9]) A *fuzzy number* is a map $\overline{\mathrm{U}}: \mathbb{R} \rightarrow [0, 1]$ that is normal, convex, and has compact cuts. This means that the α-cuts are given by $\overline{\mathrm{U}}_\alpha = \{x \in \mathbb{R}: \overline{U}(x) \geq \alpha\}$ if $\alpha \in (0, 1)$, $\overline{\mathrm{U}}_0 = cl\{x \in \mathbb{R}: \overline{U}(x) > 0\}$, where cl denotes the closure of the corresponding set; these are bounded and closed intervals. $\overline{U}(x)$ is interpreted as the degree of compatibility of x with the property that defines a $\overline{\mathrm{U}}$.

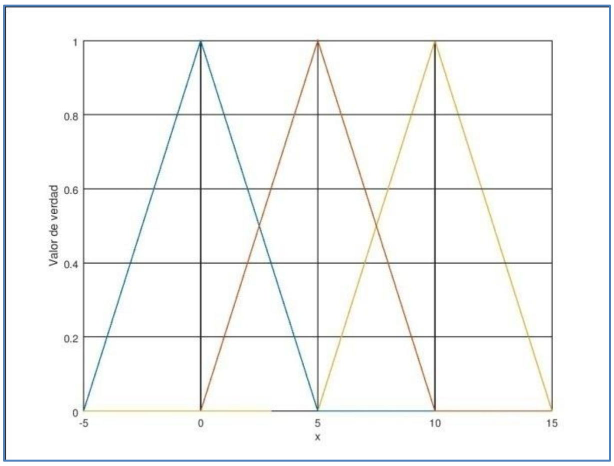

Figure 1. Three triangular fuzzy numbers. Source: [9].

A *triangular fuzzy number* is a particular case of a fuzzy number [10-15]. This is denoted by $A = Tri(a, b, c)$, and, as its name indicates, they have the shape of a triangle such that it has a null image outside the interval $[a, c]$ and reaches a maximum equal to 1 for $x = b$. Figure 1 contains three particular cases of these fuzzy numbers.

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, if\ a \le x \le b \\ \frac{c-x}{c-b}, if\ b \le x \le c \\ 0,\ otherwise \end{cases} \qquad (1)$$

Some of the properties of triangular fuzzy numbers are:

- $\overline{\mathrm{U}}_\alpha = [\alpha \mathrm{b} + (1-\alpha)\mathrm{a},\ \alpha \mathrm{b} + (1-\alpha)\mathrm{c}]$, $\alpha \in [0, 1]$,
- $\overline{\mathrm{U}}_1 = \{\mathrm{b}\}$ which is considered the value that completely satisfies the property $\overline{\mathrm{U}}$,
- $\overline{\mathrm{U}}_0 = [\mathrm{a}, \mathrm{c}]$ which are all values compatible with $\overline{\mathrm{U}}$ some truth value.

Definition 2. ([7, 9]) Let $\overline{\mathrm{U}}$ and $\overline{\mathrm{V}}$ be two fuzzy numbers. The *sum* of both is denoted by the fuzzy number $\overline{\mathrm{U}} + \overline{\mathrm{V}}$ and is defined from their α-cut as $(\overline{\mathrm{U}} + \overline{\mathrm{V}})_\alpha = [inf\overline{\mathrm{U}}_\alpha + inf\overline{\mathrm{V}}_\alpha, sup\overline{\mathrm{U}}_\alpha + sup\overline{\mathrm{V}}_\alpha]$ $\forall\alpha \in [0, 1]$.

Definition 3. ([7, 9]) Let $\overline{\mathrm{U}}$ be a fuzzy number and γ a real number. The *product* $\overline{\mathrm{U}}$ by γ, which is denoted by $\gamma\overline{\mathrm{U}}$ is a fuzzy number that is defined from its α-level as: $(\gamma\overline{\mathrm{U}})_\alpha = \begin{cases} [\gamma inf\overline{\mathrm{U}}_\alpha, \gamma sup\overline{\mathrm{U}}_\alpha], \text{if } \gamma \ge 0 \\ [\gamma sup\overline{\mathrm{U}}_\alpha, \gamma inf\overline{\mathrm{U}}_\alpha],\ \text{if } \gamma < 0 \end{cases}$ $\forall\alpha \in [0, 1]$.

Operations on triangular fuzzy numbers are shown below. Let $A_1 = Tri(a_1, b_1, c_1)$ and $A_2 = Tri(a_2, b_2, c_2)$ ([9, 16]):

- $A_1 + A_2 = Tri(a_1 + a_2, b_1 + b_2, c_1 + c_2)$.
- $A_1 - A_2 = Tri(a_1 - c_3, b_1 - b_2, a_3 - c_1)$
- $\gamma A_1 = Tri(\gamma a_1, \gamma b_1, \gamma c_1)$, if $\gamma \ge 0$; $\gamma A_1 = Tri(\gamma c_1, \gamma b_1, \gamma a_1)$, if $\gamma < 0$.

