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The Structure Of The Binary Two Fold Algebra Based On Intuitionistic Fuzzy Groups

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Abstract:

In this paper, we study the binary two-fold algebra built over intuitionistic fuzzy groups, where the two-fold binary operations and some related substructures such as binary two-fold sub-algebras of intuitionistic fuzzy groups two-fold algebras, binary two-fold centers, binary normality, binary two-fold group products. On the other hand, we illustrate many examples to explain the validity of our discussion.

Keywords: binary two-fold algebra, two-fold intuitionistic fuzzy group, binary operation, two-fold normality.

Introduction

Two-fold algebras was first presented by Smarandache in [1], where he has suggested a connection between neutrosophic sets and algebraic structures combined in a unique algebraic structure called two-fold algebra.

Although this topic is very recent, it has attracted the interest of many researchers due to its importance and generalizability of traditional algebraic structures, and also some neutrosophic structures [6-8].

We now recall some recent results that have been published on Two-fold algebras:

In [2], the concept of two-fold algebra based on a standard fuzzy number theoretical system was introduced with many interesting properties [3].

In [5], the concept of two-fold refined neutrosophic numbers was studied, where many results about these numbers were presented and handled by many theorems and examples. In [4], The concept of two-fold algebraic vector spaces and two-fold algebraic modules was proposed and studied in detail, with many interesting properties about algebraic basis and two-fold linear functions.

These results have motivated us to study for the first time the binary two-fold algebra built over intuitionistic fuzzy groups. We concentrate our study on covering substructures and binary operations on these algebras such as:

the existence of identity, the existence of the inverse, two-fold centers, and normality.

For more details about fuzzy groups, anti-fuzzy groups, and fuzzy algebras, check [9-11].

Main Discussion

Definition:

Let $(G\ ,\mu,v)$ be an intuitionistic fuzzy group, i-e. $(G\ ,.)$ is a group, and

$\mu\colon\ G\times G\to[0,1]\ ;\ \begin{cases}\mu(xy)\geq\min(\mu(x).\mu(y))\\ \mu(x^{-1})=\mu(x)\end{cases}$, $v\colon\ G\times G\to[0,1]\ ;\ \begin{cases}v(xy)\leq\max(v(x),v(y))\\ v(x^{-1})=v(x)\end{cases}$

We define the binary two-fold algebra of the intuitionistic fuzzy group $(G\ ,\mu,v)$ as follows:

$\Delta_G=\{(x_{\mu(y)},z_{v(t)})\ ;\ \ x,y,z,t\ \ \in G\}$. We define the following binary two-fold operation:

$*\colon\Delta_G\times\Delta_G\to\Delta_G$ such that:

$$(x_{\mu(y)},a_{v(b)})*(z_{\mu(t)},c_{v(d)})=((x\cdot z)_{\max(\mu(y),\mu(t))},(a.c)_{\min(v(d),v(b))})$$

Definition:

Let $(x_{\mu(y)},a_{v(b)}),(z_{\mu(t)},c_{v(d)})\in\Delta_G$, we define the relation:

$$(x_{\mu(y)},a_{v(b)})\equiv(z_{\mu(t)},c_{v(d)})\Leftrightarrow\begin{cases}x=z,a=b\\ \mu(y)=\mu(t),v(b)=v(d)\end{cases}\ ;x,y,a,b,d,z,t\in G.$$

Theorem (1):

$\equiv$ is an equivalence relation on Δ_G.

Remark:

We denote to the equivalence classes of $(\equiv)$ by $\overline{\Delta_G} = \{[(x_{\mu(y)}, a_{v(b)})] \quad ; x, a, b, y \in G\}$, where:

$[(x_{\mu(y)}, a_{v(b)})] = \{(z_{\mu(t)}, c_{v(d)}) \in \Delta_G \quad ; x = z, a = b, \mu(y) = \mu(t), v(b) = v(d) \ \}$.

Theorem (2):

1] $(*)$ is associative.

2] $(*)$ has an identity $(e_{\mu(a),}e_{v(b)})$.

3] Every $(x_{\mu(y)}, a_{v(b)})$ has an inverse.