Definition 4. ([9, 16]) Let $\overline{\mathrm{U}}$ and $\overline{\mathrm{V}}$ be two triangular fuzzy numbers $\overline{\mathrm{U}} = Tri(a_1, b_1, c_1)$ and $\overline{\mathrm{V}} = Tri(a_2, b_2, c_2)$. The *distance* between them is defined by the following equation:

$$D(\overline{\mathrm{U}}, \overline{\mathrm{V}}) = \sqrt{\frac{1}{6}[(a_1 - a_2)^2 + 4(b_1 - b_2)^2 + (c_1 - c_2)^2]} \qquad (2)$$

Definition 5. ([7, 9]) Given a random experiment modeled in a probability space $(\Omega, \mathcal{A}, \mathcal{P})$. A *fuzzy random number* (FRN) has an associated application χ defined on Ω a fuzzy number, such that $\forall\alpha \in [0, 1]$ the real maps $\inf \chi_\alpha$ and $\sup \chi_\alpha$ are real random variables. Where, $\inf \chi_\alpha(\omega) = inf(\chi(\omega))_\alpha$ and $sup\ \chi_\alpha(\omega) = sup(\chi(\omega))_\alpha$.

Definition 6. ([7, 9]) Given a random experiment and an χ associated FRN, the *mathematical expectation* of χ (Aumann type) is the fuzzy number $\bar{E}(\chi)$, if it exists, such that $\forall\alpha \in [0, 1]$ we have:

$$\left(\bar{E}(\chi)\right)_\alpha = [E(inf\chi_\alpha), E(sup\chi_\alpha)] \qquad (3)$$

Where E is the mathematical expectation of the associated random variable.

In particular, if $\tilde{x}^{(n)} = (\tilde{x}_1, \dots, \tilde{x}_n)$ is a sample of FRN observations χ, then the *sample mean* is the fuzzy number $\overline{\tilde{x}^{(n)}}$ such that $\forall\alpha \in [0, 1]$:

$$\overline{\tilde{x}^{(n)}} = \left[\frac{inf(\tilde{x}_1)_\alpha + \cdots + inf(\tilde{x}_n)_\alpha}{n}, \frac{sup(\tilde{x}_1)_\alpha + \cdots + sup(\tilde{x}_n)_\alpha}{n}\right] \quad (4)$$

Definition 7. ([7, 9]) Given a random experiment and an associated FRN χ, the ρ_2 *−variance* of χ is the real number $Var_{\rho_2}(\chi)$, if it exists, it is defined by:

$$Var_{\rho_2}(\chi) = E\left(\left[\rho_2\left(\chi, \bar{E}(\chi)\right)\right]^2\right) \quad (5)$$

2.2 T-test

T-tests are based on the distribution called this way [1, 2]. The objective of these is to determine if it can be stated that two populations have the same mean through the study of the samples obtained from both. There are several cases to apply this test. If the samples are paired, it means that elements from the same population are compared before and after being subjected to a certain treatment.

There is also a Student's t-test when the samples are independent or unpaired when we want to compare two different samples to prove that they belong to the same population, as is the case of comparing the means of an experimental group and a control group.

The statistic for unpaired tests is shown in Equation 6.

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} \quad (6)$$

Where: $s_j^2 = \frac{\sum_{i=1}^{n_j}(x_i - \bar{X}_j)^2}{n_j - 1}$ with $j = 1, 2$. In this case, it is considered that the standard deviations of both samples are different.

To perform the hypothesis test, if μ_1 and μ_2 denote the means of the two populations, then:

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$.

To perform a similar test for paired samples, the following formula is used:

$$T = \frac{\bar{X}_D}{S_D/\sqrt{n}} \quad (7)$$

Where: $\bar{X}_D$ is the mean of the differences between both samples and S_D is the standard deviation of the differences. It is taken $n - 1$ as the degree of freedom.

If both samples have different sizes and different standard deviation values, the degree of freedom of the Student's t distribution is calculated by Equation 8:

$$g.l. = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\frac{s_2^2}{n_2}\right)^2 / (n_2 - 1)} - 2 \quad (8)$$

The p-value is calculated by setting α in this case, taken $\alpha = 0.05$. The test consists of calculating $t_{1-\alpha,d.f.}$ and is decided according to the following criteria:

If $t_{1-\alpha,d.f.} > T$, the null hypothesis of equal means is not rejected, otherwise the null hypothesis of equal means is rejected.