Example:

Consider $(z_3^* = \{1,2\}.\cdot)$ the group of integers modulo 3 under multiplication, take $\mu : z_3^* \times z_3^* \to z_3^* \quad ;$

$$\mu(x) = \begin{cases} 1; x = 1 \\ \frac{1}{2}; x = 2 \end{cases} .$$

$v : z_3^* \times z_3^* \to z_3^* \ ;$

$$v(x) = \begin{cases} \frac{1}{3}; x = 1 \\ 1; x = 2 \end{cases} .$$

Δ_G
$= \{\left(1_1, 1_{\frac{1}{3}}\right), (1_1, 2_1), \left(2_{\frac{1}{2}}, 1_{\frac{1}{3}}\right), \left(2_{\frac{1}{2}}, 2_1\right), \left(1_{\frac{1}{2}}, 1_{\frac{1}{3}}\right), \left(1_{\frac{1}{2}}, 2_1\right), \left(2_1, 1_{\frac{1}{3}}\right), (2_1, 2_1), \left(1_{\frac{1}{2}}, 1_1\right), \left(1_{\frac{1}{2}}, 2_{\frac{1}{3}}\right), (2_1, 1_1), \left(2_1, 2_{\frac{1}{3}}\right)$
$, (1_1, 1_1), \left(1_1, 2_{\frac{1}{3}}\right), \left(2_{\frac{1}{2}}, 1_1\right), \left(2_{\frac{1}{2}}, 2_{\frac{1}{3}}\right)\}$.

Take: $X = \left(1_{\frac{1}{2}}, 2_1\right), Y = (2_1, 2_{\frac{1}{3}})$, then:

$X * Y = (2_1, 2_{\frac{1}{3}})$.

Definition:

Let $(x_{\mu(y)}, a_{v(b)}) \in \Delta_G$, we define:

$$\overline{(x_{\mu(y)}, a_{v(b)})} = (x_{\mu(y)}^{-1}, a_{v(b)}^{-1}).$$

Theorem (3):

Let $(x_{\mu(y)}, a_{v(b)}) \in \Delta_G$, then:

$$(x_{\mu(y)}, a_{v(b)}) * \overline{(x_{\mu(y)}, a_{v(b)})} = (e_{\mu(a),}e_{v(b)}).$$

Definition:

Let S be a non-empty subset of Δ_G, we say that it is a binary twofold sub-algebra if and only if:

$(x_{\mu(y)}, a_{v(b)}) * \overline{(z_{\mu(t)}, c_{v(d)})} \in S$ for all $(x_{\mu(y)}, a_{v(b)}), (z_{\mu(t)}, c_{v(d)}) \in S$.

Theorem (4):

Let S be a non-empty subset of Δ_G , then S is a binary two-fold subalgebra if and only if:

$S = \{(x_{\mu(y)}, a_{v(b)})\ ; x \in H,, a \in K;\ H, K \text{ are subgroups of } G\}$.

Definition:

The twofold sub-algebra $S = \{(x_{\mu(y)}, a_{v(b)})\ ; x \in H,, a \in K;\ H, K \text{ are subgroups of } G\}$ is denoted by $\Delta_G^{H,K}$.

Definition:

The binary twofold subalgebra $\Delta_G^{H,K}$ is called normal if and only

if H, K *are* normal subgroups of G.

Theorem (6):

Let $\Delta_G^{H,K}$ be a binary twofold subalgebra of Δ_G , then:

1] $\Delta_G^{H,K}$ is abelian if and only if H,K are abelian.

2] $\Delta_G^{H,K}$ is normal if and only if: $(x_{\mu(y)}, a_{v(b)}) * (z_{\mu(t)}, c_{v(d)}) * \overline{(x_{\mu(y)}, a_{v(b)})} \in \Delta_G^{H,K}$ for all $(x_{\mu(y)}, a_{v(b)}) \in \Delta_G$, $(z_{\mu(t)}, c_{v(d)}) \in \Delta_G^{H,K}$.

Definition:

We define the center of Δ_G as follows:

$Z(\Delta_G) = \{(x_{\mu(y)}, a_{v(b)}) \in \Delta_G\ ;\ (x_{\mu(y)}, a_{v(b)}) * (z_{\mu(t)}, c_{v(d)}) = (z_{\mu(t)}, c_{v(d)}) * (x_{\mu(y)}, a_{v(b)})\ ;\ \forall (z_{\mu(t)}, c_{v(d)}) \in \Delta_G\}$.

Theorem (7):

1] $Z(\Delta_G)$ is an abelian binary twofold subalgebra of Δ_G.

2] $Z(\Delta_G) = \Delta_G^{Z(G),Z(G)}$.

Theorem (8):

Let $\Delta_G^{H,L}$, $\Delta_G^{K,S}$ be binary twofold subalgebras of Δ_G,then:

1] $\Delta_G^{H,L} \cap \Delta_G^{K,S} = \Delta_G^{H \cap K, L \cap S}$

2] $\Delta_G^{H,L} * \Delta_G^{K,S} = \Delta_G^{HK,LS}$

3] if H,L are normal, then $\Delta_G^{H,L} * \Delta_G^{K,S}$ is binary twofold subalgebra of Δ_G.

Definition:

1] The binary twofold algebra of the intuitionistic fuzzy group $(G\ ,\mu,v)$, is called n-abelian if

$$(x_{\mu(y)},a_{v(b)})^n * \left(z_{\mu(t)},c_{v(d)}\right)^n = ((xz)^n_{\max(\mu(y),\mu(t))},(ac)^n_{\min(v(d),v(b))}.$$

2] it is called n-power closed if: $(x_{\mu(y)},a_{v(b)})^n * \left(z_{\mu(t)},c_{v(d)}\right)^n = \left((J)^n_{\mu(s)},(I)^n_{v(k)}\right)$ $;I,J,x,y,z,a,b,c,s,k \in G.$

Theorem (9):

Let Δ_G be the binary twofold algebra of the intuitionistic fuzzy group $(G\ ,\mu,v)$, then:

1] Δ_G is n-abelian if and only if G is n-abelian group.

2] Δ_G is n-power closed if and only if G is n-power closed group.

Proof of theorem (1):

$(x_{\mu(y)},a_{v(b)}) \equiv (x_{\mu(y)},a_{v(b)})$, that is because $\begin{cases} x = x, a = a \\ \mu(y) = \mu(y), v(b) = v(b) \end{cases}$.

If $(x_{\mu(y)},a_{v(b)}) \equiv \left(z_{\mu(t)},c_{v(d)}\right)$, then $\begin{cases} x = z, a = c \\ \mu(y) = \mu(t), v(d) = v(d) \end{cases}$, so that $\left(z_{\mu(t)},c_{v(d)}\right) \equiv (x_{\mu(y)},a_{v(b)})$.

If $(x_{\mu(y)},a_{v(b)}) \equiv \left(z_{\mu(t)},c_{v(d)}\right)$, and $\left(z_{\mu(t)},c_{v(d)}\right) \equiv (l_{\mu(k)},s_{v(n)})$, then $\begin{cases} x = l = z, a = c = s \\ \mu(y) = \mu(t) = \mu(k), v(a) = v(d) = v(n) \end{cases}$ and $(x_{\mu(y)},a_{v(b)}) \equiv (l_{\mu(k)},s_{v(n)})$.

Proof of theorem (2):

1] For $x,y,z,a,b,c,d,n,k \in G$, we have:

$(x_{\mu(y)},a_{v(b)}) * \left(\left(z_{\mu(t)},c_{v(d)}\right) * \left(l_{\mu(k)},s_{v(n)}\right)\right) = (x_{\mu(y)},a_{v(b)}) *$

$((zl)_{\max(\mu(t).\mu(k))},(cs)_{\min(v(d),v(n))}) = ((xzl)_{\max(\mu(y),\mu(t),\mu(k))},(acs)_{min(v(b),v(d),v(n))}) =$

$((x_{\mu(y)},a_{v(b)}) * \left(\left(z_{\mu(t)},c_{v(d)}\right)\right) * (\left(l_{\mu(k)},s_{v(n)}\right)\right).$

2] $(x_{\mu(y)},a_{v(b)}) * (e_{\mu(a)},e_{v(b)}) = (x_{\mu(y)},a_{v(b)}).$

3] $(x_{\mu(y)},a_{v(b)}) * (x^{-1}_{\mu(y)},a^{-1}_{v(b)}) = (e_{\mu(a)},e_{v(b)}).$

Proof of theorem (3):

It is easy and clear.