The initial hypothesis for applying the Student t-test is the normality of the data. This is convenient when the sample size is small.

2.3 Basic Notions on Neutrosophic Sets

Definition 8: ([17, 18]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow\ ]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 9: ([17, 18]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (9)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfy the condition $0 \le u_A(x) + r_A(x) + v_A(x) \le 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indeterminate and falseness of x in A, respectively. For convenience a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \le a + b + c \le 3$.

Definition 10: ([19, 20]) A *triangular single-valued neutrosophic number* (TSVNN) $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, is a neutrosophic set over ℝ, whose membership functions of truthfulness, indeterminacy, and falsity, respectively, are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \le a_3 \\ 0, \text{otherwise} & \end{cases} \quad (10)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}} & x = a_2 \\ \frac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (11)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \gamma_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{(x - a_2 + \gamma_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (12)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 11: ([19, 20]]) Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$ two triangular single-valued neutrosophic numbers and λ is any non-zero real number. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
3. Investment: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, & \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$
6. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, ∧ is a t-norm and ∨ is a t-conorm.

Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ is a TSVNN, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (13)$$

$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}})$ (14)

They are called *the score function* and *accuracy function* of ã, respectively.

Let $\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}$ be a set of n TSVNN, where $\tilde{A}_j = \langle (a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j} \rangle$ (j = 1, 2, …, n), then the weighted average of the TSVNN is calculated by the following equation:

$\tilde{A} = \sum_{j=1}^{n} \lambda_j \tilde{A}_j$ (15)

where λ_j is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

Definition 12 ([21-24]): suppose X is a space of points (objects) with a generic element in X denoted by x. An *interval-valued neutrosophic set* (IVNS) A in X is characterized by the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$.

$$A_{IVNS} = \{\langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle : x \in X\}$$

With $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$ (16)

3 Results

The study carried out consists of tests on basic arithmetic skills of addition, subtraction, multiplication, and division in 141 children from the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru during the year 2023. The children belong to the third grade of primary education.

The students were divided into two groups, experimental group 1 had 70 students, and control group 2 had 71 students, summing up the total of third-grade children at this institution. Membership in one group or another was done with the help of simple random sampling.

Then, all the children were given a test that measured the skills that they were supposed to have sufficiently mastered. Later, they were given eight weeks, two for each arithmetic operation. Group 1 was given a method where they could use some technological means in the first 20 minutes of each class and then the class continued with the traditional teaching method. The traditional teaching method was applied to the members of group 2 the entire class time and they were not allowed to perform calculations with computing means. In addition, the members of group 1 were given tasks where part of that asked them to perform calculations with some computing means or search on the Internet. For this, we had the support of the parents. After the total class sessions had passed, the test was repeated.

The evaluators of both tests were instructed to evaluate the students based on a scale of "Excellent", "Very Good", "Good", "Regular" and "Deficient". The following algorithm adapted from [9] was applied to this linguistic scale.

1. The "Excellent" evaluations were identified with the triple of fuzzy numbers $\langle Tri(15,20,25), 0, Tri(-5,0,5) \rangle$, $\langle Tri(12.5, 17.5, 22.5), Tri(-1,0,1), Tri(0,2.5,5) \rangle$ was identified with "Very Good", $\langle Tri(10,15,20), Tri(0,1,2), Tri(2.5,5,7.5) \rangle$ was identified with "Good", $\langle Tri(6,11,16), Tri(3,4,5), Tri(4,9,14) \rangle$ was identified as "Regular" and $\langle Tri(4,9,14), Tri(3,4,5), Tri(6,11,16) \rangle$ with "Deficient".
2. The T statistic of Equation 6 (or Equation 7) is calculated from the fuzzy numbers of the evaluations, for each element of the triple. For this, Equations 3 and 4 were applied.
3. The triple of fuzzy numbers obtained from the previous step is defuzzified by $\hat{x} = \overline{U}_1$ or the value where the maximum of the fuzzy number is reached. It is a particular case of the defuzzification schemes that appear in [25]. This value coincides with the three schemes that are: the minimum of $\overline{U}_1$, the maximum of $\overline{U}_1$ and the average value of $\overline{U}_1$.
4. The obtained triple of fuzzy numbers is used to perform the t-test hypothesis. The value of the degree of freedom with Equation 8 is carried out with the algebraic operations between fuzzy numbers indicated by the formula and the way of defuzzification indicated in the previous step.

The results did not show behavior according to the normal distribution, however, as indicated in [9, 26], as the samples have a size greater than 25, this test can be considered robust enough. The final results are summarized in Tables 1, 2, 3 and 4.

Tables 1 and 3 contain the results of the evaluations of the control group, before and after traditional classes, respectively. Tables 2 and 4 contain the results of the experimental group before and after, respectively.