Proof of theorem (4):

Let $(x_{\mu(y)}, a_{v(b)}), (z_{\mu(t)}, c_{v(d)}) \in S$, then: $(x_{\mu(y)}, a_{v(b)}) * \overline{(z_{\mu(t)}, c_{v(d)})} \in S$, so that $xz^{-1}, ac^{-1} \in K$ and H, K are subgroups of G, where:

$\Delta_G^{H,K} = S = \{(x_{\mu(y)}, a_{v(b)})\ ; x \in H\ ; a \in K\}$.

Proof of theorem (5):

1] If H is abelian, then $x \cdot y = y \cdot x$ for all $x, y \in H, a \cdot c = c \cdot a$ for all $x, y \in H, a, c \in K$, thus:

$(x_{\mu(y)}, a_{v(b)}) * (z_{\mu(t)}, c_{v(d)}) = (z_{\mu(t)}, c_{v(d)}) * (x_{\mu(y)}, a_{v(b)})$, thus $\Delta_G^{H,K}$is abelian.

2] Δ_G^{H}, K is normal if and only if H,K are normal, thus $\Delta_G^{H,K}$ is normal subalgebra if and only if:

$(x_{\mu(y)}, a_{v(b)}) * (z_{\mu(t)}, c_{v(d)}) * \overline{(x_{\mu(y)}, a_{v(b)})} \in \Delta_G^{H,K}$, that is because $xyx^{-1} \in H, aca^{-1} \in K$.

Proof of theorem (6):

1] For all $(x_{\mu(y)}, a_{v(b)}), (z_{\mu(t)}, c_{v(d)}) \in Z(\Delta_G)$, we have:

$(x_{\mu(y)}, a_{v(b)}) * \overline{(z_{\mu(t)}, c_{v(d)})} \in Z(\Delta_G)$, that is because for $z_{\mu(t)}, c_{v(d)} \in G$.

2] $(x_{\mu(y)}, a_{v(b)}) \in Z(\Delta_G)$, then $b, y \in G$, and $x, a \in Z(G)$, so that $Z(\Delta_G) = \Delta_G^{Z(G),Z(G)}$.

Proof of theorem (7):

1] Let $(x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{H\cap K, L\cap S}$, then $\begin{cases} x \in H \cap K \\ y \in G \end{cases}, \begin{cases} a \in L \cap S \\ b \in G \end{cases}$ and $\begin{cases} (x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{H,L} \\ (x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{K,S} \end{cases}$ so that:

$(x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{H,L} \cap \Delta_G^{K,S}$.

Conversely, if $(x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{H,L} \cap \Delta_G^{K,S}$, then $\begin{cases} (x_{\mu(y)}, a_{v(b)} \in \Delta_G^{H,L} \\ (x_{\mu(y)}, a_{v(b)} \in \Delta_G^{K,S} \end{cases}$ so that: $\begin{cases} a \in L \cap S \\ x \in H \cap K \end{cases}$

, hence

$(x_{\mu(y)}, a_{v(b)}) \in \Delta_G^{H\cap K, L\cap S}$ and $\Delta_G^{H\cap K, L\cap S} = \Delta_G^{H,L} \cap \Delta_G^{K,S}$.

2] It can be proved by a similar argument.

Proof of theorem (8):

1] Δ_G is n-abelian if and only if $(x_{\mu(y)}, a_{v(b)})^n * (z_{\mu(t)}, c_{v(d)})^n = ((xz)^n_{\max(\mu(y),\mu(t))}, (ac)^n_{\min(v(b),v(d))})$, which is equivalent to: $x^n y^n = (xy)^n$, and G is n-abelian.

2] Δ_G is n-power closed if and only if: $(x_{\mu(y)}, a_{v(b)})^n * \left(z_{\mu(t)}, c_{v(d)}\right)^n = \left(\left(\left(l_{\mu(k)}, s_{v(n)}\right)\right)^n\right)$ $;z, c, a, bd, n, k \in G.$

This is equivalent to: $x^n y^n = l^n$, hence G is n-power closed group.

Conclusion

In this paper, we studied the binary two-fold algebra built over intuitionistic fuzzy groups, where the two-fold binary operations on these algebras are defined and studied with some related substructures such as two-fold sub-algebras of intuitionistic fuzzy groups two-fold algebras, binary two-fold centers, and binary normality.

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