Table 1. Level of mathematical problem solving by the students in the control group before the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	9	9
Regular	9	9	0	0
Good	40	40	40	40
Very good	18	18	19	20
Excellent	4	4	3	2

Table 2. Level of mathematical problem solving by the students in the experimental group before the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	10	8	17
Regular	17	36	9	18
Good	32	20	33	34
Very good	18	0	18	0
Excellent	3	4	2	1

Table 3. Level of mathematical problem solving by the students in the control group after the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	0	18
Regular	13	13	13	22
Good	14	22	31	31
Very good	40	36	27	0
Excellent	4	0	0	0

Table 4. Level of mathematical problem solving by the students in the experimental group after the experiment according to the results.

Grades	Solve addition problems	Solve subtraction problems	Solve multiplication problems	Solve division problems
Deficient	0	0	0	0
Regular	0	34	21	8
Good	53	32	20	62
Very good	13	4	25	0
Excellent	4	0	4	0

Table 5 contains the results of the algorithm described above of applying the t-test for each of the four skills.

Table 5. Results of the Student t-test for each of the skills comparing the experimental group with the control group in the post-test. According to the results of Tables 3 and 4.

Ability	Triple of Fuzzy Numbers T	$\mathbf{t_{0.95g.l.}}$	Three T defuzzified	Decision
Addition	$\langle[-27.36, 0.58, 14.55], [-8.66, 0.92, 6.08], [-14.01, -0.58, 8.42]\rangle$	$\langle 1.66, 1.66, 1.66\rangle$	$\langle 0.58, 0.92, -0.58\rangle$	H0 is not rejected
Subtraction	$\langle[-19.60, 5.97, 18.76], [13.07, -5.28, -1.39], [-15.82, -5.97, 2.71]\rangle$	$\langle 1.66, 1.66, 1.66\rangle$	$\langle 5.97, -5.28, -5.97\rangle$	H0 is rejected
Multiplication	$\langle[-22.05, 0.54, 11.84], [-8.37, -1.20, 2.62], [-11.14, -0.54, 8.20]\rangle$	$\langle 1.66, 1.66, 1.66\rangle$	$\langle 0.54, -1.20, -0.54\rangle$	H0 is not rejected
Division	$\langle[-31.69, 6.75, 25.97], [-21.10, -6.38, 4.43], [-41.97, -6.75, 3.71]\rangle$	$\langle 1.66, 1.66, 1.66\rangle$	$\langle 6.75, -6.38, -6.75\rangle$	H0 is rejected

The results of Table 5 indicate that there was significant improvement in the experimental group concerning subtraction and division. The latter is the most difficult operation. In the addition and multiplication, the effectiveness was the same in both groups.

To compare the results before and after in the experimental group, the triple of fuzzy numbers was obtained $\mathrm{T} = \langle [3.9282, 9.9361, 22.5908], [0.36124, 0.58894, 0.77630], [-0.20553, 0.33689, 1.04023] \rangle$ as a result of applying Equation 7. By defuzzifying, it is $\overline{\mathrm{T}} = \langle 9.9361, 0.58894, 0.33689 \rangle$ greater than $\mathrm{t}_{0.95,\mathrm{n}-1} = \mathrm{t}_{0.95,69} = \langle 1.66, 1.66, 1.66 \rangle$ for the truthfulness values, and is not true for indeterminacy or falsity. Therefore, the null hypothesis of equality is rejected, which means that there was an improvement in the learning of the students in this group.

Conclusions

The use of Information and Communications Technologies (ICT) today is no longer foreign to most daily activities within contemporary societies, including teaching. This article proposed to analyze the impact that ICT can have on the learning of a group of children belonging to the Primary Educational Institution No. 70656 Ricardo Palma Puno, Peru, in the year 2023. There was a control group and an experimental group. The control group received a completely traditional educational program, while the experimental group was introduced to some extra-class activities and tasks with the support of technological means. Both groups were compared to each other and it was concluded that although in the simplest skills such as addition and multiplication, no significant improvements were obtained, in the case of subtraction and division there was improvement. Furthermore, it was found that the experimental group improved significantly when the skills before and after applying the proposed teaching were compared. To reach these results, the theory of fuzzy statistical tests was used. Specifically, the Student's t-test for paired and unpaired samples on the triangular fuzzy numbers was used. To gain accuracy, the tests were applied to a triple of fuzzy numbers, one indicating truthfulness, the other indicating indeterminacy, and the third one indicating falsehood. The use of a triple of fuzzy numbers, allowed raters to use a linguistic rating scale instead of a numerical scale. This made the evaluations easier for teachers to carry out and also made it possible to obtain a greater degree of interpretability of the results. The use of neutrosophy allowed us to obtain greater accuracy.

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